Structural Engineering and Mechanics, Vol. 4, No. 4 (1996) 347-364
DOI: http://dx.doi.org/10.12989/sem.1996.4.4.347

Linear fracture envelopes for fatigue assessment of welds in bridges

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Abstract. Presently welded components are designed using S/N curves which predict only the fatigue life of the component. In order to ascertain the condition of the weld at any intermediate period of its life inspection is carried out. If cracks are detected in a weld fracture mechanics is used to find their remaining life. A procedure for assessment is developed here that can be used to verify the condition of a weld before inspection is carried out to detect cracks. This simple method has been developed using linear fracture envelopes by combining S/N curves with linear elastic fracture mechanics.

Key words: assessment; bridges, fatigue; fracture mechanics; residual strength; welded component.

1. Introduction

A large portion of the bridges in use today were built just after the second world war. Many of these bridges have been in service for more then half of their lives, considering the average life of bridges is about 70 years. The condition of these structures has deteriorated from their original state due to repeated loading during service. The deterioration is widespread and upgrading is required to make sure these structures are safe to use. In the U.S.A., it is estimated that 40% of the 578,000 bridges require upgrading with 10,000 new bridges being added to the list each year (Ahlskog 1990). In order to upgrade these bridges, assessment procedures are required to evaluate the extent of deterioration of different components. The results of the assessment can then be used to make decisions regarding repair or replacement of the component.

Some bridge components, like welded components, are more sensitive to fatigue loading compared to other components. Welds have lower fatigue strength, reduced endurance limit and lower fatigue life compared to unwelded joints. With the application of fatigue loading, cracks initiate mostly in the weld toe of the component and propagate, thus causing the strength to reduce until the component fails (Maddox 1991). Failure of welds has often caused the failure of bridges like the King's bridge in Melbourne, Australia (Rolfe and Barsom 1977). Hence development of assessment procedures for such components is important for the safe operation of a bridge.

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Welded components are designed using S/N curves. These curves can only predict the fatigue life of the component. They do not provide any information on the condition of the component at any intermediate stage of its fatigue life and hence cannot be used for assessment. In order to ascertain the conditions of the component, at present welded components are checked on site to detect cracks. If existing cracks are found, crack propagation laws are used to find the remaining life of the welds. Codes such as the BS PD 6493 and AASHTO provide information on the allowable size of cracks, material properties and crack propagation equations which has been used to find the remaining life once cracks has been detected.

In contrast to present methods of assessment which involves a site inspection, a method of assessment is developd in this study that can be used to verify the condition of the component before site checks to detect cracks are carried out. The damage due to fatigue is determined in terms of reduction in strength. Oehlers, Ghosh and Wahab (1995) showed that if the initial crack length of a component is known, fracture mechanics can be used to find the reduction in strength. The strength of the component was determined considering unstable crack propagation as the mode of failure. Fracture mechanics was then used to develop curves showing the reduction of strength throughout the fatigue life of a component subjected to constant amplitude loading. These curves of remaining or residual strength have been used here to assess the condition of a welded component at any intermediate period of its fatigue life.

It has been shown (Oehlers, Ghosh and Wahab 1995) that idealised components such as an infinitely wide plate with a centre crack, when subjected to constant amplitude loading exhibit a linear residual strength variation. However, real welded components that are subjected to constant amplitude loading show a non-linear residual strength variation. In this paper the reduction in strength has been determined for components showing both linear and non-linear variations under constant amplitude loading when a sequence of different stress ranges act on the component. It is shown that when the residual strength curve is non-linear, the reduction in strength due to an individual cyclic stress range in a sequence of cyclic stress ranges depends on the previous stress ranges to which the component has been subjected. In order to find out the reduction in strength for a given stress range, the effect of all other stress ranges must be taken into account. This makes calculation for the reduction in strength difficult and cumbersome, particularly for the case when the component is subjected to a large variety of stresses. However, when the residual strength curve is linear, the reduction in strength for each individual range in a sequence of ranges is found to be independent of all previous ranges applied. Thus the total reduction of strength due to a sequence of ranges, when the residual strength curve is linear for a given range, can be found by simply adding the damage, which is the reduction in strength caused by each range. Linear residual strength curves thus allow easy methods of assessment and a procedure has been developed here where assessment of real welds is achieved by approximating the non-linear curves into linear ones.

The residual strength method of assessment developed here is based on linear elastic fracture mechanics. It is used to determine the propagation of the initial crack with the application of fatigue loading and then to find out the residual strength. Hence, the initial crack size of the component must be known to find the residual strength variation. The initial crack size has been determined using S/N curves as discussed later in section 6.

2. Fracture failure envelope of components

The initial crack in a welded component propagates with the application of fatigue loading, thus causing the strength to decrease. Therefore, the shape of the residual strength curve for a component can be found from the variation of strength with crack length and from the rate at which the crack propagates. The fracture strength of a cracked structure can be calculated at any stage of fatigue life using the stress intensity factor which gives the magnitude of stress around a crack as follows (Irwin 1957)

$$K = M\sigma\sqrt{\Pi a} \tag{1}$$

where K is the stress intensity factor, σ is the remote stress applied, 'a' is the crack length and M is the magnification factor which depends on the type of crack and the geometry of the component. If the magnitude of the remote stress applied is increased, the value of K increases, and the component finally fails, when the value of K is equal to a material property called the critical stress intensity factor, K_c. Hence, the remote stress at which failure occurs, σ_f is given by

$$\sigma_f = \frac{K_c}{M\sqrt{\Pi a}} \tag{2}$$

If, instead of failing the component by increasing the applied remote stress, constant amplitude loading is applied, then the crack will propagate as given by Paris' law (Paris 1963). Paris' equation can be written in the following integral form

$$N = \int_a^{a_x} \frac{da}{C(M\Delta\sigma\sqrt{\Pi a})^m} \tag{3}$$

where C and m are material constants and N is the number of cycles of stress range $\Delta\sigma$ applied for a crack of length 'a' to propagate to a given crack length a_x.

The stress intensity factor gives a relationship between the residual strength and the crack length while crack propagation laws give a relation between crack length and number of cycles. These two relations have been combined to give a relationship between residual strength and number of cycles (Oehlers, Ghosh and Wahab 1995). When the value of M in Eqs. (2) and (3) is constant, a closed form solution can be obtained while when M varies with crack length, the solution has to be obtained by numerical integration. The value of M is constant for idealised elements such as an infinitely wide plate with a centre crack or a semi infinite plate with an edge crack (Irwin 1957). For such components, combining Eqs. (2) and (3) gives the number of cycles N as

$$N = D - S\left(\frac{1}{\sigma^2}\right)^{1-(m/2)} \tag{4}$$

where

$$D = \frac{\left(\dfrac{K_C^2}{\sigma_i^2 M^2 \Pi}\right)^{1-(m/2)}}{C(M\Delta\sigma)^m \Pi^{m/2}(m/2-1)},$$

$$S=\frac{\left(\dfrac{K_C^2}{M^2\Pi}\right)^{1-(m/2)}}{C(M\Delta\sigma)^m\,\Pi^{m/2}(m/2-1)},$$

where σ is the residual strength.

The constant D is defined in terms of the initial strength σ_i. The material property m is equal to 3 for common classes of steel. Substituting the value of m as 3, the relation between N and residual strength becomes linear as shown in Fig. 1. A component subjected to a loading of constant stress range $\Delta\sigma$ as shown in Fig. 1 will continue to lose strength till it fails when its strength is equal to a maximum load applied. If the failure could be prevented, then the component would continue to lose strength due to crack propagation until it reaches the endurance N_f. This point N_f is the intersection of the extrapolated linear residual strength curve with the number of cycles axes and is termed the asymptotic endurance (Oehlers 1995). The asymptotic endurance N_f is given in the following equation by substituting σ as zero in Eq. 4.

$$N_f=\frac{2\sigma_i}{CM^2\Delta\sigma^3K_C\Pi} \tag{5}$$

where σ_i is the initial strength.

In the case of a real weld, the value of M in Eqs. (2) and (3) is not constant, and varies with the crack length. The values of M for a stiffener weld has been plotted against the ratio of the crack length 'a' over the thickness 't' as shown in Fig. 2 (Albrecht and Yamada 1977). Since M varies, the residual strength variation is no longer a straight line and a solution for residual strength variation must be found from numerical integration. The residual strength variation can be found by taking increments in crack length and finding out the number of cycles required by the crack to propagate from one increment to another. A typical result is shown in Fig. 3 where a curve of residual strength on the Y-axis and number of cycles on the X-axis for a stiffener weld is non-linear in shape.

Thus for constant amplitude loading, a linear residual strength curve is obtained for idealised

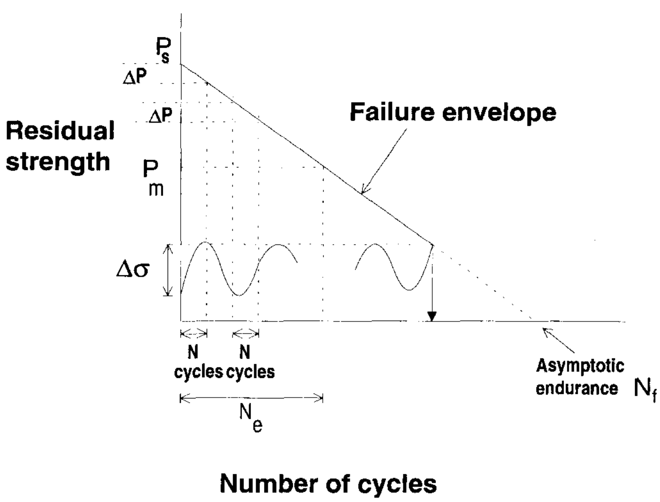

Fig. 1 Linear residual strength variation.

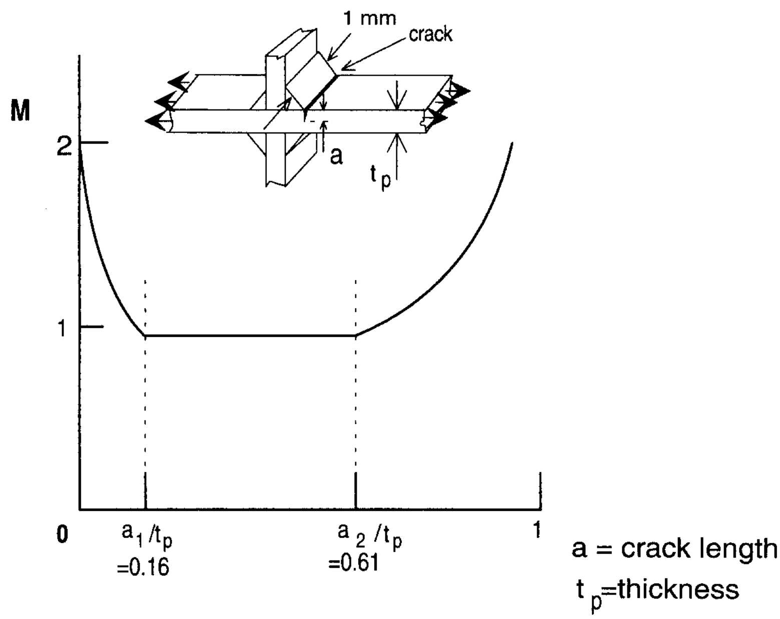

Fig. 2 Stress intensity factor for transverse stiffener.

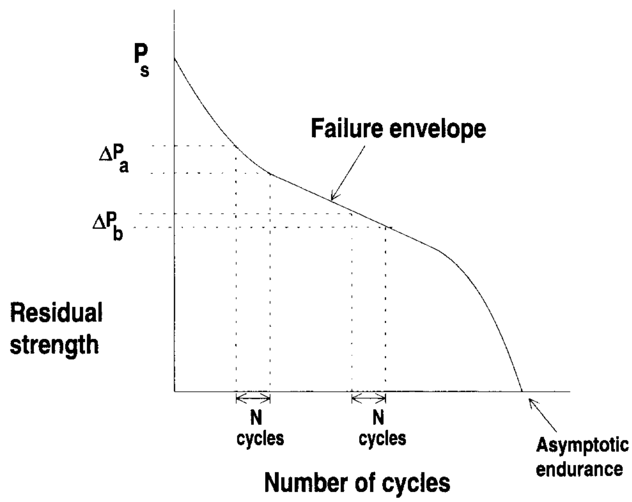

Fig. 3 Non-linear residual strength envelope.

welds and a non-linear curve is obtained for real welds. Consider the fatigue life to be divided into several equal blocks of N cycles each, both for the linear and non-linear curve as in Figs. 1 and 3. In the case of a linear curve the reduction in strength ΔP for any two given blocks is the same as shown in Fig. 1. Hence when the curve is linear, a given block of stress ranges would cause the same reduction in strength irrespective of when the block is applied. However for a non-linear curve the reduction in strength, ΔP_a and ΔP_b for any two given blocks is not equal, as shown in Fig. 3. The reduction in strength for a block in a non-linear curve thus depends on whether the block is applied early or late in the fatigue life i.e., the period of application. Linear curves subjected to several equal blocks are therefore easy to assess since if we know the reduction in strength for one block we can find the reduction due to any number of blocks. In the case of a non-linear curve, the reduction must be found individually for each block.

In practise, however, the component will rarely be subjected to equal blocks of the same stress ranges. Both the number of cycles and the magnitude of the stress ranges will vary from one block to another.

3. Sequence of loading

When blocks of different stress ranges are applied to a weld, the residual strength curves would show a non-linear variation within each block of cycles in the case of a real weld and a linear variation in the case of an idealised weld. An investigation has been carried out here by applying two blocks of stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$ with varying number of cycles to both a real and an idealised weld.

3.1. Real weld

The investigation of a real weld is carried out by using the simplest type of a non-linear failure envelope, a quadratic curve of the form $Y=mX^2+C$ which can be obtained by substituting $m=4$ in Eq. (4). The component is considered to have an initial crack length of a_i and an initial strength of P_s. Fig. 4 shows the residual strength variation if the component is subjected to a single stress range $\Delta\sigma_1$ or $\Delta\sigma_2$ of fatigue loading throughout its life. The curve *EJ* gives the variation for the stress range $\Delta\sigma_1$ while the curve *EK* gives the variation for the stress range $\Delta\sigma_2$. The asymptotic endurance for the curve *EJ* is taken as $(N_f)_3$ and for the curve *EK* as $(N_f)_4$.

The equation of the curve *EJ* can be obtained by substituting the boundary conditions $Y=0$, $X=(N_f)_3$, and $X=0$, $Y=P_s$ as

$$Y=-\frac{P_s}{(N_f)_3}X^2+P_s \tag{6}$$

Similarly the equation of the curve *EK* can be obtained by substituting the conditions $Y=0$, $X=(N_f)_4$ and $X=0$, $Y=P_s$ as

$$Y=-\frac{P_s}{(N_f)_4^2}X^2+P_s \tag{7}$$

Let the first block of fatigue loading applied be of N_3 cycles with a stress range $\Delta\sigma_1$. Let the initial crack propagate due to the application of this block to the crack length a_x so that the residual strength decreases to $(P_m)_3$. The reduction in strength of the component can be given using the curve *EJ*. Substituting N_3, $(P_m)_3$ in Eq. (6) we get

$$\frac{P_s-(P_m)_3}{P_s}=\frac{N_3^2}{(N_f)_3^2} \tag{8}$$

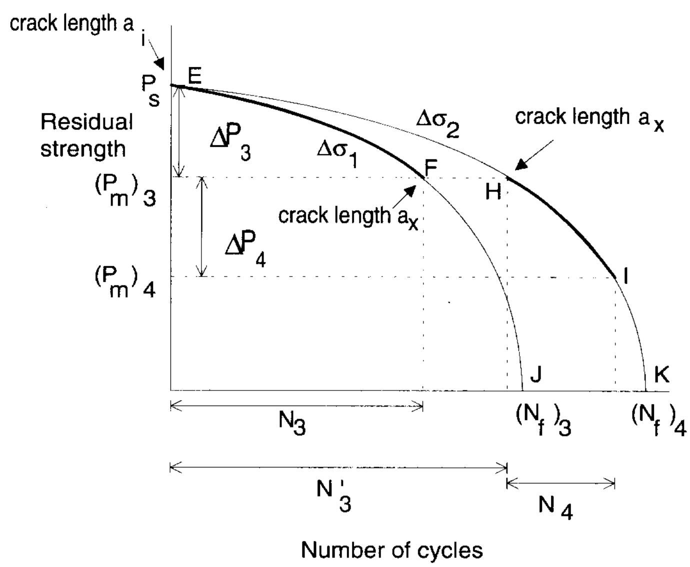

Fig. 4 Quadratic residual strength curve with variable ranges.

Denoting $P_s-(P_m)_3$ by ΔP_3, Eq. (8) can be written as

$$\frac{\Delta P_3}{P_s}=\frac{N_3^2}{(N_f)_3^2} \tag{9}$$

Hence, after application of the first block of loading, the component has lost ΔP_3 of its strength and the condition of the component is represented by the point F in Fig. 4. The component at this stage is subjected to the second block of fatigue loading consisting of N_4 cycles of the stress range $\Delta\sigma_2$. The reduction in strength due to this stress range is given using the curve EK. Fig. 4 shows that the point H in the curve EK corresponds to the same residual strength $(P_m)_3$ as the point F in curve EJ and also to the same crack length a_x. Thus, application of the stress range $\Delta\sigma_2$ causes the strength to reduce along the curve EK starting from the point H. The number of cycles corresponding to the point H in curve EK is denoted as N'_3. Substituting the coordinates of the point H, $(P_m)_3$ and N'_3 in Eq. (7) we get

$$(P_m)_3=-\frac{P_s}{(N_f)_4}N'^2_3+P_s \tag{10}$$

If, after application of the block of loading of N_4 cycles, the component has reached the position I in Fig. 4 corresponding to a residual strength of $(P_m)_4$, substituting the coordinates of I in Eq. (7) we get

$$(P_m)_4=-\frac{P_s}{(N_f)_4^2}(N'_3+N_4)^2+P_s \tag{11}$$

Subtracting Eq. (11) from Eq. (10) gives the reduction in strength $(P_m)_3-(P_m)_4$

$$\frac{(P_m)_3-(P_m)_4}{P_s}=\frac{N_4^2-2N'_3N_4}{(N_f)_4^2} \tag{12}$$

Denoting the reduction in strength $(P_m)_3-(P_m)_4$ as ΔP_4 Eq. (12) can be written as

$$\frac{\Delta P_4}{P_s}=\frac{N_4^2-2N'_3N_4}{(N_f)_4^2} \tag{13}$$

It can be seen from Fig. 4 that N'_3 in Eq. (13) is the number of cycles required by a component with an initial crack length a_i to increase in size to a crack length a_x corresponding to the strength $(P_m)_3$ when subjected to a single stress range $\Delta\sigma_2$. Application of the first block of the single stress range $\Delta\sigma_1$ of N_3 cycles also causes the initial crack length a_i to increase in size to a crack length a_x. The value of both N'_3 and N_3 can be given by Eq. (3). In both cases, the crack length propagates from a_i to a_x. However, the stress range applied in the case of N'_3 is $\Delta\sigma_2$ while $\Delta\sigma_1$ the stress range in the case of N_3. The values of the magnification factor M and the constant m in Paris' equation is the same for both cases. Dividing the value of N'_3 obtained using Eq. (3) by the value of N_3 also obtained using Eq. (3) we get

$$N'_3=\left(\frac{\Delta\sigma_1}{\Delta\sigma_2}\right)^3\times N_3 \tag{14}$$

Substituting N'_3 in Eq. (14) into Eq. (13) gives

$$\frac{\Delta P_4}{P_s}=\frac{N_4^2-2\left(\frac{\Delta\sigma_1}{\Delta\sigma_2}\right)^3 N_3N_4}{(N_f)_4^2} \tag{15}$$

It can be seen from Eq. (15) that the reduction of strength due to the stress range $\Delta\sigma_2$ depends not only on the corresponding N_4 cycles but also on the N_3 cycles applied earlier. Hence for the quadratic residual strength curve, when a number of blocks of different stress range are applied, the reduction in strength for a given block depends on the stress ranges and number of applications of the previous blocks.

3.2. Idealised weld

Let us now consider the case of an idealised component being subjected to two different stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$. As in the case of a real weld the component is considered to have an initial crack length a_i and an initial strength P_s. Fig. 5 shows the residual strength variation if the component was subjected to a single stress range $\Delta\sigma_1$ or $\Delta\sigma_2$ of fatigue loading throughout their lives. The curve AX gives the variation for the stress range $\Delta\sigma_1$ while the curve AZ gives the variation for the stress range $\Delta\sigma_2$. Let the corresponding asymptotic endurances be $(N_f)_1$ and $(N_f)_2$.

The equation of the linear curve AX can be obtained after substituting the boundary points $Y=0$, $X=(N_f)_1$ and $X=0$, $Y=P_s$ as

$$Y=-\frac{P_s}{(N_f)_1}X+P_s \tag{16}$$

while the equation of the curve AZ can be obtained by substituing $Y=0$, $X=(N_f)_2$ and $X=0$, $Y=P_s$ as

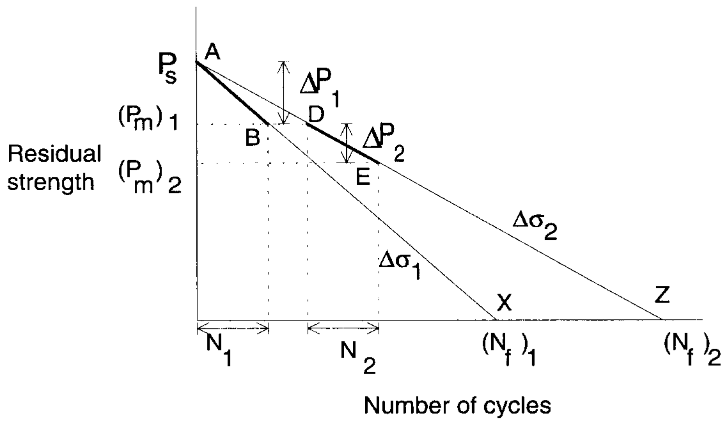

Fig. 5 Linear residual strength curve with variable ranges.

$$Y = -\frac{P_s}{(N_f)_2} X + P_s \tag{17}$$

Let the first block of fatigue loading applied be of N_1 cycles so that the strength of the component reduces by ΔP_1. Substituting the reduction in strength ΔP_1 and the number of cycles N_1 in Eq. (16) we get

$$\frac{\Delta P_1}{P_s} = \frac{N_1}{(N_f)_1} \tag{18}$$

After losing ΔP_1 of its strength due to the application of the first block of loading the condition of the component is represented by the point B in Fig. 5. The component at this stage is subjected to a second block of fatigue loading consisting of N_2 cycles of the stress range $\Delta\sigma_2$. The reduction in strength due to the second block of loading is calculated using the residual strength curve AZ. Fig. 5 shows that the point D in curve AZ corresponds to the same residual strength and hence crack length as the point B in AX. Thus, application of the stress range $\Delta\sigma_2$ causes the strength to reduce along AZ starting from the point D. Hence, if the reduction in strength due to the application of stress range $\Delta\sigma_2$ is given as ΔP_2, substituting ΔP_2 and N_2 in Eq. (17) we get

$$\frac{\Delta P_2}{P_s} = \frac{N_2}{(N_f)_2} \tag{19}$$

Adding Eqs. (18) and (19) we get

$$\frac{\Delta P_1}{P_s} + \frac{\Delta P_2}{P_s} = \frac{N_1}{(N_f)_1} + \frac{N_2}{(N_f)_2} \tag{20}$$

The equation above shows that the reduction in strength for a given stress range is solely dependent on the number of cycles of the stress range applied and the corresponding asymptotic endurance. Therefore the reduction in strength for a linear residual strength variation is independent of the sequence of loading.

As shown by the previous investigation for real welds with non-linear variations, the damage for each stress range depends on the previous history of applied stress ranges. Hence, with an increase in the number of stress ranges, calculating the reduction in strength due to the later stress ranges would get increasingly complex. Also, if the order of application of the stress ranges is altered, this would mean that the residual strength variation would alter considerably and would have to be recalculated, since each stress range depends on all the previous ranges applied. However, for idealised welds with linear curves, the change in strength for a given stress range does not depend on the previously applied stress range. The damage due to a stress range being applied is discrete and independent of other load applications. The total reduction is a summation of the reduction for each individual stress range. Any change in the order of application of the stress ranges does not alter the total reduction in strength.

Therefore linear curves are much easier to deal with and an assessment procedure has been developed here that is based entirely on linear curves. The basic equations are developed for an idealised weld which shows a linear residual strength variation for a given stress range. The procedure is then extended to cover real welds where the non-linear curve for a given stress range is approximated into several linear sections.

4. Fundamental equations of a linearised system

An assessment procedure has been developed here for a linear residual strength curve that can be used for assessment of idealised welds. However, since real welds are also assessed using linear curves, the assessment procedure developed also provides the fundamental equations for assessing a real weld.

When the curve is linear, the reduction in strength due to two different blocks of loading is given by Eq. (20). A general damage accumulation equation is formed by extending Eq. (20) to accommodate X blocks of ranges. If the reduction in strength for stress ranges $\Delta\sigma_1$, $\Delta\sigma_2$, $\cdots$, $\Delta\sigma_x$ applied for N_1, N_2, $\cdots$, N_x cycles is given by ΔP_1, ΔP_2, $\cdots$, ΔP_x, then the total damage can be given as

$$\frac{\Delta P_1}{P_s}+\frac{\Delta P_2}{P_s}+\cdots+\frac{\Delta P_x}{P_s}=\frac{N_1}{(N_f)_1}+\frac{N_2}{(N_f)_2}+\cdots+\frac{N_x}{(N_f)_x} \tag{21}$$

or

$$\frac{\sum_{i=1}^{i=x}\Delta P_i}{P_s}=\sum_{i=1}^{i=x}\frac{N_i}{(N_f)_i} \tag{22}$$

It is to be noted that the right hand side of Eq. (22) is similar or Miner's damage model except that here the asymptotic endurance is used while in Miner's model the endurance of the component is used.

Eq. (22) gives the reduction in total strength of a component by summing up the reduction caused by each stress range. However, in actual practise, a weld used in a bridge will be subjected to an extremely large number of stress ranges and to calculate the reduction in strength of the component for each individual stress range will be extremely cumbersome. An efficient method is to calculate an equivalent single stress range which will cause the same reduction in strength as all the stress ranges put together. Eq. (22) can be written in terms of the stress ranges applied by substituting the value of the asymptotic endurance $(N_f)_i$ from Eq. (5) and rearranging to give

$$\sum_{i=1}^{i=x} \Delta P_i = \frac{P_s C M^3 \Pi^{3/2} \sqrt{a_i}}{2} \sum_{i=1}^{i=x} N_i (\Delta \sigma_i)^3 \tag{23}$$

where P_s, C, M and a_i are constants for the given component. Hence, the reduction in strength for a given component depends on the summation of the number of cycles times the cube of the stress range. This reduction in strength can also be written in terms of a single equivalent stress range $\Delta\sigma_e$ as

$$\sum_{i=1}^{i=x} \Delta P_i = \frac{P_s C M^3 \Pi^{3/2} \sqrt{a_i}}{2} (\Delta \sigma_e)^3 \sum_{i=1}^{i=x} N_i \tag{24}$$

Equating Eqs. (23) and (24) gives $\Delta\sigma_e$ as

$$(\Delta \sigma_e)^3 = \frac{\sum_{i=1}^{i=x} N_i (\Delta \sigma_i)^3}{\sum_{i=1}^{i=x} N_i} \tag{25}$$

The equivalent stress range $\Delta\sigma_e$ can now be used to find the residual strength variation of the welded component having a linear residual strength variation. Thus, for a linear curve as shown in Fig. 1, the residual strength P_m after application of N_e number of cycles can be given in terms of the static strength P_s and the asymptotic endurance N_f for the equivalent stress range $\Delta\sigma_e$ as follows

$$\left(1 - \frac{P_m}{P_s}\right) = \frac{N_e}{N_f} \tag{26}$$

Eq. (26) can be modified to give the increase in crack length. The stresses P_m and P_s can be substituted in place of σ_f in Eq (2). The values of P_m and P_s can thus be obtained in terms of their corresponding crack lengths a_m and a_i. Thus, Eq. (26) can be written as

$$\left(1 - \frac{M_i}{M_m} \sqrt{\frac{a_i}{a_m}}\right) = \frac{N_e}{N_f} \tag{27}$$

where a_i is the inital crack length (corresponding to strength P_s) and M_i the corresponding magnification factor, a_m is the crack length (corresponding to the strength P_m) and M_m the corresponding magnification factor after application of N_e cycles. The variation of stress σ_y at which the component yields can also be found out as the crack progresses. Defining the crack length a_m in terms of the stress at which the component yields and substituting in Eq. (24) we get

$$\left[1 - \frac{M_i}{M_m} \sqrt{\frac{a_i}{t_p\left(1 - \frac{\sigma_y}{f_y}\right)}}\right] = \frac{N_e}{N_f} \tag{28}$$

where t_p the thickness of the component and f_y is the yield stress of the material.

5. Assessment of real components

The ease with which an assessment procedure has been derived for a single linear fracture envelope is extended here to non-linear envelopes by approximating them into several linear portions. Each linear portion is considered as an individual linear curve which predicts the changes in strength, crack length and yield strength for a period of the fatigue life of the component. A general assessment procedure for a given type of component has been developed using a unit stress range. The general procedure is then adopted for the given load pattern.

The non-linear failure envelope is determined by taking small increments in crack length and numerically integrating to find the number of cycles required to propagate between each increment. The curve has been drawn for a unit stress range of 1 N/mm^2 as shown in Fig. 6. The number of linear portions into which the non-linear curve is broken would depend on the shape of the curve in Fig. 2. In the example of a stiffener weld, the M-curve as shown in Fig. 2 can be separated into three distinct regions. The value of M decreases in the first region, remains constant in the second region and increases in the third region. A typical residual strength curve for the stiffener weld would also have three distinct regions based on the M curve as shown in Fig. 6. The three portions have been linearised as AB, BC and CD. The crack lengths a_1 and a_2 at which the shape of the residual strength curve changes are the same crack lengths at which the M curve in Fig. 2 changes shape.

Consider the linear portion AB as shown in Fig. 6. The initial crack length of the component a_i and the material property K_c is used in Eq. (2) to determine the static strength of the component, P_s. A constant value of M for the linear portion AB, denoted as M_1, is found by calculating the slope of AB. The slope of AB is found by dividing the reduction in strength by the number of cycles between the points A and B in Fig. 6. This slope calculated is equated to the slope S in Eq. (4). Since we are dealing with common steel, the value of m in Eq. (4) is taken as 3 so that the equation represents a linear variation. Also since the non-linear curve was drawn

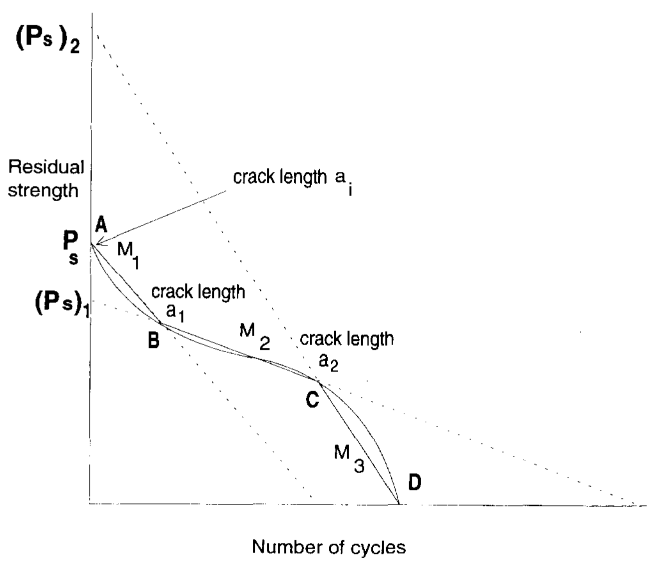

Fig. 6 Linearising a non-linear residual strength curve.

for a unit stress range of 1 N/mm^2, the stress range for calculating the slope S is also taken as 1 N/mm^2.

The magnification factor M_2 for the portion BC in Fig. 6 is calculated in a similar manner to AB. The slope of BC is determined by dividing the reduction in strength by the number of cycles between the points B and C. The value of the slope of BC is equated to slope S in Eq. (4) calculated for a stress range of 1 N/mm^2. The equivalent initial static strength corresponding to the linear portion BC is denoted as $(P_s)_1$ and is calculated using Eq. (4), substituting N as the number of cycles between A and B, the magnification factor M_2 and the static strength corresponding to the point B.

The magnification factor M_3 for the third portion CD is found in a similar manner to the portions AB and BC. The equivalent static strength $(P_s)_2$ corresponding to the portion CD is also calculated.

The static strength, and the magnification factor calculated for each of the three portions is now used in an assessment procedure for the given loading pattern. The welded component will be subjected to a variety of stress ranges which can be represented by a single effective stress range. The static strengths and the magnification factors being known, the asymptotic endurance for the linear portions can be found for the effective stress range using Eq. (5). Fig. 7 shows the linearised curve for the given load pattern with the asymptotic endurance calculated for the linear curves AE, HK and GD using the effective stress range. The linearised residual strength curve occupies only a portion of the linear curves AE, HK and GD as shown by the bold line in Fig. 7. The number of cycles N_i and N_j which separates the linear portions correspond to the crack lengths a_1 and a_2 which separates the curve for the magnification factor M in Fig. 2 into three distinct regions.

The static strength and asymptotic endurance calculated for each of the linear portions can be used in Eq. (26) to find the reduction in residual strength P_m after application of N_e cycles. Thus, if the number of applications N_e is less than N_i cycles, the residual strength P_s and the asymptotic endurance N_{fA-E} for the linear curve AE is used for calculations. If the number of applications is between N_i and N_j cycles, the residual strength P_{s1} and the asymptotic endurance N_{fH-K} for the linear curve HK is used to find the reduction in strength. Finally, if the number of applications is more than N_j, then the static strength P_{s2} and the asymptotic endurance N_f for the linear curve GD is used for calculations. For each portion, the deterioration in strength being known, the change in crack length and yield strength can be calculated using Eqs. (27) and (28).

In order to carry out the assessment of a welded component as discussed, the stress ranges and their frequencies acting on the component have to be obtained at the design point in a bridge to calculate the equivalent range. The constant C in Paris' equation is specified as 3×10^{-13} in the BS PD 6493 1991. The initial crack length for the component is determined from S/N curves given in standard codes as described in the following section.

6. Adapting S/N data

The procedure for calculating the initial crack length is shown in Fig. 8 (Ghosh, Oehlers and Wahab 1995). First a very small crack of length a_s is assumed to exist in the structure

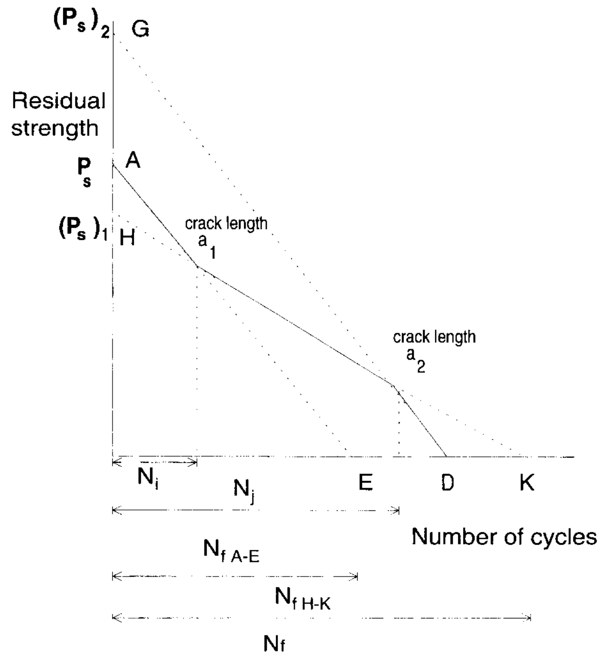

Fig. 7 Assessment using a linearised curve.

and the residual strength curve is determined. The fatigue life of the component N_e that can be obtained from S/N curves given in standard codes such as the BS 5400 1980 or the EURO-CODE 3 1992 and is then measured backwards from the asymptotic edndurance N_f to find the point L on the X-axis. The corresponding residual strength A at endurance L gives the inital strength of the component and Eq. (2) can be used to calculate the initial crack length.

7. Example

The assessment procedure developed has been illustrated here with an example. A 9.5 mm thick stiffener weld, as shown in Fig. 2, whose fatigue life can be obtained from standard codes has been assessed. The figure also shows the variation of the magnification factor M for a crack emanating from the toe of the weld, as obtained by Albrecht and Yamada (1977). The material properites of the component include a tensile strength of 412 N/mm^2; a yield strength of 250 N/mm^2; a fracture toughness of 1400 N/mm$^{3/2}$; the value of C in Eq. (3). of 3×10^{-13}; and the value of m as 3.

7.1. Converting S/N data

The procedure of assessment adopted uses S/N curves to determine the initial crack length. The variation has been determined for an assumed initial crack length of 0.1 mm and a unit stress range of 1 N/mm^2 by numerical integration in Fig. 9. The non-linear residual strength curves is used to determine the initial crack length of the component as dicussed in section 6. Thus the fatigue life of a stiffener weld is calculated from the codes and is measured backwards

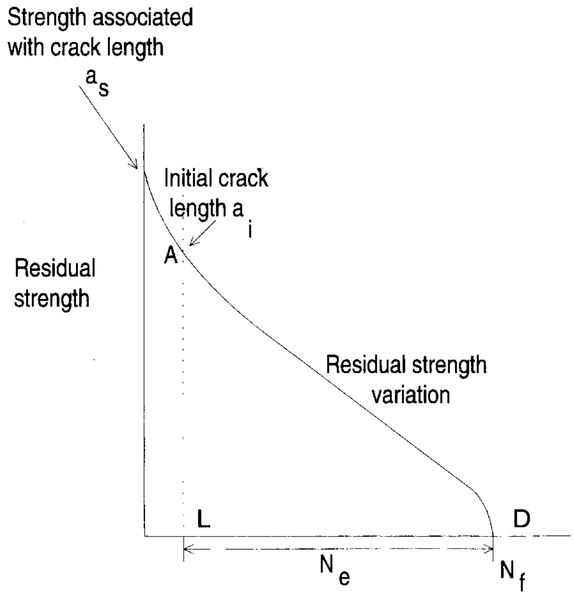

Fig. 8 Initial crack from codes.

from the asymptotic endurance D to find the point L on the X-axis. The corresponding initial crack length is calculated as 0.22 mm.

The initial strength is given by the point A in Fig. 9. The residual strength variation of the component due to a unit stress range is given by the curve AD. In order to assess the component the non-linear curve between A and D is linearised. It can be seen from Fig. 2 that the variation of the magnification factor M can be separated into three distinct regions by the crack lengths 1.6 mm and 5.3 mm. The corresponding residual strength curve of a stiffener weld can therefore be linearised into three portions. AB, BC and CD as shown in Fig. 9. In the schematic diagram in Fig. 10 it is shown that AB can be considered to be a portion of the linear residual strength variation AE of an idealised weld, BC can be considered to be a portion of the linear residual strength variation HK of an idealised weld, and GD can be considered to be a portion of the linear residual strength variation of an idealised weld. The initial strengths and the magnification factor M for the curves AE, HK and GD has been calculated using the method described in section 5. Thus the initial strength and magnification factor for the curve AE has been calculated as 1003 N/mm^2 and 0.80, for the curve HK as 1057 N/mm^2 and 0.97 and for the curve GD as 5155 N/mm^2 and 2.27.

These values of the initial strength and magnification factor has been used to determine the asymptotic endurance of the component using Eq. (5). The asymptotic endurances of AE, HK, and GD in Fig. 10 are calculated as $2.37\times10^{12}/(\Delta\sigma_e)^3$, $2.17\times10^{12}(\Delta\sigma_e)^3$ and $1.512\times10^{12}/(\Delta\sigma_e)^3$ where $\Delta\sigma_e$ is the effective stress range corresponding to the given load pattern. The values of the initial strength and asymptotic endurance can be substituted in Eq. (26) to determine the residual strength variation along AE, HK and GD. Thus for a component which follows the line AE in Fig. 10, the residual strength is given by the equation

$$1-\frac{P_m}{1003}=\frac{2.37\times10^{12}N_e}{(\Delta\sigma)^3} \tag{29}$$

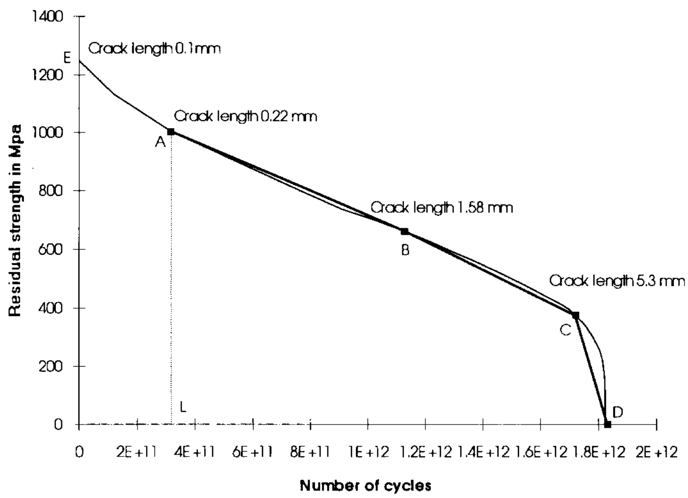

Fig. 9 Linearising the non-linear variation of a stiffener weld.

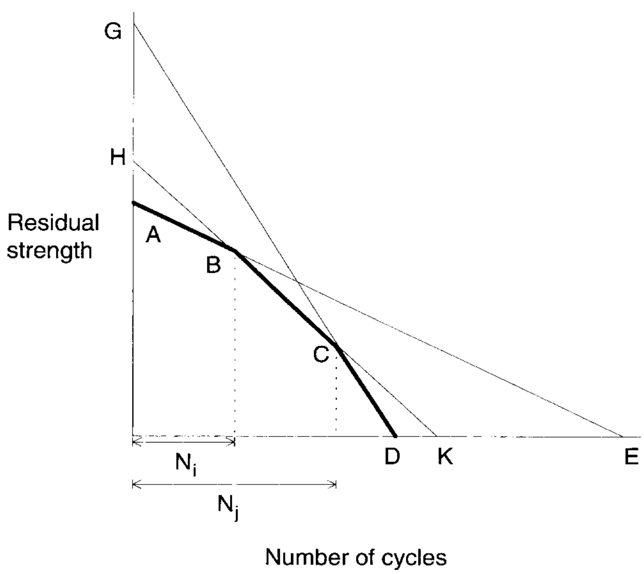

Fig. 10 Linearised residual strength variation of a stiffener weld.

and for the line *HK* the residual strength variation is given by the equation

$$1-\frac{P_m}{1057}=\frac{2.17\times10^{12}N_c}{(\Delta\sigma)^3} \tag{30}$$

while for the line *GD* the residual strength variation is given by the equation

$$1-\frac{P_m}{5541}=\frac{1.512\times10^{12}N_c}{(\Delta\sigma)^3} \tag{31}$$

Table 1 Stresses acting on stiffener weld

	(1)	(2)	(3)	(4)	(5)
Stress range in N/mm^2	30	20	15	10	7.5
Frequency in 10^6	35.5	47.25	35.5	47.25	23.5

7.2. Assessment

The above equations can be used to calculate the residual strength variations for any load pattern. Here the stiffener weld has been assessed for a loading pattern given in Table 1.

The effective stress range for the above load pattern is calculated using Eq. (25) as 20 N/mm^2. Substituting this value in Eqs. (29), (30) and (31), we can determine the corresponding residual strength after any number of applications for each of the curves *AE*, *HK* and *GD* in Fig. 10. It can be seen that the residual strength variations *AE*, *HK* and *GD* of the idealised welds intersect in such a manner so that the lower bound of these curves, *ABCD* gives the actual residual strength variation. Hence for any number of cycles, of the three values of residual strength obtained using *AE*, *HK* and *GD*, the lowest value gives the actual residual strength. Thus substituting the value of N_e as 100 million cycles in the Eqs. (29), (30) and (31) it is found that the curve *HK* gives the lowest value of residual strength as 655 N/mm^2. Thus the value of N_e of 100 million is between N_i and N_j in Fig. 10. The crack length of a component subjected to 100 million cycles is given using Eq. (27) as 1.7 mm and the yield strength using Eq. (28) as 240.3 N/mm^2.

8. Conclusions

Presently welded components are designed using S/N curves, which give the endurance of a component. Assessment of components at any intermediate period of their lives is carried out by undertaking site checks to detect cracks. If cracks are detected, then linear elastic fracture mechanics is used to find out their remaining lives. A simple method of assessment of welded components is developed here which helps to verify the condition of the weld before more expensive inspection is carried out to detect damage. The procedure combines S/N curves, which at present are used for design, with linear elastic fracture mechanics. The procedure can serve as a useful tool for practising engineers to be used alongside present methods of design to predict remaining strength, remaining life and crack length at any intermediate period of its life. It has been shown that for residual strength curves that are linear for a given stress range, the reduction in strength for an individual stress range in a sequence of ranges depends on the previous stress ranges applied. Hence if we use a non-linear curve for assessment we will require to determine the damage cycle by cycle which is too cumbersome. Therefore even though actual welds in service show non-linear variations, such failure envelopes has been linearised to provide a simple method of assessment.

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