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Optimal cross-section and configuration design of cyclic loaded elastic-plastic structures

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Abstract. This paper describes a continuum variational formulation for design optimization of nonlinear structures in the elastic-plastic domain, where unloading and reloading of the structures are allowed to occur. The Total Lagrangian procedure is used for the description of the structural deformation. The direct differentiation approach is used to derive the sensitivities of the various structural response measures with respect to the design parameters. Since the material goes into the inelastic range and unloading and reloading of the structure are allowed to occur, the structural response is path dependent and an additional step is needed to integrate the constitutive equations. It can be shown, consequently, that design sensitivity analysis is also path-dependent. The theory has been discretized by the finite element technique and implemented in a structural analysis code. Mathematical programming approach is used for the optimization process. Numerical applications on trusses are performed, where cross-sectional areas and nodal point coordinates are treated as design variables. Optimal designs have been obtained and compared by using two different strategies: a two level strategy where the levels are defined accordingly the type of design variables, cross sectional areas or node coordinates, and optimizing simultaneously with respect to both types of design variables.

Key words: optimal design; non-linear; structures

1. Introduction

Aim of the structural engineering is to design structures meeting prescribed specifications and being optimal with respect to some criteria. The structural weight has been an important criteria for many structures. Methods of Design Sensitivity Analysis (*DSA*) have been developed in order to calculate accurately the design gradients of the cost function and constraints we need for the optimization problem.

If plastic properties are taken in account, then it is possible to achieve additional material savings. Since most of real loads are cyclic, optimal design of elastic-plastic structures with unloading and reloading is of considerable practical importance. The real structural behavior leads

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also to the consideration of the material hardening.

For cyclic loading, the structural response is path-dependent. So they are the design gradients of this response. Methods of *DSA* have been reviewed for structures with path-dependent material and applied only to cross-sectional design variables (Tsay 1988, Arora and Cardoso 1989, Valido 1992, etc.).

This paper presents a variational continuum formulation for *DSA* of nonlinear structures, including path-dependent material nonlinearities. The direct differentiation method is used for *DSA*. Finite element technique is used in discretizing the theory and integrating it into a structural analysis-nonlinear programming code. To deal with optimal design of elastic-plastic strain hardening structures, a piecewise linear model of the stress-strain law is considered, where the response obeys either linear hardening law or elastic one, the latter corresponding to loading or unloading. Sample trusses are used as numerical examples. Design variables are cross-sectional areas as well as configuration of the structures. Optimization by levels and simultaneous are performed *w.r.t.* both types of design variables.

2. Nonlinear structural analysis

We shall use Total Lagrangian description for the nonlinear equilibrium. Matrix and tensor notations will be used. To identify the quantities in the deformed configuration of the bodies, a left superscript indicates the configuration in which the quantity occurs, a left subscript indicates the reference configuration. For details of the notation and analysis procedures, Bathe (1982) should be consulted. The equilibrium equation for the body at the time t (load level t) is

$$\int {}_0^t\boldsymbol{S}\cdot\delta_0^t\varepsilon \; {}^0dV - \int {}_0^t\boldsymbol{f}\cdot\delta^t\boldsymbol{u} \; {}^0dV - \int {}_0^t\boldsymbol{T}^0\cdot\delta^t\boldsymbol{u} \; {}^0d\Gamma_T = 0 \tag{1}$$

where ${}_0^t\boldsymbol{S}$, ${}_0^t\varepsilon$, ${}_0^t\boldsymbol{f}$, ${}_0^t\boldsymbol{T}^0$, ${}^t\boldsymbol{u}$, ${}^t\boldsymbol{u}^0$ are respectively 2nd Piola-Kirchhoff stress tensor, Green-Lagrange strain tensor, body force, prescribed surface traction, displacement and prescribed displacements, ${}^0V$ and ${}^0\boldsymbol{\Gamma}$ are the initial volume and its boundary, ${}^0\Gamma_T$ is the traction specified boundary, ${}^0\Gamma_u$ is the displacement specified boundary, δ refers to arbitrary variation of the state fields, and '$\cdot$' refers to standard tensor product. A general constitutive law is taken as

$${}_0^t\boldsymbol{S} = \boldsymbol{\Phi}\,({}_0^\tau\varepsilon,\; 0\leq\tau\leq t,\; \boldsymbol{b}) = \int_0^t {}_0^\tau\dot{\boldsymbol{S}}\, d\tau \tag{2}$$

where $\boldsymbol{b}$ is here the design representing material parameters as initial yield stress and hardening. For path-dependent problems $\boldsymbol{\Phi}$ takes an integral form. In numerical implementations, however, the explicit form of $\boldsymbol{\Phi}$ is not needed. Only an incremental stress-strain relation is required. To solve Eq. 1 for nonlinear response, we use incremental decomposition and linearization to get the incremental virtual work equation

$$\int {}_0\boldsymbol{S}\cdot\delta_0\boldsymbol{e} \; {}^0dV + \int {}_0^t\boldsymbol{S}\cdot\delta_0\eta \; {}^0dV = \int {}_0\boldsymbol{f}\cdot\delta\boldsymbol{u} \; {}^0dV + \int {}_0\boldsymbol{T}^0\cdot\delta\boldsymbol{u} \; {}^0d\Gamma_T \tag{3}$$

and the incremental constitutive law

$${}_0\boldsymbol{S} = {}^t\boldsymbol{\Phi}_{,\varepsilon}\cdot{}_0\boldsymbol{e} \tag{4}$$

where $\boldsymbol{u}$, ${}_0S$, ${}_0\varepsilon={}_0\boldsymbol{e}+{}_0n$, ${}_0f$, and ${}_0\boldsymbol{T}$ are the increments of the state fields, and

$$
\begin{aligned}
&{}_0\boldsymbol{e}={}_0\alpha(\boldsymbol{u}^T);\ {}_0\eta=(1/2)[({}_0\nabla\boldsymbol{u}^T)({}_0\nabla\boldsymbol{u}^T)^T];\ \delta_0\boldsymbol{e}={}_0\alpha(\delta\boldsymbol{u}^T);\\
&{}_0\alpha=(1/2)\{[{}_0\nabla(\,)]+[{}_0\nabla(\,)]^T+[{}_0\nabla(\,)][{}_0\nabla{}^t\boldsymbol{u}^T]^T+[{}_0\nabla{}^t\boldsymbol{u}^T][{}_0\nabla(\,)^T]\};\\
&\delta_0\eta={}_0\gamma(\boldsymbol{u}^T,\ \delta\boldsymbol{u}^T);\ {}_0\gamma=(1/2)\{[{}_0\nabla(\delta\boldsymbol{u}^T)]\ [{}_0\nabla\boldsymbol{u}^T]^T+[{}_0\nabla\boldsymbol{u}^T][{}_0\nabla(\delta\boldsymbol{u}^T)]\}
\end{aligned}
\tag{5}
$$

The tangent or instantaneous modulus tensor of the material at load instant t, ${}^t\boldsymbol{\Phi}_{,\varepsilon}={}^t\boldsymbol{E}$, is dependent on the stress and strain history and is considered continuously differentiable except at the points corresponding to yielding, unloading and reloading. For numerical implementation, we assume a piecewise linear stress-strain law with corners at initial yield points and at unloading and reloading points.

3. Lagrange-Eulerian description concept in design

The undeformed configuration is taken as reference during the analysis process. However, during the design process, this configuration changes with the design and another configuration has to be selected as referential. Let us call this referential as design reference volume or control volume $\bar{V}$ with boundary $\bar{\Gamma}$. When the analysis is performed, we have a Lagrangian description of the deformation. When t is fixed and a design step is performed, we have an Eulerian description of the design variation (Cardoso 1987, Cardoso and Arora 1988). The mapping of the control configuration onto the undeformed configuration is defined as

$$
\begin{gathered}
{}^0dV=J\ d\bar{V};\qquad {}^0d\Gamma=\bar{J}\,d\bar{\Gamma};\qquad {}^t\boldsymbol{u}={}^t\boldsymbol{u}({}^r\boldsymbol{x});\qquad {}_0^t\varepsilon={}_0^t\varepsilon({}^rx);\qquad {}_0^t\boldsymbol{S}={}_0^t\boldsymbol{S}({}^r\boldsymbol{x});\\
J=|\boldsymbol{X}|;\qquad \boldsymbol{X}=\frac{\partial({}^0x,\ {}^0y,\ {}^0z)}{\partial({}^rx,\ {}^ry,\ {}^ry)};\qquad \bar{\boldsymbol{X}}=\boldsymbol{X}^{-1};\qquad \bar{J}=J\|\bar{\boldsymbol{X}}^T\boldsymbol{n}\|;\qquad {}_0\nabla=\bar{\boldsymbol{X}}^T{}_r\nabla
\end{gathered}
\tag{6}
$$

where J is the Jacobian and $\bar{J}$ is the area metric with $\boldsymbol{n}$ as the unit surface normal. For oriented bodies such as bars or beams, J and $|\boldsymbol{X}|$ may be different from each other if we use volume integrals throughout the sensitivity analysis. After transformation, the domains of performance and equilibrium integrals become free of design, and there is no distinction between shape and nonshape problems. The control volume concept gets translated quite naturally into isoparametric finite element discretization, where the parent coordinates are the reference coordinates (Cardoso 1987, Cardoso and Arora 1988).

4. Cost and constraint functionals

Cost and constraint functionals are functionals defined by the optimization problem that require *DSA*. They represent measures on stresses, displacements, reaction forces, weight, compliance, etc. We may take a general performance functional as

$$
\Psi=\int G({}_0^t\boldsymbol{S},\ {}_0^t\varepsilon,\ {}^t\boldsymbol{u},\ \boldsymbol{b})\ dV(\boldsymbol{b})+\int h({}^t\boldsymbol{u},\ {}_0^t\boldsymbol{T}^0,\ \boldsymbol{b})\ d\Gamma_T(\boldsymbol{b})+\int g({}_0^t\boldsymbol{T},\ {}^t\boldsymbol{u}^0,\ \boldsymbol{b})\ d\Gamma_u(\boldsymbol{b})
\tag{7}
$$

where the domain is also dependent on the design.

5. Design sensitivity analysis

Consider the total design variation $\bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \tilde{\delta}\Psi$ of the performance functional Ψ of the Eq. (7), where $\bar{\bar{\delta}}$ and $\tilde{\delta}$ indicate explicit and implicit design variations. The direct differentiation approach is an efficient and easy implemented technique of *DSA* for path-dependent problems. It is based on satisfying the structural equilibrium after design variations. Vanishing the total design variation of the equilibrium equation, we get an auxiliary equation to solve for the implicit design variation of the state fields in terms of equilibrium explicit variations. We will call primary structure and auxiliary structure respectively to the original structure and the structure corresponding to the auxiliary equation. Taking design variations for the Eq. (1), after transformation indicated in Eqs. (6), we have the auxiliary structural equilibrium in control coordinates at load level t as

$$\int \tilde{\delta}_0^t \boldsymbol{S} \cdot \delta_0^t \varepsilon \; J\, d\bar{V} + \int {}_0^t\boldsymbol{S} \cdot {}_0\gamma(\bar{\delta}^t \boldsymbol{u}^T,\ \delta^t \boldsymbol{u}^T)\; J\, d\bar{V} =$$

$$\int \{\bar{\bar{\delta}}({}_0^t\boldsymbol{f}J) \cdot \delta^t \boldsymbol{u} - \bar{\bar{\delta}}({}_0^t\boldsymbol{S}J) \cdot \delta_0^t \varepsilon - {}_0^t\boldsymbol{S}J \cdot \bar{\bar{\delta}}(\delta_0^t \varepsilon)\} d\bar{V} - \int \bar{\bar{\delta}}({}_0^t\boldsymbol{T}^0\bar{J}) \cdot \delta^t \boldsymbol{u} \; d\bar{\Gamma}_T \tag{8}$$

where

$$\delta_0{}^t \varepsilon = {}_0\alpha(\delta^t \boldsymbol{u}^T), \qquad \tilde{\delta}(\delta_0{}^t \varepsilon) = {}_0\alpha(\tilde{\delta}(\delta^t \boldsymbol{u}^T)) + {}_0\gamma(\bar{\delta}^t \boldsymbol{u}^T,\ \delta^t \boldsymbol{u}^T), \qquad \tilde{\delta}^t \boldsymbol{u} = \bar{\delta}^t \boldsymbol{u}$$

and virtual primary fields have been used as auxiliary fields. The explicit design variation of the strain field depends on the design transformation gradient $\boldsymbol{X}$ of Eqs. (6). The explicit design variation of the stress field depends on $\boldsymbol{X}$ and on the material parameters of the stress-strain law. Comparing Eqs. (8) and (3), they have the same stiffness, i.e., the stiffness of the auxiliary structure is the tangential stiffness of the primary structure at load level t where sensitivity is required. The shape of the auxiliary structure is the deformed shape of the primary structure. Other terms for the auxiliary structure are

$$\bar{\bar{\delta}}({}_0^t\boldsymbol{f}J)/J; \quad \bar{\bar{\delta}}({}_0^t\boldsymbol{S}J)/J; \quad \bar{\bar{\delta}}(\delta_0{}^t \varepsilon); \quad \bar{\bar{\delta}}({}_0^t\boldsymbol{T}^0\bar{\mathbf{J}})/\bar{\mathbf{J}};$$

respectively body force, initial stress and strain and prescribed surface traction.

For monotonically varying loads, the auxiliary problem of Eq. (8) has to be solved only once at the time (load level) t for the auxiliary fields. For cyclic loading, the constitutive law of Eq. (2) indicates the path-dependence of the auxiliary fields:

$$\tilde{\delta}_0^t \boldsymbol{S} = \tilde{\delta}\left(\int_0^{t'} {}_0^\tau\dot{\boldsymbol{S}}\, d\tau\right) + \tilde{\delta}({}_0^t\boldsymbol{S} - {}_0^{t'}\boldsymbol{S}) \tag{9}$$

Since Eq. (2) is only uniquely defined between each pair of loading and reloading points, the auxiliary problem has to be solved at the times (load levels) t' corresponding to each of these points during the deformation process, and the solution has to be memorized until the next loading or reloading point. It should be noted that sensitivities *w.r.t.* cross-section areas are not continuous at the yielding points. However they are continuous at the same points when the design is configuration. After the auxiliary fields $\bar{\delta}^t \boldsymbol{u}$, $\tilde{\delta}_0{}^t \varepsilon$ and $\tilde{\delta}_0^t \boldsymbol{S}$ are obtained, we need only to calculate $\bar{\delta}_0^t \varepsilon = \bar{\bar{\delta}}_0^t \varepsilon + \tilde{\delta}_0^t \varepsilon$, $\bar{\delta}_0^t \boldsymbol{S} = \bar{\bar{\delta}}_0^t \boldsymbol{S} + \tilde{\delta}_0^t \boldsymbol{S}$, $\delta_0^t \boldsymbol{T} = \delta({}_0^t\boldsymbol{S} \cdot \boldsymbol{n})$, and substitute into the following performance functional design variation

$$\delta\Psi=\int\{(G_{,S}\cdot\bar{\delta}_0{}^tS+G_{,\varepsilon}\cdot\bar{\delta}_0{}^t\varepsilon+G_{,u}\cdot\bar{\delta}^t u+G_{,b}\cdot\delta b)J+G\bar{\delta}J\}d\bar{V}+$$
$$\int\{(g_{,T}\cdot\bar{\delta}_0{}^t T+g_{,u}\cdot\bar{\delta}^t{}_u{}^0+g_{,b}\cdot\delta b)\bar{J}+g\,\bar{\delta}\bar{J}\}d\bar{\Gamma}_u+$$
$$\int\{(h_{,T}\cdot\bar{\delta}_0^t T^0+h_{,u}\cdot\bar{\delta}\,{}^t u+h_{,b}\cdot\delta b)\bar{J}+h\,\bar{\delta}\bar{J}\}d\bar{\Gamma}_T \tag{10}$$

6. Discretization

Finite element technique is used to discretize Eqs. (8)-(10) for numerical evaluation of the design sensitivities of stiffnesses, forces and response quantities. They are calculated at element level (sensitivity elements) and assembled in order to get the design sensitivity analysis model for the entire structure. The isoparametric concept is used for mapping the control configuration onto the undeformed configuration of each element. Details of discretization for different types of sensitivity elements have been presented (Cardoso 1987, Arora and Cardoso 1989). Since the numerical examples in this paper will be adressed to the optimal weight design of truss structures with displacement and stress constraints, we perform the discretization only for this type of elements and response quantities.

Truss elements may be considered as a particular case of the three-dimensional solid if the parametric mapping is given as

$$^0dV=J\,d\bar{V};\quad J={}^0A\,{}^0S_{,r};\quad d\bar{V}=\bar{A}\,dr=1.\mathrm{dr};\quad X=X={}^0s_{,r} \tag{11}$$

where ${}^ts(r)$ is the arc length at load level t at the material point ${}^t\boldsymbol{x}(r)$ given by

$${}^ts(r)=\sum_{k=1}^{N}h_k(r)\,{}^ts^k \tag{12}$$

Note that in this case $J\neq|\boldsymbol{X}|$

The position and displacement of any point of the continuum are given in terms of the nodal coordinates and nodal displacements, with interpolation functions matrix $\boldsymbol{H}$, as

$${}^t\boldsymbol{x}(r)=\boldsymbol{H}(r)\,{}^t\boldsymbol{X};\qquad {}^t\boldsymbol{u}(r)=\boldsymbol{H}(r)\,{}^t\boldsymbol{U};\qquad \boldsymbol{u}(r)=\boldsymbol{H}(r)\,\boldsymbol{U} \tag{13}$$

where

$$\begin{gathered}{}^t\boldsymbol{X}=[({}^t\boldsymbol{x}^1)^T\,({}^t\boldsymbol{x}^2)^T\,\cdots\,({}^t\boldsymbol{x}^N)^T]^T;\qquad {}^t\boldsymbol{x}^k=[\,{}^tx^k\;\;{}^ty^k\;\;{}^tz^k\,]\\ \boldsymbol{H}=[\boldsymbol{H}_1\;\boldsymbol{H}_2\;\cdots\;\boldsymbol{H}_N];\qquad \boldsymbol{H}_k=h_k(r)\boldsymbol{I}_3\end{gathered} \tag{14}$$

The strain vector for a truss element contains only one component along the coordinates. This strain may be written, respectively for initial and parametric coordinates, as

$${}_0^t\varepsilon={}_0^t\boldsymbol{u}_{,s}\cdot{}^0\boldsymbol{x}_{,s}+(1/2)({}_0^t\boldsymbol{u}_{,s}\cdot{}^0\boldsymbol{x}_{,s})^2=(\bar{X})^2\,{}^t\boldsymbol{u}_{,r}\cdot{}^0\boldsymbol{x}_{,r}+(1/2)(\bar{X})^4\,({}^t\boldsymbol{u}_{,r}\cdot{}^0\boldsymbol{x}_{,r})^2 \tag{15}$$

where $\bar{X}=X^{-1}$. Now, if we substitute Eqs. (13) into Eq. (15), we have

$${}_0^t\varepsilon=\left({}_0^t\boldsymbol{B}_{L0}+\frac{1}{2}{}_0^t\boldsymbol{B}_{L1}\right){}^t\boldsymbol{U} \tag{16}$$

where the strain displacement matrices (displacement independent and displacement dependent,

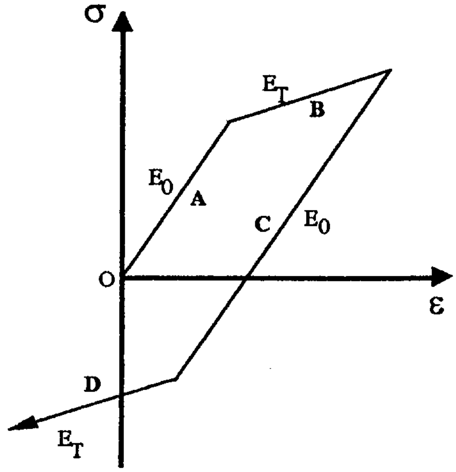


Fig. 1 Stress-Strain law
A-Elastic state
B-Plastic state after initial yielding
C-Unloading after initial yielding
D-Reverse plastic state after initial yielding
E-Reloading after reverse plastic state

respectively) are

$$ {}_0^t\boldsymbol{B}_{L0} = (\bar{X})^2\, {}^0\boldsymbol{X}^T \boldsymbol{H}_{,r}{}^T \boldsymbol{H}_{,r}; \qquad {}_0^t\boldsymbol{B}_{L1} = {}^t\boldsymbol{U}^T\, {}_0^t\boldsymbol{B}_{L0}{}^T\, {}_0^t\boldsymbol{B}_{L0} \tag{17} $$

Since material parameters have not been considered as design, the explicit design variations needed for various response quantities in Eqs. (8)-(10) with respect to initial cross-section area ${}^0A$ and nodal point coordinates ${}^0X$ are

$$ \bar{\delta}X = \sum_{k=1}^{N} h_{k,r}\, \bar{\delta}\, {}^0s^k; $$
$$ \bar{\delta}\bar{X} = -(\bar{X})^2\, \bar{\delta}X; \qquad \bar{\delta}J = {}^0A\, \bar{\delta}X + X\bar{\delta}\, {}^0A; $$
$$ \bar{\delta}\, {}_0^t\boldsymbol{B}_{L0} = [-2(\bar{X})^3\, \bar{\delta}X\, {}^0\boldsymbol{X}^T + (\bar{X})^2\, \bar{\delta}\, {}^0\boldsymbol{X}^T]\boldsymbol{H}_{,r}{}^T\boldsymbol{H}_{,r} \tag{18} $$

We may note that $\bar{\delta}^0$ A and $\bar{\delta}^0x^k$ are the only variations we need to calculate all the response variations.

7. Numerical examples

Optimal minimum weight (volume) design with respect to undeformed cross-sectional area and configuration of truss structures are obtained in the present section. Sensitivity capability is added to a specific finite element nonlinear analysis program in order to supply the nonlinear programming code *ADS* (Vanderplaats 1987) for optimization. Sensitivities of cost and constraints have been verified by finite difference approach before the optimization phase. The method of feasible directions is used to obtain the optimum solutions. A personal computer *IBM AT-486* has been used to run the applications. Materially nonlinear-only problems are presented. Piecewise linear constitutive law for elastic-plastic material with kinematic hardening (Young

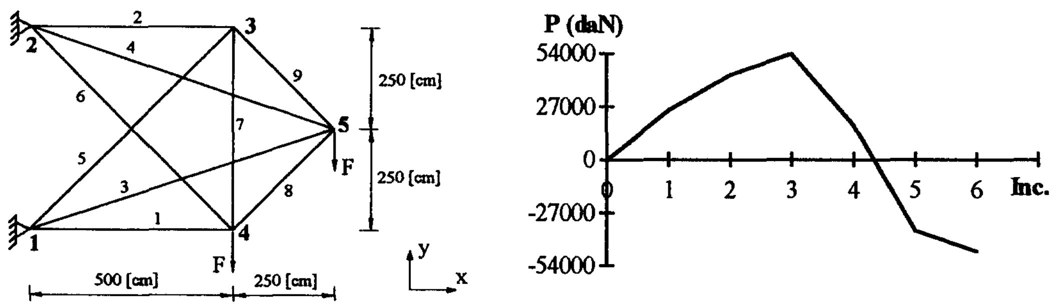


Fig. 2 Nine-bar truss structure and loading

Table 1 Nine-bar optimal designs

Design Variable	Initial Design	Status of the Material in I.D.	case 1 Final Design	case 1 Status of the Material in F.D.	case 2 Final Design	case 2 Status of the Materia l in F.D.	case 3 Final Design	case 3 Status of the Material in F.D.
A1 (cm^2)	20	−C	17.39	−D	20.36	D	19.78	D
A2	20	D	17.44	D	19.89	−D	20.14	−D
A3	20	−C	15.81	D	14.59	D	20.20	D
A4	20	A	15.79	D	14.11	−D	18.55	−D
A5	20	−D	16.31	D	12.33	D	7.90	D
A6	20	−D	16.41	−D	13.83	−D	7.98	−D
A7	20	A	17.11	A	7.23	−D	17.43	A
A8	20	A	17.97	A	12.54	−C	16.46	A
A9	20	A	17.97	A	14.04	C	16.76	A
y3 (cm)	500		493.9		372.2		313.5	
y4	0		4.6		137.2		182.8	
y5	250		249.8		246.7		237.1	
Active Constraints*	–		15		15		6,15	
Max. Violation(%)	–		0.4		0.4		0.42	
Optimal Vol.(cm^3)	104049		86363		68787		72167	

*Constraints numbers: 1-9 for element stress, 10-15 for horizontal and vertical displacements of nodes 3-5, respectively.

modulus/tangent modulus=2) and cyclic loading is considered as shown in Fig. 1.

Constraints are maximum point stresses of truss elements and node displacements for all examples. Optimal designs were performed with three different strategies indicated as

case 1-simultaneous cross sectional areas and nodal coordinates optimization
case 2-optimization by levels: [nodal coordinates [areas]]
case 3-optimization by levels: [areas [nodal coordinates]]

In **case 1**, scaling has been used for the cross-sectional areas to get the same order of values for the sensitivities of the objective function *w.r.t.* both types of design variables.

7.1. Nine-bar ground truss optimal design

A nine-bar truss structure and its cyclic loading are represented in Fig. 2. For this structure,

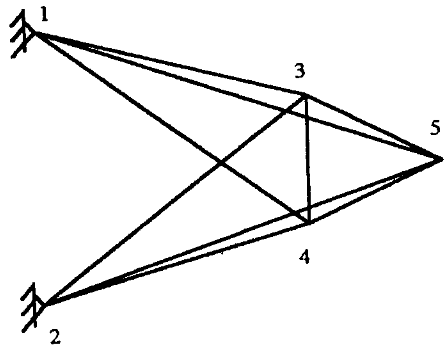
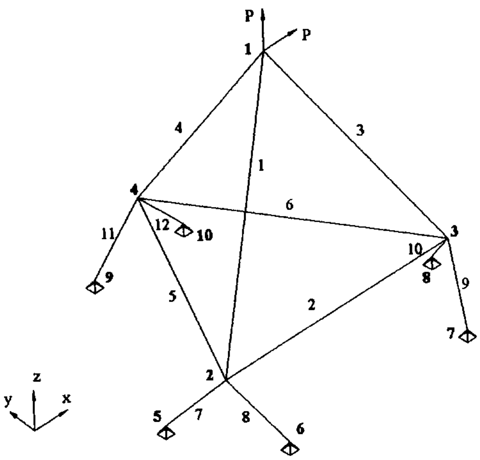


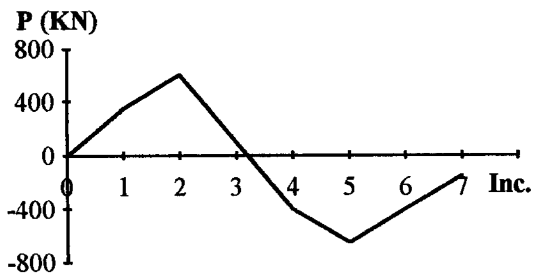
Fig. 3 Best optimal configuration



(a)

Node	X(cm)	Y(cm)	Z(cm)
1	0	0	1016.5
2	−500	−287.7	200
3	500	−287.7	200
4	0	577.4	200
5	−600	−115.5	0
6	−400	−461.9	0
7	400	−461.9	0
8	600	−115.5	0
9	−200	577.4	0
10	200	577.4	0

(b)



(c)

Fig. 4 Three-dimensional twelve-bar truss structure and loading

the material is defined by Young modulus=210 GPa and Initial yielding=240 MPa. Constraints are 360 Mpa for maximum point stresses of truss elements and 4 cm for horizontal and vertical displacements of nodes 3-5. Table 1 shows the initial and optimal designs and Fig. 3 shows the configuration for the best results.

Table 2 Twelve-bar optimal design

Design Variable	Initial Design	Status of the Material in I.D.	case 1 Final Design	case 1 Status of the Material in F.D.	case 2 Final Design	case 2 Status of the Material in F.D.	case 3 Final Design	case 3 Status of the Material in F.D.
A1 (cm^2)	30	−C	24.66	−E	24.77	−E	24.32	−E
A2	30	A	13.28	A	7.9	−C	24.53	A
A3	30	A	13.26	C	8.8	E	24.63	A
A4	30	A	13.26	A	8.0	A	24.98	A
A5	30	A	13.26	E	6.7	A	23.11	A
A6	30	A	13.26	A	6.5	A	23.11	A
A7	30	A	25.31	−C	18.75	−E	24.22	A
A8	30	A	25.25	A	26.01	−C	30.00	−E
A9	30	A	25.41	A	21.12	A	25.99	A
A10	30	A	25.21	A	18.97	A	24.22	A
A11	30	A	25.33	A	18.75	A	24.31	A
A12	30	A	25.26	A	18.75	A	24.31	A
x1 (cm)	0		65.7		−145.8		24.7	
x2	−500		−484.2		−315.2		−219.7	
x3	500		484.1		328.2		218.9	
y1	0		1.3		−207.4		−48.1	
y4	577.4		554.7		410.9		219.2	
z1	1016.5		990.1		500		500	
Active Constraints*	–		1, 12		3		1, 8	
Max. Violation (%)	–		0.4		0.4		0.4	
Optimal Vol. (cm^3)	230844		131704		74466		130603	

*Constraints numbers: 1-12 for element stress, 13-15 for x, y and z displacement of node 1.

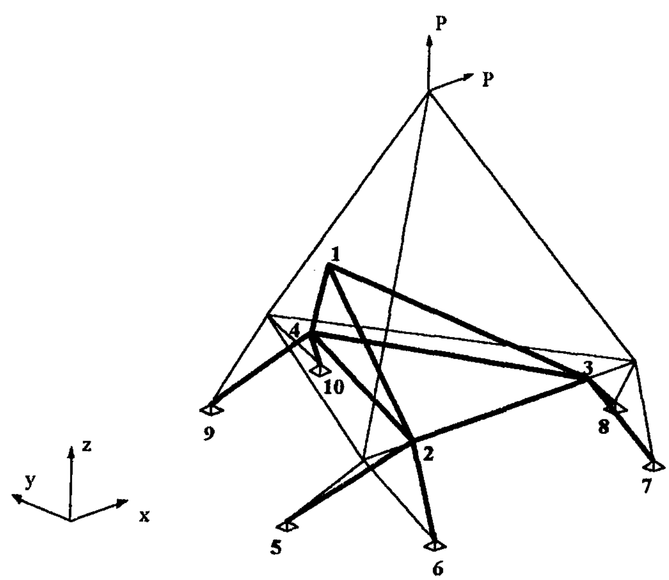


Fig. 5 Best optimal configuration

7.2. Three-dimensional twelve-bar truss optimal design

A three-dimensional twelve-bar truss structure and its cyclic loading are represented in Fig. 4. For this structure, the material is defined by Young modulus=210GPa and Initial yielding=240 MPa. Constraints are 360 MPa for maximum point stresses of truss elements and 6 cm for x, y and z displacements of node 1. Table 2 shows the initial and optimal designs and Fig.5 shows the configuration for the best results.

8. Concluding remarks

Optimal design of structures with path-dependent static response have been presented. The direct differentiation approach and a Lagrangian-Eulerian description of the deformation-design variation process is used to perform a unified viewpoint of sensitivity analysis, by a continuum formulation, with respect to dimensional and configuration problems.

For problems with path-dependent response, design sensitivities are also path-dependent and have to be evaluated and memorized at unloading and reloading points for further accumulation. Design sensitivities *w.r.t.* cross-sectional areas are discontinuous at the yielding points.

The theory is discretized by finite elements for numerical implementation. Constraints must be imposed along the deformation history. If the number of unloading and reloading points is large, a maximum point technique may be suggested.

It is observed that the second strategy (**case** 2-optimization by levels: [nodal coordinates [areas]]) gave the best results for both examples. The differences among the three cases is chiefly due to the different contribution the two types of design variables have to the objective. When configuration design is firstly performed (**case** 3), this has small influence in the objective function because member lengths have small sensitivity *w.r.t.* nodal positions. Next, when cross-section design is performed, some constraints are already active and there is little "room" to vary the areas. In **case** 2, since cross-sectional areas have large influence in the objective function, when design is firstly performed *w.r.t.* the areas, the objective is already substantially reduced before the configuration variation is started. In **case** 1 it happens that the design space is not the same when compared with the **cases** 2 or 3 and local optimum is searched.

Further developments of the theory and applications have been done for dynamic transient loading and are to be submitted for future publication.

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