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Dynamic analysis of structures in frequency domain by a new set of Ritz vectors

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Abstract. The accurate dynamic analysis of structures is usually performed by a fine finite element discretization with very large number of degrees of freedom. Apart from modal analysis, one can reduce the number of final equations by assuming the deformed shape of the structure as a linear combination of independent Ritz vectors. The efficiency of this method relies heavily on the vectors selected. In this paper, a new set of Ritz vectors is proposed. It is primarily proved that these vectors are linearly independent. Subsequently, various two and three-dimensional examples are analyzed based on the proposed method. In each case, the results are compared with the ones obtained based on usual Ritz and modal analysis methods. It is finally concluded that the proposed method is very effective and efficient method for dynamic analysis of structures in frequency domain.

Keywords: Ritz vectors; structural dynamics; frequency domain analysis; DOF reduction methods; linearly independent vectors; convergence of Ritz method

1. Introduction

The Ritz method is a well-known approach for reducing the number of equations in dynamic analysis of structures (Rossit and Ciancio 2008, Jeong 2006). Although, it is a relatively old technique, it captured the attention of researchers once again in the context of finite element method in the 1980's (Wilson *et al.* 1982, Bayo and Wilson 1984, Arnold and Citerley 1985, Wilson and Bayo 1986, Leger *et al.* 1986, Joo and Wilson 1987, Joo *et al.* 1989). Some of these researches tried to improve efficiency of the method (Joo and Wilson 1987, Joo *et al.* 1989, Ibrahimbegovic and Wilson 1990) and others proposed new applications for the method; such as dynamic analysis of large structures (Arnold and Citerley 1985), wave propagation (Bayo and Wilson 1984) and sub-structuring technique (Wilson and Bayo 1986).

In this method, a few vectors are generated initially which are referred to as Ritz vectors. Subsequently, the response of structure is written as a linear combination of these independent vectors, which leads to a reduced number of differential equations (i.e., in time domain) or fewer numbers of linear equations if the analysis is carried out in the frequency domain. Appropriate selection of these vectors is the critical step of this approach, which strongly influences its efficiency.

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The first set of Ritz vectors is introduced by Wilson *et al*. (1982) that is referred to as "Wilson vectors" in this study. These mass-orthogonalized Ritz vectors are much less expensive to generate than normal mode shapes. They showed that the method employing these vectors has better performance than the eigenvector superposition approach. The method can be applied to multi-input loading problems. Such applications are discussed in Wilson (2000). Independently, Nour-Omid and Clough proposed Lanczos vectors that are essentially identical to Wilson algorithm when applied with partial reorthogonalization (Nour-Omid and Clough 1984). In this method, the equations of motion are transformed to reduced tridiagonal form through Lanczos vectors (derived from the Lanczos algorithm) and the response of structure is expressed in Lanczos co-ordinates. Subsequently, these researchers used the block Lanczos algorithm for multi-input loading cases (Nour-Omid and Clough 1985). In addition, Kim *et al.* have discussed the Lanczos method and introduced Modified Lanczos co-ordinates to efficiently solve the dynamic equation of motion of structures under multi-input loads (Kim *et al.* 2003).

Wilson and Lanczos algorithms utilize the load pattern for computing the first vector. Thus, resulting vectors are referred to as load dependent vectors in the literature. Later on, Xia and Humar (1992) and Gu *et al.* (2000) proposed a new set of vectors that take into account the frequency content of the loading. They call these as frequency dependent Ritz vectors. An extensive review of Ritz vectors and their applications for dynamic analysis of structures can be found in Rose (2006). These vectors have also been employed by researchers in nonlinear structural dynamic investigations (Wilson 2000, Spiess and Wriggers 2005).

In this paper, a new set of Ritz vectors are proposed, and the convergence of the method is investigated thoroughly by various examples. Proposed vectors are well distinguished from other alternatives, which are employed by other researchers. Furthermore, the new Ritz vectors can be complex valued if the analysis is decided to be carried out in frequency domain. In the following, generation and concepts of the new Ritz vectors is presented in details.

2. Usual Ritz vectors (Wilson Vectors)

The construction of the usual Ritz vectors is briefly reviewed in this section. Most of the previous studies have utilized vectors that are similar to those which are presented herein (Wilson *et al.* 1982, Bayo and Wilson 1984, Arnold and Citerley 1985, Wilson and Bayo 1986, Leger *et al.* 1986, Joo and Wilson 1987, Joo *et al.* 1989). However, there might be slight differences among them.

Let us now consider the dynamic equilibrium equation of a structural system, which can be written as

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{p}(s,t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices of the system, respectively. Furthermore, $\mathbf{r}$ is the relative nodal displacements vector. For most cases, the time varying loading can be expressed as a summation of spatial load vectors multiplied by functions of time

$$\mathbf{p}(s,t) = \sum_j \mathbf{f}_j(s) g_j(t) = \mathbf{f}(s)\mathbf{g}(t) \tag{2}$$

In the above equation, $\mathbf{f}_j(s)$ indicates spatial distribution of load case, and $g_j(t)$ represents its time variation function. It is noted that it would be sufficient to consider only one vector in the case of earthquake loading corresponding to one of the global coordinate system directions. In this case, $\mathbf{f}_1$ includes the components of mass in that direction for different nodes, and $g_1(t)$ is negative of corresponding earthquake acceleration. The usual Ritz vectors are evaluated based on the following algorithm for this case:

The first vector is obtained by solving the underneath equation

$$\mathbf{K}\mathbf{x}_1^* = \mathbf{f} \tag{3}$$

The other vectors are calculated by the following recursive equation

$$\mathbf{K}\mathbf{x}_i^* = \mathbf{M}\mathbf{x}_{i-1} \quad i = 2, \ldots, n \tag{4}$$

In each step, the computed vector should become orthogonal with respect to mass matrix

$$\mathbf{x}_i = \mathbf{x}_i^* - \sum_{j=1}^{i-1} c_j \mathbf{x}_j \tag{5a}$$

$$c_j = \mathbf{x}_j^T \mathbf{M} \mathbf{x}_i^* \tag{5b}$$

and normalized thereafter such that

$$\mathbf{x}_j^T \mathbf{M} \mathbf{x}_j = 1 \tag{6}$$

In some references (e.g., Wilson *et al.* 1982), the calculated vectors are also orthogonalized with respect to stiffness matrix. This can be done by solving an eigenvalue problem corresponding to the projected mass and stiffness matrices, and employing the extracted eigenvectors.

Derivation of Ritz vectors based on the above algorithm have an interesting physical meaning (Wilson *et al.* 1982).

3. The new proposed Ritz vectors

The proposed algorithm will be presented herein, which is used to obtain the new set of Ritz vectors. Subsequently, some fundamental questions will be discussed which are arisen in dynamic analysis of structures employing these vectors.

Let us consider dynamic equation of a structural system under earthquake loading corresponding to one of the global coordinate directions

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = g_1(t)\mathbf{f}_1(s) \tag{7}$$

It must be mentioned that the analysis is assumed to be carried out in frequency domain for the present study. Therefore, the new Ritz vectors will be presented in this context herein. However, they can also be used in time domain analysis by slight modifications.

Eq. (7) can be written in the frequency domain as follows

$$[-\omega^2\mathbf{M} + \mathbf{K}(1 + 2\beta i)]\mathbf{r}(\omega) = G_1(\omega)\mathbf{f}_1(s) \tag{8}$$

where $\mathbf{r}(\omega)$ is the amplitude of relative nodal displacement vector for a harmonic excitation with

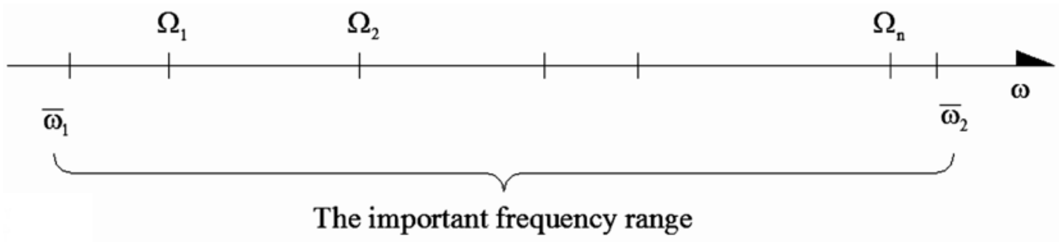

Fig. 1 Positions of frequencies corresponding to Ritz vectors (Ω_i)

frequency ω

$$\mathbf{r}(t) = \mathbf{r}(\omega)e^{i\omega t} \tag{9}$$

and $G_1(\omega)$ is the Fourier transform of $g_1(t)$. Moreover, it is assumed that damping of the structure is of hysteretic type. It means that

$$\mathbf{C} = \frac{2\beta}{\omega}\mathbf{K} \tag{10}$$

where β is the hysteretic damping constant.

The new Ritz vectors are solutions of Eq. (8) for some selected frequencies and considering unit-amplitude excitation function (i.e., $G_1(\omega) = 1$)

$$[-\Omega_i^2\mathbf{M} + \mathbf{K}(1+2\beta i)]\mathbf{X}_i^R = \mathbf{f}_1(s) \tag{11}$$

In the above relation, $\mathbf{X}_i^R$ is the i^{th} Ritz vector and Ω_i is the corresponding selected frequency. It is worth to mention that these frequencies are chosen within the important frequency range (Fig. 1). This range might be known by experience or it can be judged based on Fourier amplitude of ground excitation. However, the exact values of these frequencies in the decided range could be arbitrary in general.

The generated vectors are stored in a matrix as

$$\mathbf{X}^R = [\mathbf{X}_1^R \ \mathbf{X}_2^R \ \dots \ \mathbf{X}_n^R] \tag{12}$$

where n is the number of utilized vectors. Then, response of the structure is written as a combination of these vectors

$$\mathbf{r} = \mathbf{X}^R\mathbf{Y} \tag{13}$$

Where $\mathbf{Y}$ denotes the vector of participation factors. Substituting Eq. (13) into (8) and pre-multiplying both sides of that relation by $(\mathbf{X}^R)^T$ leads to

$$\mathbf{K}_d\mathbf{Y} = \mathbf{F}^R \tag{14}$$

Where matrix $\mathbf{K}_d$ and vector $\mathbf{F}^R$ are defined as below

$$\mathbf{K}_d = (\mathbf{X}^R)^T[-\omega^2\mathbf{M} + \mathbf{K}(1+2\beta i)](\mathbf{X}^R) \tag{15a}$$

$$\mathbf{F}^R = (\mathbf{X}^R)^T G_1(\omega)\mathbf{f}_1(s) \tag{15b}$$

The reduced system of Eq. (14) is solved for any excitation frequency ω and the corresponding $G_1(\omega)$. Thereafter, the calculated vector $\mathbf{Y}$ is utilized to obtain the response through Eq. (13).

3.1 Convergence of the algorithm

The fundamental question arisen is whether the proposed algorithm leads to independent vectors. This would guarantee its convergence to the exact solution as the number of vectors increases. Moreover, the matrix $\mathbf{K}_d$ must not be singular and the solution of Eq. (14) is obtainable for any excitation frequency ω under those circumstances.

In the following, it will be proved that these vectors are independent for any arbitrary selected frequencies as long as they are different and not repeated.

Prior to this, another citable property of new Ritz vectors can be stated as below. New Ritz vectors are response vectors of the structure for initial selected frequencies. Therefore, response calculated through (13) is exact and equal to response of Eq. (8) for those frequencies (Ω_is). This will be clearly seen in the results of analyzed models presented later on section 5.

3.2 Proof of linear independency of new Ritz vectors

Mode shapes of the damped system can be evaluated based on the following equation

$$[-\omega_i^2\mathbf{M}+\mathbf{K}(1+2\beta i)]\mathbf{X}_i = \mathbf{0} \tag{16}$$

where $\mathbf{X}_i$ is the i^{th} mode shape and ω_i is the corresponding natural frequency. The latter quantities are complex valued, while the mode shapes are real valued vectors for the hysteretic type of damping matrix. However, they could also be complex valued in general. Assume that all active mode shapes (M modes) and the squares of natural frequencies of the structure are stored in the following matrices

$$\mathbf{X} = [\mathbf{X}_1 \;\; \mathbf{X}_2 \;\; \dots \;\; \mathbf{X}_M] \tag{17a}$$

$$\Lambda = Diag[\omega_1^2 \;\; \omega_2^2 \;\; \dots \;\; \omega_M^2] \tag{17b}$$

It should be noted that active modes are not perpendicular to $\mathbf{f}_1(s)$ (i.e., $\mathbf{X}_i^T\mathbf{f}_1(s)\neq 0$), Furthermore, the number of active modes depends on the excitation direction. The orthogonality relations of the above eigenvalue problem (16) can be written as follows

$$\mathbf{X}^T\mathbf{M}\mathbf{X} = \mathbf{I} \tag{18a}$$

$$\mathbf{X}^T\mathbf{K}\mathbf{X}(1+2\beta i) = \Lambda \tag{18b}$$

Based on the proposed algorithm, the new Ritz vectors are solutions of Eq. (11). Let us now presume that the i^{th} Ritz vector is written as a linear combination of mode shapes and this relation is substituted in (11) as follows

$$[-\Omega_i^2\mathbf{M}+\mathbf{K}(1+2\beta i)]\mathbf{X}\mathbf{Y}_i = \mathbf{f}_1(s) \tag{19}$$

Pre-multiplying both sides of this equation by $\mathbf{X}^T$ and utilizing the orthogonality relations (18) leads to

$$[-\Omega_i^2\mathbf{I}+\Lambda]\mathbf{Y}_i = \mathbf{F} \tag{20}$$

Where $\mathbf{F}$ is defined as

$$\mathbf{F} = \mathbf{X}^T\mathbf{f}_1(s) \tag{21}$$

The vector of participation factors $\mathbf{Y}_i$ may be easily obtained through relation (20) due to the fact that resulting left hand side matrix is diagonal in that equation. Thus, the j^{th} component of that vector may be written as follows

$$Y_{ji} = \frac{F_j}{-\Omega_i^2 + \omega_j^2} \tag{22}$$

Thereafter, the i^{th} Ritz vector could alternatively be evaluated as below

$$\mathbf{X}_i^R = \frac{F_1}{-\Omega_i^2 + \omega_1^2}\mathbf{X}_1 + \frac{F_2}{-\Omega_i^2 + \omega_2^2}\mathbf{X}_2 + \ldots + \frac{F_M}{-\Omega_i^2 + \omega_M^2}\mathbf{X}_M \tag{23}$$

Considering the above equation, it is obvious that the following relations holds between the modal matrix and the Ritz matrix defined in (12)

$$\mathbf{X}^R = \mathbf{XZ} \tag{24}$$

where matrix $\mathbf{Z}$ is defined as

$$\mathbf{Z} = \begin{bmatrix} \frac{F_1}{-\Omega_1^2 + \omega_1^2} & \frac{F_1}{-\Omega_2^2 + \omega_1^2} & \cdots & \frac{F_1}{-\Omega_M^2 + \omega_1^2} \\ \frac{F_2}{-\Omega_1^2 + \omega_2^2} & \frac{F_2}{-\Omega_2^2 + \omega_2^2} & \cdots & \frac{F_2}{-\Omega_M^2 + \omega_2^2} \\ & \vdots & \vdots & \\ \frac{F_M}{-\Omega_1^2 + \omega_M^2} & \frac{F_M}{-\Omega_2^2 + \omega_M^2} & \cdots & \frac{F_M}{-\Omega_M^2 + \omega_M^2} \end{bmatrix} \tag{25}$$

It is also well known that the mode shapes are independent vectors. Therefore, the linear dependency of Ritz vectors could only occurs if the determinant of matrix $\mathbf{Z}$ vanishes. This determinant can be written in the following form

$$|\mathbf{Z}| = F_1F_2\ldots F_M \times \begin{bmatrix} \frac{1}{-\Omega_1^2 + \omega_1^2} & \frac{1}{-\Omega_2^2 + \omega_1^2} & \cdots & \frac{1}{-\Omega_M^2 + \omega_1^2} \\ \frac{1}{-\Omega_1^2 + \omega_2^2} & \frac{1}{-\Omega_2^2 + \omega_2^2} & \cdots & \frac{1}{-\Omega_M^2 + \omega_2^2} \\ & \vdots & \vdots & \\ \frac{1}{-\Omega_1^2 + \omega_M^2} & \frac{1}{-\Omega_2^2 + \omega_M^2} & \cdots & \frac{1}{-\Omega_M^2 + \omega_M^2} \end{bmatrix} \tag{26}$$

The second term of above expression is known as the Cauchy determinant and has a simple equivalent expression (Prasolov and Ivanov 1994). Utilizing that, determinant of matrix $\mathbf{Z}$ can be written in a more simplified form as

$$|\mathbf{Z}| = \underbrace{F_1F_2\ldots F_M}_{I} \times \underbrace{\frac{\prod_{i,j=1\ i<j}^{M}(-\Omega_i^2+\Omega_j^2)(\omega_i^2-\omega_j^2)}{\prod_{i,j=1}^{M}(-\Omega_i^2+\omega_j^2)}}_{II} \tag{27}$$

The above expression has two distinct parts, *I* and *II*. The Ritz vectors are easily shown to be independent if both of these parts are proven to be non-zero. The first part can become zero only if some F_is vanish. In other words considering Eq. (21), expression $\mathbf{X}_i^T\mathbf{f}_1(s)$ becomes zero for some mode shape vectors, $\mathbf{X}_i$s. this is not possible due to the fact that only active modes are considered in modal matrix $\mathbf{X}$ (17a) as explained previously (i.e., $\mathbf{X}_i^T\mathbf{f}_1(s)\neq 0$).

For part *II* to vanish, two Ω_i or two ω_i must be equal. The first condition is impossible because the vectors are calculated at different frequencies. Moreover, two ω_i are equal only when the structure has two equal natural frequencies. However, under those circumstances, it is well known that one of those two modes can be considered as inactive mode for the assumed excitation direction, similar to what occurs in modal analysis. Therefore, the new Ritz vectors are independent as long as their number does not exceed the number of active modes for the assumed excitation direction.

Considering the above facts, the new Ritz method utilizing *M* vectors that are calculated at arbitrary frequencies will produce accurate response of the structure in the whole frequency spectrum. However, for computing response in a finite frequency band (i.e., important frequency range), it is much more efficient to select frequencies near that range (e.g., uniformly distributed in that range).

4. Analyzed cases and basic parameters

A computer program is developed based on the theory presented in previous sections. Two models of concrete dams are considered, and they are analyzed by utilizing the new set of Ritz vectors. In each case, the accuracy of the method is examined against the direct analysis results, which can be considered as exact. Moreover, rate of convergence is compared against usual Ritz and modal analysis approaches.

4.1 Models

As mentioned, two models of concrete dams are considered for analysis based on the theories presented above. These are a two-dimensional gravity dam example and a three-dimensional concrete arch dam case. The first case relates to Pine Flat concrete gravity dam, which is referred to as model A. A typical section of this dam is considered assuming plane strain condition. The section is discretized by 36 isoparametric 8-node finite elements (Fig. 2).

The second case corresponds to Morrow Point concrete arch dam, which is referred to as model B. An idealized symmetric model of this dam is considered. The geometry of the dam may be found in Hall and Chopra (1983). The dam is discretized by 40 isoparametric 20-node finite elements in this latter case (Fig. 3) similar to previous studies (Lotfi 2005, 2007).

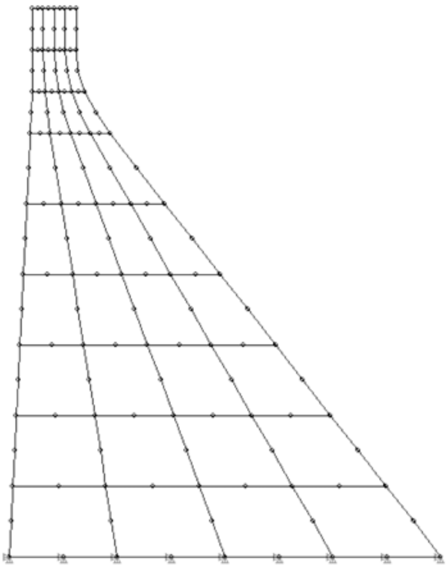

Fig. 2 Finite element discretization of Pine Flat dam (Model A)

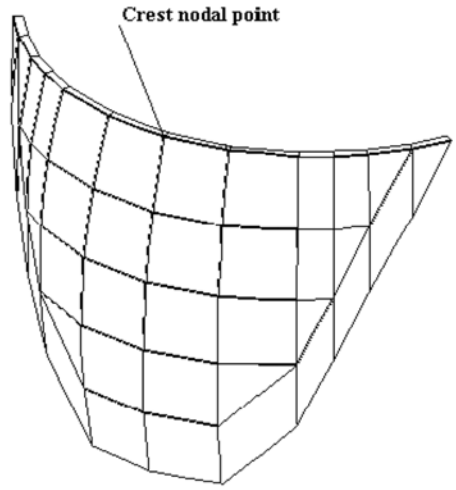

Fig. 3 Finite element mesh of Morrow Point dam (Model B)

In both cases, the foundation is assumed rigid and the empty reservoir condition is considered to simplify the problem. Harmonic support excitation is applied as loading in horizontal direction (excitation in stream direction for case B) with frequencies up to 4.5 times the fundamental natural frequency of the structure in each case (first symmetric natural frequency for case B).

4.2 Basic parameters

In both cases, the dam concrete is assumed homogeneous with isotropic linearly viscoelastic behavior and the following main characteristics:

Elastic modulus (E_d)	= 27.5 GPa
Poisson's ratio	= 0.2
Unit weight	= 24.8 kN/m^3
Hysteretic damping factor (β_d)	= 0.05

5. Results

As mentioned above, the frequencies utilized to calculate the new Ritz vectors, can be selected within the important frequency range. In the present study, this range is considered as zero up to 4.5 times the value of the first natural frequency of the dam (or first symmetric natural frequency in the case of concrete arch dam). The new Ritz vectors are calculated in frequencies that are equally spaced in the above-mentioned frequency range.

In presented graphs, vertical axis shows the horizontal acceleration of specific points in each case. These are dam crest nodal point located at upstream face for case A, and the mid-crest node for model B. Moreover, horizontal axis shows excitation frequency normalized by the first natural frequency of the dam in the case of model A, and the first symmetric natural frequency of the dam in case of model B.

The results for Pine Flat dam is discussed first and ones for Morrow Point dam will be presented subsequently.

In Fig. 4, the response for Pine Flat dam is depicted for two cases in which 3 and 5 new Ritz vectors are used, respectively. The responses for direct method are also shown for comparison purposes in each case. As mentioned, the direct method results can be considered as exact. It is

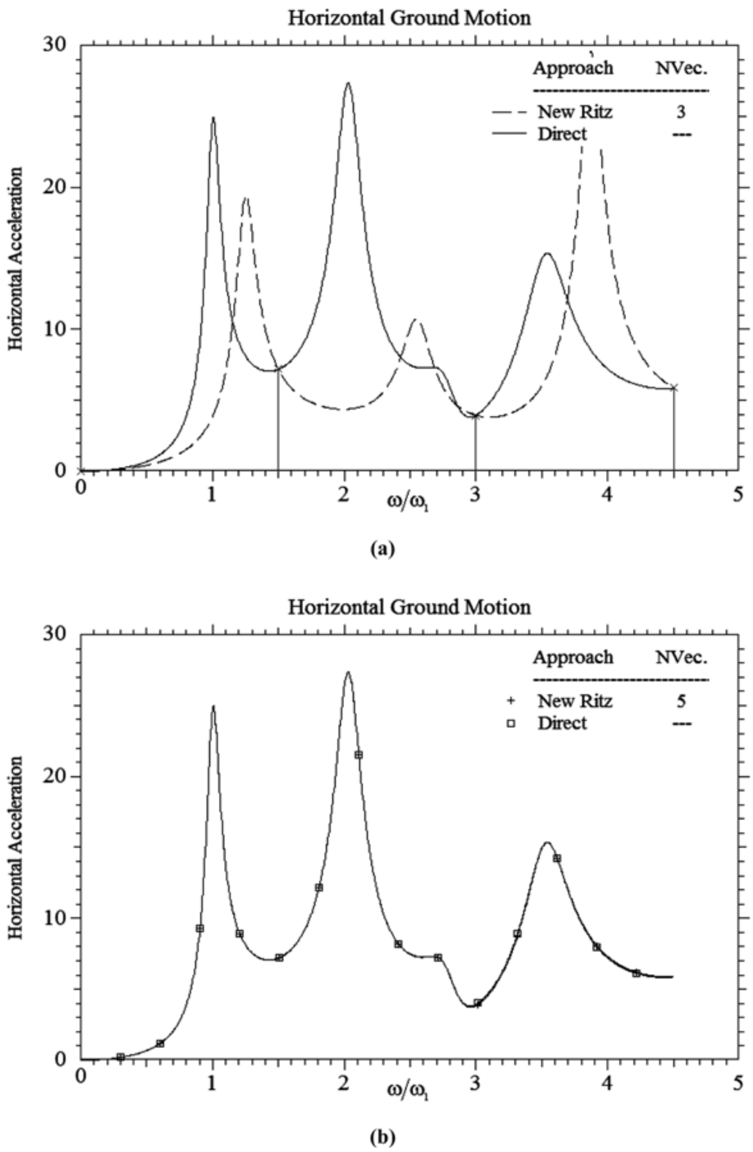

Fig. 4 Response of Model A based on new Ritz method (a) 3 Vectors, (b) 5 Vectors, (vertical lines shows the location of new Ritz vectors corresponding frequencies)

observed that response related to 3 new Ritz vectors has significant error except at frequencies corresponding to selected frequencies used for calculation of these vectors. This latter observation was expected based on theoretical concept as mentioned before. However, it is now quite apparent and verified numerically herein (Fig. 4(a)). It should be also mentioned that vertical lines are drawn on these plots to emphasize the location of these selected frequencies.

The results for 5 new Ritz vectors are illustrated in Fig. 4(b). It is noted that accuracy has increased incredibly and it matches very well with results corresponding to direct method. This reveals that although 3 new Ritz vectors is not enough to obtain accurate results in this case, the

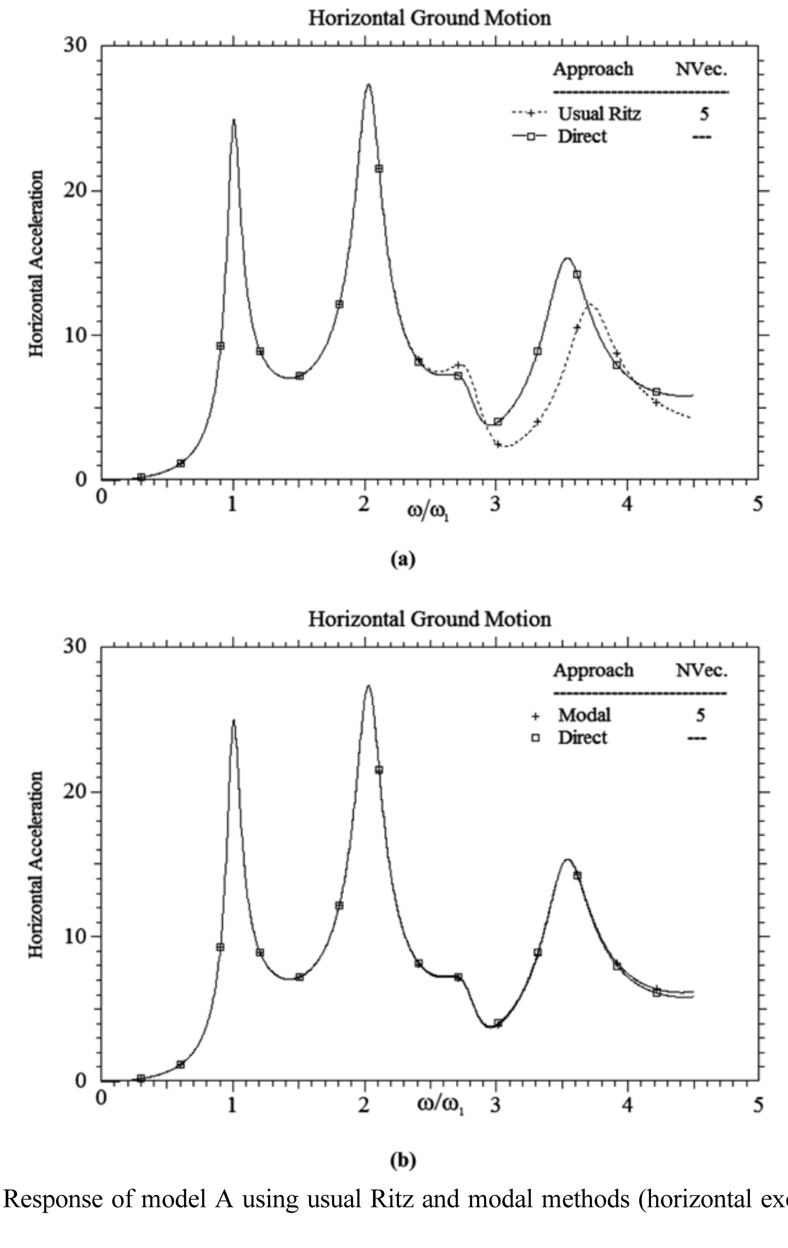

Fig. 5 Response of model A using usual Ritz and modal methods (horizontal excitation)

response is quite precise when 5 vectors are utilized. This proves that the proposed method converges very quickly for this concrete gravity dam example.

In the next stage, it was decided to have similar comparisons for the usual Ritz method, and the modal approach. Therefore, the same numbers of vectors (i.e., 5 vectors) are also considered for both of these methods. The results are presented in Fig. 5.

It is noted that the usual Ritz method and modal approach agree very well in comparison to exact response (i.e., direct method result) for the normalized frequency up to about 2.5 and 3.8, respectively. However, it deviates from the true response for higher frequencies. Therefore, it is

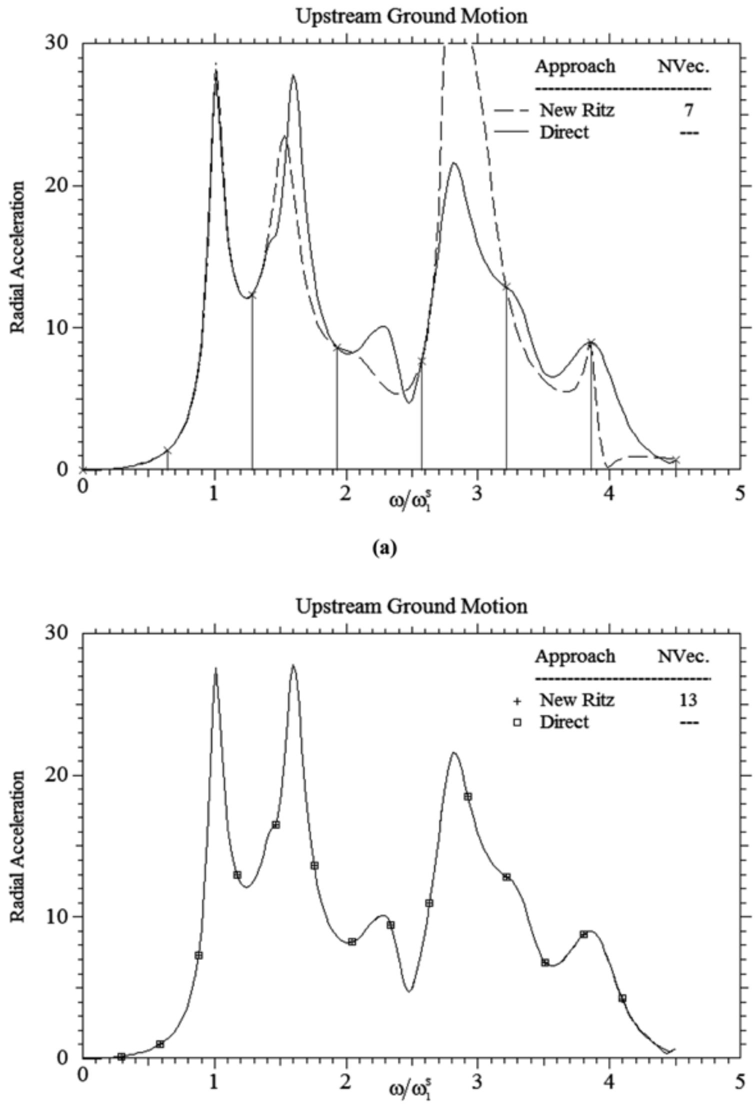

Fig. 6 Response of Model B based on new Ritz method (a) 7 Vectors, (b) 13 Vectors (vertical lines shows the location of new Ritz vectors corresponding frequencies)

quite apparent that the new Ritz method has produced more accurate response in a wider range of frequency in comparison to usual Ritz method, or even modal approach for the same number of chosen vectors.

Subsequently, the Morrow point arch dam is considered. In this example, the response is obtained for two cases of 7 and 13 new Ritz vectors. The results are presented in Fig. 6. In the first case (i.e., 7 vectors), it is observed that response does not have good accuracy except for normalized frequencies up to about 1.4, and also the selected frequencies utilized for calculation of these vectors. Of course, exact results are actually obtained for these selected frequencies as it is inherent based on the theoretical concept of this technique.

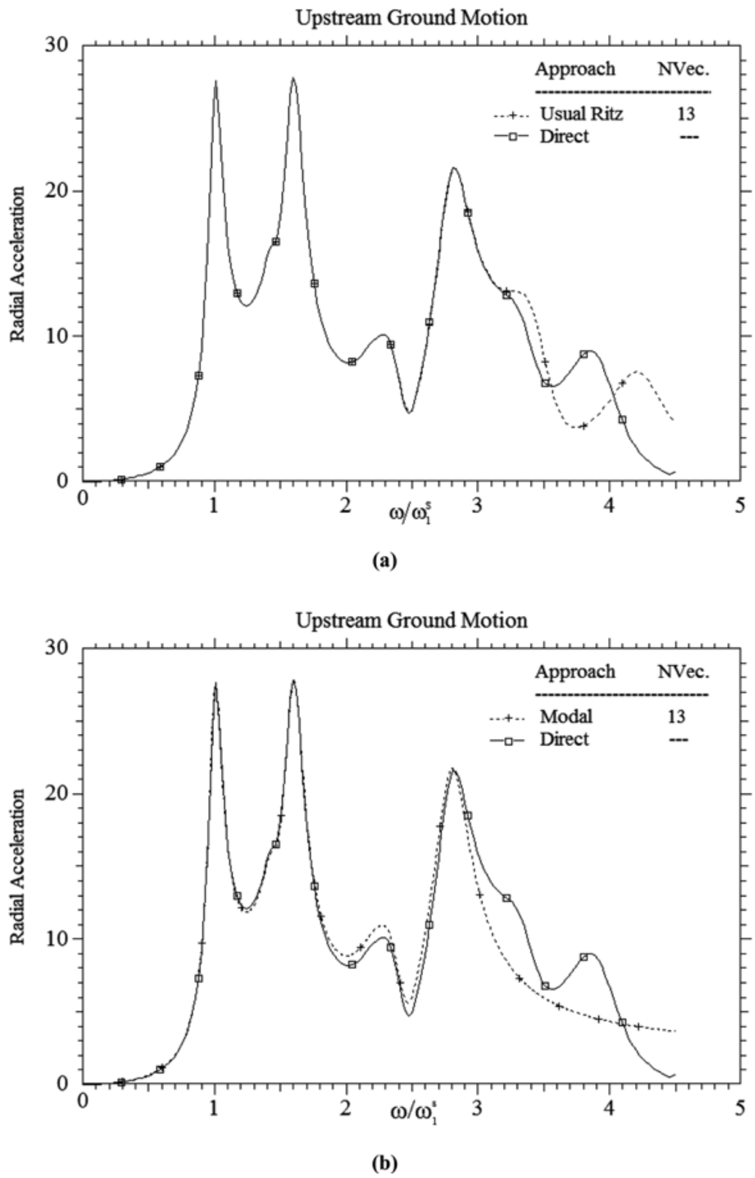

Fig. 7 Response of model B using usual Ritz and modal methods (horizontal excitation)

For the second case (i.e., 13 vectors), it is noted that excellent results are obtained for the whole frequency range considered. Therefore, this shows again that the new Ritz method converges very quickly to true response.

Finally, a similar comparison is carried out for the usual Ritz method and the modal approach based on this latter number of vectors (i.e., 13 vectors) which had produced very accurate results for new Ritz method. This is presented in Fig. 7.

It is observed that both the usual Ritz method and modal approach agree quite well in comparison to exact response for normalized frequencies less than 3.2 and 2.9, respectively. However, accuracy is diminished for normalized frequencies higher than above-mentioned values.

Once again, it proves that the new Ritz method produces accurate response in a wider range of frequency for the same number of vectors employed in comparison to usual Ritz method, or modal approach.

6. Conclusions

A new type of Ritz vectors was proposed for dynamic analysis of different systems. The generation of these vectors was explained initially. It was also proved that this technique yields independent vectors. A computer program was developed based on theories explained. By employing this tool, a study was carried out for the new Ritz method. Two examples were considered for this purpose, a typical concrete gravity dam and a concrete arch dam. Overall, the following conclusions can be extracted from this investigation:

- It is observed that response for low number of new Ritz vectors has very poor accuracy similar to usual Ritz method or modal approach. However, it converges much faster to the true response in comparison to these other two techniques. Moreover, the new Ritz method produces exact results for certain selected frequencies utilized for calculation of these vectors, no matter how small the number of vectors are chosen. This is easily apparent based on the theory of this method. Furthermore, it is proved numerically and verified for both examples considered in this study.
- The new Ritz method would produce accurate response in a wider range of frequency in comparison to usual Ritz method or modal approach for the same number of chosen vectors. This was quite apparent for the two examples considered in this study. Although, this needs to be studied more thoroughly, it seems to be true in general or in most practical cases.
- Finally, it must be mentioned that one of the main advantages of the present technique seems to be when the considered system includes a semi-infinite medium. This occurs in many practical examples of soil-structure or fluid-structure systems. Such a system was not considered in the present study. However, it would be an interesting topic for future studies and further challenges for the proposed method.

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