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Identification of progressive collapse pushover based on a kinetic energy criterion

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Abstract. The progressive collapse phenomenon is generally regarded as dynamic. Due to the impracticality of nonlinear dynamic computations for practitioners, an interest arises for the development of equivalent static pushover procedures. The present paper proposes a methodology to identify such a procedure for sudden column removals, using energetic evaluations to determine the pushover loads to apply. In a dynamic context, equality between the cumulated external and internal works indicates a vanishing kinetic energy. If such a state is reached, the structure is sometimes assumed able to withstand the column removal. Approximations of these works can be estimated using a static computation, leading to an estimate of the displacements at the zero kinetic energy configuration. In comparison with other available procedures based on such criteria, the present contribution identifies loading patterns to associate with the zero-kinetic energy criterion to avoid a single-degree-of-freedom idealisation. A parametric study over a family of regular steel structures of varying sizes uses non-linear dynamic computations to assess the proposed pushover loading pattern for the cases of central and lateral ground floor column failure. The identified quasi-static loading schemes are shown to allow detecting nearly all dynamically detected plastic hinges, so that the various beams are provided with sufficient resistance during the design process. A proper accuracy is obtained for the plastic rotations of the most plastified hinges almost independently of the design parameters (loads, geometry, robustness), indicating that the methodology could be extended to provide estimates of the required ductility for the beams, columns, and beam-column connections.

Keywords: progressive collapse; 2D frames; pushover loads; non-linear dynamic computations; zero kinetic energy configuration

1. Introduction

Various procedures for analysing progressive collapse related issues can now be found in the literature for different types of structures, materials, and collapse scenarios (Ellingwood and Leyendecker 1978, Gilmour and Virdi 1998, Hyung-Jen and Krauthammer 2003, Kaewkulchai and Williamson 2004, Agarwak *et al.* 2006, Val and Val 2006, USA-GSA 2003, USA-DOD 2005, Vlassis 2007, Izzuddin *et al.* 2008, Ruth *et al.* 2006, Powell 2004, USA-GDHS 2003, Dusenberry and Hamburger 2006, Khandelwal *et al.* 2008, Khandelwal *et al.* 2009, Fu 2009, Kim and Kim 2009, Yagob *et al.* 2009, Weerheijm *et al.* 2009, Gong *et al.* 2009, Almusallan *et al.* 2010, Yuan *et*

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al. 2011). They can be classified according to different criteria. A first distinction can be made between computational strategies aiming at a full structural analysis (Khandelwal *et al.* 2008, Khandelwal *et al.* 2009, Fu 2009, Kim and Kim 2009, Galal and Sawy 2009, Almusallan *et al.* 2010), contributions aiming at a more detailed representation of connection details (Liu 2010a, b, Lee *et al.* 2009), studies combining these two aspects (Khandelwal *et al.* 2008), or studies trying to formulate simplified analysis frameworks (Yuan *et al.* 2011). Different other criteria can be used two distinguish the approaches. Some contributions analysed the behaviour of global 3D structures (Fu 2009, Galal and Sawy 2009), while others focusing on specific aspects of progressive collapse used 2D simulations (Khandelwal *et al.* 2008, Khandelwal *et al.* 2009, Kim and Kim 2009, Lee *et al.* 2009, Kim *et al.* 2009). As another distinction, catenary effects were incorporated in some of the global studies (Khandelwal *et al.* 2008, Fu 2009, Lee *et al.* 2009), while recent contributions or codes still keep a geometrically linear approach (Dusenberry and Hamburger 2006, Kim and Kim 2009, Grierson *et al.* 2005). Finally, some contributions investigated more particularly retrofitting approaches (Galal and Sawy 2009) or the effect of bracing elements (Khandelwal *et al.* 2009).

Progressive collapse is generally regarded as a dynamic phenomenon, requiring dynamic computations, since the sudden failure of a vertical support element leaves the structure in an out-of-equilibrium situation, thereby triggering a dynamic process. Most of the recent investigations use dynamic non linear computations (Khandelwal *et al.* 2008, Fu 2009, Galal and Sawy 2009), or propose to account for them with equivalent quasi static procedures (Izzuddin *et al.* 2008, Kim and Kim 2009). In comparison with a quasi-static computation under similar loads, a dynamic computation was shown to induce stress redistribution over larger portions of the structure, as well as larger deflections and plastic hinge rotations (Kaewkulchai and Willamson 2004). The various progressive collapse mitigation procedures indeed usually try to take such effects into account (Khandelwal *et al.* 2008). Since nonlinear dynamic computations require a wider expertise and are time consuming, and since practitioners may not have the corresponding tools, simplified analysis frameworks have been a subject of interest as in Yuan *et al.* (2011). An interest also arises for procedures designed to account for dynamic inertial effects during progressive collapse through equivalent quasi-static computations. Several such procedures may be found in the literature, some being based on dynamic amplification factors to be applied to the loads during the design process (USA-GSA 2003, USA-DOD 2005, Ruth *et al.* 2006, Grierson *et al.* 2005), others proposing procedures based on energetic evaluations for the internal and external forces (Vlassis 2007, Izzuddin *et al.* 2008, Powell 2004, Dusenberry and Hamburger 2006). In all cases, equivalent quasi-static procedures designed to mimic inertial dynamic effects are often referred to as pushover analysis. The “pushover” terminology is very common in seismic design, where practitioners often use such quasi-static equivalent analyses (USA-GDHS 2003). In the context of progressive collapse, however, the source of the structural demands is completely different from seismic design, so that the loading patterns used for a pushover-like analysis in progressive collapse simulations should be defined following a different approach which is the purpose of the present contribution.

In the progressive collapse mitigation issued from various sources (USA-DOD 2005, Izzuddin *et al.* 2008, Eurocode 2006, Mohamed 2006), a procedure is specified, which allows disproportionate collapse to be avoided by tying horizontal and vertical components. The tying requirements are specified, and are intended to allow catenary action to develop, so that the structure may bridge over the removed vertical support member. In some cases, no sectional or connection ductility considerations are provided, while it has been demonstrated that such provisions may lead to unrealistic ductility demands for the beam sections and connections (Vlassis *et al.* 2008). A simplified static equivalent progressive collapse computation methodology providing estimates of plastic rotations in the various plastic hinges that appear following the initial column loss would therefore be of interest.

Equivalent static computational procedures are essentially split in two classes. The first one uses dynamic amplification factors. For instance the DoD guidelines (USA-DOD 2003) specify linear and non-linear static procedures intended to ensure the formation of an alternate load path for the gravity loads following the removal of a vertical support member. In order to account for dynamic effects, a multiplying load factor of 2 to be applied on both dead and live loads is specified. A similar approach is adopted in the guidelines issued by the GSA (USA-GSA 2005). When section plastification is taken into account, a dynamic load factor of 2 was however shown to be strongly conservative (Vlassis 2007, Ruth *et al.* 2006). Quasi-static pushover analyses for progressive collapse situations could help analysing a reduction of this excessive security margin, while not increasing the complexity of the computational procedure; as advocated in (Ruth *et al.* 2006) where a reduced multiplying factor of 1.5 is proposed, due to the ability of the structure to dissipate kinetic energy through section plastification.

The second type of quasi-static equivalent procedures is based on a criterion allowing to estimate the displacement configuration at the instant of zero kinetic energy during the dynamic collapse process (Vlassis 2009, Izzuddin *et al.* 2008, Powell 2004, Dusenberry and Hamburger 2006). The procedure presented in (Vlassis 2007, Izzuddin *et al.* 2008) is defined such that the level of model decomposition can be chosen: the complete structure may be simulated, or a part of it with proper boundary conditions (just the concerned bays, or only the concerned bays on several floors, or even just the concerned bays on one floor). This analysis bears similarities with simplified equivalent single degree of freedom models (SDOF) (Sasani and Sagiroglou 2008). Based on the chosen model decomposition a static analysis is used to produce a load-displacement curve, where the displacement corresponds to the displacement of the top of the removed column, and the load corresponds to the resultant gravity loads applied to the simulated portion of the structure. To obtain this curve, the static pushover loading pattern consists in gradually increasing the dead and live loads on the structural model from which the initially failing element has been removed, starting from the undeformed configuration. The curve is then used as the response of an equivalent SDOF system, and energetic considerations lead to an estimation of the displacements configuration at the instant of zero kinetic energy during the dynamic process.

Another distinction between progressive collapse computational assessment procedures relates to the incorporation of catenary effects captured by large displacement formulations, and which can be used to help the structure avoid progressive collapse, provided the connections be designed with sufficient ductility. However, not all the codes or procedures in the literature actually account for large displacements. The procedures incorporating geometrical nonlinearities generally include tying force requirements (USA-DOD 2005, Izzuddin *et al.* 2008, Eurocode 2006, Mohamed 2006). The procedure based on a zero kinetic energy criterion in (Vlassis 2007, Izzuddin *et al.* 2008) also accounts for such effects. Other complete dynamical analyses were also performed with large displacement formulations (Khandelwal *et al.* 2008). Numerous authors, sometimes for simplification, recently adopted a flexural structural mode to withstand collapse, and therefore based their computational procedures on geometrically linear analyses. This is the case in the DoD specifications (USA-DOD 2005), where a flexural structural mode is specified for alternate load path analyses applied for structures requiring higher protection levels, or whenever the tying force requirements cannot be met. The procedures developed in (Grierson *et al.* 2005, Vlassis 2007) are also geometrically linear, as well as all the computational procedures specified in the GSA document (USA-DOD 2005). In (Dusenberry and Hamburger 2006), the authors describe how the zero kinetic energy criterion may be applied when geometrical linearity is assumed, as well as when catenary behaviour is accounted for; while in (Marjanishvili and Agnew 2006, Marjanishvili 2004), the recommendations leading to an analysis procedure for progressive collapse do not include catenary effects.

Even though catenary effects may be used to prevent progressive collapse, adequate tying of the beams cannot always be met, especially in the case of a lateral or penultimate column failure. Since the tying force provisions given in (USA-GSA 2003, USA-DOD 2005, Eurocode 2006) do not account for the limited section ductilities, a procedure such as the one described in (Vlassis 2007, Izzuddin *et al.* 2008) is required to ensure that the critical generalised strains in the connections are not reached, thereby requiring practitioners to perform large displacement analyses. Because of these difficulties, and since several recent contributions are based on flexural structural modes to withstand the loss of a vertical support member, the present contribution will focus on geometrically linear computational procedures, and will aim at a methodology to identify a pushover analysis procedure for progressive collapse that implicitly accounts for the dynamic aspect of the collapse.

A modified pushover procedure based on the zero kinetic energy criterion is presented. A parametric study over a family of similar structures of varying size, using non-linear dynamic computations, enables the definition and validation of the proposed pushover loading pattern. The main innovation brought here lies in the proposed loading patterns to be associated with the zero kinetic energy criterion. It is shown to provide a good estimate of a structure ability to redistribute stresses further to the loss of a vertical support element, by providing an accurate representation of the plastic hinge locations and appearance sequence. Based on geometrically linear quasi-static computations, it shows lower safety margins than obtained with a dynamic load factor of 2 (USA-GSA 2003, USA-DOD 2005). Note that only dynamic effects of inertial nature (as opposed to viscous effects) are considered here.

The paper is structured as follows: Section 2 describes what is referred to as the zero kinetic energy criterion, upon which the proposed pushover analysis procedure is based. Section 3 describes the various families of structures studied in order to both establish and validate the proposed procedure, while Sections 4 and 5 describe this procedure as well as the results for the cases of central ground floor column and lateral ground floor column removals respectively. Section 6 discusses results by comparing them with other available procedures, followed by Section 7 which concludes with future prospects.

2. Pushover analysis based on zero kinetic energy criterion

Some existing procedures are based on what is referred to here as the "zero kinetic energy criterion" to define equivalent quasi-static loading conditions (Vlassis 2007, Powell 2004, Dusenberry and Hamburger 2006). The idea is based on two assumptions, namely that (i) the structure reaches an instant t_d of zero kinetic energy, and (ii) the corresponding state constitutes the dimensioning configuration for the structure.

The main idea consists in choosing a loading pattern to be applied quasi-statically to the structure, with the aim to provide an estimate of the cumulated external forces work following the sudden column removal, as well as an estimate of the cumulated internal forces work. In a dynamic context, equality between these works indicates a vanishing kinetic energy, in which case the structure is assumed to be able to withstand the sudden column removal according to the above assumptions. The complete procedure is described in details below.

Since before the collapse triggering event occurs, the structure has no kinetic energy, then at t_d (instant of zero kinetic energy), the cumulated external forces work, as of the removal of a failing element, is equal to the cumulated internal forces work. Indeed, as long as these two energies are not equal, their inequality gives rise to the kinetic energy of the structure. This translates into

$$T_i(q_d) = f_d^t q_d \quad \text{at} \quad t_d \tag{1}$$

where T_i represents the cumulated internal forces work as of the removal of the failing element, f_d is a vector in which the forces actually applied to the structure (i.e., constant gravity loads) are collected, and q_d is a vector in which the dynamically computed displacements at the instant t_d are collected (considered here as the reference solution). The purpose of a pushover analysis is to avoid such a dynamic computation. To replace it by an equivalent quasi-static loading, let us assume that we have a loading pattern to be applied quasi-statically to the structure, represented by external forces f_s, which depend on one (or several) loading factors represented generically by the parameter μ in the following equations. Let us call q_s the vector in which the displacements resulting from this quasi-static computation are collected. These displacements naturally depend on μ. Assuming that there exists some value μ^* of μ such that q_s is a good approximation of q_d, Eq. (1) can be rewritten

$$T_i(q_s(\mu^*)) = f_d^t q_s(\mu^*) \tag{2}$$

When a quasi-static computation is performed, the works of the external and internal forces are equal all through the computation, so that in the quasi-static equivalent computation, the cumulated work of internal forces can be evaluated as

$$T_i(q_s(\mu^*)) = \int_0^{q_s(\mu^*)} f_s(\mu^*)^t dq_s(\mu^*) \tag{3}$$

Finally, transposing (3) into (2) yields

$$\int_0^{q_s(\mu^*)} f_s(\mu^*)^t dq_s(\mu^*) = f_d^t q_s(\mu^*) \tag{4}$$

where, as explained above, f_d merely represents the forces actually applied to the structure during the collapse event, i.e., the gravity loads. Solving Eq. (4) for μ^* therefore provides a vector q_s which is an approximation of the vector q_d (i.e., the reference solution). This resolution is achieved by performing a quasi-static computation, at the beginning of which, the left hand term in Eq. (4) (cumulated internal work) is smaller than the right hand one (cumulated external work performed by dynamically applied forces with the approximated displacements q_s). When a value of μ is reached such that both terms are equal, the approximation q_s is found. If the structure cannot withstand any further increase of μ before this event, it is then assumed that the structure cannot withstand an instantaneous column removal, and should be redesigned in order to avoid the collapse. Alternatively, if plastic rotations limitations are not incorporated during the computation, it may occur that the structure can withstand collapse through stress redistribution. In this latter case, the procedure allows to assess the ductility demands in order to achieve the stress redistribution as expected.

The main benefit of such a procedure stems from the fact that it suffices to perform a quasi-static computation with increasing values of μ in order to solve Eq. (4) for μ. However, the loading pattern remains to be defined. Indeed, in all generality, separate load factors could be applied for every degree of freedom, so that Eq. (4) contains as many unknowns μ as there are degrees of freedom. In order to restrict Eq. (4) to a single equation with a single unknown, a loading pattern parametrised with a single load amplification factor μ needs to be defined. Various choices may lead to such a situation. For instance, the factor μ may be applied to all the dead and live loads on all bays. Alternatively, the dead and live loads may be applied as a first step (at which point Eq. (4) will not yet be satisfied), after which the factor μ is applied only to additional loads consisting the of portion of dead and live loads acting on the bays directly concerned by the removed column. The objective is to identify a loading pattern such that a good agreement is found between dynamic reference results and the quasi-static equivalent computation, according to some comparison criterion. As will be shown, the accuracy of the obtained results varies strongly according to the chosen loading pattern. In the present contribution such a loading pattern to be applied will be identified to obtain accurate results when compared with dynamic computation results.

3. Parametric study and reference solutions

3.1 Sets of studied structures

In order to establish and validate a general loading pattern to associate with the zero kinetic energy criterion, a sufficient number of structures needs to be studied. In the present section, the various sets of considered structures are described. They consist in two-dimensional beam-column frames with regular bay lengths and floor heights. Varying floor and bay numbers are considered, as well as varying bay lengths, loads and structural resistance levels.

The structures described here are based on those which can be found in (Kaewkulchai and Williamson 2004), from which the initial values of the bay lengths, yield moments, cross sections, inertias, loads and density are taken. Fig. 1 shows the generic pattern for all sets of structures. Floor and bay numbers vary from four to ten (i.e., 49 possible combinations), while five different bay lengths are considered. On Fig. 1, M_p is the yield moment of both beams and columns, Ω_c, I_c, Ω_b and I_b are respectively the cross section area and inertia of the columns and beams respectively, E is the Young Modulus, ρ is the density, P is the load uniformly applied on all floors and α is explained below.

Since different bay lengths and loads need to be considered, the structures often have to be redesigned. Due to the large number of studied structures, an automated re-design procedure is required, yet simply increasing (or decreasing) the yield moment M_p is not sufficient, since the cross sections inertias and areas need to vary accordingly. The coefficient α (expressions on the right in Fig. 1) is used to easily achieve the consistency between the yield moment values and the cross sections and inertias. When α is set to 1, the obtained values are equal to those found in (Kaewkulchai and Williamson 2004). Assuming that the yield moment is then multiplied by a factor α, the corresponding new cross sections and inertias need to be defined. For both beams and columns, the simplifying assumption is made that the cross section is I-shaped, and that only the flanges contribution to the yield moment and inertia is taken into account. Thus, the yield moment

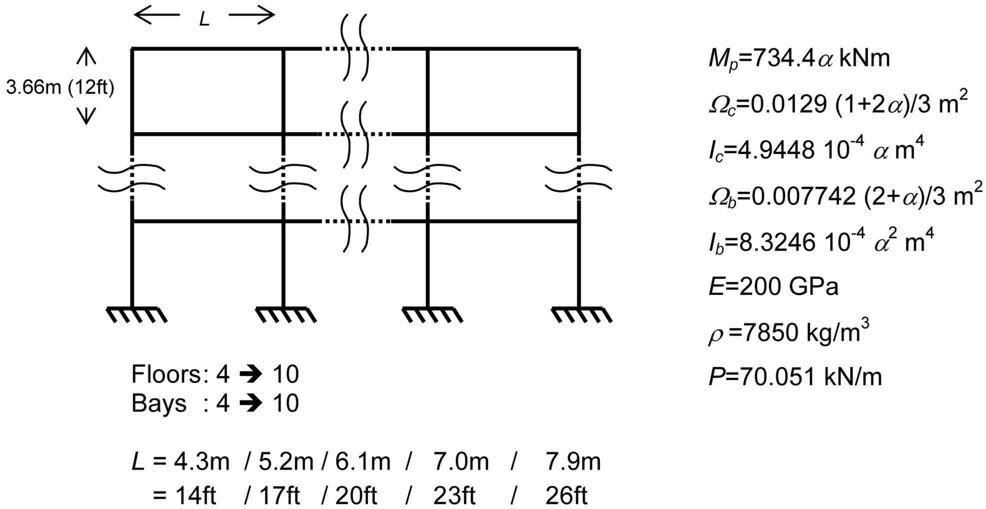

Fig. 1 Generic geometrical pattern for all structures

depends linearly on the flanges area, as well as on the web's length. For the columns, the assumption is made that the increase in yield moment is achieved through an increase in the flanges sections. For the beams, however, the assumption is made that the increase in yield moment is achieved through an increase of the web's length. Assuming finally that the initial areas of the web and each flange is roughly similar (α =1), then when the yield moment changes from M_p to αM_p, the cross section and inertia changes are given by

$$\Omega_c = \left(\frac{1+2\alpha}{3}\right)\Omega_{c,i} \quad I_c = \alpha I_{c,i} \quad \Omega_b = \left(\frac{2+\alpha}{3}\right)\Omega_{b,i} \quad I_b = \alpha^2 I_{b,i} \tag{5}$$

where the i subscript refers to the initial values. As a result, in order to generate the various sets of studied structures we have:

- For a fixed bay length L and a load P, 49 structures resulting from the combinations of floor and bay numbers varying between four and ten.
- Five different regular bay lengths (see values in Fig. 1).
- As seen later, various load levels will be considered. In order to have an automated re-design procedure for the new loads (or change in bay length), a resistance coefficient appears in the data set, the value of which is used to increase or decrease the yield moments, while keeping the cross section and inertia values consistent with these changes.

As suggested by the value of the density ρ (along with the use of I-shaped sections), the structures considered here are made of steel. A discussion focuses on the reasons for this choice, as well as on whether the procedures developed here may be generalized to structures made of other materials.

3.2 Reference solutions obtained by dynamic nonlinear computations

In order to obtain the reference solutions to which the quasi-static pushover analyses results will be compared, non-linear dynamic computations are conducted. All dynamic computations are performed with a Hilber-Hughes-Taylor procedure with 5% numerical damping (Géradin and Rixen 1992). Since the load P is a gravitational load, the corresponding mass needs to be taken into account. Assuming that the mass leading to this load P is uniformly applied along the beams, the density used for the columns is ρ (as specified in Fig. 1), while the density used for the beams is adapted as

$$\rho_b = \rho + \frac{P}{9.81\Omega_b} \tag{6}$$

The sudden removal of a column is simulated by the sudden release of the resultant end forces of the removed column. A first static analysis is therefore performed for the complete model (i.e., no removed column), to determine the end forces of the ground column to be removed. Subsequently, the model with the removed column is analysed statically with the end forces of the removed column and the dead and live loads applied, to obtain the displacement configuration at the onset of the column removal. The dynamic part of the analysis then starts from this configuration, with the gravity loads maintained constant. Equal but opposite end forces are applied at $t = 0$ at the top end node of the removed column, thus simulating the sudden column removal. A similar dynamic simulation strategy was also used in (Lee *et al.* 2009, Sasani and Sagiroglou 2008, Marjanishvili and Agnew 2006, Sasani 2008).

Note that for the sake of comparison with previous works, the computations this paper, are performed using geometrically linear formulation. In order to account for section plastification and stress redistribution, lumped plastic hinges with plastification in bending using a rigid-perfectly plastic behaviour law are used to connect linear elastic Euler-Bernoulli beam elements. An extension to plastification with interactions between bending and axial straining may however be considered, although the implementation is then more complex since composite yield surfaces have then to be identified, as well as the numerical length of the plastic hinge. This constitutive setting is similar to the one in (Menchel *et al.* 2009). No strain-based criterion is considered (i.e., no limit is imposed on the plastic rotations during computation), and since a geometrically linear formulation is used, it is assumed that these hinge elements provide the correct plastic strains, enabling to estimate the required connection ductility.

In the following sections, comparisons between pushover analyses results and reference solutions are performed. Although various quantities may be compared, it is proposed to systematically compare the plastic rotations for all plastic hinges detected by the dynamic analysis for the considered structure. In spite of the fact that within a geometrically linear analysis the plastic rotations would not reach values such that particular ductility requirement should be checked, obtaining a satisfactory accuracy for these quantities ensures that the plastic hinge locations are detected correctly, and in the correct sequence by the quasi-static procedure.

4. Central ground floor column removal

4.1 Proposed loading pattern

In this section, all structures are analysed for the case of a central ground floor column removal (i.e., the column located on the structure's axis of symmetry in the case of an odd number of columns, or the one closest to the symmetry axis when the number of columns is even). The

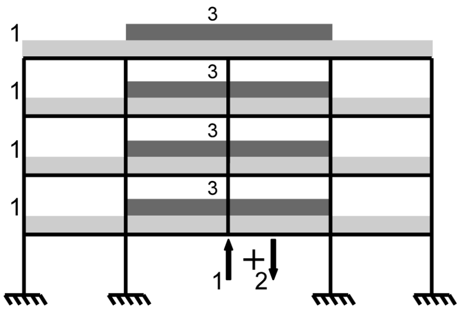

Fig. 2 Load pattern for central ground floor removal

loading pattern offering the best compromise between accuracy and generality for all structures is the following. A first static analysis is used to determine the resultant end forces at the top of the removed column. Following the column removal, the proposed loading pattern is applied in three steps :

1. Apply the dead and live loads and the end forces at the top of the failing column (marked 1 on Fig. 2).
2. Maintain the dead and live loads constant, and gradually decrease the end forces of the removed column by adding increasing equal-but-opposite end forces (marked 2 on Fig. 2), until these end forces disappear. This quasi-static computation allows the computation of both terms in Eq. (4) (see the Appendix for more details).
3. Start gradually increasing the dead and live loads on the two bays adjacent to the removed column (marked 3 on Fig. 2), maintaining the loads on the other bays constant. This quasi-static computation allows the continued computation of both terms in Eq. (4) (as explained in the Appendix). Keep on increasing these loads until the zero kinetic energy criterion approximated by Eq. (4) is verified.

4.2 Plastic hinges rotations comparison

Having analysed 49 structures for five different bay lengths, for different load levels as well as for different structural resistance levels (coefficient α in Section 3.1), an extensive presentation of the results is prevented by obvious length restrictions. In order to better classify the results, let us first define an indicator of structural strength in the context of this paper. The plastic reserve strength indicator (henceforth written as PRSI) of a structure with respect to the removal of a given column will be defined here as the maximal load factor which can be applied to the dead and live loads when applied statically on the structure from which the considered column has been removed. It can be estimated by a computation with the following steps:

1. Remove a column form the model
2. Apply the dead and live loads statically, multiplying them by an increasing factor F_r
3. When stresses can no longer be redistributed having followed this loading pattern, F_r is the PRSI of the structure.

The PRSI is assumed to offer a measure of the resistance reserve of a structure with respect to the loads applied to it, and for the removal of a specific column. Note that this definition is by no means intended as general and is only used in the context of the present work. The loading

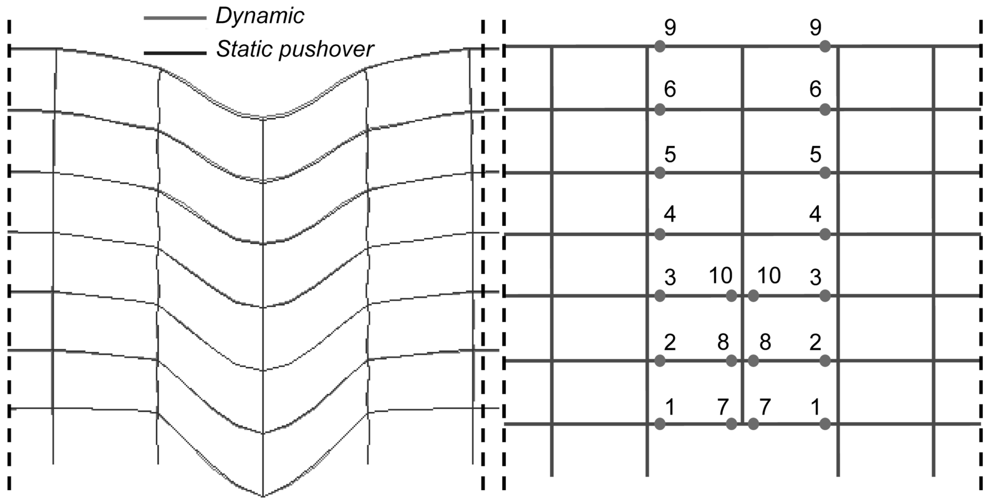

Fig. 3 Deformed structures obtained with a dynamic and the pushover analyses (left), plastic hinge locations and plastic rotation magnitude sequence (right)

sequence used to define F_r are not the loading pattern proposed for the pushover analyses.

Fig. 3 compares, as illustration, the deformed structures corresponding to the instant of minimal kinetic energy during the 'real' dynamic process (reference solution in red), and the one obtained with the pushover procedure proposed here (blue). The figures correspond to a structure with 8 bays and 7 floors. A good correspondence between the dynamic and the pushover procedure deflections can be observed. Note that the displacements have been artificially enhanced for the plot. Fig. 3 also shows the plastic hinge locations, the numbers indicating plastic rotation magnitudes: '1' corresponds to the largest plastic rotation, '10' to the lowest. Identical plastic hinge locations and plastic rotation sequences are obtained with the dynamic analysis and with the pushover procedure for this case.

Table 1 illustrates quantitatively the plastic rotations obtained by the pushover and their relative errors w.r.t. dynamic computations, for structures with seven floors, and bay numbers ranging from four to ten. All structures were calibrated to have a PRSI of 1.64, and a bay length of 6.1 m. The results in each column of the table are sorted by decreasing plastic rotations magnitude based on the dynamic computation. In the second part of the table, each cell is divided into two parts : the left one provides the relative error on the plastic rotation, the right one provides the position of the plastic hinge in the sequence detected by the pushover analysis. The caption 'nil' refers to a hinge which plastified during the dynamic computation, but not during the pushover analysis.

The comparison criterion, as explained in Section 3.2, consists in comparing the plastic rotations, with a view to ensuring a correct location and sequence detection by the pushover analysis. Table 1 shows a good agreement for the hinges with the larger plastic rotations (first few cells in each column). The mean relative error for the first six plastic rotations for all 49 structures with a 6.1 m bay length is of 8.1% with a standard deviation of 5.9%. Mainly, the table shows that the plastic hinges are detected in nearly all cases, and that the sequence appearance is well estimated, in spite of the larger errors for the hinges showing the lower plastic rotations. The alternate load path taken by the loads following the column removal is therefore well represented by the pushover procedure,

Table 1 Relative errors on the plastic rotations in %, for seven floor structures, central ground floor column removal, bays numbers ranging from 4 to 10, robustness = 1.64, bay length = 6.096 m

Number of bays	4		5		6		7		8		9		10	
Plastic rotations in mrad (reference solution)	6.3		6.3		6.1		6.1		6.2		6.2		6.2	
	6.3		6.1		6.1		6.1		6.2		6.1		6.2	
	5.3		5.3		5.3		5.3		5.3		5.3		5.3	
	5.3		5.3		5.3		5.2		5.3		5.3		5.3	
	3.8		3.8		4.2		3.9		4.0		3.8		3.7	
	3.8		3.8		4.2		3.8		4.0		3.7		3.7	
	2.5		2.6		2.7		2.8		2.9		2.5		2.6	
	2.5		2.4		2.7		2.7		2.9		2.3		2.6	
	1.7		1.8		1.7		2.0		2.0		1.8		1.7	
	1.7		1.8		1.7		1.8		2.0		1.7		1.7	
	1.7		1.7		1.7		1.8		1.7		1.6		1.7	
	1.7		1.6		1.7		1.7		1.7		1.5		1.7	
	1.6		1.6		1.5		1.4		1.4		1.2		1.1	
	1.6		1.4		1.5		1.3		1.4		1.1		1.1	
	0.45		0.50		0.44		0.50		0.62		0.34		0.33	
	0.45		0.29		0.44		0.27		0.62		0.33		0.33	
	0.14		0.25		0.32		0.23		0.27		0.15		0.12	
	0.14		0.24		0.32		0.05		0.27		0.07		0.12	
			0.19		0.10				0.15					
					0.10				0.15					
					0.10									
					0.10									
Plastic rotations relative errors (%)	-1.5	1	-0.8	1	2.9	1	2.4	1	1.6	1	1.4	1	1.4	1
	-1.5	2	2.8	2	2.9	2	3.2	2	1.6	2	1.6	2	1.4	2
	-1.2	3	-0.8	3	0.2	3	1.2	3	1.2	3	1.3	3	1.5	3
	-1.2	4	2.0	4	0.2	4	1.9	4	1.2	4	1.3	4	1.5	4
	13.4	5	18.3	5	5.9	5	13.5	5	12.0	5	17.8	5	18.3	5
	13.4	6	14.6	6	5.9	6	14.8	6	12.0	6	18.5	6	18.3	6
	37.9	7	40.2	7	31.8	7	26.6	7	23.7	7	45.1	7	36.4	7
	37.9	8	42.7	8	31.8	8	32.5	8	23.7	8	56	8	36.4	8
	53.8	9	41.4	11	40.6	11	44.7	9	40.9	9	38.8	11	63.5	9
	53.8	10	61.2	9	40.6	12	35.3	11	40.9	10	71.2	9	63.5	10
	14.3	11	15.5	13	68.9	9	51.4	10	41.6	11	46.3	12	43.5	11
	14.3	12	20.9	14	68.9	10	40.7	12	41.6	12	83.2	10	43.5	12
	38.2	13	68.4	10	29.9	13	39	13	41.2	13	56.2	13	70.9	13
	38.2	14	53.9	12	29.9	14	48.3	14	41.2	14	70.2	14	70.9	14
	132	15	110	15	Nil		108	15	65.1	15	195	16	203	15
	132	16	261	16	Nil		9.8	17	65.1	16	203	15	203	16
	129	17	19.1	19	226	15	334	16	120	17	69.2	18	120	17
	129	18	170	17	226	16	419	18	120	18	270	17	120	18
			64.7	18	193	19			74.7	19				
					193	20			74.7	20				
					470	17								
					470	18								

allowing for a correct design of the structure. The degree of validity of the approximate pushover procedure may however only be considered satisfactory provided the same level of accuracy on the plastic rotations is kept for variations of the design variables, i.e., PRSI, bay lengths, or load levels. Such variations are therefore scrutinised next.

The dynamic computations providing the reference solutions can be performed for structures of various PRSI levels, provided a minimal value of the PRSI is kept. Indeed, when solving the equation of motion, computations do not necessarily stop when the tangent stiffness becomes non-invertible; and unloading may occur due to the dynamic oscillations, so that not all hinges are necessarily in a 'plastic' loading state simultaneously. As a result, for lower levels of robustness inducing the appearance of a higher number of plastic hinges during the dynamic computations, it may become impossible to find a static load pattern inducing the appearance of all hinges : the formation of a mechanism would prevent the static computation from reaching the point where all 'dynamic' hinges are detected. For instance, for the removal of a central column, it is not possible to statically induce the appearance of all dynamically detected plastic hinges for PRSI levels lower than 1.52.

This implies that a practitioner designing a structure for progressive collapse using such a quasi-static pushover analysis procedure would produce designs with robustness levels that are not lower than 1.52. For such a design, the pushover analysis shows that Eq. (4) is verified at the onset of the formation of a mechanism (beyond which point the load controlled pushover analysis cannot be carried). Assuming that safety margins are already adopted for the loads and material resistance, there would be no need to design the structure with an extra safety margin. However, a practitioner would generally produce a final design with some additional safety margin, since the various beams and columns need to be chosen from an existing producer's catalogue.

Table 2 shows the mean relative error and the standard deviation for the first six hinges (six largest plastic rotations), for sets of 49 structures with varying PRSI levels, and a bay length of 6.1 m. The second row of Table 2 (Resistance reserve) shows the mean relative difference between the load factor for which, using the pushover loading pattern, Eq. (4) is verified, and the one for which a mechanism is formed. It is shown that a good accuracy on the plastic rotations is maintained, which implies that the pushover analysis provides accurate results whatever the realistic robustness level for the final design.

Construction codes in different countries or organisations may specify different service loads for progressive collapse situations. For instance, the progressive collapse mitigation guidelines issued by (USA-GSA 2003) specify a load factor of 25% on the service loads, while the guidelines developed by (USA-DOD 2005) specify a load factor of 50%. Obviously, the higher the load, the stronger the structure should be, but the robustness levels (i.e., loads versus structural resistance) should remain roughly the same for the final designs. Table 3 compares the accuracy of the plastic rotations obtained by the pushover procedure for different designs corresponding to different levels of service

Table 2 Mean relative errors on the six largest plastic rotations in %, for varying PRSI levels

PRSI	1.52	1.58	1.64	1.69
Resistance reserve (%)	0.59	2.2	4.0	6.0
Mean relative error on the plastic rotations (%)	6.6	6.9	8.1	8.3
Standard deviation (%)	4.6	5.3	5.9	6.7

Table 3 Mean relative errors on the six largest plastic rotations in %, for varying load levels

Load (% of *P*)	76	93	96.5	100	103.5	107	124
Mean relative error on the plastic rotations (%)	5.8	7.4	7.8	8.1	8.4	8.9	9.1
Standard deviation (%)	4.1	5.3	5.6	5.9	6.1	6.2	6.7

Table 4 Mean relative errors on the six largest plastic rotations in %, for varying regular bay lengths

Bay length in m (ft)	4.3 (14)	5.2 (17)	6.1 (20)	7.0 (23)	7.9 (26)
Mean relative error on the plastic rotations (%)	11.4	9.8	8.1	6.5	5.3
Standard deviation (%)	7.0	6.1	5.9	5.2	4.2

loads. It shows the mean relative error and the standard deviation for the six largest plastic rotations, for sets of 49 structures for various percentages of the load *P* specified in Fig. 1 (to which all previously presented results correspond). All structures are designed to have a PRSI level of 1.64.

All results presented so far correspond to structures with regular bay lengths of 6.1 m. Table 4 shows the mean relative error and the standard deviation for the six largest plastic rotations, for sets of 49 structures for various regular bays lengths, as indicated in Fig. 1.

For the specific loading pattern associated to the case of a central ground floor column failure, these results show that the plastic rotations obtained from the pushover analysis and the non-linear dynamic reference solutions are in good agreement for the plastic hinges showing the larger plastic rotations. Furthermore, this accuracy remains reasonably independent of the PRSI level of the final design, of the specified load levels and of the size and geometrical shape of the building in the considered set of structures. The pushover procedure thus yields correct plastic hinge locations and appearance sequences. When associated with the proposed loading pattern and the zero kinetic energy criterion, it allows to properly estimate the ability of the structure to redistribute loads further to the loss of a column, irrespectively of the above mentioned design parameters.

5. Lateral ground floor column removal

5.1 Proposed loading pattern

Applying the same loading scheme for the case of a lateral column removal yields less accurate, all plastic rotations being overestimated (the smallest relative error then varies between 20 and 30 percents). To further improve these results, a different loading pattern is proposed for the case of a ground floor lateral column failure, followed by a discussion relative to the accuracy of the results with respect to the design parameters.

The loading pattern offering the best compromise between accuracy and generality for all structures is the following. A first static analysis is used to determine the resultant end forces at the top of the removed column. The proposed loading pattern is applied in three steps depicted in Fig. 4:

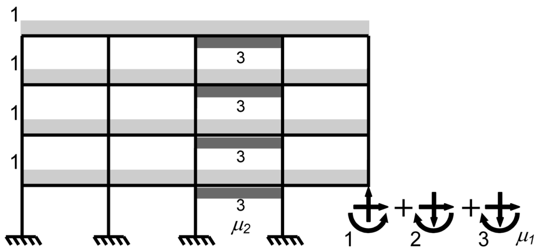

Fig. 4 Load pattern for lateral ground floor removal

1. Apply the dead and live loads and the identified end forces at the top of the removed column.
2. Maintain the dead and live load constant, and gradually decrease the end forces of the removed column (by adding increasing equal but opposite end forces for instance), until these end forces vanish. This quasi-static computation allows the computation of both terms in Eq. (4) (see the Appendix for more details).
3. When the end forces disappear, keep on adding the opposite end forces, multiplying them by an increasing factor μ_1. Simultaneously, decrease the dead and live loads applied to the adjacent bay (i.e., penultimate bay), by adding a uniform load equal but opposite to the dead and live loads, multiplied by a factor μ_2. This quasi-static computation allows the continued computation of both terms in Eq. (4). Follow this loading pattern until the zero kinetic energy criterion approximated by (4) is verified.

The dead and live load decrease on the adjacent bay specified in step 3 enables, through a lever arm effect, a reduction of the overestimations of the plastic rotations in the lateral bay.

In order to apply step 3, it is necessary to specify the ratio R between the relative increase rates of μ_1 and μ_2, defined as

$$R = \frac{d\mu_2}{d\mu_1} \tag{7}$$

This ratio has an influence on the accuracy of the obtained results. To find the optimal ratio R potentially different for each structure, a Newton-Raphson procedure is used, with the condition that the error on the hinge showing the largest plastic rotation vanishes. A parametric study by varying one single parameter at a time (floor number, bay number, robustness, bay length…), is next used to analyse the different optimal values obtained for each structure. This allows assessing whether the optimal ratios are close to one another, so that the mean value of these optima may be used for all structures; or conversely whether a relationship between R and one of the design parameters needs to be established.

5.2 Plastic hinges rotations comparison

Varying the floor and bay numbers from four to ten, for structures with a regular bay length of 6.1 m, under the load P (see Fig. 1) and for a PRSI of 1.467, leads to a mean value for the optimal ratios R of 0.277. Table 5 gives the plastic rotations and their relative errors for structures with seven floors, and bay numbers ranging from four to ten, all pushover analyses being performed with the above mentioned mean value for the ratio R. The results are sorted by decreasing plastic rotations magnitude obtained from the dynamic computation. It can be concluded that using the

Table 5 Relative errors on the plastic rotations in %, for seven floor structures, lateral ground floor column removal, bays numbers ranging from 4 to 10, robustness = 1.467, bay length = 6.096 m

Number of bays	4		5		6		7		8		9		10	
Plastic rotations in mrad (reference solution)	12.6		12.2		12.1		11.7		11.5		11.5		11.5	
	10.5		10.1		10.3		9.9		9.9		9.8		9.8	
	9.4		9.2		9.4		9.3		9.3		9.3		9.2	
	8.5		8.6		8.6		8.6		8.6		8.6		8.5	
	7.7		8.1		8.2		8.1		7.9		8		7.9	
	7.2		7.7		7.5		7.6		7.7		7.7		7.7	
	5.6		5.5		5.3		5.5		5.6		5.6		5.6	
	5.5		5.2		5.3		5.2		5.3		5.4		5.4	
	3.5		3.7		3.6		3.7		3.7		3.7		3.7	
	1.8		2.1		2.3		2.4		2.4		2.4		2.4	
	1.5		1.8		2.3		2		1.5		1.9		1.8	
	1.5		1.4		1.3		1.2		0.6		1.2		0.8	
	0.5		0.2		0.4		0.3		0.2		0.1		0.1	
Plastic rotations relative errors (%)	5.5	1	1.6	1	-1.9	1	-0.8	1	-1.5	1	-2.6	1	-3.5	1
	13.6	2	11.2	2	4.8	2	5.7	2	4.1	2	3.3	2	2.3	2
	21.0	3	16.1	3	9.9	3	8.4	3	6.6	3	5.4	3	4.4	3
	24.2	4	16.1	4	11.7	4	9.1	4	7.5	4	6.2	4	5.3	4
	30.3	5	16.5	5	12.2	5	10.6	5	11	5	8	5	8.6	5
	32.1	6	16.7	6	16.1	6	11.5	6	7.8	6	5.9	6	4.5	6
	30.8	7	18.6	8	14.1	8	4.9	8	-0.8	8	-4.6	8	-7.0	8
	27.7	8	25.6	7	16.2	7	14.2	7	10.0	7	5.8	7	4.6	7
	61.1	9	41.5	9	35.1	9	28.9	9	24.3	9	22.4	9	21.1	9
	172	10	118	10	90.5	10	77.4	10	71.6	10	67.7	10	66.4	10
	201	11	128	11	75.9	11	89.1	11	152	11	91.9	11	103	11
	201	12	181	12	179	12	197	12	465	12	184	12	291	12
	360	13	749	13	206	13	214	13	264	13	526	13	508	13

Table 6 Mean values of the optimal ratios *R*, for varying regular bay lengths

Bay length in m (ft)	4.3 (14)	5.2 (17)	6.1 (20)	7.0 (23)	7.9 (26)
Mean value of the optimal ratios *R*	0.273	0.278	0.277	0.274	0.274

Table 7 Mean relative errors on the three largest plastic rotations in %, for varying load levels

Load (% of P)	77	92	96	100	104	108	123
Mean relative error on the plastic rotations (%)	5.3	6.0	6.5	6.5	7.1	7.2	8.2
Standard deviation (%)	4.2	4.9	5.1	5 .0	5.5	5.4	5.9

mean value of the optimal ratios R for all structures of this set allows for the detection of all plastic hinges, and yields accurate results for the first few ones (the mean relative error for the three largest plastic rotations for all 49 structures of this set is of 6.9%, with a standard deviation of 5.3%). This means that R can be assumed independent from the number of floors and bays.

A variation of bays length is scrutinised in Table 6 showing the corresponding mean values of the optimal ratios R. These mean values are very close to one another, their standard deviation being less then one percent of their mean value. This implies that if their global average (= 0.275) had been used for the pushover analyses of all structures of the five sets (five bay lengths), the results would have been almost unaffected, and the accuracy level indicated by Table 5 would have been maintained. Again, this means that R can be considered independent from the design loads.

Table 7 shows the mean relative error and the standard deviation for the three largest plastic rotations, for sets of 49 structures for seven different fractions of the load P specified in Fig. 1. All structures have a PRSI level of 1.467. The mean value of the optimal ratios R calculated for structures submitted to 100% of P was used for all structures of all seven sets, showing that R can be considered independent from the design load.

All the results presented so far in this section were obtained for structures with a fixed PRSI of 1.467. This value leads to mechanisms having nearly formed at the end of the pushover analyses, which means that the structures have almost no resistance reserve left when Eq. (4) is verified. It should therefore be expected that a practitioner using the mean value specified above (Table 6) for the load ratio R would design structures with a PRSI level close to 1.467. It is however not possible to predict the exact PRSI of the final design, as explained above since it depends among others on the available beams in a producer's catalogue.

For the removal of a central column, the accuracy of the pushover results was found almost independent of the PRSI level. For the removal of a lateral column however, the results show that if the mean value of the ratios in Table 6 is used, the accuracy of the results varies significantly with the PRSI, even for quite small variations of this indicator. The optimal ratio R of the loading scheme therefore depends on the PRSI level. Fig. 5 shows the mean optimal ratios obtained for sets of structures with varying PRSI levels (for a fixed bay length of 6.1 m). The points on Fig. 5 indicate an almost linear relationship, with a best fit straight approximation (dotted line) given in Eq. (8). Using ratios given by Eq. (8) allows to recover the accuracy level indicated by Tables 5 to 7.

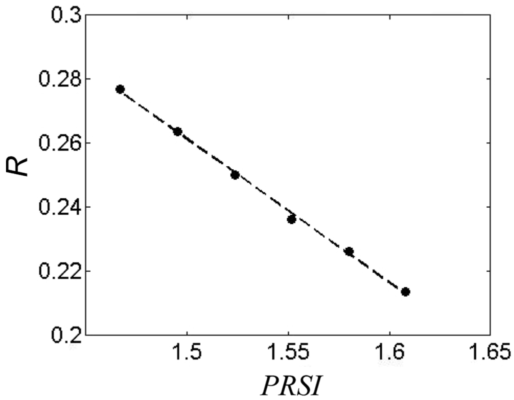

Fig. 5 Ratio R between μ_1 and μ_2 as a function of the PRSI level

$$R = -0.448\mathrm{PRSI} + 0.934 \tag{8}$$

The use of Eq. (8) therefore leads to the following design process for a lateral ground floor column removal:

1. Use the pushover analysis described in Section 5.2 with a ratio R_{init} of 0.275 to design the structure.
2. Calculate the PRSI for the obtained design, as well as the corresponding new ratio $R_{updated}$ given by Eq. (8).
3. If the new ratio R_2 is lower than R_1, the structure needs to be checked and potentially redesigned: return to step 1 using the new ratio R_2. If not, the structure is well designed as far as moments redistribution is concerned, but a more optimal design can be obtained by restarting from step one, using the new ratio.
4. Repeat steps 1 to 3 until $R_{updated}$ is equal to or larger than R_{init}.

For the specific pushover loading pattern associated to the case of a lateral ground floor column failure, comparisons with the plastic rotations obtained through the non-linear dynamic reference solutions show a good agreement for the plastic hinges with the larger rotations. The parameter on which the pushover loading pattern depends, the load ratio R defined in Eq. (7), is independent of the specified loads and of the bay lengths, but depends on the PRSI level of the final design, and Eq. (8) provides the required relationship.

6. Discussion

The results obtained here can first be compared to the general prescriptions given in design codes. For instance, both the (USA-GSA 2003) and (USA-DOD 2005) have issued guidelines for progressive collapse mitigation. The GSA guidelines describe in details what is referred to as a static linear procedure, although static non-linear and dynamic non-linear computations are allowed. The DoD describes procedures which match these three categories as well.

A direct comparison between the pushover procedure presented here and (USA-GSA 2003, USA-DOD 2005, Kim and Kim 2009) is impossible because both sets of guidelines allow for the failure of portions of the structure, referred to as the allowable collapse region. This region usually corresponds to the bays directly concerned by the failing column. Since in the present approach, structures are designed so that the concerned bays do not collapse, the acceptability criterion is not the same and it is difficult to compare the resulting safety margins.

However, the loading pattern specified in (USA-GSA 2003, USA-DOD 2005) is similar to the one in the definition used for the PRSI level (first remove the failing column from the model, then apply the dead and live loads). In both guidelines documents, a load factor of 2 to be applied to both dead and live loads is specified for static computations, to account for dynamic inertial effects. Yet, the examples shown here indicate that PRSI levels around and close to 1.5 are sufficient. If dynamic computations are performed, even lower PRSI levels could still allow the structures to prevent collapse, provided the higher sectional (and connection) ductility demands can still be met. The dynamic load factor of 2 specified in (USA-GSA 2003, USA-DOD 2005) thus appears highly conservative as already commented elsewhere based on the analysis of specific structures.

A second comment needs to be raised concerning the PRSI dependency of the identified loading schemes. The PRSI is used, in the present context, as measure of a structure's strength with respect to the loads it is submitted to (for a given missing column), with a view to relate this parameter to the pushover loads. The pushover loads could indeed be expected to be dependent on a measure of the structure's robustness. For example, a structure such that no plastic hinge appears following a column loss (i.e., linear elastic behaviour) has a dynamic load factor of roughly 2 (and therefore a PRSI larger than 2). On the other hand, structures which do show plastification following the column loss, such as those studied here, have lower PRSI's and lower dynamic load factors. It is therefore important to link these two notions. For the case of a central column removal, the procedure's accuracy is independent of the PRSI, thus showing that the zero kinetic energy criterion naturally takes into account the influence of the latter, leading to a corresponding amplification of the static loads. In order to maintain a good accuracy for a lateral column failure, it was necessary to integrate the influence of the PRSI on the parameters of the computational procedure, specifically the ratio R introduced by Eq. (7) in Section 5. This implies that an approximation is introduced when a single dynamic load factor is specified, constant for all structures, as is the case for the (USA-GSA 2003, USA-DOD 2005) procedures.

Finally, the systematic nature of the parametric study presented here, which incorporates load, bay length and PRSI variations, implies that the studied structures had to be redesigned very often. The automation of the design process was achieved through the introduction of a single coefficient in the data set (α coefficient, Section 3.2.1), relating the plastic moments to the inertias and section areas. The question is therefore raised of the extension of the pushover procedure presented here for other construction materials, such as reinforced concrete as performed in Almusallam *et al.* (2010). The same comment can be made for the incorporation of catenary effects which will be examined in a subsequent work. However, the obtained results is a strong encouragement to extend the study to other materials, as well as to three dimensional structures.

7. Conclusions

In order to study dynamic inertial effects during structural progressive collapse, a zero-length plastic hinge element accounting for inelastic rotations was used. Non-linear dynamic computations were carried out on various sets of two-dimensional beam-column structures to provide reference solutions. An equivalent pushover static analysis, based on carefully selected static equivalent loading schemes, coupled with a zero kinetic energy criterion was proposed to simulate the inertial dynamic effects without the need to perform full non-linear dynamic computations. In the case of a central ground floor column failure, the load pattern to apply is independent of design parameter, i.e., the number of floors, number of bays, bay length, loads and of a proposed measure of the structural strength reserve. The use of this load pattern for the case of lateral ground floor column failure, however, leads to slightly less accurate results. To further improve the accuracy in this case, a different specific load pattern is proposed, the main difference with the central ground floor column failure loading pattern being that it depends on the PRSI (plastic reserve strength indicator) level of the final design.

Due to the identification of specific load patterns, the obtained results offer a very good accuracy in the detection of the plastic hinge locations and appearance sequences, and therefore by extension, the structure's ability to redistribute loads following the loss of a vertical support member is well represented. This feature of the structural behaviour is key for the structural design in the geometrically linear range, where ductility requirements would probably not play a major role. Since the pushover procedure is designed for geometrically linear analyses, it is important to design the structure in accordance with this assumption when using the pushover procedure described here.

The analysis of the results shows that the dynamic load factor of 2 specified by the GSA and DoD (USA-GSA 2003, USA-DOD 2005) appears to be strongly conservative, as is also reported in (Vlassis 2007, Izzuddin *et al.* 2008, Ruth 2006).

Finally, the zero kinetic energy criterion is expressed mathematically through one equation with as many unknowns as there are degrees of freedom in the model. Such a weak constraint cannot be expected to yield accurate results on its own. In order to obtain accurate results, it needs to be coupled to a proper loading pattern. It can also be argued that there is nothing particular about the zero kinetic energy configuration, and the proper loading pattern only serves to cover up the deficiencies of the zero kinetic energy criterion. One might therefore as well look for the right loading pattern and associated loading level, and not use the zero kinetic energy criterion as the indication of when the quasi-static pushover analysis should be stopped.

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Appendix - Cumulated internal and external forces work

The computation of both terms in Eq. (4), during the application of the loading patterns identified in Sections 4 and 5, is described below.

Let us call f_d a vector in which the nodal loads corresponding to the loads actually applied to the structure (i.e., the gravity and service loads) are collected. These loads are constant through the dynamic process.

Since the static pushover analysis is a non-linear one, a stepwise application of the loads is necessary. The subscript n in the following terms and equations refers to the step n of the stepwise load application inherent to any static non-linear analysis (i.e., not to stages 1, 2 or 3 of the loading patterns specified in Sections 4 and 5). Let us call $f_{s,n}$ a vector in which the nodal loads corresponding to the static pushover procedure (see Fig. 2 and 4) at the load step n are collected.

Stage 1 in the loading patterns described in Sections 4 and 5 does not yet correspond to the dynamic collapse process, so that at the end of this step, the cumulated internal and external forces works are considered equal to zero. At the beginning of Stage 2 of the loading patterns, we therefore have

$$\begin{aligned} T_{i,0} &= 0 \\ T_{e,0} &= 0 \end{aligned} \tag{9}$$

where $T_{i,0}$ and $T_{e,0}$ respectively represent the cumulated internal and external forces works as of the removal of the failing element at the end of Stage 1 of the loading patterns, and correspond respectively to the left and right terms in Eq. (4).

At the end of step n of the stepwise application of the pushover loads, we then have

$$\begin{aligned} T_{i,n} &= T_{i,n-1} + (q_{s,n} - q_{s,n-1})\frac{(f_{s,n} + f_{s,n-1})}{2} \\ T_{e,0} &= T_{e,n-1} + (q_{s,n} - q_{s,n-1})\, f_d \end{aligned} \tag{10}$$

where $q_{s,n}$ represents a vector in which the displacements obtained at step n are collected, and $q_{s,0}$ corresponds to the displacements obtained at the end of Stage 1 of the loading patterns.