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A note on buckling and vibration of clamped orthotropic plates under in-plane loads

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Abstract. The present work deals with obtaining the critical buckling load and the natural frequencies of clamped, orthotropic, rectangular thin plates subjected to different linear distributed in-plane forces. An analytical solution is proposed. Using the Ritz method, the dependence between in-plane forces and natural frequencies are estimated for various plate sizes, and some results are compared with finite element solutions and where possible, comparison is made with previously published results. Beam functions are used as admissible functions in the Ritz method.

Keywords: critical buckling; vibration; orthotropic plate; in-plane force; rectangular, clamped; Ritz method

1. Introduction

Exact solutions for the free vibration and buckling of rectangular plates based on the classical thin plate theory are known for plates with two opposite edges simply supported when the loads are applied at these edges. Most of them for isotropic plates of uniform thickness subjected to constant in-plane loads. Leissa and Kang (2001, 2002), found exact solutions for linearly varying loads, Xiang and Wang (2002) and Xiang and Wei (2004) found exact solutions for stepped plates.

Approximate solutions are also available: Dickinson (1971) has used the Ritz method to study the free lateral vibrations of simply supported rectangular plates subjected to various in-plane force conditions. Bassily and Dickinson (1972) and Kaldas and Dickinson (1981) proposed a series composed of multiplications of beam functions in the Ritz method for vibrating rectangular plates subjected to arbitrary in-plane stresses. In 1978 Laura and his co-workers determined the fundamental frequency of transverse vibrations of rectangular plates subjected to in plane-loads. (Diez *et al.* 1978). In a paper by Dickinson (1978) a simple approximate formula for the natural frequencies of flexural vibration of isotropic plates, originally developed by Warburton using characteristic beam functions in Rayleigh's method, is modified to apply to orthotropic plates and extended to include the effect of uniform, direct in-plane forces.

Later Gorman (1990) used the superposition method subdividing the clamped orthotropic plate

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problem into building blocks and superposing the resulting solutions. Shi (1990), developed a numerical solution technique by means of the boundary element method for flexural vibration and buckling analysis of orthotropic plates. Wang *et al.* (2006) reproduced the exact values of Leissa and Kang (2002) using the Differential Quadrature method. Tripathy and Suryanarayan (2008), used a flexibility function approach to solve buckling and vibration characteristics of weld-bonded rectangular plates. Singhatanadgid and Sukajit (2011) investigated the use of a vibration correlation technique to identify the buckling load of a rectangular thin plate, they proposed that the buckling load can be determined experimentally using the natural frequencies of plates under in-plane loading.

In the present study, rectangular orthotropic clamped plates, with distributed normal forces of linear variation applied to all edges are considered.

As it is known, laminated composite panels are increasingly required in modern technological applications. Consequently, modeling the structure as an orthotropic element is worthy of consideration. Certainly, fiber reinforced composite materials are used as structural materials, in shipbuilding, aircraft, automotive industries and in civil engineering. Plates of high-strength fiber reinforced concrete are used in many technological situations (Ramadoss and Nagamani 2009).

However, to our best knowledge, there are not many results available in the literature for the case of clamped orthotropic rectangular plates subjected to different linear distributed in-plane forces.

An approximate solution is obtained by means of the Ritz method and beam functions are used in the deflection approximation, since they satisfy the essential boundary conditions at the outer edge of the plate.

In some cases an independent solution is also obtained using a finite element code (ALGOR 23.1 2009). Numerical results are presented for critical loads and natural frequencies for fully clamped plates and where possible, comparison is made with previously published results.

2. Approximate solution

The small deflection motion of a vibrating thin rectangular orthotropic plate loaded by in-plane normal forces varies harmonically with time t

$$w = We^{i\omega t}$$

where ω is the circular natural frequency of the plate and W is the deflection amplitude. Fig. 1.

The functional associated with the orthotropic vibrating plate is given by

$$\begin{aligned} J[W] = {} & \frac{1}{2}\left[\frac{b}{a^3}\int_{A_n} -D_1 N_x\left(\frac{\partial W}{\partial x}\right)^2 dxdy - \frac{1}{ab}\int_{A_n} D_1 N_y\left(\frac{\partial W}{\partial y}\right)^2 dxdy\right] \\ & + \frac{1}{2}\left[\frac{b}{a^3}\int_{A_n} D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 dxdy + \frac{2\nu_2}{ab}\int_{A_n} D_1 \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} dxdy \right. \\ & \left. + \frac{a}{b^3}\int_{A_n} D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2 dxdy + \frac{4}{ab}\int_{A_n} D_k\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 dxdy\right] \\ & - \frac{1}{2}\rho ab\omega^2 \int_{A_n} hW^2 dxdy \end{aligned} \quad (1)$$

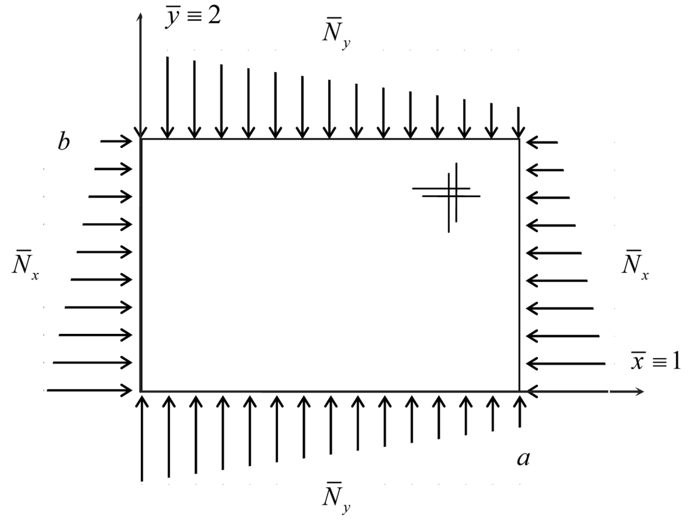

Fig. 1 Clamped orthotropic rectangular plate under linear distributed in-plane forces

where a, b, are the plate dimensions and h is the plate thickness; $D_1 = E_1 h^3/12(1-\nu_1\nu_2)$, $D_2 = E_2 h^3/12(1-\nu_1\nu_2)$, and $D_k = G_{12}h^3/12$ are bending and twisting rigidities for principal directions of elasticity for orthotropic plates, E_1 and E_2 are Young's moduli, G_{12} is the shear modulus and the Poisson's ratios are: ν_1 and ν_2. Poisson's ratios and Young's moduli are related by the reciprocal relation: $\nu_1 E_2 = \nu_2 E_1$. ρ is the material density, $x = \bar{x}/a$; $y = \bar{y}/b$ are the spatial dimensionless coordinates and N_x, N_y are the compressive in-plane forces per unit width written in dimensionless form. The forces are distributed according to a linear law.

The principal directions of elasticity 1 and 2 coincide with the directions of axes x and y.

The deflection W is assumed as a solution of the form

$$W \cong W_a(x,y) = \sum_{q=1}^{Q}\sum_{l=1}^{L} A_{ql} X_q(x) Y_l(y) \tag{2}$$

where A_{ql} are undetermined coefficients and $X_q(x)$, $Y_l(y)$ are beam functions for clamped-clamped conditions

$$\begin{aligned} X_q(x) &= \cosh(k_q x) - \cos(k_q x) + r_q[\sin(k_q x) - \sinh(k_q x)] \\ Y_l(y) &= \cosh(k_l y) - \cos(k_l y) + r_l[\sin(k_l y) - \sinh(k_l y)] \end{aligned} \tag{3}$$

with

$$r_q = \frac{\cos(k_q) - \cosh(k_q)}{\sin(k_q) - \sinh(k_q)}; \quad r_l = \frac{\cos(k_l) - \cosh(k_l)}{\sin(k_l) - \sinh(k_l)} \tag{4}$$

where k_q and k_l are obtained from the characteristic equation

$$\cos(k)\cosh(k) = 1 \tag{5}$$

Applying the Ritz minimization condition for each of the generalized coordinates A_{ql}, it is obtained

$$\frac{\partial J[W_a]}{\partial A_{ql}} = 0, \text{ with } q = 1,2,\ldots,Q;\ l = 1,2,\ldots,L \tag{6}$$

which is a set of $(Q \times L)$ linear homogeneous equations.

Using non dimensional variables, Eq. (6) becomes

$$\frac{ab}{D_1}\frac{\partial J[W_a]}{\partial A_{ql}} = 0 \tag{7}$$

The in-plane forces are described by the linear expressions

$$N_x(y) = \frac{\overline{N}_1 b^2}{D_1}(1-cy) = N_1(1-cy); \quad N_y(x) = \frac{\overline{N}_2 b^2}{D_1}(1-dx) = N_2(1-dx) \tag{8}$$

where $N_1 = \dfrac{\overline{N}_1 b^2}{D_1}$; $N_2 = \dfrac{\overline{N}_2 b^2}{D_1}$; *c; d* are constants.

The frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}\,a^2\omega_i$ are determined with the minimization condition (7) for different values of N_1/N_{crit}, where $N_{crit} = \overline{N}_{crit}b^2/D_1$ is the critical buckling value, for each ratio N_2/N_1.

The buckling problem is treated simply by equating the frequency expression derived by the present analytical method to zero.

3. Numerical results

In the numerical analysis, the *Mathematica* software was employed to obtain the results; its eigenvalue solver was used to compute the natural frequencies and the critical loads.

Tables 1 to 4 exhibit buckling and vibration dimensionless results for square and rectangular orthotropic plates for loads only in *x*-direction ($N_x \neq 0, N_y = 0$). Fig. 2. Uniform and linear varying forces are considered.

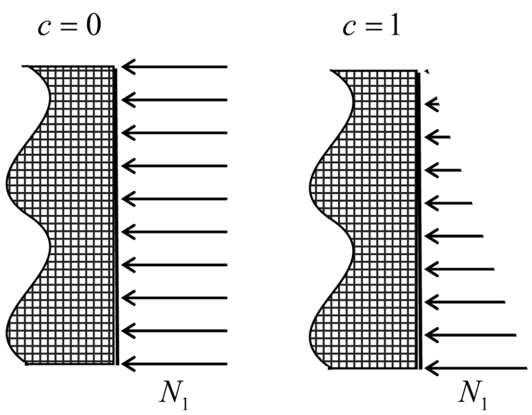

Fig. 2 Linear distributed in-plane forces in *x*-direction: $N_x(y) = N_1(1-cy)$

Table 1 Frequency coefficients $\Omega_i = \sqrt{\rho h/D_1} a^2 \omega_i$ for clamped orthotropic square plates, $a/b = 1$, subjected to uniform distributed in-plane forces. $N_x = N_1$; $N_y = 0$. Critical buckling: $N_{crit} = \bar{N}_{crit} b^2/D_1 = 148.251$. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N_1/N_{crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	45.6390	83.4289	101.124	139.749	142.014	184.870	
	45.6426	83.4394	101.132	139.778	142.034	184.883	FEM
	45.640	83.440	---	139.76	142.00	184.88	Gorman (1990)
0.30	39.3549	70.2159	98.6213	125.606	132.536	183.560	
0.50	34.3101	59.6288	96.8971	113.315	127.465	180.356	
	33.9112	59.3743	96.4218	113.128	127.191	180.085	FEM
0.80	23.9052	38.0563	91.8567	94.2093	119.384	167.814	
0.95	14.8504	19.1376	79.1148	92.8101	115.090	156.589	
1	0	9.42795	74.4531	92.3340	113.616	152.482	
	0.10453	9.38576	74.5186	92.3389	113.650	152.605	FEM

The following Tables 3 and 4 present the orthotropic rectangular plate, with $a/b = 2$, and the same two cases of distributed in-plane loads.

Table 2 Frequency coefficients $\Omega_i = \sqrt{\rho h/D_1} a^2 \omega_i$ for clamped orthotropic square plates, $a/b = 1$, subjected to linear distributed in-plane forces. $N_x = N_1(1-y)$; $N_y = 0$. Critical buckling: $N_{crit} = \bar{N}_{crit} b^2/D_1 = 281.778$. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N_1/N_{1crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	45.6390	83.4289	101.124	139.749	142.014	184.870	
	45.6426	83.4394	101.132	139.778	142.034	184.883	FEM
	45.640	83.440	---	139.76	142.00	184.88	Gorman (1990)
0.30	39.6547	70.7304	98.7504	126.114	132.951	183.624	
0.50	34.8384	60.3707	97.1121	113.820	128.237	181.603	
	34.8353	60.3785	97.1192	113.855	128.269	181.677	FEM
0.80	24.8888	38.8481	91.1870	94.9349	120.832	167.352	
0.95	16.2482	19.6268	77.6335	93.4356	116.942	154.457	
1	0	11.2387	72.5095	92.9846	115.610	149.864	
	0	11.2048	72.5815	92.9899	115.643	150.005	FEM

Table 3 Frequency coefficients $\Omega_i = \sqrt{\rho h/D_1} a^2 \omega_i$ for clamped orthotropic rectangular plates, $a/b = 2$, subjected to uniform distributed in-plane forces. $N_x = N_1$; $N_y = 0$. Critical buckling: $N_{crit} = \bar{N}_{crit} b^2/D_1 = 119.580$. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N_1/N_{crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	137.035	168.852	222.347	297.234	360.964	393.005	
	137.036	168.856	222.359	297.250	360.961	393.098	FEM
	137.032	168.800	222.337	---	360.907	393.038	Gorman (1990)
0.30	130.933	148.741	188.145	252.636	341.514	358.844	
0.50	126.328	133.068	161.017	217.709	302.264	357.415	
	126.326	133.065	161.027	217.744	302.342	357.418	FEM
0.80	101.166	101.942	121.929	151.724	231.400	339.844	
0.95	52.5642	65.9825	111.971	116.06	187.069	297.740	
1	0	36.7066	99.9207	113.760	170.362	282.398	
	0	36.7491	99.9315	113.762	170.477	282.565	FEM

Table 4 Frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}a^2\omega_i$ for clamped orthotropic rectangular plates, $a/b = 2$, subjected to linear distributed in-plane forces. $N_x = N_1(1-y)$; $N_y = 0$. Critical buckling: $N_{crit} = \overline{N}_{crit}b^2/D_1 = 231.536$. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N_1/N_{crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	137.035	168.852	222.347	297.234	360.964	393.005	
	137.036	168.856	222.359	297.250	360.961	393.098	FEM
	137.032	168.800	222.337	---	360.907	393.038	Gorman (1990)
0.30	131.126	149.335	189.038	253.629	342.476	358.916	
0.50	126.676	134.068	162.349	218.904	303.073	357.534	
	126.674	134.064	162.357	218.937	303.149	357.435	FEM
0.80	102.746	103.636	122.429	151.909	229.945	337.005	
0.95	53.8684	65.9847	111.306	116.424	182.774	291.555	
1	0	32.4642	99.4352	113.904	164.756	274.679	
	0	36.7491	99.9315	113.762	170.477	282.565	FEM

Tables 5 and 6 show the frequency parameters of biaxially compressed clamped orthotropic plates. The first six frequency coefficients of transverse vibration and the critical buckling have been computed, for three different cases of in-plane forces with $Q \times L = 225$.

Table 5 Frequency $\Omega_i = \sqrt{\rho h/D_1}a^2\omega_i$ coefficients of clamped orthotropic square plates, $a/b = 1$, subjected to linear distributed in-plane forces. $N_x = N(1-y)$; $N_y = N$. Critical buckling: $N_{crit} = \overline{N}_{crit}b^2/D_1$ = 112.350. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N/N_{crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	45.6390	83.4289	101.124	139.749	142.014	184.870	
	45.6426	83.4394	101.132	139.778	142.034	184.883	FEM
	45.6400	83.4400	---	139.760	142.000	184.880	Gorman (1990)
0.30	38.3966	76.1610	92.1626	131.444	134.591	175.113	
0.50	32.5893	70.8275	85.6338	125.594	129.324	168.286	
	32.5921	70.8417	85.6467	125.632	129.355	168.314	FEM
0.80	20.7662	61.8144	74.7125	116.261	120.848	157.483	
0.95	10.4287	56.6813	68.5682	111.300	116.301	151.790	
1	0	54.8474	66.3884	109.597	114.733	149.845	
	0	54.8684	66.4089	109.645	114.779	149.890	FEM

Table 6 Frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}a^2\omega_i$ for clamped orthotropic rectangular plates, $a/b = 2$, subjected to linear distributed in-plane forces. $N_x = N(1-y)$; $N_y = N$. Critical buckling: $N_{crit} = \overline{N}_{crit}b^2/D_1$ = 83.6822. $D_2/D_1 = 2$, $\nu_2 = 0{,}3$, $D_k/D_1 = 0{,}85$

N/N_{crit}	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	
0	137.035	168.852	222.347	297.234	360.964	393.005	
	137.036	168.856	222.359	297.250	360.961	393.098	FEM
	137.032	168.800	222.337	---	360.907	393.038	Gorman (1990)
0.30	115.189	146.405	199.451	274.039	333.551	369.590	
0.50	97.6930	129.093	182.439	257.270	313.855	350.836	
	97.7352	129.215	182.637	257.525	313.885	350.891	FEM
0.80	62.1556	97.0345	153.090	229.546	281.566	320.027	
0.95	31.1828	75.7653	135.864	214.188	263.855	303.387	
1	0	67.1312	129.580	208.791	257.666	297.623	
	0	67.6081	130.129	209.407	257.741	297.763	FEM

Table 7 Critical buckling $N = N_{crit} = \overline{N}_{crit}b^2/D_1$ for clamped orthotropic rectangular plates, subjected to different linear distributed in-plane forces. $D_2/D_1 = D_k/D_1 = 1/2$ and $\nu_2 = 0.3$

	$N_{crit} = \dfrac{\overline{N}_{crit}b^2}{D_1}$					
$\lambda = \dfrac{a}{b}$	$N_x = N$; $N_y = 0$	$N_x = N(1-y)$; $N_y = 0$	$N_x = N(1-2y)$; $N_y = 0$	$N_x = N$; $N_y = N$	$N_x = N(1-y)$; $N_y = N$	$N_x = N(1-2y)$; $N_y = N$
0.40	278.853	466.043	754.479	264.919	301.578	343.711
0.50	192.196	337.846	610.187	157.115	200.370	232.062
2/3	123.334	235.318	502.075	90.660	119.657	140.290
1.00	88.695	170.058	440.299	46.122	59.983	74.328
1.50	77.785	148.662	388.013	29.128	34.070	40.379
2.00	70.573	136.391	370.420	24.305	26.750	29.602
2.50	67.747	130.775	362.833	22.384	23.850	25.461
3.00	66.659	128.867	358.814	21.453	22.431	23.470

The results agree with those obtained by the finite element method (FEM), for N/N_{crit} = 0, 0.50 and 1. Those calculations have been performed using a well known finite element code, Algor (2009). The mesh employed for the square plate has 10000 (100 × 100) quadrilateral elements and for the rectangular one that has 20000 (200 × 100) quadrilateral elements.

Table 7 presents the dimensionless critical buckling loads of rectangular orthotropic plates for various cases of in-plane-loads and a range of plate aspect ratios: $0.40 \le a/b \le 3$.

The critical buckling coefficients have been computed with $Q \times L = 225$.

Table 8 shows the frequency parameters of square clamped plates under hydrostatic in-plane forces, having various degrees of orthotropy. It is seen that the present results agree well with fundamental frequency coefficients given by Dickinson (1978). These results were obtained with $Q \times L = 30 \times 30$.

Table 8 Natural frequency coefficients of square clamped plates under hydrostatic loads, $N_x = N_y = N$, for various relations of orthotropy

$N_x = N_y$	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
			$D_2/D_1 = 1$;	$D_k/D_1 = 0.85$;	$\nu_2 = 0.30$	
0	39.6993 39.696*	80.0811	80.0811	125.0430	139.4600	140.1900
$4.\pi^2$	25.0910 25.082*	64.5644	64.5644	110.2270	122.8750	123.9100
			$D_2/D_1 = 2$;	$D_k/D_1 = 0.85$;	$\nu_2 = 0.30$	
0	45.6386 45.642*	83.4274	101.1220	139.7410	142.0110	184.8650
$4.\pi^2$	33.6351 33.638*	68.5811	89.2944	125.8145	126.5466	172.7617

Table 8 Continued

$N_x = N_y$	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
			$D_2/D_1 = 4$;	$D_k/D_1 = 0.85$;	$\nu_2 = 0.30$	
0	55.5693 55.566*	89.4221	133.5592	145.8528	164.9011	218.1049
$4.\pi^2$	46.1901 46.186*	75.6921	124.8269	130.0883	153.8170	204.8409
			$D_2/D_1 = 1$;	$D_k/D_1 = 0.35$;	$\nu_2 = 0.30$	(isotropy)
0	35.9852 35.985*	73.3939	73.3939	108.2166	131.5808	132.2049
$4.\pi^2$	28.5729 28.573*	65.2244	65.2244	99.6993	123.0059	123.7737
			$D_2/D_1 = 2$;	$D_k/D_1 = 0.35$;	$\nu_2 = 0.30$	
0	42.3969 42.400*	76.8437	95.8804	124.6758	133.9704	176.4467
$4.\pi^2$	36.2961 36.340*	69.0445	89.7686	117.3172	125.5817	168.2671
			$D_2/D_1 = 1$;	$D_k/D_1 = 0.10$;	$\nu_2 = 0.30$	
0	33.9120 33.910*	69.6829	69.6829	98.4322	127.3822	127.8355
$4.\pi^2$	30.1519 30.149*	65.4649	65.4649	93.7736	123.0127	123.5229

*Dickinson (1978).

4. Conclusions

Frequency of vibration and critical buckling load often constitute an important part of design for thin plates or structures made of thin plates.

The paper presents analytical results for free vibrations of clamped orthotropic rectangular plates subjected to linearly varying in plane forces, by the use of the Ritz method. Letting the natural frequency be zero here led to critical buckling condition. Numerical calculations were performed for the first six natural frequencies for different in-plane loads and examples of critical buckling loads are presented.

As it was expected, the frequency coefficients decrease as the in-plane force ratio N/N_{crit} increases, and the fundamental frequency approaches to zero as N/N_{crit} approaches to 1.

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References

ALGOR 23.1 (2009), *Linear Dynamic Analysis*, Algor Inc., Pittsburgh, PA.

Bassily, S.F. and Dickinson, S.M. (1972), "Buckling and lateral vibration of rectangular plates subjected to in-plane loads-a Ritz approach", *J. Sound Vib.,* **24**(2), 219-239.

Dickinson, S.M. (1971), "Lateral vibration of rectangular plates subject to in-plane forces", *J. Sound Vib.,* **16**(4) 465-472.

Dickinson, S.M. (1978), "The buckling and frequency of flexural vibration of rectangular isotropic and orthotropic plates using Rayleigh's method", *J. Sound Vib.,* **61**(1), 1-8.

Diez, L., Gianetti, C.E. and Laura, P.A.A. (1978), "A note on transverse vibrations of rectangular plates subject to in-plane normal and shear forces", *J. Sound Vib.,* **59**(4), 503-509.

Gorman, D.J. (1990), "Accurate free vibration analysis of clamped orthotropic plates by the method of superposition", *J. Sound Vib.*, **237**(5), 901-914.

Kaldas, M.M. and Dickinson, S.M. (1978), "Vibration and buckling calculations for rectangular plates subject to complicated in-plane stress distributions by using numerical integration in a rayleigh-ritz analysis", *J. Sound Vib.,* **75**(2), 151-162.

Leissa, A.W. and Kang, J.H. (2001), "Exact solutions for the free vibrations and buckling of rectangular plates with linearly varying in-plane loading", *Proceedings of the ASME International Mechanical Engineering Congress and Exhibition*, New York.

Leissa, A.W. and Kang, J.H. (2002), "Exact solutions for vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses", *Int. J. Mech. Sci.,* **44**(9), 1925-1945.

Ramadoss, P. and Nagamani, K. (2009), "Behavior of high-strength fiber reinforced concrete plates under in-plane and transverse loads", *Struct. Eng. Mech.,* **31**(4), 371-382.

Singhatanadgid, P. and Sukajit, P. (2011), "Experimental determination of the buckling load of rectangular plates using vibration correlation technique", *Struct. Eng. Mech.,* **37**(3), 331-349.

Shi, G. (1990), "Flexural vibration and buckling analysis of orthotropic plates by the boundary element method", *Int. J. Solids Struct.*, **26**(12), 1351-1370.

Tripathy, B. and Suryanarayan, S. (2008), "Vibration and buckling characteristics of weld-bonded rectangular plates using the flexibility function approach", *Int. J. Mech. Sci.,* **50**, 1486-1498.

Wang, X., Gan, L. and Wang, Y. (2006), "A differential quadrature analysis of vibration and buckling of an SS-C-SS-C rectangular plate loaded by linearly varying in-plane stresses", *J. Sound Vib.,* **298**(1-2), 420-431.

Xiang, Y. and Wang, C.M. (2002), "Exact buckling and vibration solutions for stepped rectangular plates", *J. Sound Vib.,* **250**(3), 503-517.

Xiang, Y. and Wei, G.W. (2004), "Exact solutions for buckling and vibration of stepped rectangular Mindlin plates", *Int. J. Solids Struct.*, **41**, 279-294.