Structural Engineering and Mechanics, Vol. 38, No. 4 (2011) 503-516 503
DOI: http://dx.doi.org/10.12989/sem.2011.38.4.503

A homogenization approach for uncertainty quantification of deflection in reinforced concrete beams considering microstructural variability

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(Received April 22, 2010, Accepted April 6, 2011)

Abstract. Uncertainty in concrete properties, including concrete modulus of elasticity and modulus of rupture, are predicted by developing a microstructural homogenization model. The homogenization model is developed by analyzing a concrete representative volume element (RVE) using the finite element (FE) method. The concrete RVE considers concrete as a three phase composite material including: cement paste, aggregate and interfacial transition zone (ITZ). The homogenization model allows for considering two sources of variability in concrete, randomly dispersed aggregates in the concrete matrix and uncertain mechanical properties of composite phases of concrete. Using the proposed homogenization technique, the uncertainty in concrete modulus of elasticity and modulus of rupture (described by numerical cumulative probability density function) are determined. Deflection uncertainty of reinforced concrete (RC) beams, propagated from uncertainties in concrete properties, is quantified using Monte Carlo (MC) simulation. Cracked plane frame analysis is used to account for tension stiffening in concrete. Concrete homogenization enables a unique opportunity to bridge the gap between concrete materials and structural modeling, which is necessary for realistic serviceability prediction.

Keywords: uncertainty; concrete; deflection; homogenization; RVE; Monte Carlo method.

1. Introduction

Prediction of deflection in reinforced concrete (RC) structures is a critical requirement for satisfactory performance of concrete members. Many researches (Jokinen and Scanlon 1985, Gardner 1990, Ghali *et al*. 2000) have discussed the difficulty to improve accuracy of models that estimate deflection of concrete structures. Branson's approach (1977) has been accepted as the basis for calculating a reduced concrete stiffness of a cracked concrete section to estimate immediate deflection and long-term deflection. In spite of the critique to the approach for its incapability to capture concrete deflection with accuracy (Gilbert 1999, Ghali and Azarnejad 1999, Reda Taha and Hassanain 2003), both the American Code ACI 318-08 (2008) and the Canadian Code CSA A23.3-M04 (2004) utilize this approach. Many researchers have also shown and recommended the use of

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the mean curvature method to yield more accurate deflection predictions. The mean curvature method was adopted by the well-known CEB-FIP Model Code 90 (MC-90) (1993).

Since deflection of RC beams is affected by concrete properties, by directly affecting the structural stiffness of the element and by indirectly defining the moment redistribution due to cracking, it is important to incorporate uncertainty of concrete properties in deflection calculations for robust prediction of RC deflection. Researchers have suggested the need to consider uncertainty in modeling concrete properties related to deflection prediction. Zundelevich *et al*. (1974) applied the principles of error propagation to predict the variation in the final deflection of prestressed concrete elements using separate measures of the elastic and the long-term deflections at different loading stages. The study showed that a measured coefficient of variation (COV) of 12% in time-dependent deflection resulted in COV of 10% in the total deflection. Thompson and Scanlon (1988) noted that code deflection computations would only provide an estimate of the mean deflection. The probability to exceed this mean deflection would be about 50% if normal distribution of slab deflection was assumed. Fling (1992) showed that concrete properties incorporated in deflection calculation (i.e., modulus of elasticity, modulus of rupture, and time-dependent parameters) have wide scatter in their values. Scanlon and Pinheiro (1992) compared the current deterministic approach to deflection control to a probabilistic approach for design for safety. They suggested that the best practical probability limit can be generated using a measure of the associated damage to serviceability due to excessive deflections. Reda Taha and Hassanain (2002) showed that a 20% uncertainty of cracking strength can result in 30% uncertainty in predicted short term deflection. Choi *et al*. (2004) applied Monte Carlo simulation to evaluate the variability of deflections due to uncertainty in material properties and dimensions involved in the computation of deflections. The researchers showed that a wide variation range of deflection exists as a result of the variability in design parameters, applied moment/cracking moment ratio, reinforcement ratio and live load/dead load ratio. Kim and Reda Taha (2009) examined the propagation of random uncertainties in deflection of a continuous one-way RC slab from uncertainties in modulus of elasticity and modulus of rupture and showed that robustness to uncertainty can be used as a measure to examine the significance of modeling parameters.

The properties of concrete, including compressive strength, modulus of elasticity and modulus of rupture, have a relatively large variation as compared with other structural materials due to the inherent uncertainty in concrete microstructure. As a composite material, concrete characteristics might be determined by numerical simulation using composite homogenization techniques. Composite homogenization is based on realization of the interaction of the microstructure constituents to predict composite properties. A number of homogenization techniques to model particulate composites have been developed (Ju and Chen 1994, Torquato 2002, Jain and Ghosh 2008, Qui and Li 2009, Vorel and Šejnoha 2009, Kim and Muliana 2010, Milani and Benasciutti 2010, Kalali and Kabir 2010). The homogenization process is developed by selecting a representative volume element (RVE) of the composite microstructure to simulate the mechanical characteristics of the composite. The RVE satisfies the scale separation principle (Bear and Bachmat 1990) that the characteristic length of the heterogeneity is less than the characteristic size of the RVE. The basic premise of modeling the RVE is to utilize a robust averaging technique to determine apparent and intrinsic properties of the composite materials (Dormieux *et al*. 2006). It is proposed that this RVE would rather be considered a statistical volume element (SVE) due to stochastic aspect of RVE (Ostoja-Starzewskib 2006).

In this study, uncertainty in deflection of RC beams, which are propagated from uncertainties of concrete characteristic properties, is quantified using Monte Carlo (MC) simulation. The variations of concrete properties including concrete modulus of elasticity and modulus of rupture are predicted from a homogenization model considering randomly dispersed aggregates in concrete with uncertain mechanical characteristics of the microstructural phases of concrete. A concrete RVE is developed using the finite element (FE) method. The proposed RVE recognizes concrete as a three phase composite material incorporating cement paste, aggregate and interfacial transition zone (ITZ). It is important to note that the RVE used in this study is limited to extract the macro properties of concrete, which are modulus of elasticity and tensile strength. The deflection calculation is predicted using cracked plane frame analysis. As a result, a variation in deflections on RC beams is quantified by probabilistic analysis with the random concrete properties extracted from RVE. Moreover, it can be realized that concrete homogenization approach can be a robust method to bridge the gap between concrete materials and structural modeling for realistic serviceability prediction.

2. Methods

A two-dimensional (2D) RVE for concrete is developed to determine the modulus of elasticity and

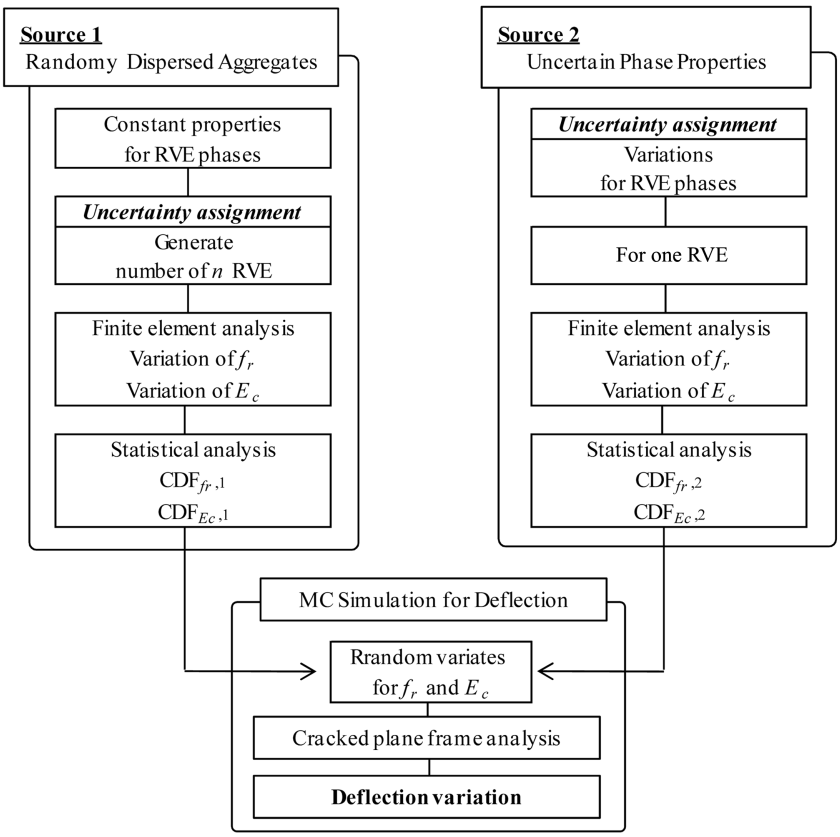


Fig. 1 Schematic representation of the proposed process for quantification of deflection uncertainty in RC beam due to microstructural variability

the modulus of rupture of homogenized concrete. Random uncertainties of concrete characteristics including modulus of elasticity and modulus of rupture are obtained using FE modeling of the RVE. Two sources of random uncertainty were considered to generate the random uncertainty in the RVE. First, uncertainty was generated by considering random generation of RVE. This random generation represents the randomly dispersed aggregates in concrete. The second source of uncertainty was generated by considering the uncertain mechanical properties of the RVE phases including the cement paste, the aggregates and the ITZ. The uncertainty in the modulus of elasticity and modulus of rupture is represented in the form of cumulative probability density function (CDF). MC simulation is performed for deflection calculation of a RC beam by cracked plane frame (CPF) analysis. A schematic representation of this procedure is shown in Fig. 1.

2.1 Concrete homogenization

Concrete was successfully modeled using this approach where aggregate was assumed randomly dispersed in the cement paste (Kurukuri 2005). Here, we follow this assumption in performing concrete homogenization. At that length scale, concrete is considered a three phase material: cement paste, aggregate (coarse and fine) and ITZ. Selection of the three phases was based on their unique microstructural features and intrinsic properties (Mehta and Monterio 2006). The RVE characteristics include the volume fraction of cement paste and aggregate, size distribution of the aggregate and the constitutive models of the cement paste and the aggregate. A 2D finite FE model of the RVE is developed to predict the constitutive relationship and the fundamental characteristics of concrete. For the selection of RVE size, it was proposed that RVE-based homogenization is analogous to specimen testing in lab to get the material properties (Hashin 1983). Kurukuri (2005) studied the effect of the dimension ratio of RVE matrix and inclusion on the prediction of macro-scale properties of concrete and it was shown that the predicted modulus of elasticity of a concrete has a variation of 2% for the range of the dimension ratio between 2.5 and 17.5. Many researchers (Le Pape 2009, Wu 2010) reported successful use of RVE for predicting macro-scale properties of concrete as summarized in Table 1. Therefore, it is necessary to consider the dimensional ratios recommended for the RVE validity while considering the accepted accuracy for the application in hand and the associated computational cost. In this study, considering the largest aggregate size of 12 mm, RVE size is selected as 100 mm × 100 mm which gives the dimension ratio 8.3. The 100 mm × 100 mm RVE is a FE model including 250,000 elements developed under ANSYS® FE

Table 1 The dimension ratio of RVE to aggregates in this study compared with those used by others

References	Smallest RVE size, *a*	Largest inclusion size, *d*	ratio, *a*/*d*
This study	100 mm	12 mm	8.3
Hashin (1983)*	100 to 150 mm	10-25 mm	8 to 15
Kurukuri (2005)**	60 to 210 mm	12 mm	5 to 17.5
Le Pape (2009)	150 mm	10 mm	15
Wu *et al*. (2010)	100 mm	15 mm	6.7

*Based on the discussion (Hashin 1983): “RVE-based homogenization is analogous to specimen testing in lab to get the material properties.”
**There are no units for the sizes of *a* and *d* in original study.

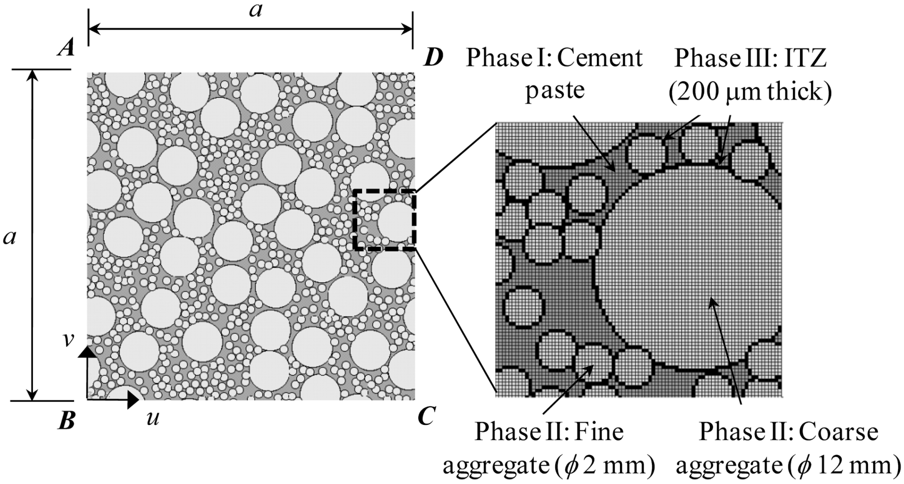


Fig. 2 Finite element (FE) model representing the three phase RVE model

environment as shown in Fig. 2. It is notable that the reason to use 2D RVE for concrete instead of three-dimensional (3D) RVE is to reduce computation time with the numerous elements for modeling the relatively small phase ITZ of concrete rather than the other aggregates of concrete. The RVE elements were modeled using PLANE42 (SAS 2003), which is a plane element with 4 nodes, each having two degrees of freedom (DOF); translations in the x and y directions at each node. Elements are randomly assigned within each microstructural phase. This allows for dealing with the RVE as an isotropic material. To satisfy the spatial periodic boundary conditions of the RVE, which means that the RVE deforms identical to its neighbors (Smit *et al*. 1998), the following boundary conditions are applied to the RVE as

$$u_{CD} - u_{AB} = \delta, \quad v_{AD} - v_{BC} = 0, \quad u_B = v_B = 0 \tag{1}$$

where u and v are the horizontal and the vertical displacements at the boundaries of the RVE as shown in Fig. 2 respectively. δ is the applied displacement at the edge CD of the RVE. Considering plane stress condition of the RVE, that is the normal stress to plane and two out-of-plane stresses are zero, the constitutive model of the RVE defined as

$$\begin{bmatrix} f_u \\ f_v \\ f_{uv} \end{bmatrix} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \dfrac{1-\upsilon}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \gamma_{uv} \end{bmatrix} \tag{2}$$

where f_u, f_v, ε_u and ε_v are the normal stresses and strains of u and v directions. f_{uv} and γ_{uv} are the shear stress and strain in plane. E and υ are the modulus of elasticity and Poisson's ratio of the homogenized concrete. The average normal stresses $f_{u,ave}$ and $f_{v,ave}$ by unit axial load (u-direction) in the RVE are determined using the FE analysis. The axial strains ε_u and ε_v are calculated as one over a (RVE size in Fig. 2) and zero, respectively, with the boundary conditions in Eq. (1). Therefore, E and υ of the homogenized concrete are determined as

$$E = \frac{f_{u,ave}(1-\upsilon^2)}{a}\delta \tag{3}$$

$$\upsilon = \frac{f_{u,ave}}{f_{v,ave}} \tag{4}$$

To get the constitutive models of the RVE for both tension and compression, the displacement is applied to the RVE until the average strain of the ITZ phase approaches the maximum elastic strain of tension and compression respectively.

Linear elasticity bounds are used here to avoid the challenge in computing the strain energy function tensors when microstructural homogenization of nonlinear phases is considered (Khisaeva *et al*. 2006). Such complexity might be necessary to consider for realistic modeling of heterogeneous microstructures as concrete if accurate stress states are required. Since our intention is to estimate the probabilistic uncertainty of concrete characteristics, the use of linear homogenization techniques shall provide results with reasonable accuracy.

2.2 Monte Carlo simulation

Monte Carlo (MC) simulation has been proven efficient by many researchers for modeling uncertainty in complex systems (Hall and Strutt 2003, Çavdar *et al*. 2008, 2009, Muzzammil 2008, Beer and Spanos 2009, Pellissetti 2009, Schuëller 2009, Kim *et al.* 2010, Milani and Benasciutti 2010, An *et al.* 2011) and for calibration of structural concrete design codes (Nowak and Szerzen 2003). Here MC simulation is used for numerical evaluation of RC deflection uncertainty propagated from uncertainties in concrete properties. The uncertainties in concrete properties are determined from two independent sources. Therefore, two probability density functions (PDF) for a concrete property will be obtained from random generation of a RVE and from assignment of variations in mechanical properties of RVE phases as schematically presented in Fig. 1. To generate random variates for a concrete property considering the two sources of uncertainty, two cumulative probability density functions (CDF) of the corresponding PDF are used. Considering σ-algebraic of variations, the i-th random variate of modulus of rupture f_r^i is generated as

$$f_r^i = \Gamma_{fr,1}^{-1}(u_1^i) + \Gamma_{fr,2}^{-1}(u_2^i) - \left(\frac{c_{fr,1}+c_{fr,2}}{2}\right) \tag{5}$$

where $\Gamma_{fr,1}^{-1}$ and $\Gamma_{fr,2}^{-1}$ are the inverse of two CDF, which are numerically generated from FE analysis of RVE. u_1^i and u_2^i are the i-th standard uniform variates (generated uniformly between 0 to 1) for the corresponding CDF generated independently. If one standard uniform variate is used for Eq. (5), the resulting variation will be the algebraic summation of two variations, not σ-algebraic of variations. $c_{fr,1}$ and $c_{fr,1}$ are the mean values of the corresponding CDF. The procedure is repeated for the modulus between each of elasticity. The full positive correlation between f_r and E_c is considered to account for the relationship between each of those variables and concrete compressive strength (ACI 318-08 2008, CSA A23.3-M04 2004). This can be done by sharing the same set of standard uniform variates for generating random variates of f_r and E_c (Kim and Reda Taha 2009). The i-th deflection of a RC beam Δ^i is then computed as

$$\Delta^i = F(E_c^i, f_r^i, C) \tag{6}$$

where F is a RC deflection calculation method, which is based here on cracked plane frame analysis (Ghali and Favre 2002). C represents constant parameters for deflection calculations. By generating a large number of random variates for each variable, the probability distribution of RC deflection can be estimated.

2.3 Cracked plane frame analysis

Cracked plane frame analysis is used here to model deflection of RC beams. This method incorporates axial deformation and curvature to calculate deflection (Ghali and Favre 2002). The transformed section properties, the cross sectional area A, first and second moment S and I for a section, are calculated with respect to a reference axis (arbitrary axis in a section). The axial strain ε and curvature ψ with respect to the reference axis are then calculated as

$$\varepsilon = \frac{IN-SM}{E(AI-S^2)} \tag{7}$$

$$\psi = \frac{-SN+AM}{E(AI-S^2)} \tag{8}$$

where N and M denote normal force and bending moment respectively. E is modulus of elasticity of the transformed section, steel being that of concrete. The stiffness matrix of the cracked plane frame analysis accounting for tension stiffening is evaluated using the mean curvature method. The mean axial strain and curvature can be expressed as

$$\psi_m = (1-\lambda)\psi_{uc} + \lambda\psi_{cr} \tag{9}$$

$$\varepsilon_m = (1-\lambda)\varepsilon_{uc} + \eta\varepsilon_{cr} \tag{10}$$

where subscript uc and cr denote uncracked and fully cracked sections respectively. λ is a dimensionless coefficient, which is determined by considering the extent of cracking. For a local element as shown in Fig. 3, flexibility matrix F is evaluated using virtual work as

$$F_{ij} = \int_0^l N_{ui}\varepsilon_{m,j}dx + \int_0^l M_{ui}\psi_{m,j}dx \tag{11}$$

where l is element length. N_{ui}, and M_{ui} are the normal force and the bending moment when a unit force acts to the i-th degree of freedom (DOF). $\varepsilon_{m,j}$ and $\psi_{m,j}$ are the mean axial strain and the mean curvature evaluated at the sections along the element (shown in Fig. 3 by broken lines) when a unit

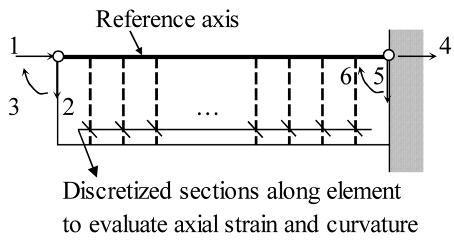


Fig. 3 Description of the degree of freedom (DOF) to generate cracked plane frame element

force acts to the j-th degree of freedom (DOF). The stiffness matrix is then determined by inverse of the flexibility matrix. Full details of CPF analysis can be found elsewhere (Ghali and Favre 2002).

3. Case study

For a case study, the reinforced concrete beam for deflection test by Christiansen (1988) was considered. The beam had 7.5 meters span, 280 mm thick and 170 mm wide. The reinforced steel areas for compression and tension are 452 mm^2 respectively. The bottom concrete cover to the reinforcement is 31 mm. The beam is subjected to a distributed load of 1.143 kN/m and two concentrated load of 2.27 kN. For this beam, the test result of the instantaneous deflection at the mid-span was measured as 14.2 mm. The concrete compressive strength f_c' of 26 MPa was reported for the tested RC beam. The beam is shown schematically in Fig. 4.

To generate the RVE model for this concrete, a concrete mix with an average compressive strength of 26 MPa after Kim (2009) was considered. This concrete mix is presented in Table 2. Two aggregate particle sizes of 2 mm and 12 mm considered to simulate fine and coarse aggregate particles (Phase II) respectively. The thickness of ITZ (Phase III) is modeled as 200 mm. The volume fractions of fine and coarse aggregates are determined by considering their specific gravity as 2.7 and 2.65 respectively. The volume fraction of ITZ is calculated by assuming that ITZ is surrounding all aggregate particles with thickness of 200 mm. The volume fraction of ITZ is then approximated as 10%. The remaining volume fraction of concrete 21% is considered as cement paste (Phase I).

The elastic constitutive model in tension and compression were used for cement paste (Phase I) and aggregates (Phase II). The tensile strength and stiffness of Phase III is then determined through iterative process such that the RVE of concrete has an elastic tensile strength of $0.62\sqrt{f_c'}$ MPa, while the compressive strength and stiffness of Phase III is determined through iterative process such that the RVE of concrete has a compression modulus of elasticity of $4730\sqrt{f_c'}$ MPa. The two selections for the modulus of rupture and modulus of elasticity are therefore in agreement with most

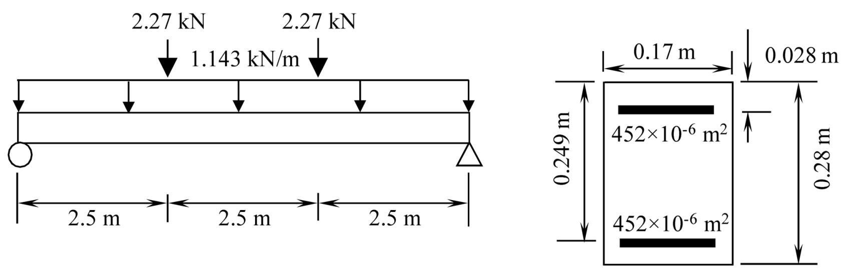


Fig. 4 The beam setup of the deflection test by Christiansen (1988) for a case study

Table 2 Mixture ingredients of concrete (kg/m^3)

Cement	Silica fume	Water	Fine aggregate	Coarse aggregate
236	79	192	700	1143

Table 3 Volume fractions and mechanical properties of RVE phases

Phase	Phase I	Phase II		Phase III
Materials	Cement paste	Fine aggregate	Coarse aggregate	ITZ
Volume fraction	21%	25%	44%	10%
f_c'	30 MPa	70 MPa		10.5 MPa
$^*\varepsilon_c$	0.01	0.001		0.00015
f_r	30 MPa	70 MPa		1 MPa
$^{**}\varepsilon_t$	0.01	0.001		0.001

*ε_c: the maximum compressive elastic strain.
**ε_t: the maximum tensile elastic strain.

design codes (ACI 318-08 2008, CSA A23.3-M04 2004). The volume fractions and the mechanical properties of each phase of RVE are presented in Table 3.

4. Results and discussion

Finite element (FE) analysis of the RVE is conducted until the average strain of Phase III (ITZ) approaches its peak elastic strain. Applying tension and compression displacement to the RVE, the constitutive model shown in Fig. 5 is extracted. The modulus of rupture of the homogenized concrete is then determined from the tension (-) part of the constitutive model, while the modulus of elasticity of the homogenized concrete is determined from the compression (+) part of constitutive model. The strain distributions due to tensile stress in the RVE are shown in Fig. 6. By marking the elements of the RVE whose strains are over the maximum elastic tensile strain of ITZ 0.001 as black color, the initiating of micro cracks at ITZ can be observed in Fig. 6(a) at the RVE strain of 0.00008 when the ITZ cracking is initiated (Point I in Fig. 5). The micro crack distribution at failure of the RVE is also shown in Fig. 6(b) at the RVE strain of 0.00027 when the RVE is assumed to fail (Point II in Fig. 5). This might be extended to simulate more realistic crack propagation using a homogenization model that considers a cohesive contact zone between the

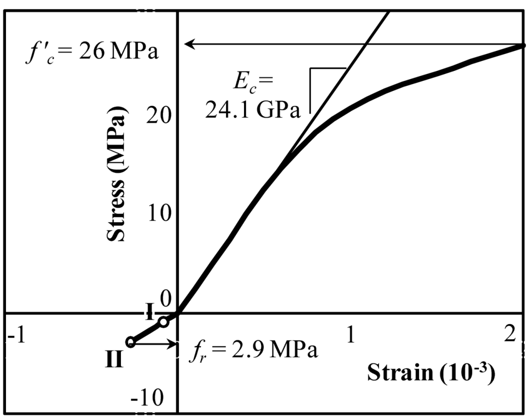
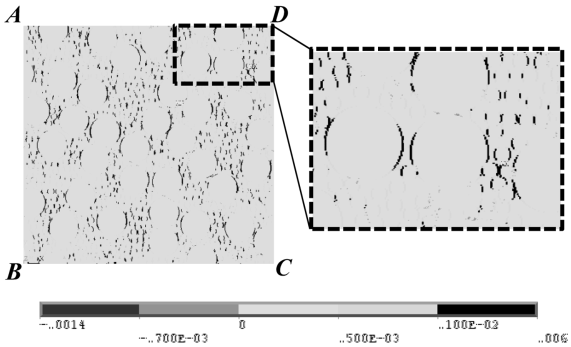
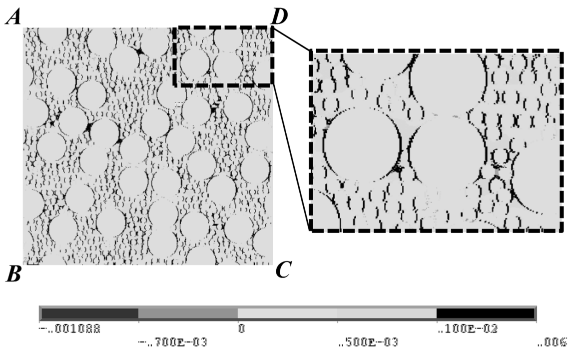


Fig. 5 Constitutive model generated using the RVE



(a) the RVE strain of 0.00008 when the ITZ cracking is initiated (Point I in Fig. 5)



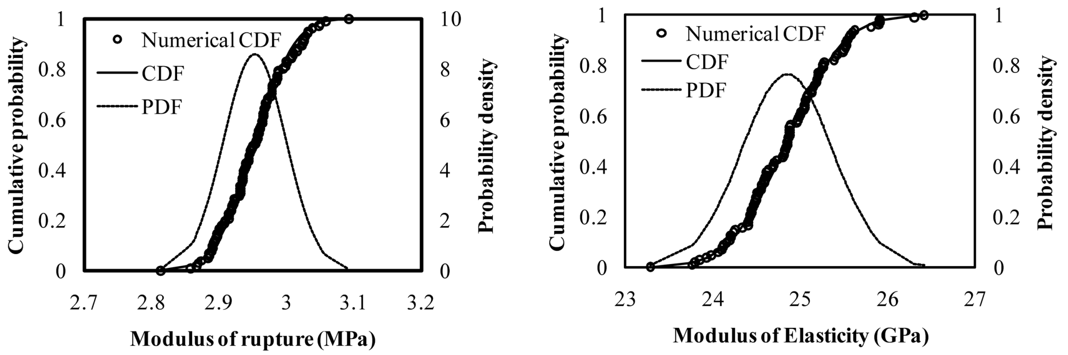
(b) the RVE strain of 0.00027 when the RVE is assumed to fail (Point II in Fig. 5)

Fig. 6 Strain distribution due to tensile stress showing the cracking status of the ITZ

aggregate particles and the cement paste. This extended approach is beyond the scope of this paper.

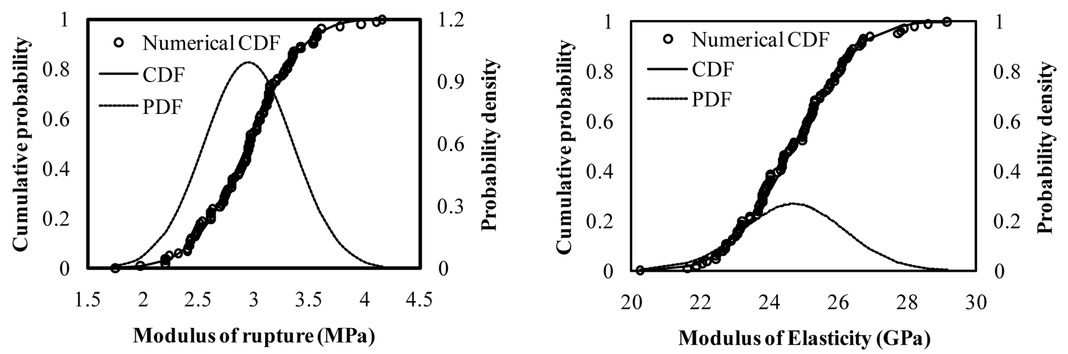
To quantify the variations of modulus of rupture and modulus of elasticity, the uncertainty associated with the randomly dispersed aggregates is considered first. One hundred RVE is generated randomly for the given volume fractions of microstructural phases and geometries. Using the 100 results of modulus of rupture and modulus of elasticity, the CDF of modulus of rupture (average 2.95 MPa with standard deviation 0.05 MPa) and modulus of elasticity (average 24.9 GPa with standard deviation 0.5 GPa) are evaluated as shown in Figs. 7(a) and (b) respectively.

Second, the uncertainty in the modulus of rupture and modulus of elasticity propagated from considering uncertain microstructural phase characteristics is considered. A single RVE, with the median of f_r and E_c, is selected among the 100 RVE generated at the first step. The modulus of elasticity of the three microstructural phases are assumed as normally distributed. The coefficient of variation (COV) of modulus of elasticity for aggregates, cement paste and ITZ are assumed as 5%, 10% and 20%, respectively, for case study. Probabilistic analysis is conducted for the homogenization analysis to identify the uncertainty of the modulus of rupture and modulus of elasticity of the RVE respectively. For each case, 100 FE analysis iterations are conducted. Random numbers are generated by Latin hypercube sampling method. The numerical CDF of modulus of rupture (average 2.96 MPa with standard deviation 0.4 MPa) and modulus of elasticity (average



(a) $CDF_{fr,1}$ for modulus of rupture (b) $CDF_{Ec,1}$ for modulus of elasticity

Fig. 7 CDF for (a) modulus of rupture and (b) modulus of elasticity propagated from the uncertainty source 1, randomly dispersed aggregates



(a) $CDF_{fr,2}$ for modulus of rupture (b) $CDF_{Ec,2}$ for modulus of elasticity

Fig. 8 CDF for (a) modulus of rupture and (b) modulus of elasticity propagated from the uncertainty source 2, uncertain concrete phase properties

24.7 GPa with standard deviation 3 GPa) are then determined as shown in Figs. 8(a) and (b) respectively.

Using the two CDFs for each f_r and E_c from two sources of uncertainty, the uncertainty of deflection is quantified using Monte Carlo simulation. For random sampling for f_r and E_c, 5000 random variates for each f_r and E_c were generated by Eq. (5). The deflections were calculated using the cracked plane frame analysis after Ghali and Favre (2002). The resulted deflection histogram and the relative frequency density are shown in Fig. 9 with the average of 14.6 mm and COV of 10%. The resulted COV might be considered as relatively lower than practical deflection variations. This might be attributed to the fact that the uncertainty quantification was dependent on two sources of uncertainty only. Other sources of uncertainty might include uncertainty in ITZ thickness, aggregate size distribution, quality of cementitious materials, etc. Nevertheless, the above method and case study provide a framework for quantifying uncertainty in deflection of RC members by considering sources of inherent randomness in concrete micro and macrostructure. This quantified uncertainty might be integrated with other uncertainties in loading and structural geometry to

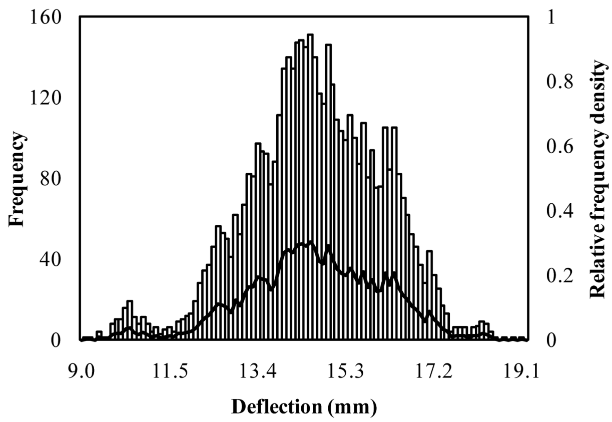


Fig. 9 Frequency histogram and relative frequency density for deflection of a case study

provide a rational estimate for uncertainty in deflection of RC members. Furthermore, using the probabilistic distribution of deflection predicted in Fig. 9, the probability of exceeding deflection threshold can be quantified by reliability analysis. This might enable probabilistic design of serviceability limit states considering realistic assumptions of the materials behaviour under service conditions. Finally, introducing homogenization techniques for modeling reinforced concrete elements shall bring forward the ability to incorporate concrete deterioration mechanisms on the structural behaviour of RC members.

5. Conclusions

A concrete homogenization approach is developed to examine the significance of random distribution and uncertain mechanical characteristics of concrete composite phases on the uncertainty of deflection of reinforced concrete beams. The inherent randomness of aggregate dispersion in concrete matrix and the uncertainty in different phase characteristics are considered. This enabled computing the uncertainty in concrete characteristics namely modulus of rupture and modulus of elasticity. The uncertainties in concrete characteristics are used to compute uncertain deflection probability distributions using Monte Carlo simulation. It is shown that the proposed approach is capable of quantifying uncertainty in RC deflection by considering linear homogenization of concrete. Further research is warranted to consider the significance on concrete heterogeneity on the proposed approach for uncertainty quantification using means of nonlinear homogenization.

Acknowledgements

The financial support by Army Research Office Grant # W911NF-08-1-0421 to study topological optimization and homogenization of composites is greatly appreciated. The support to the second and third author by Defense Threat Reduction Agency (DTRA) Grant for uncertainty quantification is greatly appreciated.

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