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Dynamics of a bridge beam under a stream of moving elements. Part 2 – Numerical simulations

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Abstract. The paper constitutes the second part of the author's study. The first part (Podworna 2010) formulates four fundamental tasks in dynamics of the bridge-track-train systems. The following cyclic moving loads are considered: a concentrated forces stream (model P), an unsprung masses stream (model M), a single-mass viscoelastic oscillators stream (model M_o) and a double-mass viscoelastic oscillators stream (model MM_o). Three problems precluding to the numerical simulations have been developed, i.e., prediction of the forced resonances, the parameters of integration of equations of motion, the output results. A computer programme was written in Pascal and numerical research in the scope of the fundamental tasks was worked out. The investigations were focused on adequacy evaluation of the moving load models, P, M, M_o, MM_o, in predicting dynamic processes in railway bridges.

Keywords: bridge-track-train system; fundamental tasks; bridge beam; moving elements streams; one-way contact; numerical simulations.

1. Introduction

In the first part of this study (Podworna 2010) the author developed a new concept of the fundamental tasks in dynamics of the bridge-track-train systems (BTT), with special attention on evaluating moving load's models adequacy. The 2D physical models of BTT systems, corresponding to the fundamental tasks, have been worked out taking into account one-way constraints between the moving unsprung masses and the track. A method for deriving the implicit equations of motion, governing vibrations of BTT systems' models, as well as algorithms for numerical integration of these equations, leading to the solutions of high accuracy and relatively short times of simulations, have been developed as well.

Recent references concerning the moving load problem (Cojocaru 2004, Garinei 2006, Yau 2007, Yau 2008, Bilello 2008, Fryba 2009, Muscolino 2009, Yau 2009, Wu 2010, De Salvo 2010, Liu 2011) have been discussed in the first part of this study (Podworna 2010). These references show that simplified analytic and analytic – numerical methods in dynamics of structures under moving loads are still developed. The writers adopt simplified models of vehicles or trains in the form of streams of forces, mass particles or oscillators without verification of adequacy of these models to reality.

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In this study the following cyclic moving loads are considered (Podworna 2010): a concentrated forces stream (task 1 – model P), an unsprung concentrated masses stream (task 2 – model M), a single-mass viscoelastic oscillators stream (task 3 – model M_o) and a double-mass viscoelastic oscillators stream (task 4 – model MM_o). The numerical simulations are precluded to prediction of the forced resonances, description of the parameters of integration of equations of motion, and description of the output results. A computer program was developed in Pascal and the numerical research in the scope of the fundamental tasks was developed. The investigations are focused on the adequacy evaluation of the moving load models, P, M, M_o, MM_o, in predicting dynamic processes in railway bridges. Numerical simulations were performed in the base of dimensionless parameters corresponding to the selected steel railway bridge of 15.00 m span length loaded by a Shinkansen high-speed train.

2. Shortened description of fundamental tasks

The fundamental tasks are as follows (Podworna 2010):

1) transient and quasi-steady-state vibrations of an Euler-Bernoulli beam under a cyclic stream of concentrated forces (model P),
2) transient and quasi-steady-state vibrations of an Euler-Bernoulli beam under a cyclic stream of concentrated unsprung masses (model M),
3) transient and quasi-steady-state vibrations of an Euler-Bernoulli beam under a cyclic stream of single-mass viscoelastic oscillators (model M_o),
4) transient and quasi-steady-state vibrations of an Euler-Bernoulli beam under a cyclic stream of a double-mass viscoelastic oscillators (model MM_o).

Assumptions in physical modelling of above noted systems, matrix equations of motion and numerical integration algorithms have been developed in the first part of this study (Podworna 2010). Among the others it was assumed that:

1) the beam – moving load system is physically and geometrically linear in respective time subintervals,
2) an Euler-Bernoulli beam is prismatic, inertial, deformable in flexure and made of viscoelastic material,
3) the load moves along the track at a constant horizontal velocity.

The considered systems are described by the following dimensional and dimensionless parameters:

l - a span length of the bridge beam [m],
m - beam mass per unit length [kg/m],
E - a Young's modulus for the beam material [Pa],
I_b - an inertia moment of the beam cross-section with respect to the horizontal central axis [m^4],
EI_b - bending stiffness of the beam [$N{\cdot}m^2$],
γ - a damping ratio for the beam,
v - moving load's horizontal velocity (service velocity) [m/s],
P - a concentrated force in model P [N],
M - a concentared unsprung mass in models M, MM_o [kg],
M_o - a concentared sprung mass in models M, MM_o [kg],
k_o - suspension stiffness for mass M_o [N/m],
c_o - a suspension damping coefficient for mass M_o [N·s/m],

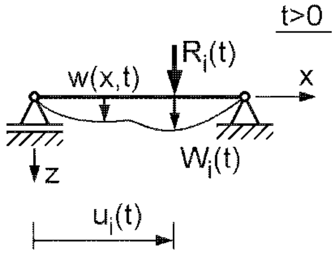

Fig. 1 The beam dynamic deflection, $w(x, t)$, and the interaction position, $R_i(t)$

k_M - contact stiffness,

b_1, b_2 - distances between concentrated moving elements representing the axial distance of trucks (b_1) and length of a repeatable vehicle ($b_1 + b_2$).

Vertical deflection of the beam is approximated with a sine series meeting Ritz's conditions, i.e. (Fig. 1)

$$w(x,t) = \mathbf{q}^T(t)\mathbf{s}(x) = \mathbf{s}^T(x)\mathbf{q}(t) \tag{1}$$

where

$$\mathbf{q}(t) = [q_1(t), q_2(t), \ldots, q_n(t)]^T$$

$$\mathbf{s}(x) = [\sin\pi\xi, \sin 2\pi\xi, \ldots, \sin n\pi\xi]^T, \quad \xi = \frac{x}{l} \tag{2}$$

while

x - an abscissa in the xz planar coordinate system,

t - a time variable,

$\mathbf{q}(t)$ - a vector of Lagrange's generalized coordinates for the beam,

$\mathbf{s}(x)$ - an approximate functions vector (a sine series).

The following values will occur in the description of the output results

$$\xi = \frac{x}{l}; \quad \tau = \frac{vt}{l}, \quad R_i(\tau); \quad i = 1, 2, \ldots, N; \quad k_M = 8 \cdot 10^8 \text{ N/m} \tag{3}$$

where ξ is a relative abscissa, $R_i(\tau)$ is an interaction between the ith moving element and the track, k_M is contact stiffness. Variable τ is dimensionless and determines the relative position of the first interaction in relation to the beam left support. The use of the τ variable makes possible to put on time histories of a given quantity for different service velocities as well as to put on the dynamic curve on a quasi-static one.

3. Prediction of the forced resonances

An Euler-Bernoulli beam representing a railway bridge and a moving elements stream representing a train consisted of repeatable rail-vehicles are geometrically bounded objects. In the case of a moving forces cyclic stream, forced resonances resulting from periodicity of the moving load in relation to the beam may occur.

The fundamental eigenperiod of a simply supported beam equals $T_1 = 1/f_1$, where (Langer 1980)

$$f_1 = \frac{\pi}{2l^2}\sqrt{\frac{EI_b}{m}} \tag{4}$$

is the fundamental natural frequency of the beam. A set of eigenperiods in decreasing order is expressed by the well-known formula

$$T_j = \frac{T_1}{j^2} = \frac{1}{j^2 f_1}, \quad j = 1, 2, \ldots, n \tag{5}$$

The fundamental period of the beam vibration excitation, resulting from a moving forces cyclic stream, amounts to $T = (b_1 + b_2)/v$. Periods of subsequent harmonic components of Fourier's series for the periodic beam excitation equal

$$\mathbf{T}_i = \frac{T}{i} = \frac{b_1 + b_2}{iv}, \quad i = 1, 2, \ldots \tag{6}$$

After considering Eqs. (5), (6), the $T_i = T_j$ forced resonance occurs at the resonance service velocity

$$v = v_{ij} = 3,6\frac{j^2}{i}(b_1 + b_2)f_1 \ \text{[km/h]} \tag{7}$$

Beam vibrations in all the tasks are transient processes that tend to steady-state processes. In the case of model P a classical task occurs in the form of forced vibrations of a periodically linear discrete system. In other cases the additional factors extending transient process duration occur, i.e.:

a) model M:
- forced and parametric excitation,
- the possibility of separation and recontact of moving masses M with the track (impact loads),

b) model M_o:
- forced and parametric excitation,
- time-varying interactions tending to the steady state simultaneously with the beam,

c) model MM_o:
- forced and parametric excitation (slighter than in model M),
- the possibility of separation and recontact of moving oscillators MM_o with the track (impact loads slighter than those in model M),
- transient vibrations of masses M_o tending to the steady state simultaneously with the beam.

4. Parameters for numerical integration of equations of motion

The dynamic process including the passage of the moving elements stream through the beam and the interval of beam free damped vibrations is simulated numerically. The simulations have been performed for the passage of the train consisted of 5 repeatable vehicles. The real time of the dynamic process amounts to

$$T_p = \frac{l + 6(b_1 + b_2)}{v} \tag{8}$$

In the dynamic response of the B-S system (beam – moving elements stream system) the oscillations with the highest frequency $\hat{f}$ occur, among the others. The Newmark average acceleration method $(\beta_N = 1/4, \gamma_N = 1/2)$ (Newmark 1959) applied in the algorithms is unconditionally stable and not affected with an amplitude error. A time step is given from the accuracy condition put on the period error for oscillations with frequency $\hat{f}$. A time step is assumed as equal to

$$h = 10^{-2}\hat{T} = \frac{0.01}{\hat{f}} \tag{9}$$

where $\hat{T}$ is a period of oscillations of frequency $\hat{f}$. This value of the time step protects technical accuracy of the numerical results. The highest frequency $\hat{f}$ depends on the considered system and can be evaluated from the set of the local frequencies of insulated subsystems, i.e.

1) the beam eigenspectrum dominant:

$$f_n = n^2 f_1 \tag{10}$$

2) the local frequency of oscillator M, k_M:

$$f_M = \frac{1}{2\pi}\sqrt{\frac{k_M}{M}} \tag{11}$$

3) the local frequency of oscillator M_o, k_o:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_o}{M_o}} \tag{12}$$

4) frequency of 10^{th} harmonic component of periodic excitation (the influence of higher components is negligible)

$$f_s = 10\frac{v}{b_1 + b_2} \tag{13}$$

Evaluation of the highest frequency $\hat{f}$ has the form:

1) model P: $$\hat{f} = \max[f_n, f_s] \tag{14}$$

2) model M: $$\hat{f} = \max[f_n, f_M, f_s] \tag{15}$$

3) model M_o: $$\hat{f} = \max[f_n, f_o, f_s] \tag{16}$$

4) model MM_o: $$\hat{f} = \max[f_n, f_M, f_o, f_s] \tag{17}$$

Number of time steps of value h amounts to $N_p = T_P/h$. The extreme values of output quantities are searched in all time steps. Parameter ε in the iteration ending condition is assumed as equal to $\varepsilon = 10^{-9}P$. The allowed number of the iterations is assumed to be equal 15.

5. Output quantities

Output quantities are selected as follows:

1) time histories of beam deflections in the selected cross-sections,

2) time histories of normal stresses in the bottom fibres of the beam in the selected cross-sections,
3) time histories of the interactions between concentrated moving elements M, M_o, MM_o and the track,
4) dynamic coefficients of the beam deflections,
5) dynamic coefficients of the normal stresses in the beam,
6) loading and unloading coefficients of concentrated moving elements M, M_o, MM_o.

The quasi-static time histories of the output quantities constitute the background for the dynamic time histories. The quasi-static solution is the same for all moving elements streams and described with the matrix equation

$$\{\mathbf{K}\}\mathbf{q}_{qs}(\tau) = P\mathbf{S}(\tau)\mathbf{1} \tag{18}$$

Solving Eq. (18), one obtains

$$\mathbf{q}_{qs}(\tau) = P\{\mathbf{K}\}^{-1}\mathbf{S}(\tau)\mathbf{1} = \frac{2Pl^3}{EI_b}\{\mathbf{d}\}^{-4}\mathbf{S}(\tau)\mathbf{1} \tag{19}$$

In Eqs. (18), (19) the following quantities occur (Podworna 2010): $\{\mathbf{K}\}$ – a beam stiffness matrix, $\mathbf{S}(\tau)$ – a matrix following moving elements' positions, $\mathbf{1}$ – a unit vector, $\{\mathbf{d}\}$ – a differentiating diagonal. Symbol $\mathbf{q}_{qs}(\tau)$ denotes generalized coordinates for the beam in the quasi-static conditions. Interaction forces in the quasi-static conditions are equal to static pressures of moving elements which amounts to *P*.

There are examined two characteristic cross-sections of the beam of abscissas $x = 0{,}50l;0{,}75l$ that result in $\xi = 0{,}50;0{,}75$. Dynamic and quasi-static time histories of the beam deflections in the examined cross-sections are equal to (*vide* Eqs. (1), (2))

$$\begin{aligned} w(\xi l, \tau) &= \mathbf{q}^T(\tau)\mathbf{s}(\xi) \\ w_{qs}(\xi l, \tau) &= \mathbf{q}_{qs}^T(\tau)\mathbf{s}(\xi) \end{aligned} \tag{20}$$

A dynamic bending moment in the cross-section at the beam abscissa *x* amounts to

$$M(x,t) = -EI_b\frac{\partial^2 w}{\partial x^2} = -EI_b\mathbf{q}^T\frac{\partial^2 \mathbf{s}}{\partial x^2} = \frac{EI_b}{l^2}\mathbf{q}^T\{\mathbf{d}\}^2\mathbf{s} \tag{21}$$

A bending index of a beam cross-section in relation to the bottom fibres equals $W_b = I_b/h_b$, where h_b is the distance of the bottom fibres from the horizontal central axis, *y*, of the cross-section. Normal stresses in the bottom fibres amount to

$$\sigma(x,t) = \frac{M(x,t)}{w_b} = \frac{h_b EI_b}{I_b\, l^2}\mathbf{q}^T\{\mathbf{d}\}^2\mathbf{s} = \frac{Eh_b}{l^2}\mathbf{q}^T\{\mathbf{d}\}^2\mathbf{s} \tag{22}$$

Based on Eq. (22) the dynamic and quasi-static time histories of normal stresses in the beam bottom fibres are described by the formulae

$$\begin{aligned} \sigma(\xi l, \tau) &= \frac{Eh_b}{l^2}\mathbf{q}^T(\tau)\{\mathbf{d}\}^2\mathbf{s}(\xi) \\ \sigma_{qs}(\xi l, \tau) &= \frac{Eh_b}{l^2}\mathbf{q}_{qs}^T(\tau)\{\mathbf{d}\}^2\mathbf{s}(\xi) \end{aligned} \tag{23}$$

Dynamic coefficients for displacements and normal stresses in the investigated beam cross-sections are defined as follows

$$\varphi_w(\xi l) = \frac{\max_\tau w(\xi, \tau)}{\max_\tau w_{qs}(\xi, \tau)}$$

$$\varphi_\sigma(\xi l) = \frac{\max_\tau \sigma(\xi, \tau)}{\max_\tau \sigma_{qs}(\xi, \tau)} \tag{24}$$

Dynamic coefficients for deflections and stresses are related to transient processes corresponding to the passage of the train composed of 5 repeatable rail-vehicles. Increasing number of vehicles may change values of these coefficients.

Dynamic time histories of interactions $R_i(\tau)$, $i = 1, 2, \ldots, N$ are calculated in the collocation points for models M, M_o, MM_o. Quasi-static time histories of the interactions for the analysed models are constant in time, i.e.

$$R_{i,qs}(\tau) = P \tag{25}$$

The loading and unloading coefficients of concentrated moving elements M, M_o, MM_o are defined as follows

$$\varphi_i = \frac{1}{P}\max_\tau R_i(\tau), \; i = 1, 2, \ldots, N$$

$$\rho_i = \frac{1}{P}\min_\tau R_i(\tau), \; i = 1, 2, \ldots, N \tag{26}$$

The loading and unloading coefficients for streams M, M_o, MM_o are defined by the formulae

$$\max \varphi_R = \max_i \varphi_i, \quad i = 1, 2, \ldots, N$$

$$\min \rho_R = \min_i \varphi_i, \quad i = 1, 2, \ldots, N \tag{27}$$

6. Dynamic analysis of the fundamental tasks

6.1 Values of system parameters and numerical integration parameters

The fundamental tasks reflect a selected railway bridge loaded with a selected high-speed train. An Euler-Bernoulli beam represents a simply supported bridge span corresponding to a single track. The bridge superstructure is made of structural steel. The beam parameters' values correspond approximately to the SB15 bridge belonging to some type-of-series (Klasztorny 2005) and are equal to

$$l = 15.00 \text{ m}, \quad m = 5000 \text{ kg}, \quad h_b = 0.80 \text{ m}$$

$$E = 206 \cdot 10^9 \text{ Pa}, \; f_1 = 7.00 \text{ Hz}, \; \gamma = 0.004$$

Inertia moment I_b is calculated from Eq. (4).

A moving elements stream approximately reflects a Shinkansen train; each vehicle is $b_1 + b_2 =$ 25.00 m long and has the truck axial distance $b_1 = 17.50$ m. The body mass equals 40 000 kg, two two-axle truck frames mass is equal to 10 000 kg, and four wheel sets mass amounts to 10 000 kg. The local natural frequency for the vehicle body amounts to $f_o = 1.00$ Hz, and local damping ratio of the suspension equals $\gamma_o = 0.10$. The maximum service velocity equals 280 km/h (Klasztorny 2005).

The four-axle rail-vehicle has been modelled with two moving elements MM_o, which parameters take the following values (Klasztorny 2005):

$$M_o = 25000 \text{ kg}, \ M = 5000 \text{ kg}, \ f_o = 1.00 \text{ Hz}, \ \gamma_o = 0.10$$

$$k_o = M_o(2\pi f_o)^2 = 987000 \text{ N/m}, \ c_o = 2\gamma_o\sqrt{k_o M_o} = 31400 \text{ Ns/m}$$

Parameters describing the simplified models P, M, M_o take the following values:

a) model P: $P = 294300$ N

b) model M: $M = 30000$ kg

c) model M_o: $M_o = 30000$ kg, $f_o = 1.00$ Hz, $\gamma_o = 0.10$

$$k_o = M_o(2\pi f_o)^2 = 1185000 \text{ N/m}, \ c_o = 2\gamma_o\sqrt{k_o M_o} = 37700 \text{ Ns/m}$$

Common parameters for models P, M, M_o, MM_o have the values:

$$P = 294300 \text{ N}, \quad k_M = 8 \cdot 10^8 \text{ N/m}, \ b_1 + b_2 = 25.00 \text{ m}, \ b_1 = 17.50 \text{ m}, \ N = 10$$

Numerical simulations have been performed for service velocity interval $v \in [80;360]$ [km/h]. In this interval the condition described by Eq. (9) if satisfied for time step $h = 10^{-5}$ s. Remaining numerical parameters have been assumed to be equal to

$$n = 10, \ \varepsilon = 0.001 \text{ N}, \ L_{it} = 15$$

where L_{it} is an admissible number of iterations for models M, M_o, MM_o.

6.2 Prediction of the resonance velocities

The resonance velocities of concentrated forces cyclic stream P were calculated from Eq. (7), where index i is a number of the beam excitation harmonic component, and j is a number of the beam eigenfrequency. Predicted values of the resonance velocities, v_{ij}, are set up in Table 1. For the considered system there are possible resonant service speeds corresponding to the beam fundamental modal system and excitation harmonic components $i = 2, 3, 4, \ldots$. Contributions of subsequent harmonic components are checked via numerical simulations of transient vibrations of the system.

Table 1 Values of resonant service speeds subset for moving load P

j	v_{ij} [km/h]						
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$
1	630	315	210	157,5	126	105	90
2	2520	1260	840	630	504	420	360

6.3 Numerical simulations

Dynamic analysis presented in this study is focused to evaluation of the adequacy of moving loads models P, M, M_o, MM_o to reality. The simulations have been performed for systems approximately reflecting the selected railway bridge and the selected high-speed train. The influence of basic factors of cyclic moving loads to dynamic responses of the system in undertaken, i.e. the influence of unsprung masses of moving elements, the influence of sprung masses of moving elements, the influence of viscoelastic suspensions of moving elements.

Based on the analytical description of fundamental tasks as well as on algorithms for numerical integration of explicit/implicit equations of motions (Podworna 2010), the author has developed a computer programme in Pascal for numerical simulation of dynamic and quasi-static processes of the considered systems. The results in the scope of dynamic coefficients for deflections and normal stresses in the selected beam cross-sections as well as the loading and unloading coefficients for streams M, M_o, MM_o are presented in Figs. 2-7.

In order to illustrate qualitative and quantitative differences in dynamic responses of the examined systems, related to models P, M, M_o, MM_o, resonance velocity $v_{31} = 210$ km/h has been chosen. The results are shown in the form of dynamic and quasi-static time histories of deflections and normal stresses at the $x = 0.50l$ cross-section (Figs. 8, 9) as well as in the form of time histories of the 8^{th} interaction (the back truck of the 4^{th} vehicle, Fig. 10).

Based on Figs. 2-5 the following conclusions can be formulated:

1) For model P one can observe forced resonances corresponding to resonant service speeds $v_{21}, v_{31}, v_{41}, v_{61}, v_{71}$. Maximum dynamic coeeficients are related to speeds v_{21}, v_{31}.
2) The resonant service speeds for model P are also valid for model M_o, while the dynamic coefficients take lower values for model M_o because of stabilizing influence of sprung masses (inertia) as well as of suspension damping influence.
3) For model MM_o one may observe a shift of the resonances to the left what results from decreasing the system eigenfrequencies (the parametric excitation effect).
4) Dynamic coefficients' diagrams for model M substantially differ, both qualitatively and quantitatively, from the diagrams corresponding to remaining models. Thus, model M is inadequate.

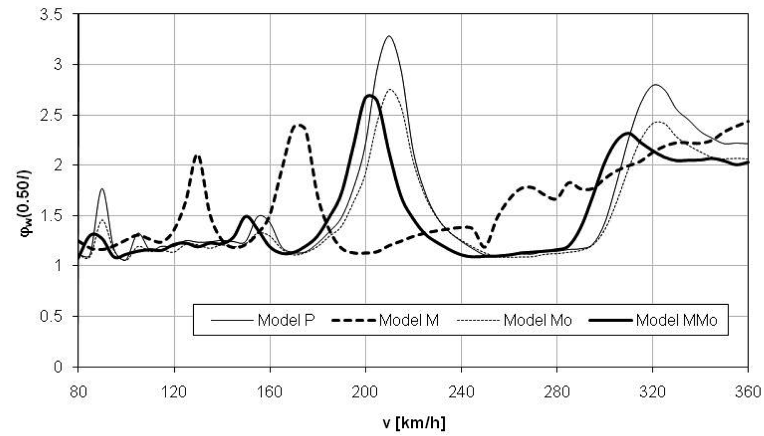

Fig. 2 The impact factor for deflection $w(0.50l, t)$

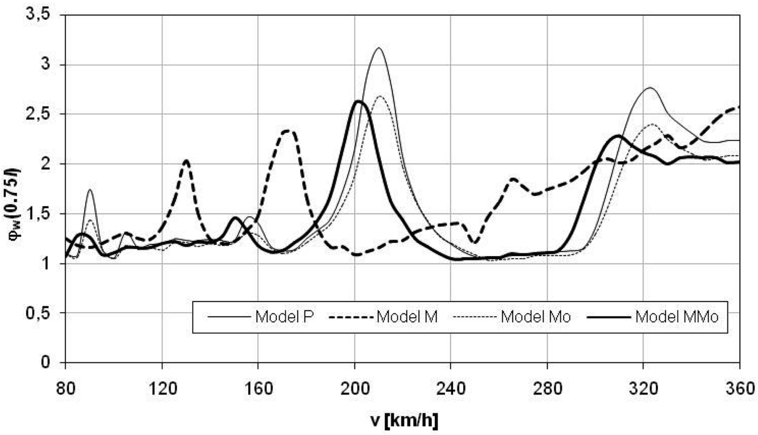

Fig. 3 The impact factor for deflection $w(0.75l, t)$

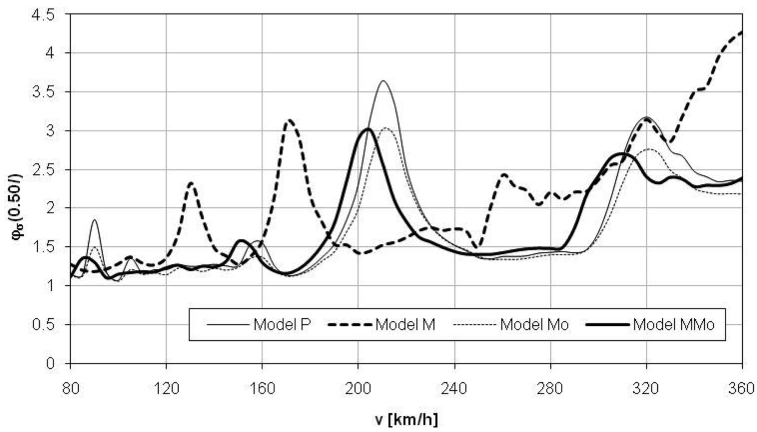

Fig. 4 The impact factor for normal stress $\sigma(0.50l, t)$

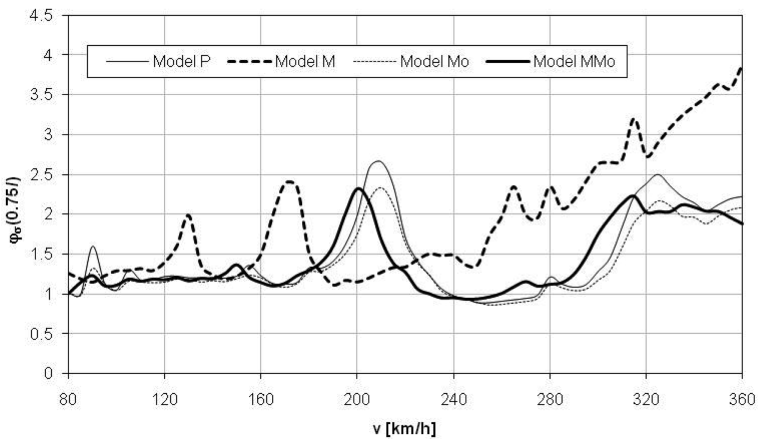

Fig. 5 The impact factor for normal stress $\sigma(0.75l, t)$

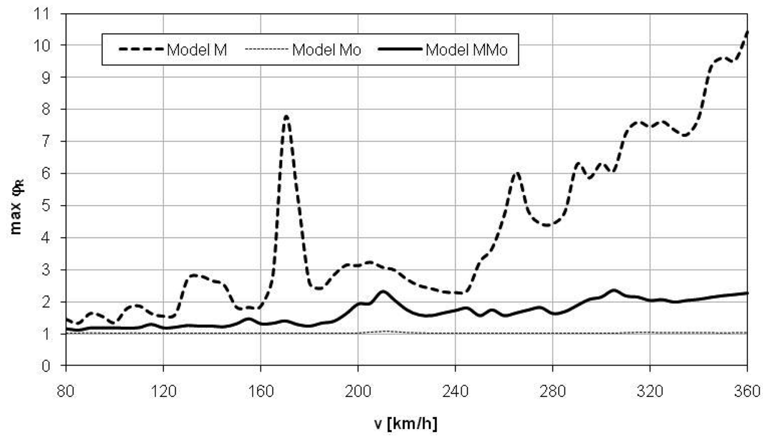

Fig. 6 The maximum dynamic interaction for the finite cyclic streams

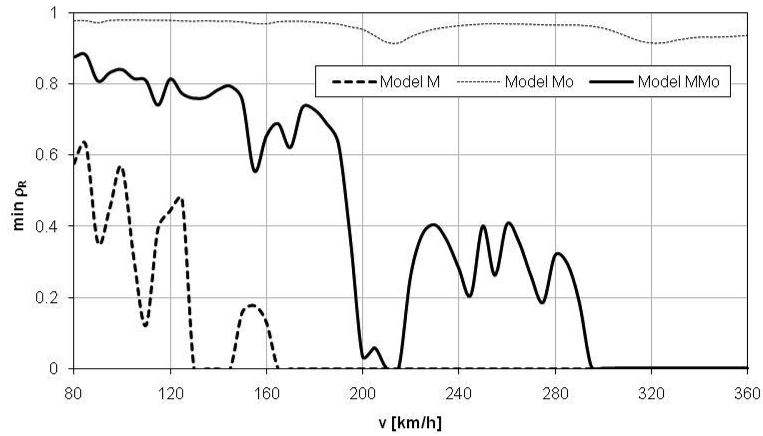

Fig. 7 The minimum dynamic interaction for the finite cyclic streams

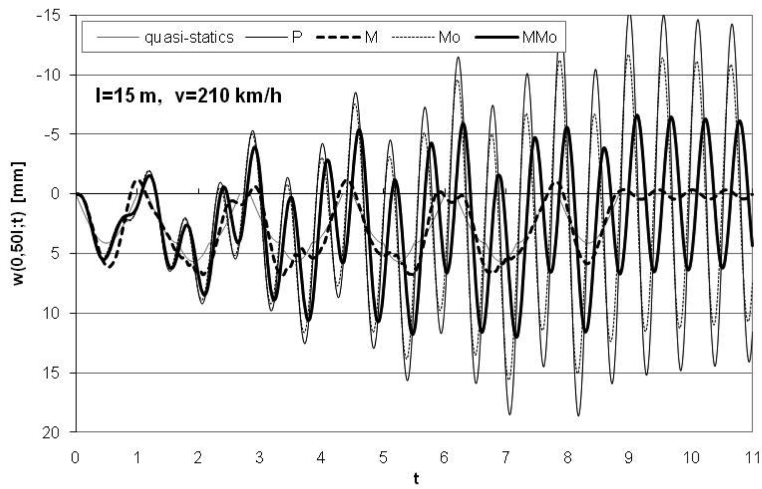

Fig. 8 Deflection $w(0.50l,\ t)$ versus relative time τ for service velocity $v = 210$ km/h

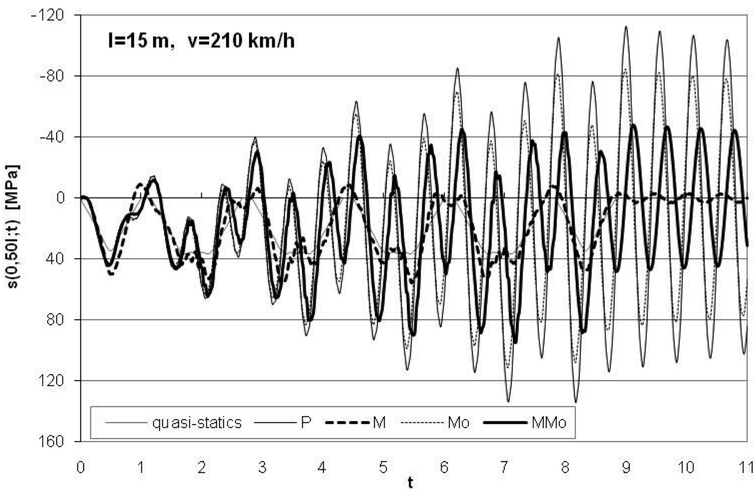

Fig. 9 Normal stress $\sigma(0.50l, t)$ versus relative time t for service velocity $v = 210$ km/h

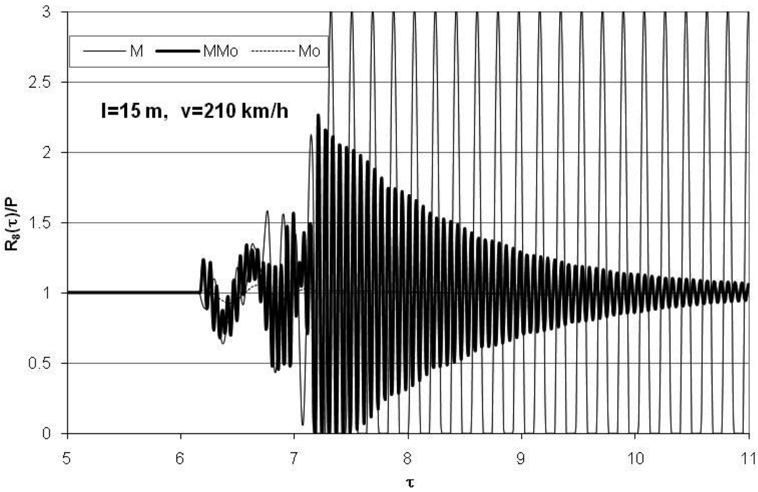

Fig. 10 Relative interaction $R_8(\tau)/P$ versus relative time τ

5) The resonant states for models M, M_o, MM_o may led to beam dynamic stresses exceeding even three times quasi-static stresses.

From diagrams presented in Figs. 6, 7 one can conclude that:

6) Loading coefficients for model M reach values >>5 for $v = 170$ km/h and for $v > 285$ km/h. Unsprung masses in model M separate from the track at $v > 130$ km/h, what is inadequate to reality.
7) Loading coefficients for model MM_o take realistic values. Unloading coefficients are safe for $v < 190$ km/h. In the intervals for $v = 210 \div 215$ km/h and $v > 295$ km/h there occur microseparations of double-mass oscillators from the track.
8) Loading and unloading coefficients for model M_o slightly differ from the values for model P what is inadequate to reality.

Figs. 8-10 give the following conclusions:

9) For model P, at $v = 210$ km/h time histories of deflections and normal stresses show the forced resonance of the 3rd harmonic excitation component to the 1st beam modal system. Increase of number of vehicles would lead to further increase of beam dynamic response.

10) For model M, at $v = 210$ km/h time histories of deflections and normal stresses do not exhibit any resonant state.

11) For model M_o, at speed $v = 210$ km/h time histories of deflections and normal stresses also show the forced resonance of the 3rd harmonic excitation component to the 1st beam modal system but with lower dynamic effects (see the explanation in conclusion 2).

12) For model MM_o, at $v = 210$ km/h one can observe going out from the forced resonance of the 3rd harmonic excitation component to the 1st beam modal system (see the explanation in conclusion 3).

13) Time histories for interaction $R_8(\tau)$ differ significantly from each other in relation to models M, M_o, MM_o. Model MM_o is closest to reality. For this model, one may observe microseparation of the oscillator from the track during crossing the beam right support and free damped vibrations of the oscillator after this crossing. Model M results in inrealistic leaping along after passing by the bridge.

7. Conclusions

The study presents the results of dynamic analysis in the range of the fundamental tasks in dynamics of railway bridges under high-speed trains. The simulations have been performed with the use of the author's computer programme. The dynamic analysis has been carried out for the system modelling approximately the steel bridge of 15.00 m span length and a Sinkansen train composed of 5 rail-vehicles.

The following final conclusions are formulated:

1) Modelling high-speed rail-vehicles requires taking into account unsprung masses, sprung masses and viscoelastic suspensions. Model MM_o having these features is rather too simplified. It is suggested to reflect wheel sets modelled separately, sprung truck frames, sprung vehicle body and viscoelastic suspensions of the 1st and 2nd stage.

2) In the case of repeatable rail-vehicles there may occur resonant states, thus a total number of vehicles as well as damping factors should be taken into account in dynamic simulations.

3) Model M exhibits large differences, both qualitative and quantitative, in comparison to model MM_o, so model M is assessed as inadequate to reality.

4) Model P is close to reality but in resonant states one may observe some shifts from the resonances and too increased values of dynamic coefficients when compared to model MM_o.

5) Model M_o gives the results also close to reality but in the resonant states some shifts from the resonances and slight increase of values of dynamic coefficients may occur when compared to model MM_o.

6) Model MM_o is the most adequate to reality in the scope of simplified train models undertaken.

7) One-way constraints between wheel sets and rails should be taken into consideration in modelling high-speed rail-vehicles. Derailment risk should be analyzed for high velocities of trains.

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