A new method for optimal selection of sensor location on a high-rise building using simplified finite element model

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Abstract. Deciding on an optimal sensor placement (OSP) is a common problem encountered in many engineering applications and is also a critical issue in the construction and implementation of an effective structural health monitoring (SHM) system. The present study focuses with techniques for selecting optimal sensor locations in a sensor network designed to monitor the health condition of Dalian World Trade Building which is the tallest in the northeast of China. Since the number of degree-of-freedom (DOF) of the building structure is too large, multi-modes should be selected to describe the dynamic behavior of a structural system with sufficient accuracy to allow its health state to be determined effectively. However, it's difficult to accurately distinguish the translational and rotational modes for the flexible structures with closely spaced modes by the modal participation mass ratios. In this paper, a new method of the OSP that computing the mode shape matrix in the weak axis of structure by the simplified multi-DOF system was presented based on the equivalent rigidity parameter identification method. The initial sensor assignment was obtained by the QR-factorization of the structural mode shape matrix. Taking the maximum off-diagonal element of the modal assurance criterion (MAC) matrix as a target function, one more sensor was added each time until the maximum off-diagonal element of the MAC reaches the threshold. Considering the economic factors, the final plan of sensor placement was determined. The numerical example demonstrated the feasibility and effectiveness of the proposed scheme.

Keywords: optimal sensor placement (OSP); finite element method (FEM); modal assurance criterion (MAC); series of multidegree-of freedom; high-rise building.

1. Introduction

Modern tall buildings are constructed with high-strength and lighter-weight materials tending to be more flexible and lightly damped than those in the past. These structures are sensitive to the effects of strong wind, earthquake, wheeled track etc. For example, when structural drifts exceed certain permissible levels, non-structural elements (partitions, cladding systems) may be damaged and

mechanical services (elevators) may not operate properly. The collapse and failure of these deficient structures even cause increasing concern with structural integrity, durability and reliability. To monitor and diagnose the performance of a structure, find the damage of the structure in time, predict the calamity that may appear and assess the security of the structure have become an inevitable requirement in civil engineering (Majumder *et al.* 2008).

Structural health monitoring (SHM) refers to the use of in-situ, continuous or regular measurement and analyses of key structural and environmental parameters under operating conditions, for the purpose of warning impending abnormal states or accidents at an early stage to avoid casualties as well as giving maintenance and rehabilitation advice (Housner et al. 1997). Remarkable progress has been made in the SHM over the last decade. Successful laboratory demonstrations have led to deployments of the integrated SHM systems in a variety of applications from civil infrastructure to transportation systems to defense-related assets (Kister et al. 2007, Efstathiades et al. 2007, Takeda et al. 2007, Kim et al. 2008). In spite of this progress, there are still challenges to address in the SHM. Deciding on an optimal sensor placement (OSP) is a critical issue in the construction and implementation of an effective SHM system. An optimal configuration can minimize the number of sensors required, enhance the accuracy and provide a robust system. Thus, sensors must be judiciously placed in order to provide adequate information for the identification of the structural behavior and reduce the overall cost of system. Previous work addressing the issue of optimally locating a given number of sensors in a structure has been carried out by several investigators. A comprehensive survey of sensors placement strategies for aerospace applications can be found in Kammer (2005), for the OSP problem in the process industry in Naimimohasses et al. (1995), for the safe operation of nuclear reactors in Oh and No (1994), and for the damage identification and the SHM can be found in Worden and Burrows (2001). Mossberg (2001) performed a theoretical study on optimal sensor configurations for parametric identification, and the results indicate a substantial increase in accuracy by the use of optimal sensor locations. Liu and Tasker (1996) proposed a perturbation-based approach to predict the optimal locations for sensors. Heredia-Zavoni et al. (1999) treat the case of large model uncertainties expected in model updating. The optimal sensor configuration is chosen as the one that minimizes the expected Bayesian loss function involving the trace of the inverse of the Fisher information matrix (FIM) for each model.

It is well known that if the number of degree-of-freedom (DOF) of a structure is too large, the multi-modes should be selected to describe its dynamic behavior of the structural system with sufficient accuracy to allow its health state to be determined effectively (Kister et al. 2007). On the other hand, due to placing the sensors only in the translational direction of the structure during the on-site test, it's necessary to reduce the DOF number that couldn't be placed (e.g., rotational direction etc.) in the analytical model. For the differentiation between translational and rotational modes, generally it can be accurately selected by the modal participation mass ratios. However it's difficult to do that for the flexible structures with closely spaced modes. For this, a new method computing the mode shape matrix of weak axis by the simplified series MDOF system is presented based on the equivalent rigidity parameter identification method. It may avoid the difficulty of accurately choosing the modes by the method. The paper is organized as follows: first, the identification method of equivalent stiffness coefficients is briefly presented followed by the introduction of the initial sensor assignment by the QR-factorization. Then the procedure using the sequential sensor placement (SSP) algorithm to solve the OSP problem is outlined. Next, the effectiveness of the proposed scheme is demonstrated via a numerical simulation study for Dalian World Trade Building. Finally, a few concluding remarks are given.

2. OSP based on simplified finite element model

2.1 Identification method of equivalent stiffness coefficients

The mass matrix of the structure is generally integrated by the FE model and the Rayleigh' damping matrix is usually used as damping matrix. Thus, the difficult problem of simplifying the model is how to identify the stiffness matrix of the structure accurately. Countless identification methods of stiffness coefficients have been developed; but to some extent these methods would be limited (Hiroyuki 2002). For example, the well known D-value method can only be used for low-to-mid-rise shear type buildings, while the shear-bending type storey model can only be applied to frame structures of 10 > H/B > 3, where H and B are respectively the height and width of the structure. Here, a kind of method called as the parameter identification method is used in which the influences of various factors in the original structure are considered and all assumptions are abandoned in order to make the simplified system more accurately coincided with the original structure (Fig. 1) (Sun *et al.* 1992).

In general, the sensors are deployed along the weak axis of the building. The dynamic equation of the FE model can be expressed as

$$[K_0]\{U_0\} = \{F_0\} \tag{1}$$

The expression of the equation of motion for a lumped mass system can be written as follows

$$[K]{U} = {F}$$
(2)

Where, $[K_0]$ stands for the stiffness matrix and $\{F_0\}$ is the load column vector which are known, $\{U_0\}$ denotes the unknown displacement column vector, [K] means the unknown stiffness matrix, $\{F\} = \{F_0\}$ and $\{U\}$ implies the mass center displacement column vector of the FE model.

Because the stiffness matrix [K] is symmetric, the number of the unknown numbers is M = n(n+1)/2, where n is the number of the structure layers. Thus, the groups of load vector, $\{F_0\}$, can give w Eq. (2) (w is the dimensional number of $\{F_0\}$), i.e., $w \cdot n$ equations, so w is at least equal to M/n = (n+1)/2. To obtain enough precision, the least square method is usually used



Fig. 1 Lumped mass representation of the tall building

to identify the rigid coefficients, where the integer of $w \ge (1.5 - 2)(n+1)/2$ is chosen. When solving this problem, it's necessary to know the unknown numbers of the stiffness matrix, which is

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \dots & k_{1n} \\ k_{22} & k_{23} & \dots & k_{2n} \\ & & k_{33} & \dots & k_{3n} \\ \text{symmetrical} & \ddots & \vdots \\ & & & & & k_{nn} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_n \\ & x_{n+1} & x_{n+2} & \dots & x_{2n-1} \\ & & x_2 & \dots & x_{3n-3} \\ \text{symmetrical} & \ddots & \vdots \\ & & & & & x_{n(n+1)/2} \end{bmatrix}$$
(3)

Thus, the unknown vector can be expressed in the following form

$$\{x\} = \{x_1 \ x_2 \ x_3 \ \dots \ x_{n(n+1)/2}\}^T$$
(4)

Eq. (2) can be modified as

$$[u] \times \{x\} = \{f\} \tag{5}$$

where, [u] is the $(n \cdot w) \times M$ matrix, which is extended by [U] and $\{f\}$ represents the $(n \cdot w) \times 1$ column vector, which is extended by $\{F\}$.

Then, Eq. (2) results in the following expression

$$[u]^{T}[u]\{x\} = [u]^{T}\{f\}$$
(6)

By the definition of $[u]^{T}[u] = [A]$ and $[u]^{T}{f} = {P}$, therefore

$$[A]\{x\} = \{P\}$$
(7)

where, [A] is the $M \times M$ symmetric matrix and $\{P\}$ denotes the $M \times 1$ column vector. Then, the stiffness coefficients K_{ij} can be obtained from Eq. (7).

2.2 Initial sensor assignments by QR-factorization

Based on the modal superposition method, the vector of the measured structural responses denoted by $\{u\}$ can be estimated as a combination of *s* mode shapes through the expression

$$\{u\} = \Phi_s\{q\} \tag{8}$$

where, Φ_s means the matrix of target mode shapes partitioned to the sensor locations and $\{q\}$ is the vector of target modal coordinates.

The vector $\{q\}$ can be computed using the least square (LS) method by solving Eq. (8), yielding

$$\{q\} = [\Phi_s^T \Phi_s]^{-1} \Phi_s^T \{u\}$$
(9)

The superscripts "-1" and "T" represent the inversion and transpose of a matrix, respectively.

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Considering the noise of sensor, a modification of the output of Eq. (8) is introduced as follows

$$\{u\} = \Phi_s\{q\} + \{v\}$$
(10)

where, the vector $\{v\}$ represents the stationary Gaussian white noise variance σ^2 .

To simplify the analytical procedure, it is assumed that the measurement noise be uncorrelated and possessed identical statistical properties of each sensor. For an efficient unbiased estimator, the covariance matrix of the estimated error is given by

$$[B] = E[(\{q\} - \{\overline{q}\})(\{q\} - \{\overline{q}\})^{T}] = \left[\frac{1}{\sigma^{2}}\Phi_{s}^{T}\Phi_{s}\right]^{-1} = [Q]^{-1}$$
(11)

In which, [Q] is the FIM, E denotes the expected value and \overline{q} means the efficient unbiased estimator of q.

To minimize [B], a suitable norm of [Q] must be maximized. Kammer (1991) suggested the spectral norm as a useful and physically meaningful matrix norm

$$\|Q\| = \left\|\Phi_s^T \Phi_s\right\| = \left\|\Phi_s^T\right\|^2 \tag{12}$$

Thereby, maximizing Φ_s^T would lead to the maximization of [B] and, thus the best state estimate $\{q\}$. It is known that the QR factorization (also called the QR decomposition) of a matrix is a decomposition of the matrix into an orthogonal matrix and a triangular matrix. Suppose the subset of candidate location corresponding to the obtained mode matrix from FE model be Φ , $\Phi \in \mathbb{R}^{n \times m}$, and generally m < n and $r(\Phi) = m$. By the QR factorization of the matrix Φ^T , the initial candidate set of sensor locations is obtained as follows

$$\Phi^{T} E = QR = Q \begin{bmatrix} R_{11} & \dots & R_{1m} & \dots & R_{1n} \\ \vdots & \dots & \ddots & \dots \\ 0 & R_{mm} & \dots & R_{mn} \end{bmatrix}$$
(13)

where, $Q \in \mathbb{R}^{m \times m}$, $R \in \mathbb{R}^{m \times n}$ and $E \in \mathbb{R}^{n \times n}$ are the permutation matrices and $|R_{11}| > |R_{22}| > \dots |R_{mm}|$.

2.3 Objective function

It is known from the structural dynamic principle that the structural inherent modes should comprise a group of orthogonal vectors at the nodes. But in fact, it's impossible to guarantee the measured modal vector are orthogonal, because of the problems of the measured freedoms less than those of model and measuring accuracy limitation. Further, it is even possible to lose many important modes owing to the too small space angles between vectors. The larger space angles among the measured modal vectors should be guaranteed while choosing measuring points in order to keep the original properties of the model if possible. Carne and Dohmann (1995) thought that the MAC is an ideal scalar constant relating the causal relationship between two modal vectors

$$MAC_{ij} = \frac{\left(\Phi_i^T \Phi_j\right)^2}{\left(\Phi_i^T \Phi_i\right)\left(\Phi_i^T \Phi_i\right)} = \frac{a_{ij}^2}{a_{ii}a_{jj}}$$
(14)

where, Φ_i and Φ_j represent the *i*th and *j*th column vectors in matrix Φ , respectively; a_{ij} is the

elements of $A = \Phi^T \Phi$. For an optimal (orthogonal) set this would be diagonal, so the size of the off-diagonal elements is an indication of fitness. In this formulation, the values of the MAC range between 0 and 1, where zero indicates that there is little or no correlation between the off-diagonal element MAC_{ij}($i \neq j$) and one indicates that there is a high degree of similarity between the modal vectors.

Hence, it is desirable to try to make the off-diagonal element in the MAC small when deploying the sensors. The objective function in each step of iteration could be given in Eq. (15)

$$f(k) = M_{axk} - M_{ax} \tag{15}$$

where, M_{ax} denotes the maximal value in MAC matrix, M_{axk} represents the maximum vale in k step iteration.

2.4 Sequential sensor placement (SSP) algorithm

For a structure that has simple geometry, or smaller number of DOF, experience and a trial-anderror approach may suffice to solve the problem. However, for a large-scale complicated structure with N_d DOFs, the number of all distinct sensor configurations involving N_0 sensors is

$$N_s = \frac{N_d!}{N_0!(N_d - N_0)!}$$
(16)

This can be an extremely large number for most cases of practical interest. Therefore, an exhaustive search over all sensor configurations for the computation of the optimal sensor configuration is largely with time consuming and prohibitive even for models with a relatively small number of DOFs in most cases. The computational issues arising in the search of the optimal sensor configuration have been addressed in the literature. A great deal of research has been conducted over the last decade on optimal sensors placement using a variety of placement techniques, such as the Genetic algorithms (GA), Worst-Out-Best-In (WOBI) algorithm, Exhaustive Single Point Substitution (ESPS) algorithm and Sequential sensor placement (SSP) algorithm (Papadimitriou *et al.* 2004).

The GA has been widely proposed as an effective alternative to the previous heuristic algorithm which is not guaranteed to give the optimal solution (Worden and Burrows 2001). Another systematic and computationally very efficient approach for obtaining a good sensor configuration for N_0 sensors is to use a SSP algorithm as follows. The computations involved in the SSP algorithm are an infinitesimal fraction of the ones involved in the exhaustive search method and can be done in realistic time, independently of the number of sensors and the number of model DOFs. The positions of N_0 sensors are computed sequentially by placing one sensor at a time in the structure at a position that results in the highest reduction in the maximum off-diagonal element of the MAC. Specifically, the position of the first sensor is chosen as the one that gives the highest reduction in the maximum off-diagonal element for one sensor. Given the optimal position of the first sensor, the position of the second sensor is chosen as one that gives the highest reduction in the maximum off-diagonal element computed for two sensors with the position of the first sensor fixed at the optimal one already computed in the first step. Continuing in a similar fashion, given the positions of (i-1) sensors in the structure computed in the previous (i-1) steps, the position of the next *i*th sensor is obtained as one that gives the highest reduction in the maximum off-diagonal element for i sensors with the positions of the first (i-1) sensors fixed at the optimal ones already

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obtained in the previous (i-1) steps. This procedure is continued for up to N_0 sensors. The SSP algorithm will give the optimal sensor configuration only in the case for which the optimal sensor positions for i sensors is a subset of the optimal sensor positions for (i-1) sensors for all i from one to N_0 . Although the iterative nature of the optimization process only seeks a suboptimal or near-optimal solution, the result is believed to be close to the optimal one. Compared to the GA algorithm, the SSP algorithms are preferred since they are found to maintain higher levels of accuracy with less computational effort than that involved in GAs (Qin *et al.* 2001, Papadimitriou *et al.* 2004). For the sake of reference, the aforementioned algorithm is termed the forward sequential sensor placement (FSSP) algorithm. The SSP algorithm can also be used in an inverse order, starting with N_d sensors placed at all DOF's of the structure and removing successively one sensor at a time from the position. This algorithm is termed as the backward sequential sensor placement (BSSP). Since the number of DOF of the structure is too large, from the computational point of view, the FSSP algorithm should be preferred than that involved in the BSSP.

The modal vector matrix comprised of the measured freedoms (The term candidate sensor location is also referred to as DOF measured by sensors) and the residual freedoms are expressed by $\Phi(n \times m)$ and $\hat{\Phi}(\hat{n} \times m)$, respectively, in which, *m* is the number of the measurable freedoms, *n* denotes the number of the measured freedoms and \hat{n} means the number of residual freedoms equal to the possibly measured freedoms minus the measured ones.

After adding the kth row of $\hat{\Phi}_{ki}$ and $\hat{\Phi}_{ki}$, the MAC matrix is become as follows

$$(MAC_{ij})_{k} = \frac{(a_{ij} + \hat{\Phi}_{ki}\hat{\Phi}_{kj})^{2}}{[a_{ii} + \hat{\Phi}_{ki}^{2}][a_{ii} + \hat{\Phi}_{ki}^{2}]}$$
(17)

where, a_{ij} is the elements of $A = \Phi^T \Phi$; $\hat{\Phi}_{ki}$ and $\hat{\Phi}_{kj}$ denote the modal vector values corresponding to node *i* and *j* at the *k*th freedom, respectively; Φ , Φ and MAC matrices should be modified each time when adding sensors to measuring group, in order to search the measurable point of $\hat{\Phi}$ which gives the highest reduction of the maximum off-diagonal elements of the MAC matrix at each computation until the maximum reaches the specified threshold value.

To sum up, the procedure of the FSSP method based on the simplified FE model is shown as follows:

(1) The simplified series MDOF system is obtained by the equivalent rigidity parameter identification method;

(2) The modal vector matrix is set up based on the simplified FE model and then the MAC matrix is ob The freedoms obtained by the QR factorization of the modal vector matrix are used as the initial candidate set of sensor locations, Then the maximum off-diagonal element M_{ax} of the MAC matrix is chosen;

(4) The residual freedoms are added one by one, e.g. adding the *k*th freedom to the measured freedoms and then computing the $(MAC)_k$ matrix of the modal vector and the maximum off-diagonal element M_{axk} ;

(5) Computing $f(k) = M_{axk} - M_{ax}$, add the freedom corresponding to the maximum of f(k) (f(k) < 0) to the sensor locations;

(6) Steps (4) and (5) should be repeated for all of the residual freedoms of the modal vector matrix until the maximum off-diagonal element reaches the specified threshold.

3. Application to Dalian World Trade building

To illustrate the proposed scheme for modal survey, the procedure for identifying an actual structure-Dalian World Trade building is as follows.

3.1 Description of building

The Dalian World Trade Building, comprising of office, commerce, finance and security parts, is a super high-rise structure. It has 4 stories under the ground level and 50 stories above. The main structure is about 201.9 m high from ground level. With the top tower, the total height is about 242 m and there is also an 8-story commercial building (local 9-story) 34.2 m in height around it. The commercial building and main office tower are linked as shown in Fig. 2 and to its south of the main tower. Up to now, the building has still been the tallest in the northeast of China. The structural system utilizes both steel and reinforced concrete (SRC), including core wall systems and perimeter steel frame coupled by outrigger trusses at two levels (the 30th and 45th floors). The plan of a standard floor is 37.4 m long by 38.3 m wide, and the floor-to-floor height is 3.8 m. The 15th, 30th and 45th floors are the refuge floors, with the height of 5.1 m (Li *et al.* 2007). The building is located near the sea, in an active area with a strong wind. Hence, monitoring the wind-induced vibration of the building is of particular importance to provide the important validation of the design procedures and an assurance of acceptable behavior of the high-rise structure.

3.2 Modal analysis

In order to provide input data for the OSP method a three-dimensional FE model of the building was built. The construction of a FE model, capable of accurately replicating the behavior of the real structure, was undertaken using the ETABS software. The analytical model was based on the structural drawings and other information, such as mass of core wall, column, core slab, office slab, curtain wall at each storey, which was provided by the architect. All the beams and columns are simulated using "Frame Elements" in the ETABS element library. The beam and column properties are input by defining the relevant cross-sectional shape from the pre-defined ETABS cross-section



Fig. 2 Vertical overview of the building

	1	1	e			
Mode	1	2	3	4	5	6
Period (s)	3.6856	3.4686	1.1875	0.8227	0.7717	0.5735
Frequency (Hz)	0.2713	0.2883	0.8421	1.2155	1.2958	1.7437

Table 1 Modal frequencies and periods calculated using FE model



library. The slab and core wall are simulated using the "Shell Element" having bending and membrane stiffness terms available in the ETABS library. The model is supported at the bottom using "Link Element". The FE model is built considering the bending and shearing deformation of the beam and column, and also the axial deformation of the column. The rigid floor assumption is used. To the 30th and 45th strengthened story, for the axial deformation of the column needs to be considered, the corresponding floors are computed as flexible floors. The model has 13,324 node elements, 90,062 frame elements and 22,967 shell elements, considering 31 section types and 8 materials' properties. The mesh representing the model has been studied and is sufficiently fine in the areas of interest to ensure that the developed forces can be accurately determined. Then, the modal analysis is carried out, and the frequencies and periods of the first 6 modes are listed in Table 1 and the mode shapes are shown in Fig. 3.

3.3 Simplification of the FE model

The mass matrix [M] is obtained by concentrating the total mass of each story on the floor of the FE model. The damping matrix of structure is expressed as the Rayleigh orthogonal damping formulation (Eq. (17)), and the damping ratio of the first two modes is chosen as 4%.

$$[C] = \alpha_1[M] + \alpha_2[K] \tag{17}$$

where, $\alpha_1 = 2\omega_1\omega_2(\xi_1\omega_2 - \xi_2\omega_1)/(\omega_2^2 - \omega_1^2)$ and $\alpha_2 = 2(\xi_2\omega_2 - \xi_1\omega_1)/(\omega_2^2 - \omega_1^2)$, in which ω_1, ω_2 and ξ_1, ξ_2 are the vibrating frequencies and damping ratios of the first and second modes, respectively.



Fig. 4 Diagonal elements of mass, stiffness and damping matrixes of the simplified model

Table 2 Comparison on relative errors of periods of two models in the weak axis

	FE Model	Simplified Model	Error (%)	
Mode 1	3.6856	3.7308	1.23%	
Mode 2	0.8227	0.8437	2.55%	
Mode 3	0.3403	0.3500	2.85%	



Fig. 5 Comparison of the first three modes of two models

Here, the diagonal elements of mass, stiffness and damping matrices are illustrated in Fig. 4. The natural frequencies of first two modes are obtained by the FE model of the original structure and simplified model and their relative errors to compare the accuracy of two models are calculated by

$$\frac{\left|T_{F}-T_{S}\right|}{T_{F}} \times 100\% \tag{18}$$

Where, T_F and T_S are the periods of the FE model and simplified model, respectively. The

results are listed in Table 2. The comparison between the FE and the simplified mode shapes for first three vibration modes is shown in Fig. 5. It can be seen from Table 2 and Fig. 5 that the errors of computational results of two models are quite small so that the simplified model can be applied to calculate the vibration responses of original structure.

3.4 Results of OSP

The OSP of the building is performed by the FSSP algorithm. Fig. 6 shows the MAC values of all of the 50 DOFs and Fig. 7 shows the MAC values of the initial placement (for the first 4 DOFs) determined by the QR factorization. As shown, the maximum off-diagonal element of the sensor placement determined by the QR factorization is close to the diagonal element and there are many higher off-diagonal elements, so the results are not perfect. Fig. 8 shows the variation curve of the maximum off-diagonal element for adding one more DOF to the initial placement. The curve shows



Fig. 6 MAC values of all of the 50 DOFs

Fig. 7 MAC values of the initial placement determined by the QR factorization



Fig. 8 Variation curve of the maximum off-diagonal element



Fig. 9 The optimal value of the MAC

Table 3 OSP on Dalian	World	Trade	building
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Sensor No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Story	50	46	18	8	28	34	11	15	43	22	5	39	4	23	7	33	37	9
Table 4 Maximum off-diagonal element of MAC of each kind of sensor placement																		
Sensor placement			All	All of the 50 DOFs				4 DOFs of QR factorization			31 DOFs obtained by SSP method			18	18 final selected DOFs			
Maximum off-diagonal element				0.1442				0.6872		0.0222				0.0458				

a decreasing trend at initial stage and then an increasing trend later, and the lowest point is the value for adding 27 DOFs, i.e., the optimal result when selecting 31 DOFs (see Fig. 9). However, it is not economic due to high cost of the data acquisition systems (sensors and their supporting instruments). As shown in Fig. 8, the maximum off-diagonal element varies gently with 18 sensors. Thus, 18 DOFs should be selected as the sensor locations and the MAC values are shown in Fig. 10. The 18 selected DOFs are listed in Table 3, where the first 4 DOFs are the results of the QR factorization. Table 4 gives the maximum off-diagonal elements of the MAC matrix for various sensor locations. As shown, after adding 14 measurement points, the maximum of off-diagonal element obtained by cumulative method is fastly reduced from 0.6872 to 0.0458. This indicates that the FSSP method is greatly effective.

4. Conclusions

The feature of the dynamic behavior of a real structure is possibly obtained only if the amount of the basic information is available. This, in turn, implies that a minimum number of sensors must be

placed on the structure under assessment. Thus, the sensors must be judiciously placed in order to provide adequate information for the identification of the structural behavior. However, modern buildings have become so complicated that with closely spaced modes, making it difficult to select important DOF based on modal participation mass ratios. The fundamental problem is how many and which DOF should be taken in the identification process. In solving this problem the due account has to be taken of economic factors, weight and physical constraints, etc., which may require a limited number of sensors being placed at accessible locations on the real structure. This paper focuses on the OSP of super high-rise buildings. A method of OSP based on the simplified models is proposed. With the case study, the following conclusions are drawn as:

(1) The calculation model of the super high-rise building is generally a bending-shear one, i.e. its rigidity matrix is full rank. The triple diagonal matrix simplified by traditional methods is not reasonable, while the identification method of equivalent rigidity coefficients is the better one.

(2) The maximum off-diagonal element of the MAC matrix can be rapidly reduced by the FSSP algorithm, which can obtain the MAC satisfying the related requirements in the condition of few sensors.

(3) The method in this paper avoids the problem in which it is difficult to choose the high order mode accurately based on the modal participation mass ratios. The numerical example demonstrated the feasibility and effectiveness of the proposed scheme. It's practical for large structures with more DOFs and coupled-space vibrations.

(4) As we know, it is unreasonable that a large number of sensors are uniformly placed, since this kind of method didn't consider any modal vector of the structure (Chung and Moore 1993). For example, it may deploy the sensor on the constrained nodes or on the nodes where the mode shape values are zero. On the other hand, if the test mode shapes are not spatially independent, test-analysis mode shape correlation using orthogonality and cross-orthogonality computations cannot be performed because the test modes and the corresponding FE modal partitions will be indistinguishable.

(5) Enough modes must be considered for the optimal locations of large structures. However how to determine the order of the monitoring modes still needs further research.

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References

Carne, T.G. and Dohmann, C.R. (1995), "A modal test design strategy for modal correlation", *Proceedings of the* 13th International Modal Analysis Conference, New York, February.

Chung, Y.T. and Moore, J.D. (1993), "On-orbit sensor placement and system identification of space station with limited instrumentations", *Proceedings of the 11th International Modal Analysis Conference*, Florida, January.

- Efstathiades, C.H., Baniotopoulos, C.C., Nazarko, P., Ziemianskib, L. and Stavroulakis, G.E. (2007), "Application of neural Networks for the structural health monitoring in curtain-wall systems", *Eng. Struct.*, **29**(12), 3475-3484.
- Housner, G.W., Bergman, L.A., Caughey, T.K., Chassiakos, A.G., Claus, R.O., Masri, S.F., Skelton, R.E., Soong, T.T., Spencer, B.F. and Yao, J.T.P. (1997), "Structural control: past, present, and future", *J. Eng. Mech.*, **123**(9), 897-971.
- Heredia-Zavoni, E., Montes-Iturrizaga, R. and Esteva, L. (1999), "Optimal instrumentation of structures on flexible base for system identification", *Earthq. Eng. Struct. D.*, **28**(12), 1471-1482.
- Hiroyuki, A. (2002), Design of Modern Highrise Reinforced Concrete Structures, Imperial College Press, England.
- Kammer, D.C. (1991), "Sensor placement for on-orbit modal identification and correlation of large space structures", J. Guid. Contr. Dynam., 14(2), 251-259.
- Kammer, D.C. (2005), "Sensor set expansion for modal vibration testing", Mech. Syst. Signal Pr., 19(4), 700-713.
- Kim, H., Kim, W., Kim, B.Y. and Hwang, J.S. (2008), "System identification of a building structure using wireless MEMS and PZT sensors", *Struct. Eng. Mech.*, **30**(2), 191-209.
- Kistera, G, Badcocka, R.A., Gebremichaelb, Y.M., Boyleb, W.J.O., Grattanb, K.T.V., Fernandoc, G.F. and Canningd, L. (2007), "Monitoring of an all-composite bridge using Bragg grating sensors", *Constr. Buil. Mater.*, 21(7), 1599-1604.
- Liu, C. and Tasker, F.A. (1996), "Sensor placement for time-domain modal parameter estimation", J. Guid. Contr. D., 19(6), 1349-1356.
- Li, H.N., Yi, T.H., Yi, X.D. and Wang, G.X. (2007), "Measurement and analysis of wind-induced response of tall building based on GPS technology", *Adv. Struct. Eng.*, **10**(1), 83-93.
- Mossberg, M. (2004), "Optimal experimental design for identification of viscoelastic materials", *IEEE T. Contr. Syst. T.*, **12**(2), 578-582.
- Majumder, M., Gangopadhyay, T.K., Chakraborty, A.K., Dasgupta, K. and Bhattacharya, D.K. (2008), "Fibre Bragg gratings in structural health monitoring-present status and applications", *Sensor Actuat. A-Phys.*, 147(1), 150-164.
- Naimimohasses, D.M., Barnett, D.M., Green, D.A. and Smith, P.R. (1995), "Sensor optimization using neural network sensitivity measures", *Meas. Sci. Technol.*, 6(9), 1291-1930.
 Oh, D.Y. and No, H.C. (1994), "Determination of the minimal number and optimal sensor location in a nuclear
- Oh, D.Y. and No, H.C. (1994), "Determination of the minimal number and optimal sensor location in a nuclear system with fixed incore detectors", *Nucl. Eng. Des.*, **152**(1-3), 197-212.
- Papadimitriou, C. (2004), "Optimal sensor placement methodology for parametric identification of structural systems", J. Sound. Vib., 278(4-5), 923-947.
- Qin, X.R. and Zhang, L.M. (2001), "Successive sensor placement for modal paring based on QR-factorization", J. Vib., Meas. Diag., 21(3), 168-173. (In Chinese)
- Sun, H.C., Qu, N.S. and Lin, J.H. (1992), Computational Structural Dynamics for High Education, Dalian University of Technology Press, China. (In Chinese)
- Takeda, S., Aoki, Y., Ishikawa, T., Takedac, N. and Kikukawad, H. (2007), "Structural health monitoring of composite wing structure during durability test", *Compos. Struct.*, 79(1), 133-139.
- Worden, K. and Burrows, A.P. (2001), "Optimal sensor placement for fault detection", *Eng. Struct.*, 23(8), 885-901.

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