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Crack-contact problem for an elastic layer with rigid stamps

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Abstract. The plane crack-contact problem for an infinite elastic layer with two symmetric rectangular rigid stamps on its upper and lower surfaces is considered. The elastic layer having an internal crack parallel to its surfaces is subjected to two concentrated loads *P* on its upper and lower surfaces trough the rigid rectangular stamps and a pair of uniform compressive stress p_0 along the crack surface. It is assumed that the contact between the elastic layer and the rigid stamps is frictionless and the effect of the gravity force is neglected. The problem is reduced to a system of singular integral equations in which the derivative of the crack surface displacement and the contact pressures are unknown functions. The system of singular integral equations is solved numerically by making use of an appropriate Gauss-Chebyshev integration formula. Numerical results for stress-intensity factor, critical load factor, Q_c, causing initial closure of the crack tip, the crack surface displacements and the contact stress distribution are presented and shown graphically for various dimensionless quantities.

Keywords: crack; contact; stress-intensity factor; elastic layer.

1. Introduction

The solution of crack and contact problems in plane elasticity has been a topic of considerable research and practice interest for a long time. This is simply doe to the frequent appearance of such problems in fracture mechanics. In studying in fracture mechanics, it is assumed that all real structures have initial cracks and failure is caused by propagation of these. Stress-intensity factor plays an important role in the field of fracture mechanics. The stress-intensity factor defines the stress field close to the tip of a crack and provides fundamental information of how the crack is going to behave. This is reason why a great deal of research has been devoted to this topic in recent years.

The result of crack and contact problems have been used to a great extent in the application of fracture mechanics and because of this, these problems attracted much attention by several investigators. Erdogan *et al.* have investigated various crack problems by using Integral transform technique (Gupta and Erdogan 1974, Nied and Erdogan 1979, Bakioglu and Erdogan 1977, Delale and Erdogan 1988, Kadioglu and Erdogan 1995, Ozturk and Erdogan 1996, Lu and Erdogan 1998, Erdogan and Dag 2001). Beghini and Bertini (1996) considered a partially closed Griffith crack in

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bending and examined the effective stress-intensity factor and contact stress of the problem. A plate with a pre-existent through crack under the action a remote bending moment and a remote in-plane force was considered by Dempsey and Shekhtman (1998). Hwu and Fan (1998) solved the punch problems by analogy with the interface crack problems. A contact-crack problem in an infinite plate loaded by remote tensile stress and a pair of compressive concentrated forces symmetrically applied on the mid plane is investigated by Chen (1999). The plane problems of stress distribution in an elastic ponderable layer with a stationary edge crack normal to the boundary plane and the stress analysis near a crack tip in an elastic layers resting on Winkler foundation were analyzed by Matysiak and Pauk (1999, 2003). The problem of bending of a cracked plate with elastic supports was solved in the two dimensional statement by Shats'kyi and Makoviichuk (2003). The partial closure problem of an infinite isotropic elastic layer with a crack parallel to its surfaces was solved Birinci and Cakiroglu (2003). Kahya *et al.* (2006) analyzed the same problem for an anisotropic infinite elastic layer. The plane problem of a layered composite containing an internal or edge crack perpendicular to its boundaries in its lower layer was analyzed by Birinci and Erdol (2004). Chattopadhyay (2005) investigated analytical solution of an orthotropic elastic plate containing cracks. A new method for the shape of a surface crack in a plate based on a given stress-intensity factor distribution was developed by Wu (2006). A simple and physically acceptable, two-dimensional analysis of stress-intensity factor for a centre-cracked plate that includes the effect of crack surface interference was presented by Albrect and Lenwari (2006).

In the present study, the plane crack-contact problem of an infinite elastic layer with two symmetric rectangular rigid stamps on its upper and lower surfaces is investigated. The elastic layer having an internal crack parallel to its surfaces is subjected to two concentrated loads P on its upper and lower surfaces through the rigid stamps and a pair of uniform compressive stress p_0 along the crack surface. It is assumed that the effect of the gravity force is neglected and the contact between the elastic layer and the rigid stamps is frictionless. The considered crack-contact problem is of interest in engineering constructions such as beams resting on supports, railway ballast and pavements in roads and runways. The problem is reduced to a system of singular integral equations in which the derivative of the crack surface displacement and the contact pressures are unknown. The system of singular integral equations is solved numerically by making use of an appropriate Gauss-Chebyshev integration formula. Numerical results for the stress-intensity factor, the crack surface displacements, the contact stress distribution and critical load factor, Q_c, causing initial closure of the crack tip are presented and shown graphically for various dimensionless quantities.

2. General expressions for displacements and stresses

Consider linear-elastic and isotropic infinite layer of thickness $2h$ with an internal crack. The geometry, coordinate system and loading cases are shown in Fig. 1. The problem will be solved under the assumptions that the contact between the layer and the rigid stamps is frictionless and the effect of the gravity force is neglected.

In the absence of body forces, the two-dimensional Navier equations may be written in the following form

$$\mu\nabla^2 u + \frac{2\mu}{\kappa - 1}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{1a}$$

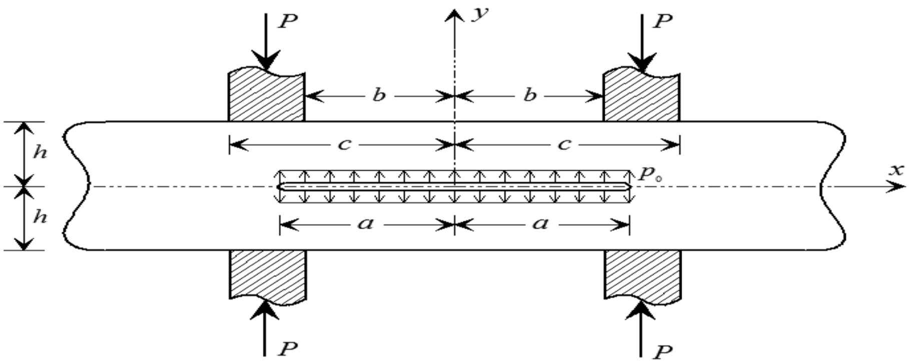

Fig. 1 Geometry and loading cases of the problem

$$\mu\nabla^2 v+\frac{2\mu}{\kappa-1}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{1b}$$

where u and v are the x and y-components of the displacement vector, μ is the shear modulus and $\kappa=(3-\nu)/(1+\nu)$ is valid for plane stress and $\kappa=3-4\nu$ for plane strain, ν is the Poisson's ratio. The use of κ instead of ν reduced two separated formulations for the plane strain and plane stress case to a single formulation.

Observing that $x=0$ and $y=0$ are the plane symmetry axes, it is sufficient to consider the problem in the region $0\le x<\infty$ and $0\le y\le h$ only. Taking advantage of symmetry condition and using Fourier transform technique, the following expressions may be written

$$u(x,y)=\frac{2}{\pi}\int_0^\infty \Phi(\alpha,y)\sin(\alpha x)d\alpha \tag{2a}$$

$$v(x,y)=\frac{2}{\pi}\int_0^\infty \Psi(\alpha,y)\cos(\alpha x)d\alpha \tag{2b}$$

where the functions Φ and Ψ are Fourier transforms of u and v, respectively. Taking necessary derivatives of Eqs. (2a) and (2b), and substituting them into Eqs. (1a) and (1b), and solving second-order differential equations, the following expressions may be obtained for the displacements

$$u(x,y)=\frac{2}{\pi}\int_0^\infty [(A+By)e^{-\alpha y}+(C+Dy)e^{\alpha y}]\sin(\alpha x)d\alpha \tag{3a}$$

$$v(x,y)=\frac{2}{\pi}\int_0^\infty\left\{\left[A+\left(\frac{\kappa}{\alpha}+y\right)B\right]e^{-\alpha y}+\left[-C+\left(\frac{\kappa}{\alpha}-y\right)D\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha \tag{3b}$$

where A, B, C and D are the unknown constants which will be determined from the boundary conditions prescribed on $y=0$ and $y=h$. Using Hooke's law and the Eq. (3a, b), the stress components of the elastic layer may be expressed as follows

$$\frac{1}{2\mu}\sigma_x(x,y)=\frac{2}{\pi}\int_0^\infty\left\{\left[\alpha(A+By)-\frac{3-\kappa}{2}B\right]e^{-\alpha y}+\left[\alpha(C+Dy)+\frac{3-\kappa}{2}D\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha \tag{4a}$$

$$\frac{1}{2\mu}\sigma_y(x,y) = \frac{2}{\pi}\int_0^\infty\left\{-\left[\alpha(A+By)+\frac{1+\kappa}{2}B\right]e^{-\alpha y}+\left[-\alpha(C+Dy)+\frac{1+\kappa}{2}D\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha \quad (4b)$$

$$\frac{1}{2\mu}\tau_{xy}(x,y) = \frac{2}{\pi}\int_0^\infty\left\{-\left[\alpha(A+By)+\frac{\kappa-1}{2}B\right]e^{-\alpha y}+\left[\alpha(C+Dy)-\frac{\kappa-1}{2}D\right]e^{\alpha y}\right\}\sin(\alpha x)d\alpha \quad (4c)$$

3. The boundary conditions and the system of integral equations

The plane crack-contact problem outlined above as shown in Fig. 1 must be solved under the following boundary conditions

$$\tau_{xy}(x,0) = 0, \qquad (0 \le x \le \infty) \quad (5a)$$

$$\tau_{xy}(x,h) = 0, \qquad (0 \le x \le \infty) \quad (5b)$$

$$\sigma_y(x,h) = \begin{cases} -p(x); & b<x<c \\ 0; & 0\le x<b,\ c\le x<\infty \end{cases} \quad (5c)$$

$$\frac{\partial}{\partial x}[v(x,0)] = \begin{cases} \varphi(x); & 0\le x<a \\ 0; & a<x<\infty \end{cases} \quad (5d)$$

$$\sigma_y(x,0) = -p_0\,, \qquad (0\le x<a) \quad (5e)$$

$$\frac{\partial}{\partial x}[v(x,h)] = 0\,, \qquad (b<x<c) \quad (5f)$$

where $p(x)$ is the unknown contact pressure under the rigid stamp, $\varphi(x)$ is the unknown function defined by the derivative of the y-component of the displacement vector. a, b and c are defined as shown in Fig. 1.

By making use of the boundary conditions (5a-5d), the unknown constants A, B, C and D appearing in Eqs. (3) and (4) may be obtained in terms of the unknown functions $p(x)$ and $\varphi(x)$. Thus, the stresses and the displacements for the elastic layer can be expressed depending on the unknown functions $p(x)$ and $\varphi(x)$ which have not yet determined.

The solution of (3) and (4) with A, B, C and D satisfies all of the boundary conditions stated by Eqs. (5a)-(5d) except the conditions (5e) and (5f). The new unkown functions $p(x)$ and $\varphi(x)$ are determined from the boundary conditions (5e) and (5f) which have not yet been satisfied. After some routine manipulations and using symmetry consideration, these conditions give the following system of singular integral equations

$$\frac{1}{\pi}\int_{-a}^{a}\left[\frac{1}{t-x}+k_1(x,t)\right]\varphi(t)dt+\frac{1}{\pi}\frac{(1+\kappa)}{2\mu}\int_b^c k_2(x,t)p(t)dt = -\frac{(1+\kappa)p_0}{4\mu}\,, \qquad (-a\le x\le a) \quad (6a)$$

$$\frac{1}{\pi}\int_b^c\left[\frac{1}{t-x}-\frac{1}{t+x}+k_3(x,t)\right]p(t)dt+\frac{1}{\pi}\frac{4\mu}{(1+\kappa)}\int_{-a}^{a}k_4(x,t)\varphi(t)dt = 0\,, \qquad (b<x<c) \quad (6b)$$

where

$$k_1(x,t) = \int_0^\infty \left\{ \frac{1}{\Delta(\alpha)}[1+e^{4\alpha h}-2e^{2\alpha h}(1+2\alpha^2 h^2)]-1 \right\} \sin\alpha(t-x)d\alpha \tag{7a}$$

$$k_2(x,t) = \int_0^\infty \frac{1}{\Delta(\alpha)}[(1-\alpha h)e^{\alpha h}-(1+\alpha h)e^{3\alpha h}][\cos\alpha(t+x)+\cos\alpha(t-x)]d\alpha \tag{7b}$$

$$k_3(x,t) = \int_0^\infty \left[\frac{1}{\Delta(\alpha)}(-1-e^{4\alpha h}+2e^{2\alpha h})+1 \right][\sin\alpha(t+x)-\sin\alpha(t-x)]d\alpha \tag{7c}$$

$$k_4(x,t) = \int_0^\infty \frac{1}{\Delta(\alpha)}[(\alpha h-1)e^{\alpha h}+(1+\alpha h)e^{3\alpha h}][\cos\alpha(t+x)-\cos\alpha(t-x)]d\alpha \tag{7d}$$

in which

$$\Delta(\alpha) = -1+e^{4\alpha h}+4\alpha h e^{2\alpha h} \tag{7e}$$

The system of singular integral Eqs. (6a) and (6b) must be solved with the following equilibrium and single-valuedness conditions, respectively

$$\int_b^c p(t)dt = P \tag{8a}$$

$$\int_{-a}^{a} \varphi(t)dt = 0 \tag{8b}$$

where P is the known compressive resultant force applied to the rigid stamp.

4. The solution of the system of integral equations

Designating the variables (x,t) on $y=0$ and $y=h$ by (x_1,t_1) and (x_2,t_2) respectively, and defining the following dimensionless quantities

$$r_1 = \frac{x_1}{a}, \qquad r_2 = \frac{2x_2}{c-b}-\frac{c+b}{c-b} \tag{9a}$$

$$s_1 = \frac{t_1}{a}, \qquad s_2 = \frac{2t_2}{c-b}-\frac{c+b}{c-b} \tag{9b}$$

$$g_1(s_1) = \frac{\mu}{p_0(1-\nu)}\varphi(t_1), \qquad g_2(s_2) = 2\frac{p(t_2)}{p_0} \tag{9c}$$

the system of integral Eqs. (6a) and (6b) may be expressed as follows

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-r}+\frac{a}{h}m_1(r,s)\right]g_1(s)ds+\frac{1}{\pi}\frac{(c-b)}{2h}\int_{-1}^{1}m_2(r,s)g_2(s)ds = -1, \quad (-1<r<1) \tag{10a}$$

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{s-r}-\frac{1}{s+r+2\frac{c+b}{c-b}}+\frac{(c-b)}{2h}m_3(r,s)\right]g_2(s)ds+\frac{1}{\pi}\frac{2a}{h}\int_{-1}^{1}m_4(r,s)g_1(s)ds = 0 \quad (-1<r<1) \quad (10b)$$

where

$$m_1(r_1,s_1) = k_1(x_1,t_1) \quad (11a)$$

$$m_2(r_1,s_2) = k_2(x_1,t_2) \quad (11b)$$

$$m_3(r_2,s_2) = k_3(x_2,t_2) \quad (11c)$$

$$m_4(r_2,s_1) = k_4(x_2,t_1) \quad (11d)$$

Note that the subscripts have been deleted in Eqs. (10a) and (10b) since the variables r_1, s_1, r_2 and s_2 all vary between −1 and +1. Similarly, the additional conditions (8a) and (8b) may be expressed as

$$\frac{1}{\pi}\frac{(c-b)}{2h}\int_{-1}^{1}g_2(s)ds = \frac{2}{\pi}Q \quad (12a)$$

$$\frac{1}{\pi}\frac{a}{h}\int_{-1}^{1}g_1(s)ds = 0 \quad (12b)$$

where Q is the load factor and it is defined as

$$Q = \frac{P}{p_0 h} \quad (13)$$

In order to solve the system of integral equations with the additional conditions, it is found that the integral Eqs. (10a), (10b) and the additional Eqs. (12a) and (12b) have an index +1 (Erdogan and Gupta 1972) because the unknown functions $p(x)$ and $\varphi(x)$ are infinite at the ends. Hence, the solution will be in the following form

$$g_1(s) = G_1(s)(1-s^2)^{-1/2}, \qquad (-1<s<1) \quad (14a)$$

$$g_2(s) = G_2(s)(1-s^2)^{-1/2}, \qquad (-1<s<1) \quad (14b)$$

where $G_1(s)$ and $G_2(s)$ are bounded in the closed interval $(-1 \le s \le 1)$. Then, using the appropriate Gauss-Chebyshev integration formula (Erdogan and Gupta 1972), Eqs. (10a), (10b), (12a) and (12b) are replaced by

$$\sum_{k=1}^{N}\frac{1}{N}\left\{\left[\frac{1}{s_k-r_i}+\frac{a}{h}m_1(r_i,s_k)\right]G_1(s_k)+\frac{(c-b)}{2h}m_2(r_i,s_k)G_2(s_k)\right\} = -1, \quad (i=1,\ldots,N-1) \quad (15a)$$

$$\sum_{k=1}^{N}\frac{1}{N}\left\{\left[\frac{1}{s_k-r_i}-\frac{1}{s_k+r_i+2\frac{c+b}{c-b}}+\frac{(c-b)}{2h}m_3(r_i,s_k)\right]G_2(s_k)+2\frac{a}{h}m_4(r_i,s_k)G_1(s_k)\right\} = 0$$

$$(i=1,\ldots,N-1) \quad (15b)$$

$$\frac{(c-b)}{2h}\sum_{k=1}^{N}\frac{1}{N}G_2(s_k) = \frac{2}{\pi}Q \tag{15c}$$

$$\frac{a}{h}\sum_{k=1}^{N}\frac{1}{N}G_1(s_k) = 0 \tag{15d}$$

where

$$s_k = \cos\left(\frac{2k-1}{2N}\pi\right), \qquad (k = 1, \ldots, N) \tag{16a}$$

$$r_i = \cos\left(\frac{i\pi}{N}\right), \qquad (i = 1, \ldots, N-1) \tag{16b}$$

Note that the system given by Eqs. (15a-15d) contains $2N$ algebraic equations for $2N$ unknowns, $G_1(s_k)$ and $G_2(s_k)$, $(k = 1, \ldots, N)$. After solving the system of $2N$ algebraic equations for $2N$ unknowns, $G_1(s_k)$ and $G_2(s_k)$, $(k = 1, \ldots, N)$, are determined numerically.

5. The stress-intensity factor and the crack surface displacement

The stress-intensity factor $k(a)$, which characterizes the strength of the stress singularity at the crack tip a, may be defined by (Gupta and Erdogan 1974)

$$k(a) = -\lim_{t\to a}\frac{\mu}{1-\nu}\sqrt{2(a-t)}\,\varphi(t) \tag{17}$$

Substituting Eq. (9c) into the Eq. (17), after some routine manipulations, the following equation is obtained for $k(a)$

$$k(a) = -p_0\sqrt{a}\,G_1(1) \tag{18}$$

where $G_1(1)$ is obtained from $G_1(s_k)$, $(k = 1, \ldots, N)$, by using the interpolation formulas given in (Krenk 1975). To facilitate the numerical calculations, Eq. (18) may be written in the following form

$$\frac{k(a)}{p_0\sqrt{a}} = -G_1(1) \tag{19}$$

The crack surface displacement may be obtained from (Gupta and Erdogan 1974)

$$v(x,0) = \int_{-a}^{x}\varphi(t)dt, \qquad (-a < x < a) \tag{20}$$

Using Eqs. (9a-c), Eq. (20) takes the following form

$$\frac{v(x,0)}{h(1-\nu)p_0}\mu = \frac{a}{h}\int_{-1}^{r}g_1(s)ds, \qquad (-1 < r < 1) \tag{21}$$

Then, using an appropriate Gauss-Chebyshev integration formula (Erdogan and Gupta 1972) and Eq. (14a), the Eq. (21) is replaced by

$$\frac{v(x,0)}{h(1-\nu)p_0}\frac{\mu}{\pi} = \frac{a}{h}\sum_{k=1}^{i-1}\frac{1}{N}G_1(s_k), \qquad (i = 2, \ldots, N) \tag{22}$$

where s_k is given in Eq. (16a). Substituting $G_1(s_k)$, $(k = 1, \ldots, N)$, obtained from Eqs. (15a-d), into Eq. (22), the crack surface displacement is computed numerically.

6. Numerical results and discussions

Some calculated results obtained from the solution of the system of linear algebraic Eqs. (15a)-(15d) described in the previous section for various dimensionless quantities such as, a/h, b/h, $(c-b)/h$ and $Q = P/p_o h$ are shown in Figs. 2-7 and Tables 1, 2. In Figs. 4-7 and Table 1, the normalized crack surface displacement $v'(x,0)$ and the normalized stress-intensity factor $k'(a)$ may be expressed in the following form

$$v'(x,0) = \frac{v(x,0)\mu}{h(1-\nu)p_0}, \qquad (-a<x<a) \tag{23}$$

$$k'(a) = \frac{k(a)}{p_0\sqrt{a}} \tag{24}$$

Figs. 2 and 3 show the normalized contact pressure $p(x)/p_0$ under the rigid stamp for various values a/h and Q. As it can be seen in Fig. 2, the normalized contact pressure becomes infinitely

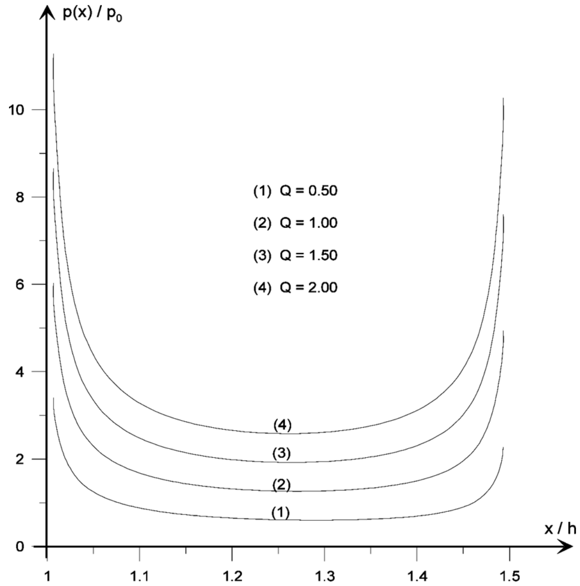

Fig. 2 Contact stress distribution under the rigid stamp for various values of the load factor Q ($a/h = 0.50$, $b/h = 1.00$, $(c-b)/h = 0.50$)

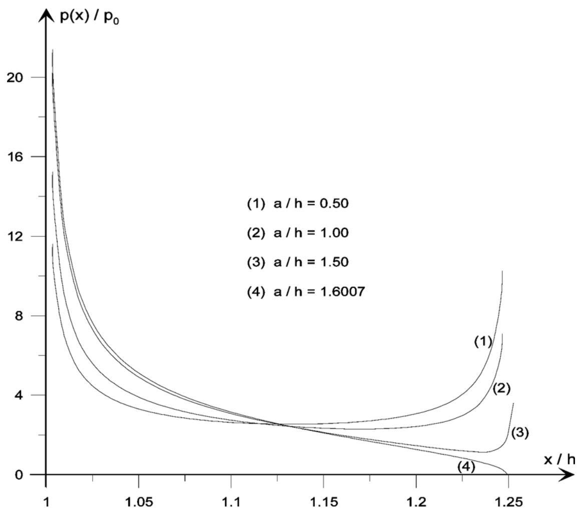

Fig. 3 Contact stress distribution under the rigid stamp for various values of a/h ($b/h = 1.00$, $(c-b)/h = 0.25$, $Q = 1.00$)

large at the corners of the rigid stamp and the contact pressure increases as the load factor Q increases.

Fig. 3 shows that the normalized contact pressure increases at the left corner of the stamp and decreases at the right corner of the stamp as the crack half-length, a/h, increases. If the crack half-

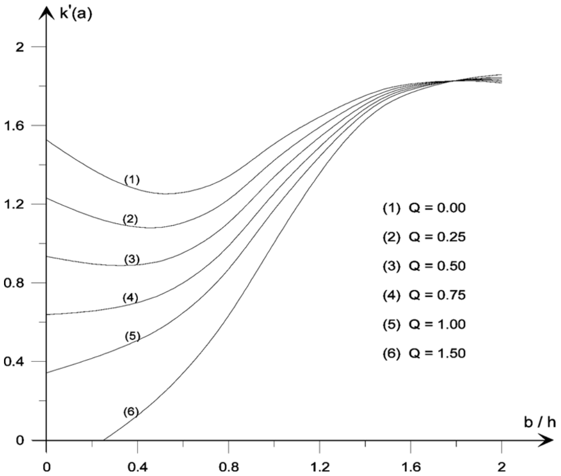

Fig. 4 Variation of the normalized stress-intensity factor $k'(a)$ with b/h for various values of the load factor Q ($a/h = 1.00$, $(c-b)/h = 1.00$)

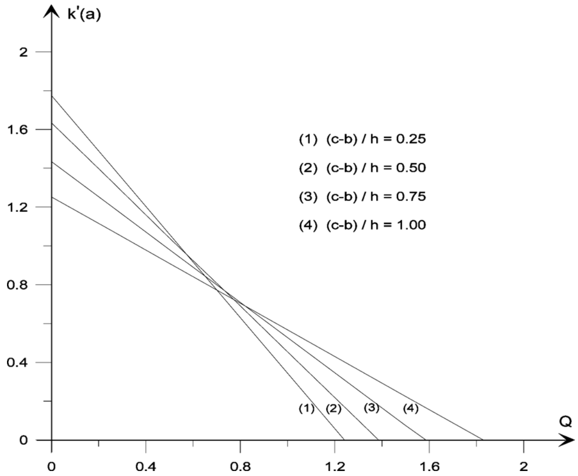

Fig. 5 Variation of the normalized stress-intensity factor $k'(a)$ with the load factor Q for various values of $(c-b)/h$ ($a/h = 1.00$, $b/h = 0.50$)

Table 1 Variation of the normalized stress-intensity factor $k'(a)$ with b/h for various values of Q ($(c-b)/h = 0.25$, $a/h = 1.00$)

Q	$k'(a) = \frac{k(a)}{p_0\sqrt{a}}$					
	$\frac{b}{h} \to 0.00$	$\frac{b}{h} \to 0.10$	$\frac{b}{h} \to 0.25$	$\frac{b}{h} \to 0.50$	$\frac{b}{h} \to 0.75$	$\frac{b}{h} \to 1.00$
0.01	1.8139	1.8101	1.8004	1.7758	1.7544	1.7601
0.25	1.3805	1.3836	1.3918	1.4179	1.4725	1.5695
0.50	0.9472	0.9572	0.9832	1.0600	1.1907	1.3790
0.75	0.5138	0.5307	0.5746	0.7021	0.9089	1.1884
1.00	0.0805	0.1042	0.1660	0.3442	0.6270	0.9979
1.0464	→ 0.00	0.0251	0.0901	0.2777	0.5747	0.9625
1.0611	-	→ 0.00	0.0661	0.2567	0.5581	0.9513
1.1015	-	-	→ 0.00	0.1988	0.5125	0.9205
1.2404	-	-	-	→ 0.00	0.3560	0.8146
1.50	-	-	-	-	0.0633	0.6168
1.5561	-	-	-	-	→ 0.00	0.5740
2.00	-	-	-	-	-	0.2357
2.3093	-	-	-	-	-	→ 0.00

length is sufficiently increased, i.e., $a/h = 1.6007$ the normalized contact pressure becomes zero at the right corner of the stamp, and for bigger values of a/h, a separation of the contacting surface of the rigid stamp and the elastic layer may take place around the right corner of the stamp.

The variation of the normalized stress-intensity factor $k'(a)$ with b/h and Q for various dimensionless quantities is shown in Figs. 4, 5 and Table 1. It can be seen in Fig. 4 and Table 1 that the normalized stress-intensity factor decreases as the load factor increases. For sufficiently big values of b/h, i.e., $b/h > 2.5$, the stress intensity factor approaches a constant asymptotic value, $k'(a) \to 1.8156$. Table 1 shows that for determined values of the critical load factor Q_c and b/h, the crack tip takes place initial closure, for example, for $b/h = 0.50$, $Q_c = 1.1015$. As b/h increases the

Table 2 Variation of the critical load factor Q_c, causing initial closure of the crack tip, with $(c-b)/h$ for various values of b/h ($k'(a) = k(a)/p_0\sqrt{a} = 0$, $a/h = 2.00$)

b/h	Q_c			
	$\frac{c-b}{h} = 0.25$	$\frac{c-b}{h} = 0.50$	$\frac{c-b}{h} = 0.75$	$\frac{c-b}{h} = 1.00$
$\to 0.00$	1.7393	1.7667	1.8187	1.8882
0.10	1.7482	1.7855	1.8454	1.9197
0.20	1.7624	1.8088	1.8753	1.9535
0.30	1.7823	1.8365	1.9085	1.9900
0.50	1.8372	1.9050	1.9859	2.0738
0.75	1.9388	2.0215	2.1112	2.2104
1.00	2.0923	2.1913	2.2949	2.4137
1.50	2.7519	2.9832	3.2419	3.5538
2.00	5.5802	7.0211	8.7954	10.8140
2.50	29.2252	48.6292	77.6594	115.6245

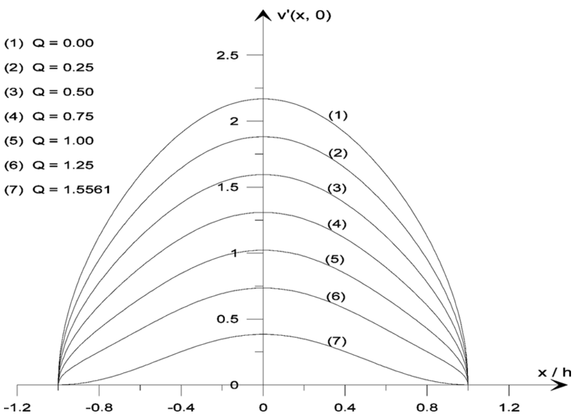

Fig. 6 Normalized crack surface displacement $v'(x, 0)$ for various values of the load factor Q ($a/h = 1.00$, $b/h = 0.75$, $(c-b)/h = 0.25$)

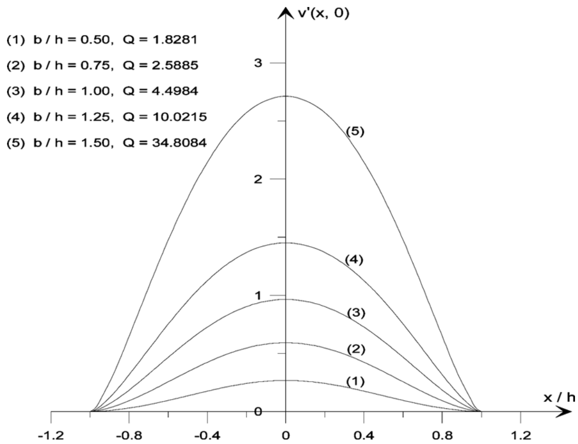

Fig. 7 Normalized crack surface displacement $v'(x, 0)$ for various values of b/h in the case of being initial closure of the crack tip ($a/h = 1.00$, $(c-b)/h = 1.00$)

closure of the crack tip occurs bigger load factor, that is, the closure of the crack tip occurs more difficult. Also, these results can be seen in Table 2 and Fig. 5 which are examined the variation of the critical load factor Q_c with $(c-b)/h$ for various values of *b/h* and the variation of the normalized stress-intensity factor $k'(a)$ with load factor Q for various values of $(c-b)/h$, respectively. It can be seen in Table 2 that the critical load factor increases as $(c-b)/h$ increases for *a/h* = 2.00.

Some sample results calculated from Eq. (22) giving the normalized crack surface displacement $v'(x,0)$ are shown in Figs. 6 and 7 for various values of Q and *b/h*. As it will be shown in Fig. 6, as expected, the normalized crack surface displacement decreases as the load factor increases for *b/h* = 0.75. But, as *b/h* increases, the normalized crack surface displacement increases although the load factor increases. This result can be seen in the Fig. 7 which is examined the normalized crack surface displacement for various values of *b/h* in the case of being initial closure of the crack tip.

7. Conclusions

In this paper, the plane crack-contact problem for an infinite elastic layer with two symmetric rectangular rigid stamps on its upper and lower surfaces is presented. It can be seen in the previous section that the crack length, the length of rigid stamp and the load factor play a very important role on the contact stress distribution, the stress-intensity factor, the crack surface displacement and the critical load factor causing initial closure of the crack tip.

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