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Response of lap splice of reinforcing bars confined by FRP wrapping: modeling approach

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Abstract. This paper presents a tri-uniform bond stress model for predicting the lap splice strength of reinforcing bar at the critical bond splitting failure. The proposed bond distribution model consists of three zones, namely, splitting zone, post-splitting zone and yielding zone. In each zone, the bond stress is assumed to be constant. The models for bond strength in each zone are adopted from previous studies. Combining the equilibrium, strain-slip relation and the bond strength model in each zone, the steel stress-slip model can be derived, which can be used in the nonlinear frame analysis of the column. The proposed model is applied to derive explicit equations for predicting the strength of the lap splice strengthened by fiber reinforced polymer (FRP) in both elastic and post-yield ranges. For design purpose, a procedure to calculate the required FRP thickness and the number of FRP sheets is also presented. A parametric investigation was conducted to study the relation between lap splice strength and lap splice length, number and thickness of FRP sheets and the ratio of concrete cover to bar diameter. The study shows that the lap splice strength can be enhanced by increasing one of these parameters: lap splice length, number or thickness of FRP sheets and concrete cover to bar diameter ratio. Verification of the model has been conducted using experimental data available in literature.

Keywords: confinement; lap splice strength; tri-uniform bond stress model; fiber-reinforced polymer; splitting failure.

1. Introduction

The strength of lap splice is very important for the development of load capacity and ductility of reinforced concrete column. The investigation of buildings constructed following the sub-standard (pre-1970) seismic design approaches by Chai *et al*. (1991) and Melek *et al*. (2003) have led to the conclusions that lap splices in sub-standard columns were typically designed as compression splices with the lap length of about 20 to 24 times bar diameter and were poorly confined by small amount of transverse steels. Moreover, a lot of reinforced concrete bridge piers in low to moderate seismic zones may have inadequate lap splice between column longitudinal bars and starter bars projected from the footing at the base of column. The failure of the lap splice may lead to low lateral strength and poor ductility in cyclic responses of sub-standard RC columns subjected to seismic excitation.

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Various methods of strengthening short lap splice length have been studied, such as external confinement by column or steel jacketing (Aboutaha *et al*. 1996, Chai *et al*. 1991) and fiber reinforced polymer sheets (Xiao and Ma 1997, Bousias *et al*. 2006, Harajli 2008). Fiber-Reinforced Polymer (FRP) sheets are now considered the state-of-the-art technology in rehabilitating and strengthening of reinforced concrete (RC) structures. Due to the confinement effect, the bond stress between reinforcing bar and surrounding concrete is enhanced, thus increasing the lap splice strength. Some reinforced concrete columns confined with FRP sheets were tested by Xiao *et al*. (1997), Ma *et al*. (2000), Harries *et al*. (2006), Bousias *et al*. (2006), and Harajli and Dagher (2008). The tests showed that FRP confinement could prevent the splitting failure prior to yielding of main bars, thus developing the strength of short lap splice into post-yield range with higher ductility.

To estimate the lap splice strength, some bond slip models have been proposed. One of the most widely used bond slip models was proposed by Eligehausen and Popov (Eligehausen *et al*. 1983) based on an experimental program at the University of California, Berkeley, but that model is applicable for bars in elastic range only. Based on Eligehausen's works, Harajli (Harajli 2006) proposed a bond slip model based on the test results of beams with lap splice length of 5 times bar diameter at the middle of the beam. The model can be used for estimating the strength of lap splices confined by both transverse steels and FRP sheets.

The bond characteristics in the post-yield range of steel bars must be known to estimate the strength of lap splice. Hassan and Hawkins (1977) have developed a model for predicting the pull-out strength of an anchored steel bar in post-yield range by assuming the distribution of bond stress along the lap splice length. An investigation of the bond-slip and stress-strain relationships in post-yield range of reinforcing bar embedded in massive concrete was conducted by Murayama *et al*. (1986), but there was no formula drawn. Another long embedment pull-out test series was conducted by Shima *et al*. (1987) to investigate the bond characteristics in post-yield range of deformed bars. Based on the experimental results, the bond stress in the yielding zone of the bar can be derived. However, these studies have not focused on the effect of FRP confinement on lap splice strength in post-yield range. In past studies, there was very scarce information on bond slip model or bond stress distribution of lap splice strengthened by FRP in the post yield range of reinforcing bars. This paper attempts to propose a model that can predict the lap splice strength of bar in both elastic and post-yield range. Not only the strength of lap-splice, but the entire steel stress-slip model can also be derived, which is useful for the nonlinear analysis of RC columns subjected to ground excitations. By incorporating the steel stress-slip model, the nonlinear analysis of RC column can be greatly enhanced to capture the lap splice failure and the additional flexibility due to slip of spliced bars. A companion paper is entirely devoted to the nonlinear modeling and analysis of RC column in details.

2. Strength of lap splice confined by FRP

2.1 Tri-uniform bond stress model

Fig. 1 shows a column with longitudinal bars spliced with the starter bars at the base of the column. When the column is subjected to an applied load, a crack occurs at the interface between column base and footing (point O). Considering an outermost bar on the tension side, the developed bar stress f_s at the starting point of the lap splice zone (point O) must be in equilibrium with bond

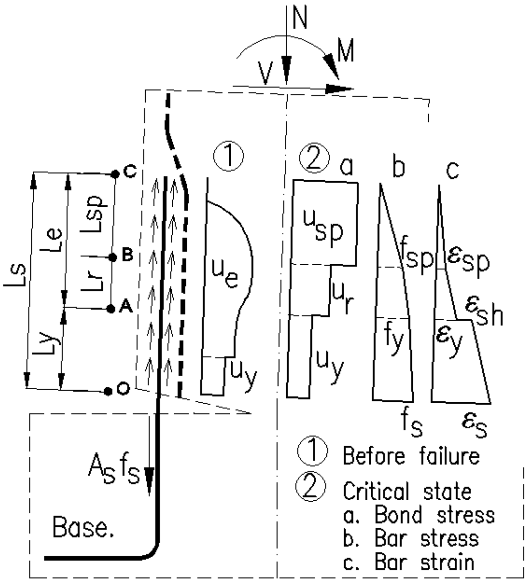


Fig. 1 Tri-uniform bond stress model

stresses on the bar surface along the lap length. The bond stress distribution along the lap splice length L_s depends on many factors such as the pull-out force $A_s f_s$, the length of the lap splice L_s, the confinement condition, and so on. An example of bond stress distribution before lap splice failure is shown in Fig. 1(1), in which u_y and u_e, are bond stresses acting on yielding zone and elastic zone, respectively.

At the critical state of splitting failure, the bond stress distribution along the lap splice length is assumed to follow the tri-uniform bond model as shown in Fig. 1(2a). In the model, the lap splice length is divided into three zones; namely, yielding zone (OA), post-splitting zone (AB) and splitting zone (BC), with bond stresses on each zone denoted by u_y, u_r and u_{sp} respectively. The lengths of yielding zone, post-splitting zone, and splitting zone are L_y, L_r and L_{sp} respectively. The sum of these lengths must be equal to the lap splice length L_s.

$$L_s = L_y + L_r + L_{sp} \tag{1}$$

The tri-uniform bond model is different from the model proposed by Sezen and Moehle (2003) for predicting the slip of bars anchored into footing in that the bond stress in the elastic zone (AC) is assumed to be composed of splitting zone (BC) and post-splitting zone (AB) while Sezen and Moehle assumed a constant bond stress equal to twice the bond strength in the yield zone. Fig. 1(2b, 2c) shows a tri-linear distribution of stress and strain in reinforcing bars. The steel stress and strain are zero at the end of lap splice (C) and increases to (f_{sp}, ε_{sp}) at point B. In the post-splitting zone (AB), bar stress and strain are increased from f_{sp} and ε_{sp} at point B to yield point f_y and ε_y at point A. At point A which separates the bar into elastic and yielding zones, the strain exhibits a discontinuous jump from ε_y to ε_{sh}, which is the strain at the onset of strain hardening. In the next section, the bond stress models are described for each zone. By combining the equilibrium equations, steel strain-slip relation and bond slip models, the steel stress-slip relation of the lap splice can be derived.

2.2 Bond stress-slip model of lap splice confined by FRP

In the splitting zone of the tri-uniform bond model, the uniform bond stress (u_{sp}) can be obtained from the bond stress-slip model of lap splice confined by FRP. Wrapping FRP around lap splice zone induces additional lateral stress, thereby increasing the bond stress of the spliced bar. In order to simulate the bond-slip behavior of spliced bars strengthened by FRP, a bond-slip model (Fig. 2) proposed by Harajli (2006) is adopted in this paper. In this model, the peak bond stress u_{sp} and the corresponding slip s_{sp} at bond splitting failure are expressed by Eq. (2) and Eq. (3) respectively. In the equations, u_m is the maximum bond stress at pullout mode of bond failure given as $u_m = 2.57\sqrt{f_c'}$; n_f and t_f are the number and the thickness of FRP sheets, respectively. The bond stress u_p on the decreasing curve is calculated using Eq. (4). The factor α_f, that represents the method of wrapping, is expressed by Eq. (5), in which N_f is the number of FRP strips with equal width b_f and for full wrapping, $\alpha_f = 1$. In the model, the parameter α indicates the starting point of degradation stage of bond stress for both plain and confined concrete. The parameter β indicates the starting point of residual bond strength for the plain concrete. In this paper, $\alpha = 0.7$ and $\beta = 0.65$, respectively. The parameter u_f is the residual bond strength in the pull-out failure mode.

$$u_{sp} = 0.75\sqrt{f_c'}\left(\frac{c + 56\dfrac{E_f\alpha_f n_f t_f}{E_s n_s}}{d_b}\right)^{2/3} \le u_m \tag{2}$$

$$s_{sp} = s_1 e^{3.3\ln\left(\frac{u_{sp}}{u_m}\right)} + s_o \ln\left(\frac{u_m}{u_{sp}}\right) \tag{3}$$

$$u_p = u_{sp}\left(0.5 + 46\frac{E_f\alpha_f n_f t_f}{E_s c n_s}\right) \le u_{sp} \tag{4}$$

$$\alpha_f = \frac{N_f b_f}{L_s} \tag{5}$$

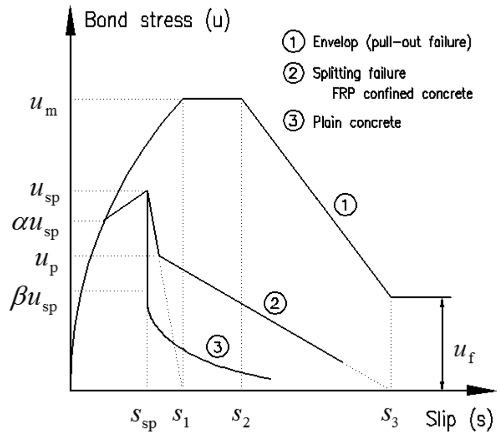
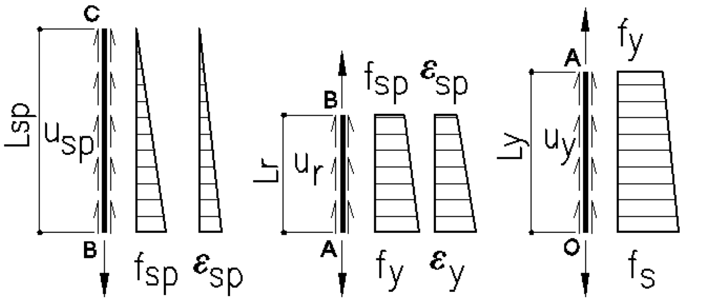


Fig. 2 Bond stress-slip model by Harajli (2006)



(a) Splitting zone (b) Post-splitting zone (c) Yielding zone

Fig. 3 Free body diagram

Other notations in the above equations are as follows: E_f, E_s are the modulus of elasticity of FRP sheets and transverse steels respectively; n_s is number of lap splices in tension; c is the concrete cover depth, s_0, s_1 and s_2 are local slip parameters that are computed from the clear distance between ribs on the reinforcing bar c_0. The details of the model can be found in Harajli (2006)'s paper.

2.3 Equilibrium and strain-slip condition

Fig. 3 illustrates the free-body diagrams of the splitting zone (Fig. 3(a)), post-splitting zone (Fig. 3(b)) and yielding zone (Fig. 3(c)). In each zone, the equilibrium equation between bond stress and bar stress can be derived. The slip of the bar can be calculated by integrating the strain along the lap splice length. The steel stress (f_s) and the slip (s) can thus be calculated by

$$f_s = (u_{sp}L_{sp} + u_r L_r + u_y L_y)P \tag{6}$$

$$s = \int \varepsilon_s dx + s_0 \tag{7}$$

Where, P is the bar perimeter; ε_s is the strain in steel bar; and s_0 is the free slip (if any) at the end of lap splice. If the lengths of the three zones are known, it is possible to calculate the steel stress versus slip relation. In the next section, the equilibrium condition and the steel strain-slip relation will be described for each zone.

2.3.1 Splitting zone

A free body diagram and distribution of bar stress and bar strain along this zone are shown in Fig. 3(a). The bond and bar stresses have to satisfy Eq. (8).

$$u_{sp}(\pi d_b)L_{sp} = f_{sp}\frac{\pi d_b^2}{4} \leftrightarrow f_{sp} = \frac{4u_{sp}L_{sp}}{d_b} \tag{8}$$

To ensure that the splitting bond stress reaches the bond strength, the slip at point B has to attain the value of s_{sp} as shown in Eq. (9).

$$s_B = \frac{\varepsilon_{sp}L_{sp}}{2} = s_{sp} \leftrightarrow \frac{f_{sp}L_{sp}}{2E_s} = s_{sp} \tag{9}$$

By substituting Eq. (8) into Eq. (9), the expression for calculating the length of splitting zone can be derived in Eq. (10) in which the splitting bond strength u_{sp} and the corresponding slip s_{sp} are given in Eq. (2) and Eq. (3) respectively.

$$L_{sp} = \sqrt{\frac{E_s d_b}{2} \frac{s_{sp}}{u_{sp}}} \tag{10}$$

2.3.2 Post splitting zone

Fig. 3(b) shows the free body diagram and distribution of bar stress and strain along this zone. The bond and bar stresses have to satisfy the equilibrium expressed by Eq. (11).

$$u_r(\pi d_b)L_r = (f_y - f_{sp})\frac{\pi d_b^2}{4} \leftrightarrow L_r = (f_y - f_{sp})\frac{d_b}{4u_r} \tag{11}$$

To compute the length of the post splitting zone (L_r), the uniform bond stress u_r has to be determined. Fig. 4 shows how to calculate the bond stress on post-splitting zone based on the bond stress-slip model of lap splice strengthened by FRP wrapping. In the figure, the relationship between post splitting bond stress (u_r) and corresponding slip is represented by a polyline m-n-q. An example of calculating bond stress u_r for a specified slip s_A is illustrated in Fig. 4.

In Fig. 4, c_0 is the distance between the ribs of a reinforcing bar. Depending on the slip at point A, the bond stress can be computed following the line m-n-q. Expressions (12a) and (12b) are equations of these lines. In these equations, s_p is the slip at bond stress u_p which is calculated by Eq. (4).

$$u_r = \frac{0.15c_0 - s_A}{0.15c_0 - s_{sp}} u_{sp} \qquad \text{If} \quad s_A \le s_p \tag{12a}$$

$$u_r = \frac{c_0 - s_A}{c_0 - s_p} u_p \qquad \text{If} \quad s_A \ge s_p \tag{12b}$$

$$s_p = s_{sp} + \left(1 - \frac{u_p}{u_{sp}}\right)(0.15c_0 - s_{sp}) \tag{13}$$

The slip at point A is calculated from bar strain distribution along this zone as follows.

$$s_A = s_{sp} + \frac{(\varepsilon_{sp} + \varepsilon_y)L_r}{2} \tag{14}$$

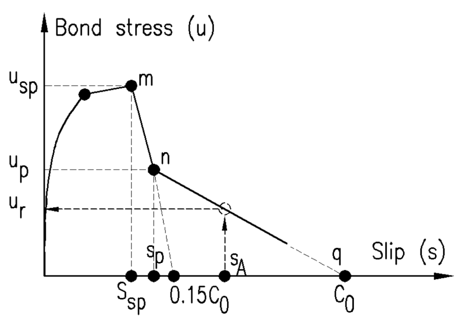


Fig. 4 Calculation of bond stress for a specific slip s_A

Substituting u_r and s_A from Eqs. (12a, b) and (14) into Eq. (11), and rearranging the terms, Eqs. (15a) and (15b) for determining the length L_r of the post splitting zone are derived. It is noted that the length of post splitting zone will be determined from Eq. (15a) if the length is less than L_n given in Eq. (16), otherwise it will be obtained from Eq. (15b).

$$\frac{\varepsilon_{sp}+\varepsilon_y}{2(0.15c_0-s_{sp})}L_r^2-L_r+\frac{(f_y-f_{sp})d_b}{4u_{sp}} = 0 \tag{15a}$$

$$\frac{\varepsilon_{sp}+\varepsilon_y}{2(c_0-s_p)}L_r^2-\frac{c_0-s_{sp}}{c_0-s_p}L_r+\frac{(f_y-f_{sp})d_b}{4u_p} = 0 \tag{15b}$$

$$L_n = \frac{2(s_p-s_{sp})}{\varepsilon_{sp}+\varepsilon_y} \tag{16}$$

2.3.3 Yielding zone

A free body diagram and distribution of bar stress and bar strain along this zone are shown in Fig. 3(c). The bond and bar stresses have to satisfy equilibrium expressed by Eq. (17)

$$u_y(\pi d_b)L_y = (f_s-f_y)\frac{\pi d_b^2}{4} \leftrightarrow L_y = (f_s-f_y)\frac{d_b}{4u_y} \tag{17}$$

To compute the length of yielding zone, the bond stress u_y has to be determined. However, there is no model of bond and slip relationship proposed for the yielding bars. In an effort to calculate the bond slip of bars anchored into the basement, Sezen and Moehle (2003) used a bi-uniform bond stress model to simulate the slip behavior of bars in yielding range. They assumed a constant bond stress of $0.5\sqrt{f_c'}$ along the yielding portion of the bar. An experimental program on pull-out test was conducted to make an assessment on bond characteristics in post-yield range of deformed bar by Shima *et al.* (1987). In the test, the embedment length was set to $50d_b$ (d_b = 19 mm which was so sufficiently long that no free end slip occurred even if the bar reached yielding during pull-out. Fig. 5 shows the experimental results of three types of steel with different nominal yield strengths 300, 500 and 700 MPa, referred to as SD30, SD50 and SD70 respectively. As observed from Fig. 5, it is found that the bond stress acting along the yielding zone tends to be uniformly distributed and is equal to $u_y = 0.25f_c'^{2/3}$.

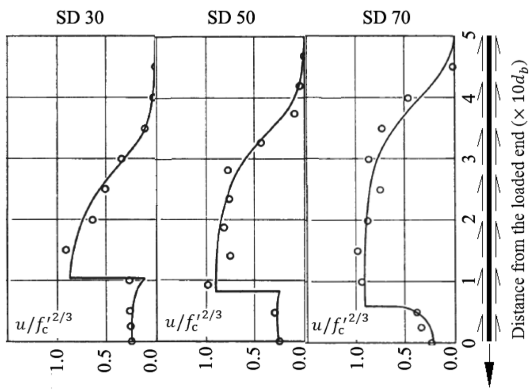


Fig. 5 Experimental result by Shima *et al.* (1987)

From these researches, the bond stress in yielding zone is assumed to be $u_y = 0.25 f_c'^{2/3}$, based on Shima *et al*. (1987) experimental data. Since wrapping FRP around column bar lap splice increases the compressive strength of concrete, the confined compressive strength of concrete f'_{cc} is used to calculate the bond stress in the yielding zone.

$$u_y = 0.25 f_{cc}'^{2/3} \tag{18}$$

Substituting Eq. (18) into Eq. (17), we obtain Eq. (19) for calculating the length of yielding zone.

$$L_y = (f_s - f_y)\frac{d_b}{f_{cc}'^{2/3}} \tag{19}$$

2.4 Explicit equation for lap splice strength (both elastic and post-yield range)

For a reinforced concrete column with lap splice zone confined by FRP, and with known parameters including the length of lap splice, the concrete cover depth of reinforcing bar, the properties of concrete and steel materials, the characteristics of FRP sheets, and others, it is possible to determine the length of splitting zone, post-splitting zone and yielding zone as functions of the number and thickness of FRP sheets $n_f t_f$ by using Eqs. (10), (15a,b) and (19), respectively. Then the steel stress f_s and corresponding slip s can be computed by Eqs. (6) and (7), respectively. Thus, a steel stress-slip relation can be derived and the lap splice strength is determined as the maximum stress from the calculated relation.

However, rather than constructing the entire steel stress-slip relation, it is also possible to derive explicit equations for the lap splice strength. For known parameters as mentioned above, the post-yield strength of lap splice $[f_s]$ can be calculated from Eq. (20) which is obtained by imposing the condition of Eq. (1) that the total length of the three zones must be equal to the length of lap splice L_s. It is noted that in Eq. (20), the variable L_r can be obtained by solving the Eq. (15a) or Eq. (15b).

$$[f_s] = f_y + \left(L_s - L_r - \sqrt{\frac{E_s d_b}{2}\frac{s_{sp}}{u_{sp}}}\right)\frac{f_{cc}'^{2/3}}{d_b} \tag{20}$$

In case that the bar stress at point B reaches the yield stress ($f_{sp} = f_y$), or the lap splice length is not sufficient to develop the post-splitting zone, the bond stress distribution along the splice length will consist of two zones, that is, yielding zone and splitting zone. In this case, a free slip s_0 must exist at the end point (C) of lap splice length to compensate the slip at point B in order to develop

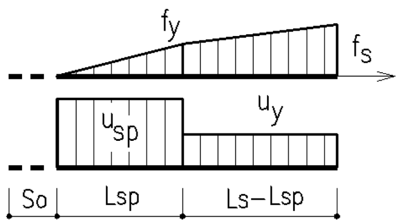


Fig. 6 Free slip at the end of lap splice length

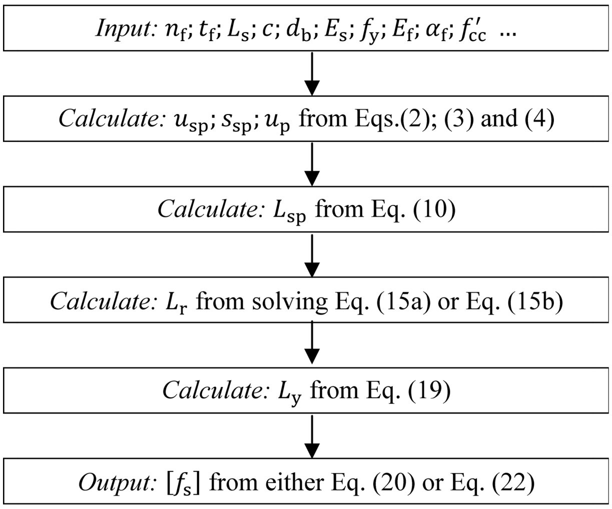


Fig. 7 Calculation steps for lap splice strength

bond splitting strength u_{sp} (Alsiwat and Saatcioglu 1992). Fig. 6 shows the bond and bar stress distribution along lap splice length. The lap splice strength can be estimated by Eq. (21) and finally by Eq. (22), in which L_{sp} is the length of splitting zone which can be computed from Eq. (8) by replacing the terms f_{sp} with yield stress f_y.

$$[f_s] = \frac{4}{d_b}(u_y L_s + (u_{sp} - u_y)L_{sp}) \tag{21}$$

$$[f_s] = \frac{f_{cc}'^{2/3}}{d_b}\left(L_s + \frac{f_y d_b}{f_{cc}'^{2/3}} - 0.25\frac{f_y d_b}{u_{sp}}\right) \tag{22}$$

Fig. 7 shows the flowchart of computational procedure to predict the post-yield strength of lap splice confined by FRP.

It is noted that the discussion above concerns the post-yield strength of lap splice. However, the lap splice strength in the elastic range of reinforcing bar can also be computed by applying one or two parts of the tri-uniform bond stress model. There are two circumstances regarding the strength of lap splice in the elastic range. The first circumstance occurs when the lap splice length is sufficient so that bond stress can be developed to the splitting strength ($L_s \geq L_{sp}$). In this case, the bond stress distribution along the lap splice length will consist of two zones, that is, splitting zone and post-splitting zone. The elastic strength of lap splice is given by Eq. (15c) if $L_s - L_{sp} < L_n$, or Eq. (15d) if $L_s - L_{sp} \geq L_n$. Eqs. (15c) and (15d) are derived from Eqs. (15a) and (15b) by replacing f_y with $[f_s]$. The second situation occurs when the lap splice length is insufficient for the bond stress to reach the splitting strength ($L_s < L_{sp}$). In this case, there is a free slip at the end of lap splice (point C in Fig. 1). There is only the splitting zone in the bond stress distribution and the elastic strength of the lap splice can be calculated by Eq. (23).

$$[f_s] = f_{sp} + \frac{4(L_s - L_{sp})u_{sp}}{d_b}\left(1 - \frac{(\varepsilon_{sp} + \varepsilon_y)(L_s - L_{sp})}{2(0.15c_0 - s_{sp})}\right) \tag{15c}$$

$$[f_s] = f_{sp} + \frac{4(L_s - L_{sp})u_p}{d_b}\left(\frac{c_0 - s_{sp}}{c_0 - s_p} - \frac{(\varepsilon_{sp} + \varepsilon_y)(L_s - L_{sp})}{2(c_0 - s_p)}\right) \tag{15d}$$

$$[f_s] = \frac{4u_{sp}L_s}{d_b} \tag{23}$$

2.5 Calculation procedure for required FRP thickness to reach a desired stress

In the previous section, the equations for predicting the lap splice strength have been derived in both elastic and post-yield ranges of steel bars at the splitting failure. For design purpose, it may be useful to derive equations for computing the required FRP thickness or the amount of FRP sheet for a given desired lap splice strength (or steel stress). Basically, the equations for the required FRP thickness express the inverse relation of the equations for predicting the strength. However, as observed in Eq. (20), the right hand-side of the equation contains a complicated expression for $n_f t_f$, thus, it is very difficult to convert Eq. (20) into the form that explicitly calculates the thickness of FRP as a function of the lap splice length. Alternatively, the required thickness or the number of FRP sheets can be determined by applying the calculation produce as shown in Fig. 8 based on trial and error procedure. The key mechanism of the proposed calculation procedure is to adjust the FRP thickness $n_f t_f$ so that the condition given in Eq. (1) that the sum of lengths of the three zones is equal to the lap splice length is satisfied.

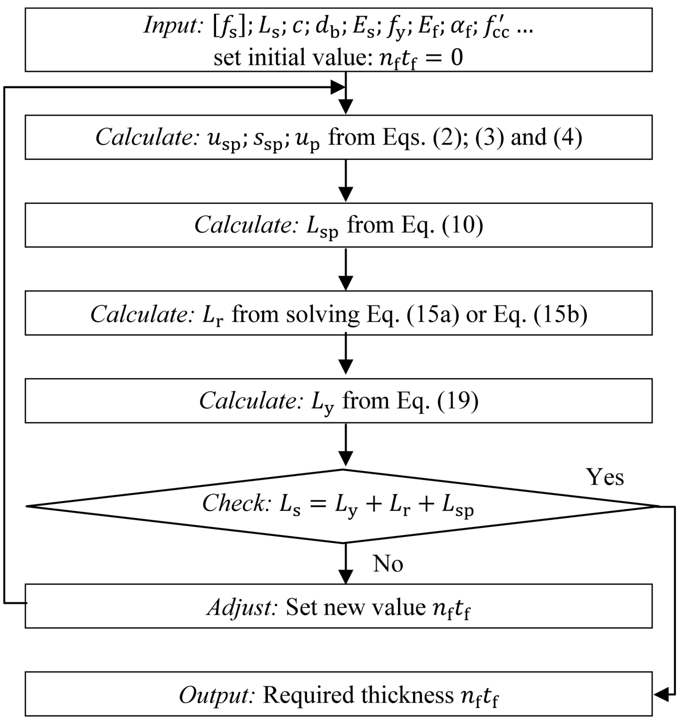


Fig. 8 Calculation procedure for determining the required $n_f t_f$

3. Verification of lap splice strength

In this section, the proposed model for computing the strength of lap splice is verified with experimental data available in literature. It should be noted that there is very little experimental data readily for verification at the constitutive level. On the other hand, there are quite a number of cyclic tests on RC column with lap splices strengthened by FRP (Xiao *et al*. 1997, Harajli *et al.* 2008, Bousias *et al*. 2006). In this paper, the verification of the proposed tri-uniform bond stress model will be conducted at the constitutive level. The verification at the structural level through the cyclic test data of columns will be performed in the companion paper. Here, six RC columns strengthened by FRP sheets and tested by Harajli *et al*. (2008) were adopted for verification. In the test, the peak strength of reinforcing bars was measured. This section applied the proposed tri-uniform bond stress model to calculate the lap splice strength in these columns through explicit equations given in Eq. (20) or Eq. (22). Table 1 shows the comparison between the experimental peak bar stresses and the analytical lap splice strength computed by the model. The verification shows a good agreement between experimental results and analytical ones with a difference within the range of 10%. It should be noted that there was a strange result in experimental data of column C14FP2 confined with 2 FRP layers which had lower strength than that of C14FP1 confined with 1 FRP layer. The analysis, however, predicts a correct trend of increasing lap splice strength with increasing number of FRP layers.

Bousias *et al*. (2006) conducted a test of reinforced concrete columns to investigate the effect of FRP confinement on the lap splice strength. The columns with lap splice length of 15 and 30 times bar diameter were adopted for verification. No direct data on the lap splice strength were reported in the test. Instead, the column lateral strengths were back calculated to obtain the lap splice strength (bar stress at peak load) using the material properties, cross section dimensions and reinforcing details reported in Bousias *et al.* (2006). In these calculations, the elastic modulus of steel was assumed to be 200 GPa. The columns that failed in post-yield range were not used in this verification because the post-yield characteristics of steel bars such as hardening strain, stiffness of yield plateau and hardening range have not been reported in Bousias *et al.* (2006). Table 2 shows a comparison between the experimental results and the analytical ones. As can be seen, the analytical predictions are close to the experimental ones with the average difference of around 5%.

Table 1 Comparison between analytical and experimental splice strengths of columns tested by Harajli *et al.* (2008)

Column	L_s/d_b	c/d_b	f_y	FRP sheets	Exper. $[f_s]/f_y$	Analysis $[f_s]/f_y$	Diff. %
C14FP1	30	1.4	550	1	1.02545	1.05254	2.64
C14FP2	30	1.4	550	2	1.00909	1.11709	10.70
C16FP1	30	2.1	528	1	1.00758	1.03449	2.67
C16FP2	30	2.1	528	2	1.00947	1.09220	8.20
C20FP1	30	1.0	617	1	0.74554	0.71756	-3.75
C20FP2	30	1.0	617	2	1.00000	0.90210	-9.79
						Average	1.78
						S.D	7.57

Table 2 Comparison between analytical and experimental splice strengths of columns tested by Bousias *et al.* (2006)

Column	L_s/d_b	c/d_b	f_y	FRP sheets	Exper. $[f_s]/f_y$	Analysis $[f_s]/f_y$	Diff. %
R-0L1	15	1.17	514	0	0.407	0.372	9.516
R-2L1	15	1.72	514	2	0.706	0.679	3.991
R-5L1	15	1.44	514	5	0.773	0.766	0.950
R-0L3	30	1.44	514	0	0.901	0.866	4.055
						Average	4.63
						S.D	3.57

4. Parametric investigation

The proposed tri-uniform bond stress model for predicting the strength of lap splice confined by FRP is examined as per parametric study to investigate the effect of influencing parameters, such as the ratio of lap splice length to bar diameter L_s/d_b, the ratio of concrete cover to bar diameter c/d_b, the amount of transverse reinforcements and the number and thickness of FRP layers. The hypothetical column section is 200 mm wide × 400 mm deep as shown in Fig. 9. The section is reinforced longitudinally by 8 ϕ14 (deformed bar with 14 mm diameter) and transversely by ϕ8 spaced at 200 mm. The yield strength f_y of bars is 550 MPa and the elastic modulus E_s is 1.96 × 10^5 MPa. The unconfined concrete compressive strength f_c' is 39 MPa. The FRP sheet is 0.13 mm thick and the elastic modulus E_f is 2.30 × 10^5 MPa.

4.1 Strength of lap splice length L_s = 20d_b, 25d_b, 30d_b and 35d_b

The ratio of concrete cover to bar diameter c/d_b varies from 1.0 to 2.4. For each value of c/d_b, the number of FRP sheets is changed in order to assess the effect of FRP thickness on the strength of lap splice. Fig. 10 to Fig. 13 show the relationship between lap splice strength and the number of FRP sheets for four selected values of L_s/d_b. As can be observed in Fig. 10 to Fig. 13, for the columns confined by the same amount of FRP, the lap splice strength is higher in the columns with larger concrete cover. It is also found that the rate of strength increase with respect to the number of FRP layers is faster in the pre-yield range than in the post-yield range. This would be expected because the bond strength in the yielding zone is normally smaller than the elastic zone. In pre-yield range, the strength increase varies linearly with the number of FRP layers. In post-yield range, the

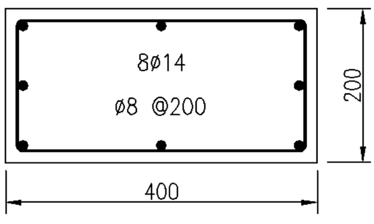


Fig. 9 Column section used in parametrical investigation

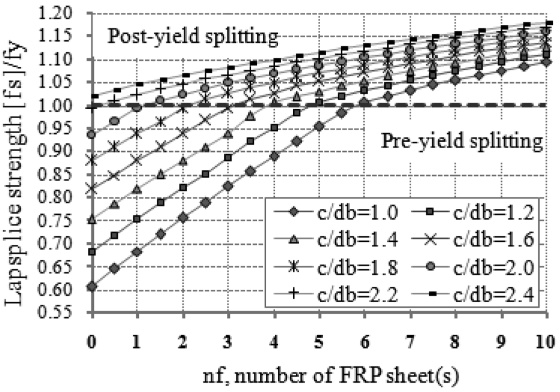


Fig. 10 Strength of lap splice ($L_s/d_b = 20$)

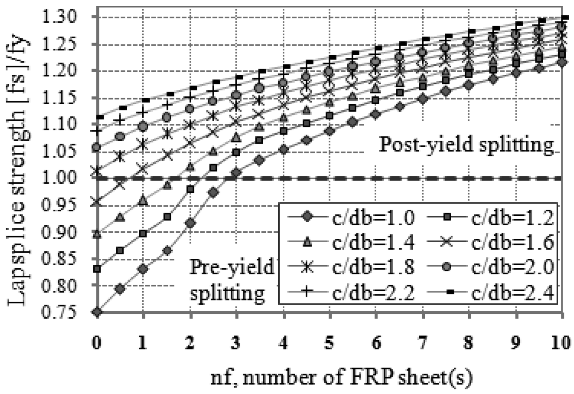


Fig. 11 Strength of lap splice ($L_s/d_b = 25$)

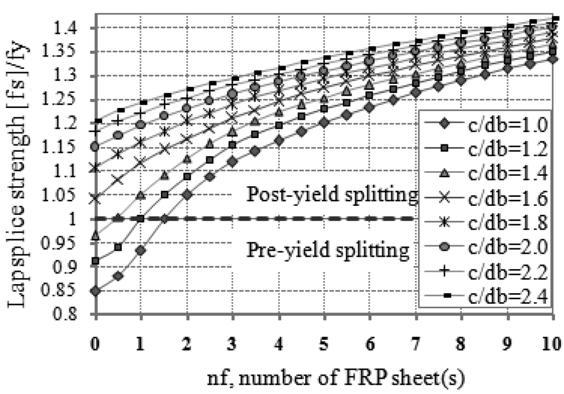


Fig. 12 Strength of lap splice ($L_s/d_b = 30$)

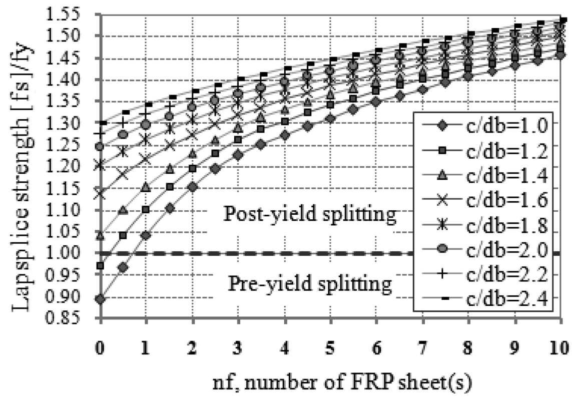


Fig. 13 Strength of lap splice ($L_s/d_b = 35$)

strength increases in a nonlinear decreasing rate owing to the yielding in the bars. These figures are useful for design because the number of FRP layers to develop a required strength for a given lap splice length and ratio of concrete cover to bar diameter can be obtained. It should be noted that the developed lap splice strength predicted by the model is associated with the splitting failure mode only. In reality, the bar may fracture before reaching the post-yield splitting strength and the actual strength of the bar is therefore governed by the yielding strength.

4.2 Required number of FRP sheets to develop a specified lap splice strength

Fig. 14 shows a relationship between a required number of FRP sheets and lap spliced length (L_s) in order to develop the lap splice strength of $1.25f_y$. Here the factor 1.25 is selected to ensure an adequate excess of strength over the bar nominal yield strength. In the graph, eight values of ratio of concrete cover to bar diameter c/d_b are plotted versus the lap splice length of $20d_b$, $25d_b$, $30d_b$ and $35d_b$. As can be seen, the required number of FRP sheets reduces as the lap splice length increases. At the same lap splice length, the required number of FRP layers is reduced as the ratio of concrete cover to bar diameter increases. As can be seen from the graph, the nonlinear relation

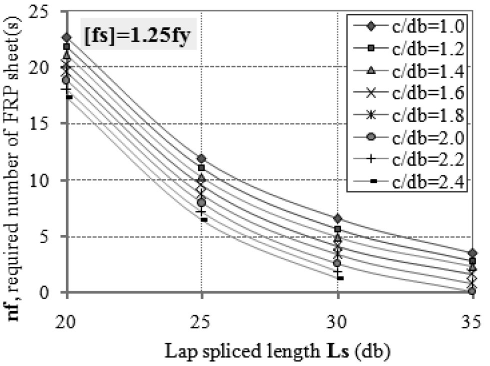


Fig. 14 Required FRP sheets versus lap spliced length

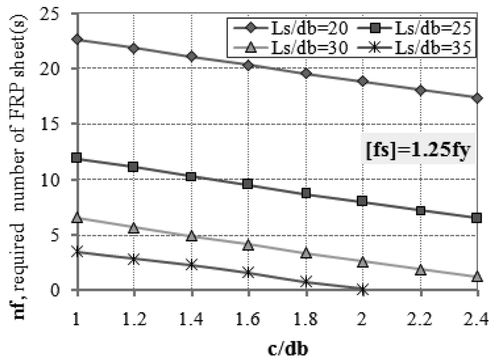


Fig. 15 Required FRP sheets versus c/d_b

between the required FRP layers versus lap splice length is evident. For instance, at $c/d_b = 1.6$, the increase in L_s/d_b from 20 to 30 (1.5 times) results in the decrease in the required number of FRP layers from 20 to 5 (about 4 times). The reduction rate is faster for the short lap splice length. It can be also seen that for any values of L_s/d_b, increasing the lap splice length by $5d_b$ can decrease the number of FRP layers by two times. This shows a high impact of lap splice length to bar diameter ratio on the required number of FRP layers. This prediction seems to qualitatively agree with Bousias's experimental results (Bousias *et al*. 2006). Bousias tested a series of columns with variable lengths of lap splice 15, 30, $45d_b$ and with two and five layers of FRP sheets in each column. They found that FRP significantly increased the strength of lap splice when the length of lap splice was relatively short, say $15d_b$, but did not show a marked capacity improvement in case of longer lap splice (30-$45d_b$).

This implies that the required FRP sheets are highly sensitive to lap splice length when lap splice length is relatively short and not so in case of a longer lap spice length.

Fig. 15 shows the relationship between the required number of FRP sheets to develop $1.25f_y$ strength and ratio of concrete cover to bar diameter for four selected values of L_s/d_b. Basically, the data in Fig. 15 are the same as those in Fig. 14. It can be seen that the required number of FRP layers decreases linearly with the increase in c/d_b. This is rooted in Eq. (2) in which the terms c/d_b and $n_f t_f$ both appear as linear terms inside the parenthesis. Thus, when c/d_b changes linearly, the $n_f t_f$ must also change linearly in an inverse manner to keep the same bond strength u_{sp}. Physically, the terms $n_f t_f$ and c/d_b both refer to the same confining effect. Thus, when the confinement provided by concrete or c/d_b increases, the required confinement from FRP is proportionally decreased.

5. Conclusions

The tri-uniform model for bond stress distribution at critical splitting failure of lap splice in the post-yield range of steel bar was proposed to estimate the lap splice strength confined by FRP sheets in the post-yield range of reinforcing bar. The model consists of three zones, namely, splitting, post-splitting and yielding zones. The bond strength or bond stress-slip relation for the confined lap splice is adopted from previous studies. By imposing the equilibrium condition, strain-slip relation and bond-slip relation in each zone, the entire steel stress-slip relation can be derived, which can be used in the nonlinear frame analysis. Explicit equations for predicting the strength of confined lap splices in both pre-yield and post-yield ranges are derived. The calculation procedure for estimating the required thickness and number of FRP sheets are also outlined which is useful for design purpose. The model has been verified against experimental data available in literature. It is found that the required number of FRP layers to develop a desired strength has a nonlinear relation with the lap splice length and almost a linear relation with the ratio of cover to bar diameter. The length of lap splice is verified to be a main influencing parameter for the strength development.

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Notations

A_s : Area of a reinforcing bar
b_f : Width of a FRP strip
c : Concrete cover depth.
c_0 : Distance between the ribs of the reinforcing bar
d_b : Diameter of reinforcing bar
E_f : Modulus of elasticity of FRP
E_s : Modulus of elasticity of reinforcement
$[f_s]$: Lap splice strength
f_s : Stress of longitudinal reinforcement
f_{sp} : Bar stress at the start of splitting zone
f_y : Yield stress of longitudinal reinforcement
f_c' : Compressive strength of unconfined concrete
f_{cc}' : Compressive strength of confined concrete
L_n : Length of post-splitting zone if slip of point B is equal to s_p
L_s : Length of lap splice
L_y : Length of yielding zone
L_r : Length of post splitting zone
L_{sp} : Length of splitting zone
n_f : Number of transverse FRP layers
N_f : Number of partial FRP strips
n_s : Number of lap splices in tension
s_0 : Local slip factor
s_1 : Local slip at peak pullout mode failure
s_{sp} : Local slip at bond splitting failure
s_p : Local slip at corresponding post-yield stress
t_f : Thickness of one FRP layer
u_e : Bond stress in the elastic zone
u_m : Maximum bond stress at pullout mode
u_p : Post splitting bond stress
u_r : Bond stress in post splitting zone
u_{sp} : Bond stress in splitting zone
u_y : Bond stress in yielding zone
α_f : Factor of partial wrapping
ε_s : Strain of reinforcement
ε_{sh} : Strain of reinforcement at start of strain hardening
ε_{sp} : Strain of reinforcement at start of splitting zone
ε_y : Strain of reinforcement at yielding