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Modeling of progressive collapse of a multi-storey structure using a spring-mass-damper system

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Abstract. A simple mechanical model is proposed to demonstrate qualitatively the pancake progressive collapse of multi-storey structures. The impact between two collapsed storeys is simulated using a simple algorithm that builds on virtual mass-spring-damper system. To analyze various collapse modes, columns and beams are considered separately. Parametric studies show that the process of progressive collapse involves a large number of complex mechanisms. However, the proposed model provides a simple numerical tool to assess the overall behavior of collapse arising from a few initiating causes. Unique features, such as beam-to-beam connection failure criterion, and beam-to-column connection failure criterion are incorporated into the program. Besides, the criterion of local failure of structural members can also be easily incorporated into the proposed model.

Keywords: progressive collapse; impact; virtual spring; virtual damper.

1. Introduction

Progressive collapse occurs when a structure has its loading pattern or boundary conditions changed such that some members are loaded beyond their intended capacities. The residual structure is then forced to seek alternate load paths to redistribute the out-of-balance loads from damaged members. As a result, other neighboring members surrounding the residual structure may also fail shedding some applied loads. The redistribution of loads is a dynamic process and will continue until a new equilibrium position is reached by the residual structure, either through finding a stable alternate load path or through further shedding of loads as a consequence of collapsed members.

The literature reveals that the phenomenon of progressive collapse of buildings is receiving considerable attention in the professional engineering community (Yagob *et al*. 2009). For plane framed structures, several approaches have been proposed to include structural resistance to progressive collapse in building design (Pretlove *et al.* 1991, Virdi and Beshrra 1992, Jones 1995), namely, based on quasi-static approach or based on structural dynamics (Izzuddin *et al*. 2008, Bao *et al*. 2008, Fu 2009, Kim *et al*. 2009). As opposed to a full dynamics analysis, the benefits of a quasi-static approach are twofold. Firstly, the formulation is the same as regular static analysis. Secondly, the computer time required to obtain numerical solutions is very much reduced due to a lack of dynamic terms in the governing equations. However, as pointed out by other researchers, the

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quasi-static approach is may not be conservative because it ignores the impulsive effects of energy released in the process of overloaded members failing and impacting upon the storeys below. The energy released induces dynamic transient loads and displacements far greater than those apparent from a static analysis. From this point of view, the quasi-static approach may omit potential member failures which may occur at lower overloads. As an alternative and possibly more accurate analysis for this purpose, numerical simulations based on structural dynamics have been developed in the past two decades or so (Itoh *et al.* 1994, Isobe and Tsuda 2003, Kaewkulchai and Williamson 2004, Weerheijm *et al.* 2009). It should be noted that details of structural collapse modes are very complex indeed, and their clarification would require considerable computational effort. Moreover, such kind of extensive numerical treatment of progressive collapse may not be required by engineers at the preliminary design stage. Besides, specific threats to multi-storey buildings are often difficult to be pinned down. What engineers really want to find out is the nature of progressive collapse once initiated, and, whether the fall is self-arresting (stable case), or propagating leading to domino effect (unstable case). Yuan and Tan developed a 1-dimensional spring-mass-damper system and explored the effects of a few relevant parameters in the progressive collapse analyses (Yuan and Tan 2003). To make the problem more tractable and amendable by mechanics, this 1-dimensional model adopts some simple assumptions. For instance, in the collapse of a multi-storey structure, each storey is treated as an equivalent component including a mass, a virtual spring and a virtual damper. However, it was reported that columns and beams may not fail simultaneously in reality. An example of progressive collapse shows that a portion of the floor on the 17th storey concrete slab caved in and plummeted downwards, sparking off a domino effect that brought down the 16 slabs below (Lim and Quek 2007). Hence, it is unreasonable to treat a whole storey as one component in all cases. In this study, a new 1-dimensional mass-spring-damper model is proposed. Unlike the previous one, columns and beams can fail separately.

2. Mass-spring-damper system

Generally, multi-storey structures are complex systems. However, as shown in Fig. 1, they can be idealized as a typical frame which consists of three components, viz. columns, beams and beam-to-column connections. To study pancake progressive collapse, such a frame is described by a mass-spring-damper system given in Fig. 2.

In Fig. 2, the letters m, c and k represent mass, damping and stiffness, respectively. x denotes the vertical location of structural members in the coordinate system. The terms $\dot{x}$ and $\ddot{x}$ are the first and second derivatives of x with respect to time t. In the subscripts of these terms, the letters inside the brackets denote a particular storey, while the numerals outside the brackets are used to indicate the properties of these components. In this manuscript, 1, 2 and 3, indicate column-to-column, beam-to-column and beam-to-beam connections, respectively. As an example, the term $k_{2(i+1)}$ represents the stiffness of the beam-to-column spring located at the $(i+1)$th storey.

Based on Fig. 2, the following two dynamical equations can be established

$$\begin{aligned} m_{1(i)}\ddot{x}_{1(i)} + k_{1(i)}(x_{1(i)} - x_{1(i-1)}) + c_{1(i)}(\dot{x}_{1(i)} - \dot{x}_{1(i-1)}) + k_{2(i)}x_{2(i)} + c_{2(i)}\dot{x}_{2(i)} \\ - k_{1(i+1)}(x_{1(i+1)} - x_{1(i)}) - c_{1(i+1)}(\dot{x}_{1(i+1)} - \dot{x}_{1(i)}) = -m_{1(i)}g \qquad (1) \end{aligned}$$

$$m_{2(i)}(\ddot{x}_{1(i)}-\ddot{x}_{2(i)})+k_{3(i)}(x_{1(i)}-x_{2(i)}-x_{1(i-1)}+x_{2(i-1)})+c_{3(i)}(\dot{x}_{1(i)}-\dot{x}_{2(i)}-\dot{x}_{1(i-1)}+\dot{x}_{2(i-1)})$$
$$-k_{2(i)}x_{2(i)}-c_{2(i)}\dot{x}_{2(i)}-k_{3(i+1)}(x_{1(i+1)}-x_{2(i+1)}-x_{1(i)}+x_{2(i)})-c_{3(i+1)}(\dot{x}_{1(i+1)}-\dot{x}_{2(i+1)}-\dot{x}_{1(i)}+\dot{x}_{2(i)})$$
$$=-m_{2(i)}g \tag{2}$$

where g is the acceleration of gravity.

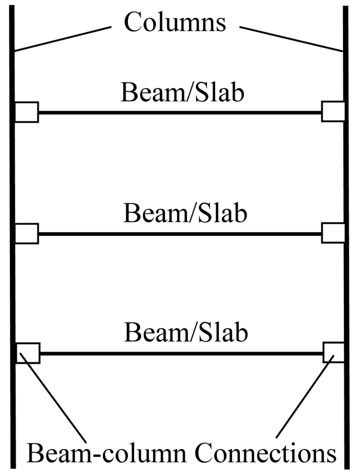


Fig. 1 A typical multi-storey frame

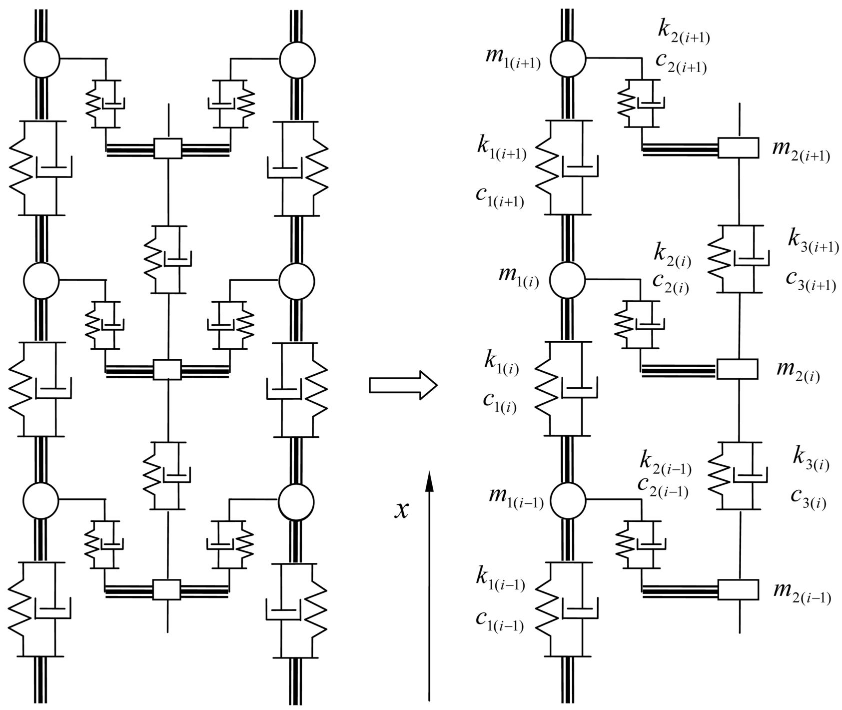


Fig. 2 A multi-storey frame is represented by a mass-spring-damper system

Assuming a structure has N storeys, Eqs. (1) and (2) can be expressed to a familiar form

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{3}$$

where $\mathbf{M} = \begin{pmatrix} {}^1\mathbf{M} \\ {}^2\mathbf{M} \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} {}^1\mathbf{C} \\ {}^2\mathbf{C} \end{pmatrix}$ and $\mathbf{K} = \begin{pmatrix} {}^1\mathbf{K} \\ {}^2\mathbf{K} \end{pmatrix}$. The terms ${}^i\mathbf{M}, {}^i\mathbf{C}$, and ${}^i\mathbf{K}$ $(i = 1, 2)$ are all $N \times 2N$ matrixes and their explicit expressions are given in Appendix I. $\mathbf{X}$ is a $2N \times 1$ vector defined by $\mathbf{X} = (x_{1(1)}\ x_{2(1)}\ x_{1(2)}\ x_{2(2)}\ x_{1(3)}\ x_{2(3)}\ \dots\ x_{1(N)}\ x_{2(N)})^T$. The term $\mathbf{F}$ is also a $2N \times 1$ vector which denotes external forces. If only gravity force is considered, $F_i = -m_{1(i)}g$ and $F_{i+N} = -m_{2(i)}g$ $(i = 1, N)$.

Generally, it is difficult to obtain the analytical solution of Eq. (3). In this study, central difference scheme is applied in time domain to convert Eq. (3) into an algebraic equation given by

$$\mathbf{M}\frac{\mathbf{X}_{(j+1)} - 2\mathbf{X}_{(j)} + \mathbf{X}_{(j-1)}}{\Delta t^2} + \mathbf{C}\frac{\mathbf{X}_{(j+1)} - \mathbf{X}_{(j-1)}}{2\Delta t} + \mathbf{K}\mathbf{X}_{(j)} = \mathbf{F}_{(j)} \tag{4}$$

where Δt is time step. The subscript of $\mathbf{X}$ indicates the number of steps in the course of time. For instance, $\mathbf{X}_{(j)}$ means the solution of $\mathbf{X}$ at time $t_{(j)}$. Based on Eq. (4), one obtains

$$\mathbf{X}_{(j+1)} = \left(\frac{\mathbf{M}}{\Delta t^2} + \frac{\mathbf{C}}{2\Delta t}\right)^{-1}\left[\mathbf{F}_{(j)} - \left(\mathbf{K} - \frac{2\mathbf{M}}{\Delta t^2}\right)\mathbf{X}_{(j)} - \left(\frac{\mathbf{M}}{\Delta t^2} - \frac{\mathbf{C}}{2\Delta t}\right)\mathbf{X}_{(j-1)}\right] \tag{5}$$

Obviously, once the solutions of $\mathbf{X}$ at time $t_{(j-1)}$ and $t_{(j)}$ are both known, $\mathbf{X}_{(j+1)}$ can be calculated. However, one finds that the term $\mathbf{X}_{(-1)}$ must be known priori the evaluation of $\mathbf{X}_{(1)}$. To overcome this numerical difficulty, $\mathbf{X}_{(-1)}$ can be obtained from Eq. (6)

$$\mathbf{X}_{(-1)} = \mathbf{X}_{(0)} - \Delta t\dot{\mathbf{X}}_{(0)} + \frac{\Delta t^2}{2}\ddot{\mathbf{X}}_{(0)} \tag{6}$$

It should be noted that Eq. (6) is derived by eliminating $\mathbf{X}_{(1)}$ from $\ddot{\mathbf{X}}_{(0)} = \mathbf{X}_{(1)} - 2\mathbf{X}_{(0)} + \mathbf{X}_{(-1)}/\Delta t^2$ and $\dot{\mathbf{X}}_{(0)} = \mathbf{X}_{(1)} - \mathbf{X}_{(-1)}/2\Delta t$.

Once the initial conditions of a multi-storey frame are given, say initial displacements, velocities and accelerations are given, $\mathbf{X}_{(-1)}$ can be evaluated directly.

3. Assumptions for stiffness and damping

In a dynamic system established for a multi-storey structure, the impact between different storeys must be considered in a progressive collapse. Unfortunately, it is very difficult to calculate the contact force since this term is closely related to the contact duration and other factors. However, this difficulty can be overcome by employing some assumptions for the stiffness and the damping terms of the equivalent dynamic system. Unlike $m_{j(i)}$ $(i = 1, N \text{ and } j = 1, 2)$, the terms $k_{j(i)}$ and $c_{j(i)}$ $(i = 1, N \text{ and } j = 1, 3)$ are not constants. They are given in Eq. (7)-Eq. (12)

$$c_{1(i)} = {}^e c_{1(i)}[1 - H(u_{1(i)} - u_{1(i)-\min})] + {}^e c_{1(i)}\alpha_{1(i)}H(u_{1(i)} - u_{1(i)-\max}) \tag{7}$$

$$k_{1(i)} = {}^e k_{1(i)}[1 - H(u_{1(i)} - u_{1(i)-\min})] + {}^e k_{1(i)}\beta_{1(i)}H(u_{1(i)} - u_{1(i)-\max}) \tag{8}$$

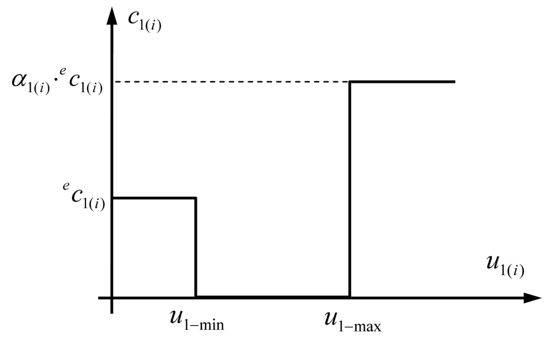


Fig. 3 Assumption for dynamic damping of equivalent column

$$c_{2(i)} = {}^{e}c_{2(i)}[1 - H(u_{2(i)} - u_{2(i)-\min})] \tag{9}$$

$$k_{2(i)} = {}^{e}k_{2(i)}[1 - H(u_{2(i)} - u_{2(i)-\min})] \tag{10}$$

$$c_{3(i)} = {}^{e}c_{3(i)}\alpha_{3(i)}H(u_{1(i)} - u_{3(i)-\max}) \tag{11}$$

$$k_{3(i)} = {}^{e}k_{3(i)}\beta_{3(i)}H(u_{3(i)} - u_{3(i)-\max}) \tag{12}$$

where $u_{j(i)}$ indicates the change of the length of the jth spring located at the ith storey. The terms ${}^{e}c_{j(i)}$ and ${}^{e}k_{j(i)}$ represent the equivalent damping and stiffness for a particular damper and spring, respectively. $H(\cdot)$ is Heaviside function, in which $u_{j(i)-\min}$ and $u_{j(i)-\max}$ are two constants. $\alpha_{1(i)}$, $\beta_{1(i)}$, $\alpha_{3(i)}$ and $\beta_{3(i)}$ are also constant factors and will be explained later.

According to Eq. (7), the damping $c_{1(i)}$ is equal to ${}^{e}c_{1(i)}$ if $u_{1(i)} \leq u_{1(i)-\min}$. However, it becomes zero once $u_{1(i)-\min} < u_{1(i)} \leq u_{1(i)-\max}$. If $u_{1(i)} > u_{1(i)-\max}$, $c_{1(i)}$ takes the value of $\alpha_{1(i)} \cdot {}^{e}c_{1(i)}$. Eq. (7) can also be described in Fig. 3. From Eq. (8), one obtains a similar interpretation for $k_{1(i)}$. In Eqs. (9) and (10), it is defined that the values of $c_{2(i)}$ and $k_{2(i)}$ are ${}^{e}c_{2(i)}$ and ${}^{e}k_{2(i)}$ respectively if $u_{2(i)-\min} < u_{2(i)}$. However, they all change to zero when $u_{2(i)} > u_{2(i)-\max}$ is satisfied. Eqs. (11) and (12) indicate that $c_{3(i)}$ and $k_{3(i)}$ are zero unless $u_{3(i)} > u_{3(i)-\max}$, under which $c_{3(i)} = \alpha_{3(i)} \cdot {}^{e}c_{3(i)}$ and $k_{3(i)} = \beta_{3(i)} \cdot {}^{e}k_{3(i)}$.

Based on Eqs. (7)-(12), the failure of the ith storey of a multi-storey structure can be classified into three categories:

(1) Only $k_{1(i)}$, the value of the stiffness of the column-to-column spring becomes zero under the condition $u_{1(i)-\min} < u_{1(i)} \leq u_{1(i)-\max}$. This is to simulate the case that the ith storey fails to resist any applied loading. At this situation, $c_{1(i)}$ is also set to zero to simulate a free-falling procedure. However, for the case when the distance between the ith and $(i-1)$th storeys approaches to a small value ($u_{1(i)} > u_{1(i)-\max}$ and $u_{1(i)-\max}$ is close the length of the column in the ith storey), which means the two storeys come into contact with each other, $k_{1(i)}$ takes on the value of $\beta_{1(i)} \cdot {}^{e}k_{1(i)}$ where $\beta_{1(i)}$ may be a large number. Meanwhile, $c_{1(i)}$ becomes $\alpha_{1(i)} \cdot {}^{e}c_{1(i)}$ where $\alpha_{1(i)}$ is large. The purpose of assigning large values to $\beta_{1(i)} \cdot {}^{e}k_{1(i)}$ and $\alpha_{1(i)} \cdot {}^{e}c_{1(i)}$ is to simulate the impact between the ith and $(i-1)$th columns. Theoretically, a stiff spring (large $\beta_{j(i)}$) can transfer the load from the upper storey to the lower storey. The large value of α is to ensure the velocities of the two storeys are tied together so that the ith and $(i-1)$th storeys stack up together after their initial contact,

(2) Only $k_{2(i)}$, the value of the stiffness of the beam-to-column spring becomes zero under the condition $u_{2(i)-\min} < u_{2(i)}$. This is to simulate the failure of the beam-to-column connection at the ith storey. In this situation, $c_{2(i)}$ is also zero. This is to guarantee that the ith slab becomes a free-falling object before it impacts onto the $(i-1)$th slab. If the ith and $(i-1)$th storeys contact each other ($u_{3(i)} > u_{3(i)-\max}$), $k_{3(i)}$ and $c_{3(i)}$ become $k_{3(i)} = \beta_{3(i)} \cdot {}^{e}k_{3(i)}$ and $c_{3(i)} = \alpha_{3(i)} \cdot {}^{e}c_{3(i)}$, respectively. Similar to above discussion, $k_{3(i)}$, a very stiff spring can be used to simulate the impact between two slabs, while $c_{3(i)}$, a damper with large damping can be used to tie the two slabs together after the impact.
(3) Both $k_{1(i)}$ and $k_{2(i)}$ fail. This is a combination of (1) and (2).

4. Numerical examples

A 9-storey frame with a local damage at the 5th story is taken as a numerical example. The consequent dynamic response of the frame will be investigated. To make the simulations more reasonable, a realistic steel frame is designed to withstand conventional dead and live loads. For simplicity, the geometric dimensions and material properties of all stories are the same. The relevant information of the structure is listed in Table 1.

In this example, an equivalent stiffness ${}^{ref}k_{(i)}$ $(i = 1, 9)$ is defined by ${}^{ref}k_{(i)} = 2E_{(i)} \cdot {}^{c}A_{(i)} / {}^{c}L_{(i)} = 29.25 \times 10^{7}$ N/m for reference where $A_{(i)}$ and $E_{(i)}$ are the area of the column's cross-section and the elastic modulus of the material, respectively. The term ${}^{c}L_{(i)}$ denotes the column length. Meanwhile, the uniformly distributed load applied to each story is set to $q = 8$ kN/m. Hence, the total mass of the ith story is assumed to be $m_{1(i)} + m_{2(i)} = 4.9 \times 10^{3}$ kg $(i = 1, 9)$. In this manuscript, it is assumed without loss of generality that $m_{1(i)}/m_{2(i)} = 0.25$, which means 20% of the whole mass of each storey is assigned to columns and the remaining 80% is assigned to slab. As a reference, a critical load for the ith story is defined to be ${}^{ref}P_{(i)-cr} = 8\pi^{2} E_{(i)} {}^{c}I_{(i)} / {}^{c}L_{(i)}^{2}$. It can be seen that ${}^{ref}P_{(i)-cr} = 1.2 \times 10^{7}$ Pa is equal to the Euler buckling load of the two columns with fixed ends. The use of ${}^{ref}P_{(i)-cr}$ is to define $P_{j(i)-cr}$, the load-bearing capacity for each type of virtual spring. Based on $P_{j(i)-cr}$, the value of $u_{j(i)-\min}$ appeared in Eqs. (7)-(12) can be calculated by $u_{j(i)-\min} = P_{j(i)-cr} / {}^{e}k_{j(i)}$. In the study, it is assumed without loss of generality that $u_{1(i)-\max} = 0.98 {}^{c}L_{(i)}$.

In total, six typical scenarios listed in Table 2 are studied. Among the case studies, the cause of the collapse in each of the first 3 scenarios is the failure of the 5th storey beam-to-beam connection. Both the columns and the beam-to-column connections are stiff, but the load-bearing capacity of the beam-to-column connections varies. In each of the last 3 scenarios, the collapse of the frame is induced by the failure of the 5th storey column. The bearing capacity of the beam-to-column

Table 1 Member and material properties of steel frame

Column	Beam
UC 152×152×23	UB 254×146×31
${}^{c}A = 29.25$ cm^{2}	${}^{b}A = 39.69$ cm^{2}
${}^{c}I = 1249.8$ cm^{4}	${}^{b}I = 4413.41$ cm^{4}
${}^{c}L = 4.0$ m	${}^{b}L = 6.0$ m
$E = 2.0 \times 10^{11} P_{a} (20^{\circ}\mathrm{C})$	$E = 2.0 \times 10^{11} P_{a} (20^{\circ}\mathrm{C})$

Table 2 Parameters used in case studies

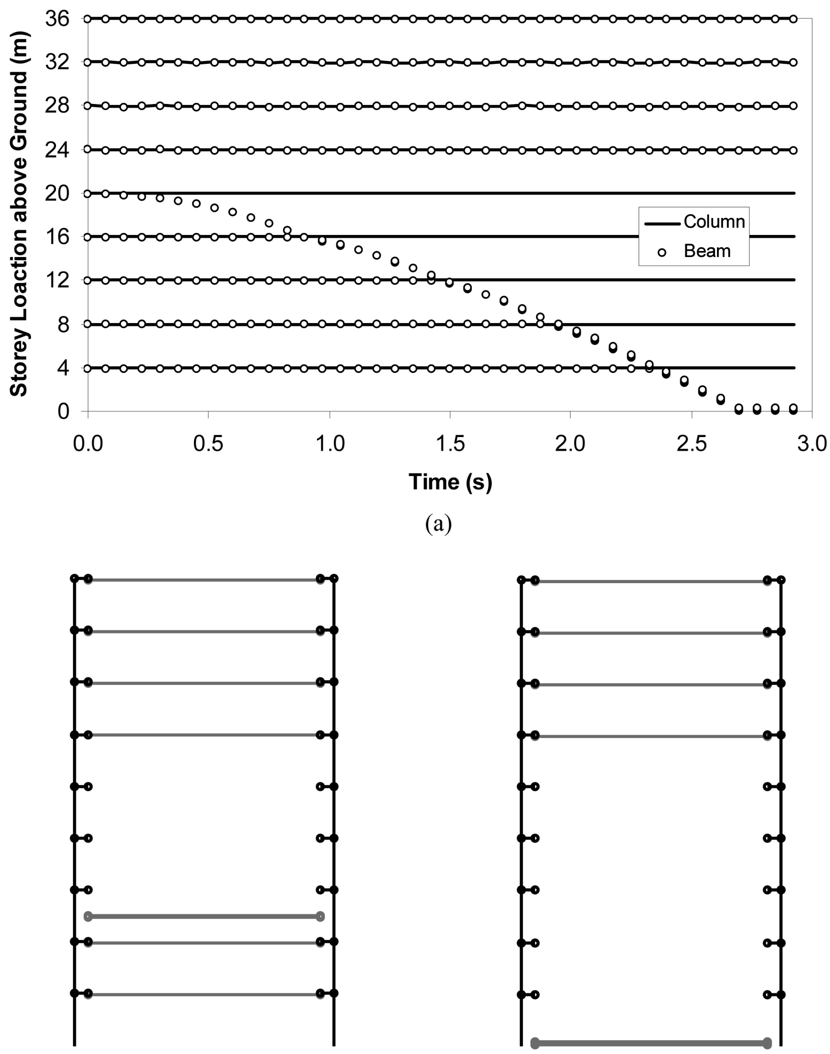
Scenarios		Parameters	
Cause of failure	Case study	$k_{j(i)}$ (i = 1, 9 and j = 1, 3)	$p_{j(i)-cr}$ (i = 1, 9 and j = 1, 3)
The 5th Storey beam-to-column connection fails	1	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = {}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = 0.1{}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$
	2	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = {}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$
	3	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = {}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = 0.2{}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$
The 5th storey column fails	4	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = 0.1{}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = 0.8{}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$
	5	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = 0.5{}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = 0.8{}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$
	6	$k_{1(i)} = {}^{ref}k_{(i)}$ $k_{2(i)} = 0.3{}^{ref}k_{(i)}$ $k_{3(i)} = 0$	$p_{1(i)-cr} = {}^{ref}p_{(i)-cr}$ $p_{2(i)-cr} = 0.8{}^{ref}p_{(i)-cr}$ $p_{3(i)-cr} = \infty$

connections is constant, but their stiffness changes.

Scenario 1: Stiff beam-to-column connection with low load-bearing capacity

The result is depicted in Fig. 4. It can be seen that all columns remain intact after the failure of the beam-to-column connection at the 5th storey. However, all the beams beneath the 5th storey collapse due to domino effect. Firstly, all columns are very strong and they can resist the impact when the 5th storey impacts onto the 4th storey. Secondly, $P_{2(i)-cr}$, the load-bearing capacity of beam-to-column connection is only 10% of the load-bearing capacity of column-column connection. It is not enough to remain intact once collapse occurs. The curves in Fig. 4 show that the 5th storey coalesces onto the 4th storey within one second after the collapse starts, then the impact causes the failure of the 4th beam-to-column connection. After that, both the two damaged storeys fall together as one block and impact onto the 3rd storey. The collapse of beams cannot be stopped once it is initialed since the impact between the upper and lower storeys becomes more serious due to the accumulation of masses as the collapse progresses.

Certainly, the load-bearing capacities of columns are affected by beams connected to them. For instance in Fig. 4(b), the columns running from the 1st to the 5th storeys become one very long



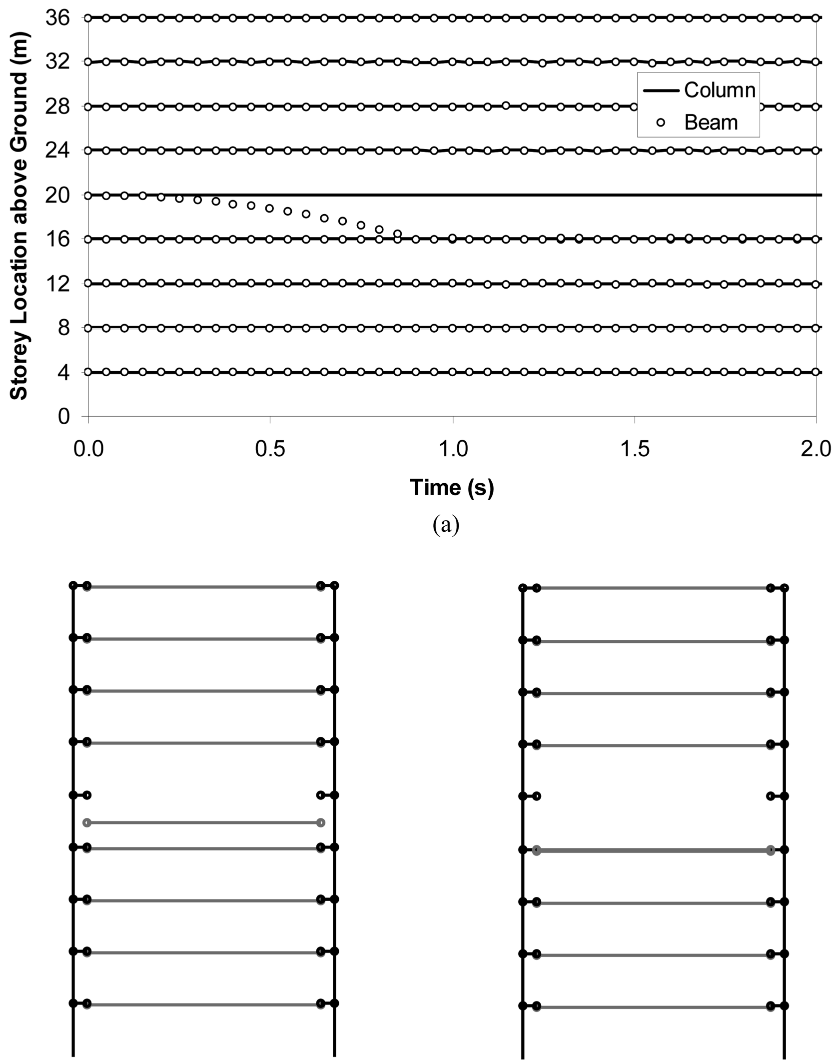
(a)

(b)

Fig. 4 Simulation of collapse for Scenario 1

column if all the beam-to-column connections beneath the 5th storey fail. In this situation, the critical load for stability of the frame is reduced significantly. Hence, the critical load for columns must be modified. In this manuscript, assuming that all beam-to-column connections between the *i*th and the *j*th storeys lose load-bearing capacities, the modified critical load for the *k*th column denoted by ${}^{mod}p_{1(k)-cr}$ is defined to be ${}^{mod}p_{1(k)-cr} = p_{1(k)-cr}/(j-i)^{\gamma}$, where $i<k<j$ and the term γ is a constant. For simplicity, γ is set to 1.5 arbitrarily. Certainly, further research is needed to determine the expression of ${}^{mod}p_{1(k)-cr}$, but this study only focuses on providing a numerical framework for simulation of progressive collapse.

Scenario 1 indicates that in the case of a beam-to-column connection failure, column failure may not occur if the load-bearing capacity of beam-to-column connections is very low. In fact, this



(a)

(b)

Fig. 5 Simulation of collapse for Scenario 2

observation has happened in reality. It was reported that the columns in a building were kept intact when all 16 floors crushed through the building in a domino effect to slam onto the ground due to the falling of a portion of the floor on the 17th storey (The Straits Times 2007). Although this real example of progressive collapse was caused by the failure of slab-column connection instead of beam-to-column connection, the reasonability of the proposed simulation is verified since slab-column connections can be modeled by the same approach.

Scenario 2: Stiff beam-to-column connection with high load-bearing capacity

Only the beam-to-column connection at the 5th storey fails at the beginning. However, as shown in Fig. 5, this does not lead to subsequent progressive collapse. Compared with Scenario 1, the stiffness

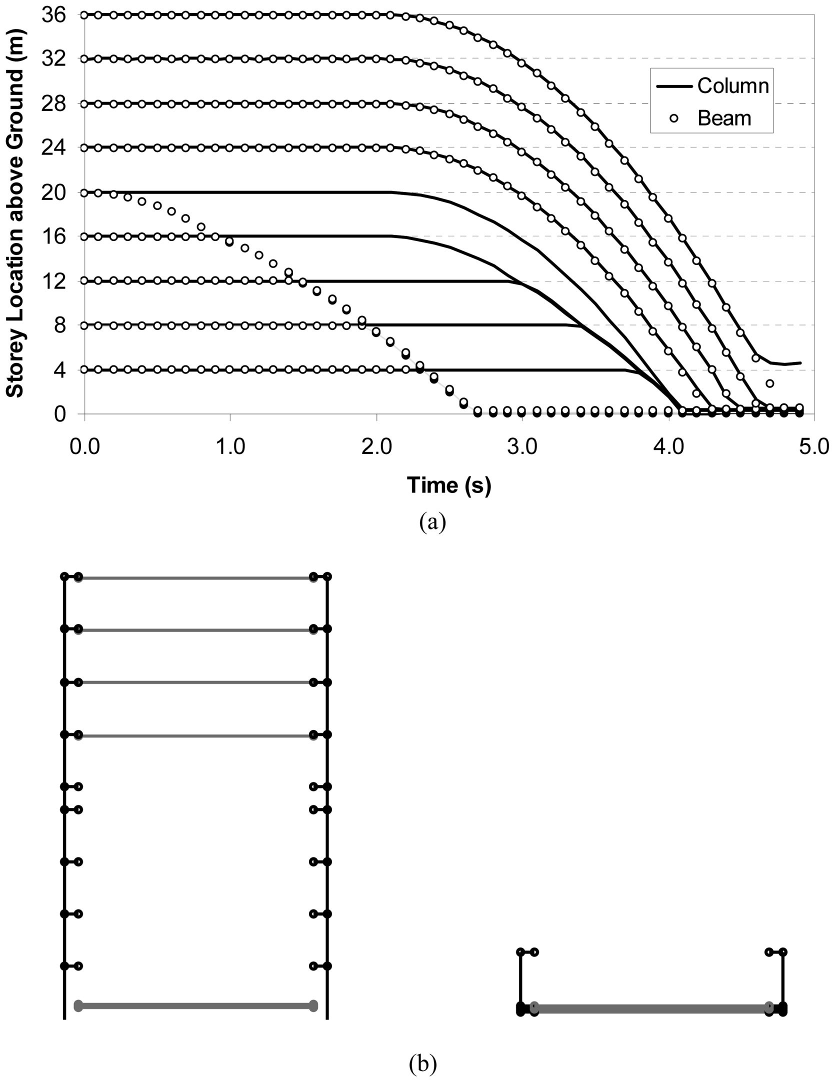
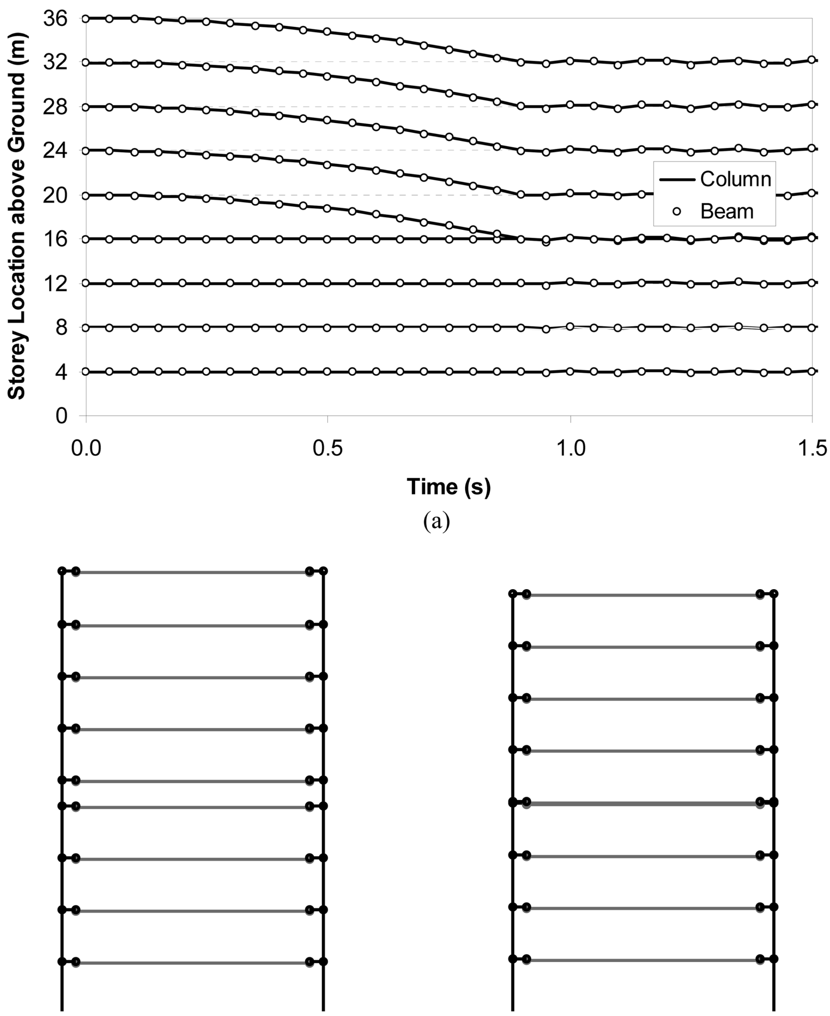


Fig. 6 Simulation of collapse for Scenario 3

of beam-to-column connection does not change, but its bearing capacity is ten times higher than that in Scenario 1. Since the beam-to-column connection is much stronger, they are not easily damaged. Hence, the falling of the 5th storey is stopped by the 4th storey. At the same time, the impact between the 5th and 4th beams is transferred to columns, but the frame can still remain intact.

Scenario 3: Stiff beam-to-column connection with moderate load-bearing capacity

Only the beam-to-column connection at the 5th storey fails at the beginning. However, as shown in Fig. 6, the columns also collapse due to the impact between beams. Compared with Scenarios 1 and 2, the stiffness of beam-to-column connection does not change, but its bearing capacity is two times higher than that in Scenario 1. After the 5th storey beam hits on the 4th storey beam, the 4th storey beam-to-column connection fails due to impact since it is not as strong as that in Scenario 2.



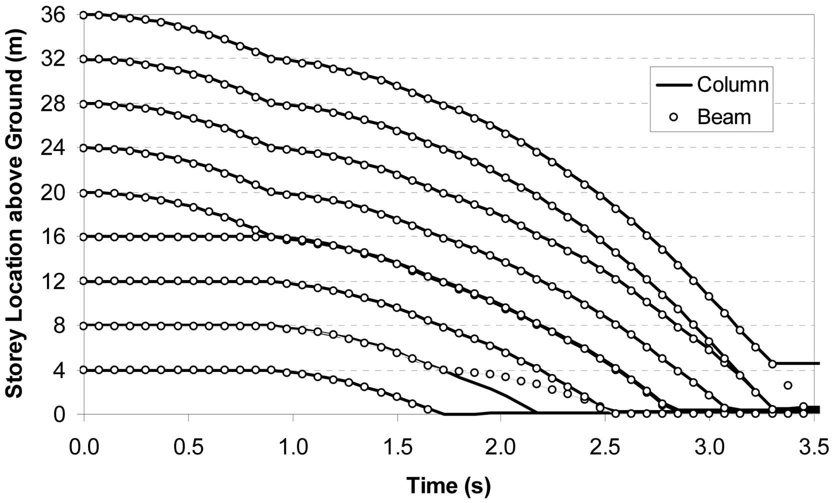
(a)

(b)

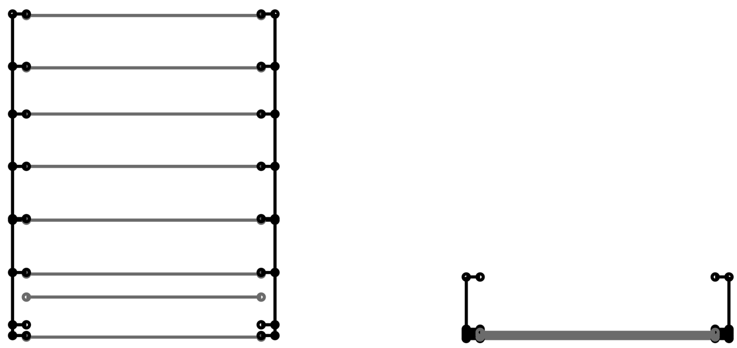
Fig. 7 Simulation of collapse for Scenario 4

Following this failure, all beams below the 4th storey experience a domino collapse. At the same time, the bearing capacities of columns are weakened progressively after the beam-to-column connections fail. Without these beams, columns at different storeys turn out to be a ultra slender member. Theoretically, this slender column has very low buckling load under compression. In Fig. 6, due to the failures of the beam-to-column connections, columns at the 3rd, 4th and 5th storeys merge to form a long column but still remain intact. After that, this long column is further weakened when the 2nd storey beam-to-column connection fails. Subsequently, the whole structure collapses when the 2nd storey impacts onto the 1st storey.

Based on the analyses about Scenario 1, 2 and 3, one concludes that in the case of a beam-to-column connection failure, the bearing capacities of beam-to-column connections dominate the ensuing structural response of the multi-storey frame. It can be seen that progressive collapse of



(a)



(b)

Fig. 8 Simulation of collapse for Scenario 5

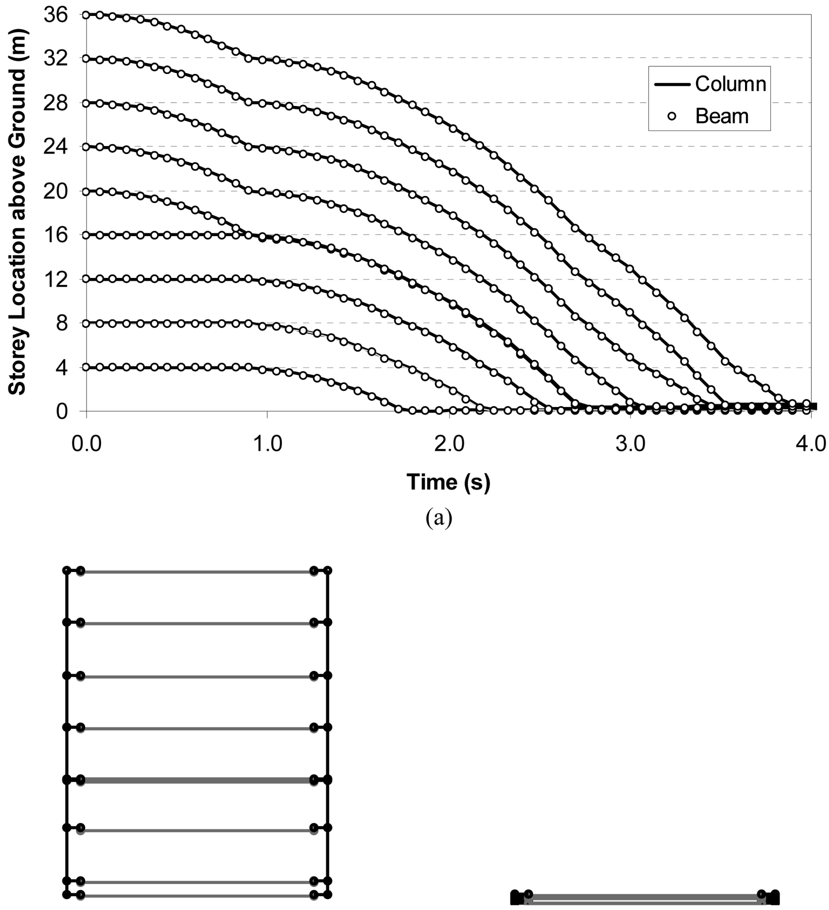
beams occur but it does not cause column failure if beam-to-column connections are very weak. Generally, as shown in Scenario 2, it is believed that beam-to-column connections with high ultimate strength are helpful to prevent progressive collapse. However, one should apply this concept restrainedly in structural design due to the observation in Scenario 3. In this scenario, the beam-to-column connections with higher ultimate strength lead to a completed collapse as compared with Scenario 1.

Scenario 4: Very flexible beam-to-column connection with high load-bearing capacity

In this scenario, the local damage is the failure of the 5th storey column. Although the beam-to-column connections are much more flexible than the columns, the curves in Fig. 7 show that the progressive collapse does not happen.

Scenario 5: Stiff beam-to-column connection with high load-bearing capacity

The local damage is the same as in Scenario 4. Compared with Scenario 4, the bearing capacity



(a)

(b)

Fig. 9 Simulation of collapse for Scenario 6

of beam-to-column connection does not change, but its stiffness is five times greater than that in Scenario 4. From Fig. 8, one finds that the whole structure collapses. It can be seen that some beam-to-column connections also fail during the collapse.

Scenario 6: Flexible beam-to-column connection with high load-bearing capacity

The local damage is the same as in Scenarios 4 and 5. However, the beam-to-column connections are stiffer than that in Scenario 4, but more flexible than that in Scenario 5. Similar to Scenario 5, the whole structure collapses due to impact as shown in Fig. 9. However, it should be noted that no beam-to-column connection fails as the collapse progresses.

Scenarios 4, 5 and 6 are used to investigate the cases in which a column fails at the beginning. Numerical results reveal that flexible beam-to-column connections are helpful to mitigate the risk of progressive collapse. It is found that stiffer beam-to-column connections, under particular bearing capacities, make the structure more critical.

5. Conclusions

This manuscript proposes a numerical model to simulate progressive collapse of multi-storey buildings. The model is established based on simplifying assumptions. Although such a model can hardly make detailed analyses at the member level, it can be used to study the overall behavior of buildings. For instance, several observations have been obtained through the numerical examples: (1) a local member failure in a multi-storey structure may lead to a global progressive collapse. (2) if the initial local damage is the falling of a beam, beam-to-column connections with high bearing capacities are not always good. (3) if the initial local damage is the failure of a column, flexible beam-to-column connections are more helpful to structural safety than stiff beam-to-column connections when they have same bearing capacities. (4) in dynamic cases, flexible structures with high strength have advantages to keep away from progressive collapse.

Obviously, the proposed model is far from a mature approach. To make it more substantial, it is suggested that the future work focuses on the following areas: (1) Develop a method to determine the ultimate loads of columns and beams under various conditions, and then incorporate the criteria into the proposed model. (2) Extend the present model to 3-dimensional problems. (3) D Propose a proper definition of failure. Failure defined in terms of critical load or displacement of single storey affects the structural response significantly. However, what kind of criteria should be employed has not been validated yet. Although quite a number of attempts have been made to define suitable unified criteria of structural failure (Colombo and Negro 2005, Stylianidis *et al*. 2009), it must be pointed out that the applicability of those criteria may not be generalized. (4) Propose a proper method to determine the key parameters to simulate the impact between different storeys. In this study, the contact dynamics is modelled by using virtual springs and dampers. By proper definition of key parameters including $k_{j(i)}, c_{j(i)}$ and $u_{j(i)-\max}$, impact behaviour such as rigid contact and plastic contact can be simulated reasonably. However, it is still necessary to develop a unified approach for the determination the values of the key parameters. Recently, a new design-oriented methodology for progressive collapse assessment of floor systems has been proposed and calibrated (Vlassis *et al.* 2009). It is possible to incorporate that approach into the present model to study the nature of impact event. (5) Experiment. Structural small scale test may be conducted to verify the current approach. Some key parameters such as damping and stiffness should be obtained through dynamic tests and should be further incorporated into the study for more realistic numerical simulations.

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