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Mode-by-mode evaluation of structural systems using a bandpass-HHT filtering approach

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Abstract. This paper presents an improved version of the Hilbert-Huang transform (HHT) for the modal evaluation of structural systems or signals. In this improved HHT, a well-designed bandpass filter is used as preprocessing to separate and determine each mode of the signal for solving the inherent mode-mixing problem in HHT (i.e., empirical mode decomposition, EMD, associated with the Hilbert transform). A screening process is then applied to remove undesired intrinsic mode functions (IMFs) derived from the EMD of the signal's mode. A "best" IMF is selected in each screening process that utilizes the orthogonalization coefficient between the signal's mode and its IMFs. Through mode-by-mode signal filtering, parameters such as the modal frequency can be evaluated accurately when compared to the theoretical value. Time history of the identified modal frequency is available. Numerical results prove the efficiency of the proposed approach, showing relative errors 1.40%, 2.06%, and 1.46%, respectively, for the test cases of a benchmark structure in the lab, a simulated time-varying structural system, and of a linear superimposed cosine waves.

Keywords: bandpass filter; Hilbert-Huang transform; modal frequency; orthogonalization coefficient; signal filtering.

1. Introduction

Since the parameters of the natural modes, i.e., the mode shape, frequency and damping, are products of a structure, of its mass and stiffness (Hearn and Testa 1991), the modal frequencies of structures have been extensively studied (Kibboua *et al*. 2008, Ni *et al*. 2007). A different approach to the modal analysis can be found in the Hilbert-Huang transform (HHT), i.e., the empirical mode decomposition (EMD) associated with the Hilbert transform (Huang *et al*. 2003, Huang *et al*. 1998). The invention of HHT has received a wide variety of attention and application in many fields in the past decade. For example, Schlurmann (2002) applied the HHT to spectral frequency-analysis of nonlinear transient water waves. Huang *et al*. (2003) applied the HHT to non-stationary financial time series analysis. Duffy (2004) illustrated the advantages of applying Hilbert-Huang transforms to signals that include synoptic as well as climatic signals. After EMD of the measured signal, intrinsic mode functions (IMFs) are obtained, each of which represents dynamic information of oscillation for the system. Using the Hilbert transform to convert the appropriate IMFs into the frequency domain in a time-frequency-amplitude display makes the HHT more sophisticated and complete.

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However, the HHT methodology is difficult to detect the non-linear and non-stationary characteristics of the signal with respect to some aspects. It was pointed out that there is a problem of IMF mode rectifications (Huang and Shen 2005). Straightforward implementation of the EMD procedure will produce mode-mixing. While relying on EMD to resolve the signal, the IMFs obtained may encompass a large frequency range, thereby making it impossible to attain a signal with a single characteristic of interest. Hence, an improved version of the HHT was proposed that uses a wavelet packet transform method to preprocess the original signal in order to resolve it into narrow frequency groups with different ranges, thereby giving each IMF a single component (Peng *et al*. 2005). Other works regarding the refinement of the HHT, for instance, was developed by Cheng *et al*. (2006), who utilized the energy difference tracking method to define and screen the IMF obtained from the EMD according to the integrity and orthogonality characteristics of each IMF. This method provides a cutoff criterion for EMD in the screening process and saves calculation time while screening IMFs. Messina and Vittal (2007) examined IMFs over a large range and utilized the EMD, along with proper orthogonal decomposition, to determine the correlation between the energy of all IMFs and the phase of the data in order to find the "best" location within the range. This work eliminates information unrelated to the system and further improves the performance of the EMD for catching the significant IMF of the observed signal in the event of a sudden change. Lin *et al*. (2008) worked on the mode-mixing problem of the IMFs in EMD when resolving the original signal, and they used a bandpass filter to separate the blended modes when necessary in a data preprocessing procedure. This data preprocessing procedure is sufficient to answer the question that if another filtering technique is required to solve the mode-mixing problem of the HHT, why not filter any noisy components from the beginning before performing the EMD. Nevertheless, there have been few generalized and convenient criterions for the selection of IMFs that best represent structural characteristics of interest, along with the applicability of such criterions to other systems or signals.

In this context, a signal filtering technique at the data preprocessing stage to alleviate the mode-mixing problem of the HHT is proposed, followed by a signal reconstruction technique, which is based on the HHT, for the modal evaluation of structural systems or signals. First, a benchmark structure in the lab (Lin and Chen 2009) is evaluated, by using a shear-building model for a theoretical estimation. Next, the resulting HHT-based filtering technique is applied to evaluate the natural frequency of the structure and compare the value obtained with the theoretical value. The filtering technique comprised the following steps (1) designing a bandpass filter for separating structural modes; (2) the modal signal reconstruction for extracting a signal with a single characteristic of interest and for filtering out noisy components to a higher degree; and (3) evaluation of modal frequency in the time-frequency domain or in the frequency spectrum. Finally, a simulated time-varying structural system and a linear sum of two cosine waves are tested using the proposed HHT-based bandpass filtering technique. Compared to other existing approaches, the proposed filtering technique is similar to the procedure of Peng *et al*. (2005) aforementioned, which solved the mode-mixing problem of the HHT at the data preprocessing stage, but differs not only in treating the modes sequentially but also in the modal signal reconstruction procedure. In addition, the proposed filtering technique distinguishes from other works, such as the previous studies using "energy" methods, by its convenient criterions for the screening process of IMFs. It also provides a refined version of the bandpass filter and wider applicability to other systems or signals, focusing on the efficiency of the mode separation, when compared to the work of Lin *et al*. (2008).

2. Evaluation of a benchmark structure using a shear building model

First, a benchmark problem of a three-story 30% scaled steel structural system (Fig. 1) subjected to one-directional excitations is evaluated. Principal signals ($\Delta t = 0.005$ s within 1201 data points) of 2 seismic excitations, i.e., 1940 El Centro (California) and 1999 Chi-Chi (TCU084, Taiwan), are used to evaluate structural responses in the lab. The structure contains a slab-beam-column connection for each floor (degree of freedom). It is 1.1 m long and 1.1 m wide, each story is 0.9 m high for a total height of 2.7 m, and the cross-sections of the beams and columns are H 100 * 50 * 5 * 7 mm. Estimates of the structural masses can be obtained from the geometry and materials of the structure for the first, second, and third floors as 530.65 kg, 530.65 kg, and 514.67 kg, respectively.

Evaluation of structural parameters, such as the natural frequency, was first conducted by considering the scaled steel structure as a shear-building model. By using Eq. (1) and Eq. (2) (Chopra 1995, Chen and Ricles 2010, Udwadia 2005), the stiffness of the structure, k, for the first, second, and third floors was calculated as 1933431.73 N/m, 1933431.73 N/m, and 1933431.73 N/m, respectively.

$$k_j = \sum_{\text{columns}} \frac{12EI}{l^3} \tag{1}$$

Fig. 1 A benchmark problem of a three-story 30% scaled steel structure

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{2}$$

where k_j is the story stiffness, $\mathbf{K}$ is the stiffness matrix, l represents the height of the floor, E represents the modulus of elasticity, and I denotes the second moment of the cross-sectional area.

By transferring the values of the mass and the calculated stiffness of each floor into Eq. (3) (Chopra 1995), the values of the structural frequencies obtained were 4.31078 Hz, 12.0441 Hz, and 17.6073 Hz for the first, second, and third modes of the structure, respectively.

$$\det\left|\mathbf{K} - \mathbf{M}\omega_n^2\right| = 0 \tag{3}$$

where "det" represents the determinant of a matrix, $\mathbf{K}$ represents the stiffness matrix, $\mathbf{M}$ denotes the diagonal mass matrix of the structure, and ω_n represents the natural frequency of the structure.

3. Evaluation of a benchmark structure using the HHT-based filtering approach

The proposed HHT-based filtering technique is applied to evaluate the natural frequency of the structure in comparison with the theoretical value determined above by using a shear-building model. The HHT (Hilbert transform after EMD) technology is a set of superior algorithms for analyzing nonlinear and nonstationary signals, but it also provides increased accuracy for analyzing linear and stationary signals (Huang *et al.* 2003, Huang *et al.* 1998).

The Hilbert transform for an arbitrary time series $X(t)$ can be represented by $Y(t)$ as

$$Y(t) = \frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{X(t')}{t-t'}dt' \tag{4}$$

where P denotes the Cauchy principal value in Eq. (4), and the associated analytic signal $Z(t)$ is described by

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} \tag{5}$$

where

$$a(t) = [X^2(t) + Y^2(t)]^{1/2}, \quad \theta(t) = \arctan\left(\frac{Y(t)}{X(t)}\right) \tag{6}$$

By using these expressions, the time-frequency-amplitude can be adequately represented, because the phase $\theta(t)$ is a function of time t; thus, frequency will be a function of time by taking the derivative of $\theta(t)$ to give the instantaneous frequency (Huang *et al.* 1998) as

$$\omega(t) = \frac{d\theta(t)}{dt} \tag{7}$$

The EMD (Huang *et al.* 1998) of the signal $X(t)$ is used to resolve the signal as

$$X(t) = \sum_{k=1}^{n} c_k(t) + r_n(t) \tag{8}$$

where $c_k(t)$ $(k = 1, 2, \ldots, n)$ are the IMFs and $r_n(t)$ is the residue of $X(t)$. After performing the Hilbert transform on each IMF by using Eqs. (4)-(8), one can express the signal $X(t)$ (Huang *et al.* 1998) as

$$X(t) = \sum_{k=1}^{n} a_k(t)\exp\left(i\int\omega_k(t)dt\right) \tag{9}$$

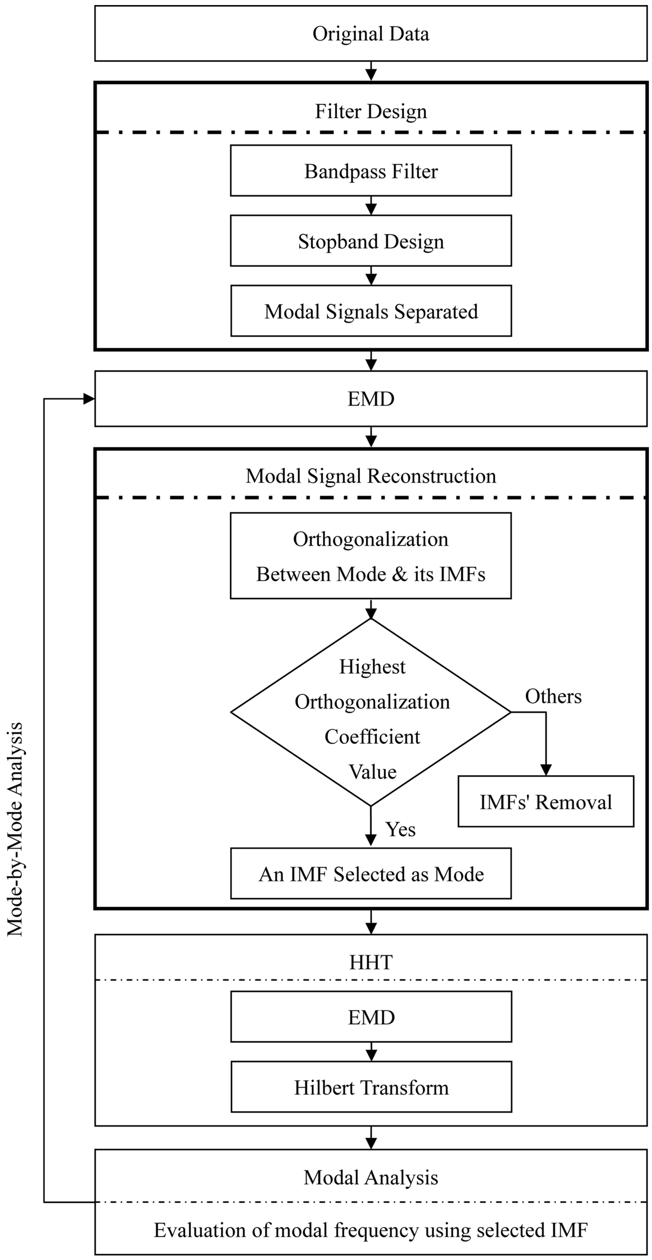

Fig. 2 The overall approach of the signal filtering, and HHT-based signal reconstruction techniques and analyses developed for the modal evaluation of systems or signals

Using Eq. (9), it is possible to represent the variable amplitude $a_k(t)$ on the time-frequency plane to yield the Hilbert spectrum $H(\omega, t)$, which can be integrated with respect to time to give the marginal spectrum as

$$h(\omega) = \int_0^T H(\omega, t)dt \tag{10}$$

The marginal spectrum provides a measure of total amplitude (or energy) contribution from each frequency value (Huang *et al.* 1998).

Because the IMFs of a signal derived from the EMD will often engender the mode-mixing problem for a relatively large range of the frequency spectrum, a designed bandpass filter is first utilized to separate the modes of the signal. Resolution of the original signal now converts to the resolution for each mode. The corresponding IMFs are obtained mode by mode. Next, IMFs for each mode can be filtered further, using the orthogonalization coefficient to select a proper IMF in order to reconstruct the modal signal. Finally, the reconstructed modal signal is used to evaluate the frequency of each mode, which can be represented in the time-frequency domain. Fig. 2 illustrates the overall approach of the signal filtering, and signal reconstruction techniques and analyses developed, including (1) the design of a bandpass filter, (2) the modal signal reconstruction technique, and (3) the evaluation of the modal frequency after employing the HHT for the reconstructed mode.

4. Design of a bandpass filter for solving the mode-mixing problem in HHT

In order to solve the mode-mixing problem in HHT and to reduce measurement errors in an experiment that might affect successful identification results, a bandpass filter was designed to filter the measured vibration data, e.g., the acceleration signal (Fig. 3) for the first floor of the structure under El Centro earthquake excitations. Measurement errors could arise from sampling errors in the response data of the structure. In particular, the first floor of the structure is connected to the base-shaking table, where nonlinear vibration could produce noise to contaminate the response data. Higher floors of the structure have the possibility of weak excitation, whereby a low noise could dominate the measured signal.

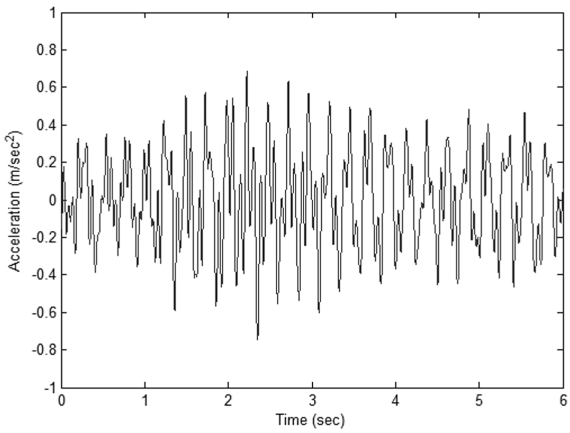

Fig. 3 Measured acceleration signal for the first floor of the benchmark structure under El Centro earthquakes

To extract each modal signal from the benchmark structure (Fig. 1), a digital bandpass filter for the structure that contains 3 modal frequencies for each degree of freedom (Chopra 1995) was

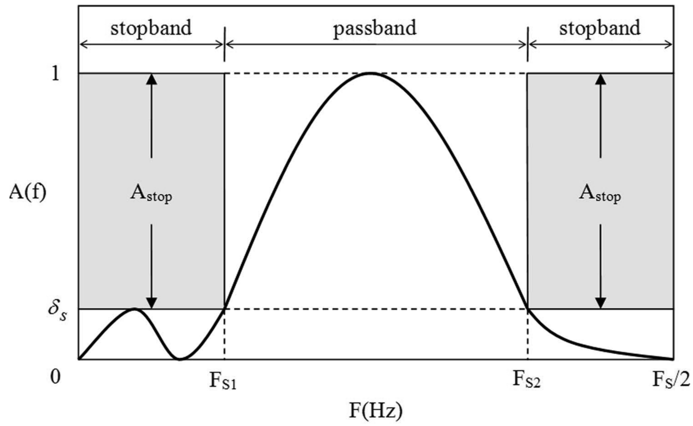

Fig. 4 Illustration of the design map of a bandpass filter ($\delta_s = 0.1$)

Table 1 Bandpass filter design specifications for the 3 modes of the benchmark structure under the 2 different earthquake excitations

Excitations	Floor	Mode	Design Specification: Cutoff Frequency (Hz) Fs1	Design Specification: Cutoff Frequency (Hz) Fs2	Band Width Fs2-Fs1 (Hz)	Sampling Frequency Fs (Hz)
El Centro	1	1	3.6168	4.4966	0.8798	200
		2	11.2414	12.5122	1.2708	
		3	17.1065	18.5728	1.4663	
	2	1	3.4213	4.7898	1.3685	
		2	9.0909	15.0538	5.9629	
		3	16.6178	18.5728	1.9550	
	3	1	3.4213	4.9853	1.5640	
		2	10.9482	12.9032	1.9550	
		3	16.4223	18.2796	1.8573	
TCU084	1	1	3.5191	4.4966	0.9775	200
		2	11.0459	13.1965	2.1505	
		3	16.7155	20.1369	3.4213	
	2	1	3.9101	4.4966	0.5865	
		2	10.8504	13.5875	2.7370	
		3	16.7155	18.6706	1.9550	
	3	1	3.9101	5.6696	1.7595	
		2	11.3392	14.0762	2.7370	
		3	16.7155	18.8661	2.1505	

designed. Fig. 4 illustrates the design map of the filter, which uses a cutoff frequency that corresponds in amplitude to 10% of the peak value ($\delta_s = 0.1$) for the signal in the frequency spectrum (Etter 1993, Matlab Toolbox). There is no transition band between the passband and the stopband for the designed filter, i.e., the cutoff frequency equals the rejection frequency. The signal was only allowed oscillations between the cutoff frequencies (i.e., Fs1 and Fs2), and amplitudes in other intervals (i.e., 0-Fs1 and Fs2-Fs/2) were set to zero so as to filter out lower and higher portions of the signal, which arise from measurement errors. Table 1 lists the corresponding design specifications for the 3 modes of the structure under the 2 earthquake excitations described above. In the last 2 columns of Table 1, the bandwidth is relatively small compared to the sampling frequency, which implies that the filter is of narrowband design for the modal signal for each floor of the structure. Such a narrow bandwidth is automatically determined according to the input signal and the corresponding cutoff frequency design, in which $\delta_s = 0.1$, and it is especially required in seismic engineering for the accurate estimates of the natural frequencies of the structure.

5. Modal signal reconstruction technique via an orthogonalization analysis

After the measured acceleration data had been bandpass filtered, modal signals of accelerations have been separated and each mode becomes available. The following step consists of the modal signal reconstruction technique to extract more accurate modes. In the past, there was relevant research in this respect, such as the revised HHT work proposed by Xun and Yan (2008), who employed the optimal IMFs in their selection process through the correlation coefficients. Bao *et al.* (2009) proposed an improved HHT algorithm, which also used the correlation coefficients as a selection criterion.

The developed signal reconstruction technique instead analyzes each mode of the measured signal and utilizes the orthogonalization coefficient (Huang *et al.* 1998), Eq. (11), to select a most appropriate IMF from the EMD result of each mode. The IMF selected represents the reconstructed modal signal. Utilizing the orthogonalization coefficient, which ranges from 0 to 0.5, checks the orthogonality between signals of interest in addition to the independency between them. Further, all positive values of orthogonalization coefficient can be conveniently listed in an orderly fashion without considering negative correlation. The orthogonalization coefficient is defined as

$$IO_{fg} = \sum_t \frac{C_f C_g}{C_f^2 + C_g^2} \tag{11}$$

where IO_{fg} denotes the index of orthogonality between the two signals, C_f and C_g.

The upper portion of Table 2 lists the orthogonalization coefficients between the modal signal and its IMFs obtained from the EMD, after filtering acceleration data for the first floor of the benchmark structure under El Centro and TCU084 earthquake conditions. The lower portion of Table 2 presents values for the comparative case after the modal signal is reconstructed. The modal signal reconstruction technique increases the orthogonalization coefficient of the selected IMF1 from 0.49 to 0.50 (highest value), thereby indicating that IMF1 best represents the modal signal. The IMF number selected is identical for each mode, each floor, and each excitation, thus, confirming the consistency and efficiency of the modal signal reconstruction technique.

Table 2 Orthogonalization coefficients between (1) the modal signal, and (2) the reconstructed modal signal, and its IMFs obtained from the EMD, after filtering acceleration data for the first floor of the benchmark structure under El Centro and TCU084 earthquakes

		First Floor's Orthogonalization Coefficient between					
	Excitations	El Centro			TCU084		
	Mode	1st	2nd	3rd	1st	2nd	3rd
		0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
Modal Signal	IMF1	**0.4999**	**0.5000**	**0.4999**	**0.4999**	**0.5000**	**0.4999**
	IMF2	-1.22E-04	-1.16E-05	-2.83E-05	2.22E-04	-3.77E-05	2.45E-05
	IMF3	-1.53E-04	-5.59E-06	-3.80E-06	-5.37E-05	-2.30E-05	-2.01E-05
	IMF4	-3.48E-05	-2.72E-06	-3.98E-07	-3.60E-05	-1.06E-05	-3.82E-06
	IMF5	-2.23E-05	-1.84E-06	1.25E-06	-3.05E-05	-9.75E-06	-1.55E-06
	IMF6	-1.80E-05	-1.50E-06	-3.83E-08	-2.11E-05	-1.06E-05	-5.59E-07
	IMF7	-3.73E-05		7.53E-07	-8.14E-06	-7.43E-06	-6.23E-07
	IMF8				-2.36E-05	-4.08E-06	6.06E-07
	IMF9					-5.85E-06	-2.58E-07
	IMF10					-5.48E-06	
Recon-structed Modal Signal	IMF1	**0.5000**	**0.5000**	**0.5000**	**0.5000**	**0.5000**	**0.5000**
	IMF2	-3.64E-05	-1.21E-05	-3.17E-06	2.97E-05	-4.08E-05	2.01E-05
	IMF3	-1.69E-05	-2.89E-06	-3.74E-07	-3.12E-06	-1.15E-05	-1.01E-05
	IMF4	-1.22E-05	-7.36E-07	-4.87E-08	-7.23E-07	-1.51E-06	-5.99E-08
	IMF5	-4.87E-06	-5.48E-07	-1.98E-09	-2.24E-07	-1.13E-06	-3.70E-08
	IMF6	-1.87E-06	-2.91E-07	-3.69E-08	4.34E-07	-1.00E-06	4.49E-08
	IMF7	-1.32E-06	-2.16E-07	-3.50E-08	-4.42E-07	-5.05E-07	-2.72E-07
	IMF8	-6.86E-07	-2.18E-07	-1.31E-08	-9.77E-07	-2.25E-07	-1.05E-07
	IMF9		-1.71E-07	-1.46E-08	4.31E-07	-2.43E-07	2.79E-08
	IMF10			-4.69E-08		-1.98E-07	-8.92E-08
	IMF11			-1.56E-08			6.82E-09
	IMF12						-9.32E-08

The effectiveness of this signal reconstruction technique can be proved by comparing the 3 Hilbert spectra (through HHT) obtained from (1) the measured acceleration signal for the first floor of the structure under El Centro earthquake conditions, for instance (Fig. 5); (2) the corresponding modal signals after bandpass filtering (left portion of Fig. 6); and (3) the reconstructed modal signals (right portion of Fig. 6). It is not surprising that the Hilbert spectrum of the original measured signal appears contaminated with noise, and that it is difficult to determine the modal frequencies. After bandpass filtering, modal frequencies can be extracted from the Hilbert spectra (left portion of Fig. 6). Incorporating modal signal reconstruction shows that the disturbance-embedded modal frequencies can be further reduced to a satisfactory degree (right portion of Fig. 6). Hence, the modal signal reconstruction technique has enhanced the efficacy of noise filtering.

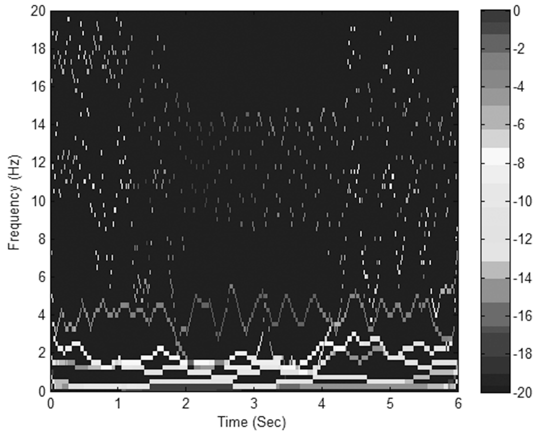

Fig. 5 Hilbert spectrum (through HHT) obtained from the measured acceleration signal for the first floor of the benchmark structure under El Centro earthquakes

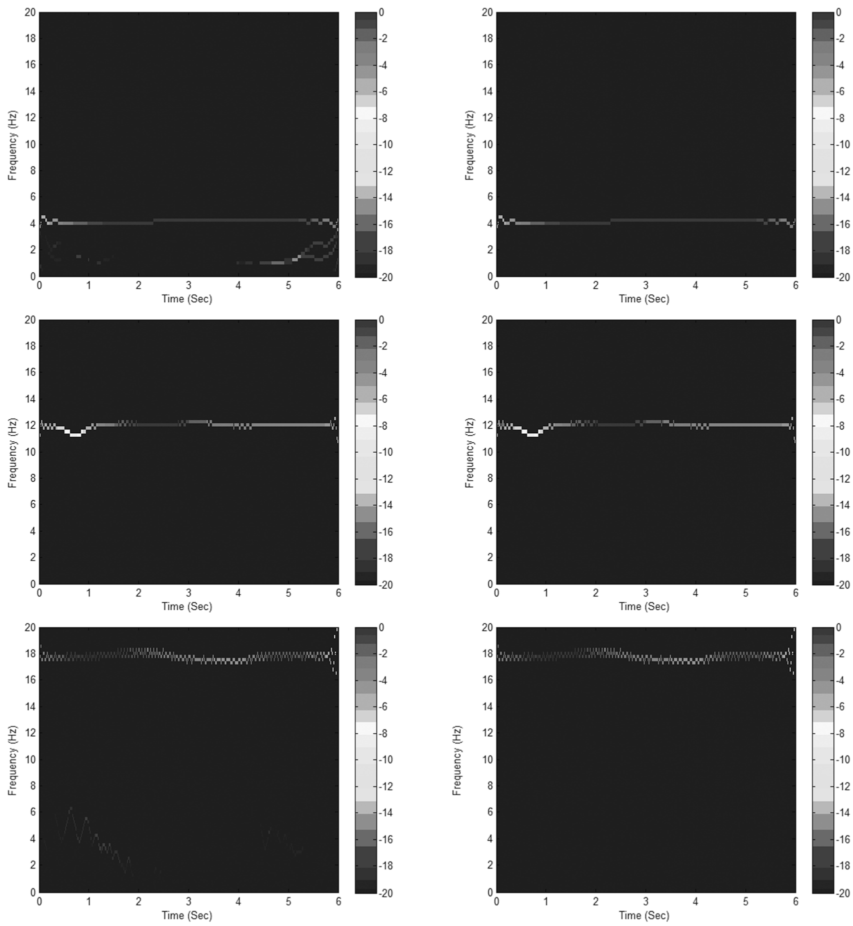

Fig. 6 Hilbert spectra (through HHT) obtained from: the modal signals after bandpass filtering (left portion) and the corresponding reconstructed modal signals (right portion) for the first floor of the benchmark structure under El Centro earthquakes

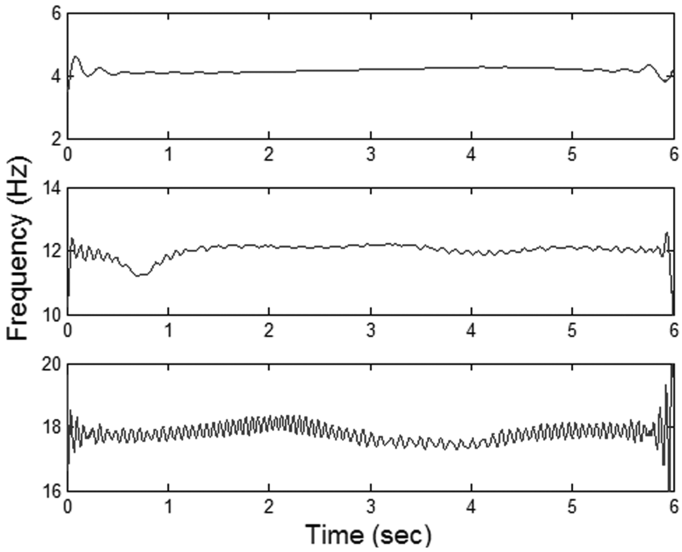

Fig. 7 The identified frequency of the reconstructed modal signal for the first floor of the benchmark structure under El Centro earthquakes (Top-down: mode 1, 2, and 3, respectively)

6. Evaluation of modal frequency represented in time-frequency domain

It is convenient to utilize the Hilbert spectrum (i.e., the time-frequency distribution of the amplitude) of the reconstructed modal signal to evaluate the range of the natural frequency of the structure, while using the correspondingly identified IMF (i.e., the time-frequency diagram) sifted from the EMD to pinpoint the value of the modal frequency. The final estimated parameter in time history is used to represent the current condition of system (Lin 2001, Lin and Betti 2004).

By comparing the identified frequency in Fig. 7 with the corresponding Hilbert spectrum (right side of Fig. 6), it appears that the steady state of the Hilbert spectrum shows the ranges of the three modal frequencies around 4 Hz, 12 Hz, and 18 Hz. When pinpointing the value of the modal frequency using the selected IMF as shown in the time-frequency diagram, the "end effect" (Huang *et al*. 1998) should be avoided; the frequency value for the selected IMF at the end of the time period should not be taken as the modal frequency because of the cubic-spline curve fitting used in EMD and of the Hilbert transform. Thus, it is convenient to choose a value for the modal frequency near the end of the time period i.e., at 5.55, 5.7, and 5.85 s (corresponding to the data points at 1110, 1140, and 1170 for 1200 overall data points), and to average the 3 values chosen as the modal frequency for the first mode at the first floor of the structure.

Table 3 lists evaluated modal frequencies for the 3 modes of the three-story scaled steel structure under El Centro and TCU084 earthquake conditions. By comparing the results of the modal frequencies evaluated with the theoretical values obtained using a shear-building model, a slight difference between the 2 approaches was obtained, with a total averaged relative error of 1.40%. However, the evaluated modal frequencies listed in Table 3 that were obtained via vibration measurements in the lab represent more accurate results than those derived using theoretical values. Hence, the HHT-based signal reconstruction approach provides reliable modal frequencies for structural assessment.

Table 3 Evaluated modal frequencies for the 3 modes of the three-story benchmark structure under El Centro and TCU084 earthquakes

Floor	Mode	Evaluated Modal Frequency (Hz)				Theoretical Value (Hz)	Relative Error	Averaged Relative Error
		Data Point	El Centro	TCU084	Average			
1	1	1110	4.1879	4.1762	4.1564	4.3108	3.58%	1.64%
		1140	4.2030	4.1012				
		1170	4.0192	4.2511				
	2	1110	12.0841	12.0004	12.0497	12.0441	0.05%	
		1140	12.0677	11.8396				
		1170	12.1347	12.1719				
	3	1110	17.6067	18.1243	17.8341	17.6073	1.29%	
		1140	17.8388	17.1051				
		1170	17.7107	18.6193				
2	1	1110	4.2320	4.2690	4.1934	4.3108	2.72%	1.68%
		1140	4.2795	4.1413				
		1170	3.9665	4.2721				
	2	1110	11.8529	11.9383	12.2847	12.0441	2.00%	
		1140	12.0022	12.1866				
		1170	13.0091	12.7189				
	3	1110	17.6927	17.9644	17.6656	17.6073	0.33%	
		1140	17.8329	17.6641				
		1170	17.5357	17.3036				
3	1	1110	4.2931	4.1518	4.2574	4.3108	1.24%	0.89%
		1140	4.3424	4.2443				
		1170	3.9456	4.5669				
	2	1110	11.9639	11.7566	11.9824	12.0441	0.51%	
		1140	11.9011	11.7634				
		1170	12.1007	12.4085				
	3	1110	17.8252	17.6772	17.7699	17.6073	0.92%	
		1140	17.9030	17.4330				
		1170	18.1708	17.6104				
Total Averaged Relative Error							1.40%	

7. Evaluation of a time-varying structure

In order to prove the effectiveness of the developed HHT-based filtering, a structure with damage subjected to a white noise excitation was simulated (Lin and Chen 2009). This simulated single degree-of-freedom (SDOF) nonlinear structure is modeled by the Bouc-Wen model in the work of Smyth *et al*. (1999), i.e.

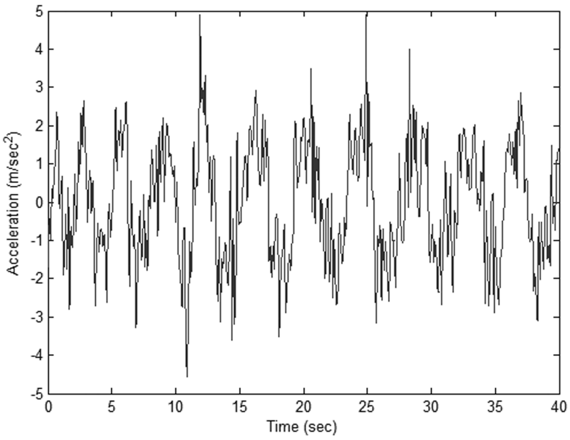

Fig. 8 The acceleration response of the simulated SDOF structure with damage

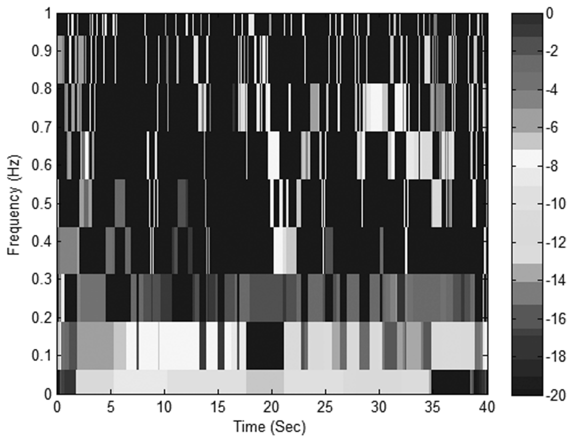

Fig. 9 Hilbert spectrum obtained from the acceleration response of the simulated SDOF structure with damage

Table 4 Bandpass filter design specifications for the simulated structure with damage

Floor	Mode	Design Specification		Band Width Fs2-Fs1(Hz)	Sampling Frequency Fs (Hz)
		Cutoff Frequency (Hz)			
		Fs1	Fs2		
1	1	0.1915	0.3889	0.1974	100

$$\dot{r} = 5\dot{u} - 0.1|\dot{u}||r|r - \dot{u}r^2 \tag{12}$$

with $m = 1$ used in Eq. (13)

$$m\ddot{u}(t) + r(u(t), \dot{u}(t), t) = -m\ddot{u}_g(t) \tag{13}$$

Structural damage is simulated by linearly reducing the stiffness from 5 N/m (with frequency 0.3559 Hz) to 4 N/m (with frequency 0.3183 Hz) during time intervals at 10 s and 15 s (sampling $\Delta t = 0.01$ s), and the acceleration response of the simulated structure with damage is shown in Fig. 8. Fig. 9 depicts the Hilbert spectrum of the acceleration response signal.

Similarly, the overall approach of the HHT-based filtering, as shown in Fig. 2, is applied to the acceleration signal. First, a bandpass filter was designed to separate structural modes for the simulated SDOF structure with damage. The associated specifications of the narrowband filter design are shown in Table 4.

Second, after filtering the simulated acceleration data, it is possible to obtain the Hilbert spectrum of the modal signal (left-hand side of Fig. 10). In this manner, the modal signal reconstruction technique, using the orthogonalization coefficient, can be applied to filter out additional noisy components, (right-hand side of Fig. 10). It is clear to see that the modal frequency was moving toward its true value, as shown in the Hilbert spectrum from left to right side (Fig. 10). Associated data for the modal signal reconstruction are provided in Table 5. Table 5 lists the orthogonalization coefficient between (1) the modal signal, and (2) the reconstructed modal signal, and its IMFs,

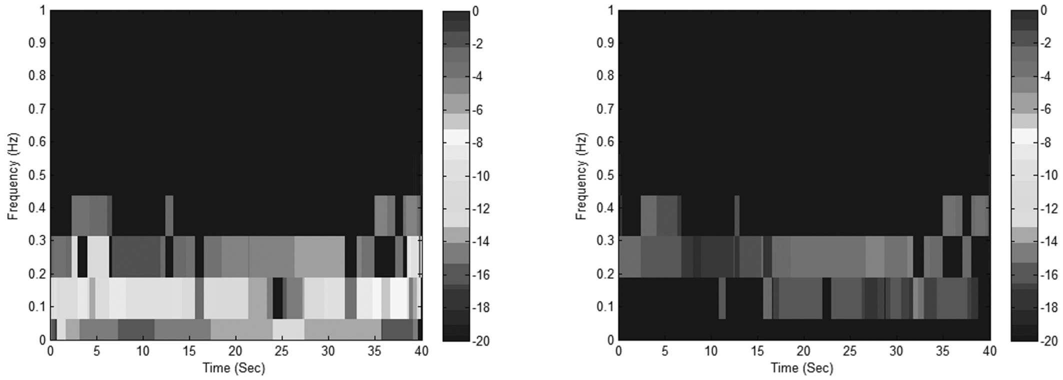

Fig. 10 Hilbert spectra (through HHT) obtained from: the modal signal after bandpass filtering (left portion) and the corresponding reconstructed modal signal (right portion) for the simulated SDOF structure with damage

Table 5 Orthogonalization coefficients between (1) the modal signal, and (2) the reconstructed modal signal, and its IMFs obtained from the EMD, after filtering acceleration data for the simulated SDOF structure with damage

	Orthogonalization Coefficient between	
	Mode	1st
		0.5000
Modal Signal	IMF1	0.4888
	IMF2	6.20E-03
	IMF3	-1.58E-03
	IMF4	-6.63E-04
Reconstructed Modal Signal	IMF1	0.4997
	IMF2	-1.04E-03
	IMF3	-4.70E-04
	IMF4	1.19E-04
	IMF5	-1.67E-05

derived from EMD, respectively, for the acceleration data of the simulated SDOF structure with damage. It is clear that the selected IMF (IMF1) with the highest orthogonalization coefficient value among all IMFs increases from 0.4888 to 0.4997 after the modal signal is reconstructed, thereby proving the efficiency of the modal signal reconstruction technique.

Finally, the modal frequency of the structure with damage in the time-frequency domain was identified and shown in Fig. 11. On comparison of the selected IMF1 in Fig. 11 with the corresponding Hilbert spectrum on the right portion of Fig. 10, it appears that the steady state of the Hilbert spectrum shows the range of the modal frequency around 0.3 Hz, while the corresponding time-frequency diagram of IMF1 also confirms a steady state condition around 0.3 Hz. When pinpointing the value of the modal frequency using the selected IMF1, the “end effect” (Huang *et al*. 1998) should be avoided. As with the previous case when evaluating the modal frequency, it is convenient to choose a value of the identified frequency near the end of the time period for IMF1 i.e., at 37, 38, and 39 s (i.e., corresponding to the data points at 3700 = 4000 * 1110/1200, 3800 = 4000 * 1140/1200, and 3900 = 4000 * 1170/1200 for 4000 overall data points) and to use the average of the 3 values chosen for representing the modal frequency of the structure.

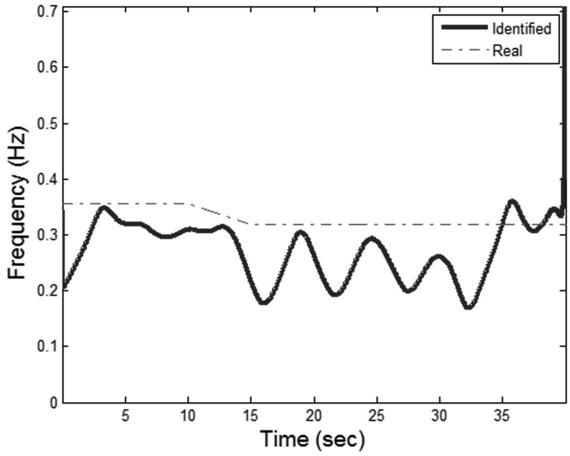

Fig. 11 The time history of the identified frequency vs. the real frequency diagram

Table 6 Evaluated modal frequency for the simulated SDOF structure with damage

Floor	Mode	Data Point	Identified Frequency (Hz)	Real Frequency (Hz)	Relative Error
1	1st	3700	0.3148	0.3183	1.12%
		3800	0.3129	0.3183	1.71%
		3900	0.3465	0.3183	8.86%
Total Averaged Relative Error					3.90%

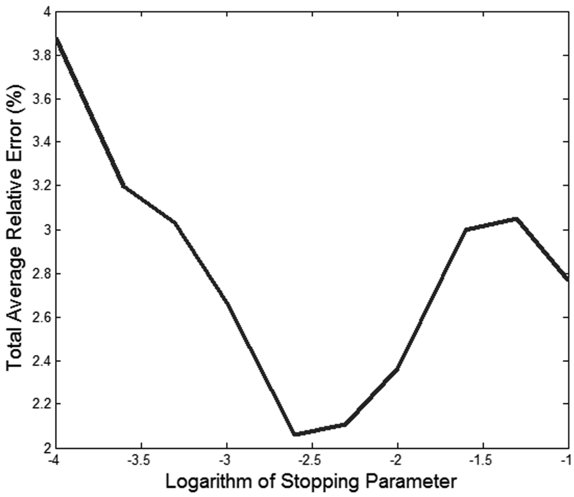

Fig. 12 Averaged relative error vs. logarithm of stopping parameter in EMD

Table 6 lists the identified frequency of the structural mode. Comparing the results of the identified frequency with the real value yields a total averaged relative error of 3.90%. Fig. 11 illustrates the corresponding time history of the identified frequency vs. the real frequency diagram. In fact, this relative error of 3.90% was the worst case. If the stopping parameter (EMD evaluation code by Gabriel.Rilling (at) ens-lyon.fr) in the sifting process of EMD is varied using scenario tests, the relative error can be reduced to 2.06%, which is shown in Fig. 12. Based on the numerical results, the developed filtering approach provides reliable modal frequency evaluation for structural assessment.

8. Evaluation of a linear sum of two cosine waves

In order to verify the wider applicability of the proposed filtering approach and its efficiency at mode separation, let us consider a test case of a linear sum of two cosine waves (Huang *et al*. 1998) described in Eq. (14)

$$x(t) = \cos\frac{2}{30}\pi t + \cos\frac{2}{34}\pi t \tag{14}$$

over t from 1-512 s, with the time interval $\Delta t = 0.1$. The real frequencies are 1/34 (0.02941) Hz and 1/30 (0.03333) Hz.

Fig. 13 shows the corresponding Fourier spectrum vs. the marginal spectrum using the proposed filtering approach, in which the symbol × indicates the real frequencies of the two modes. It is noteworthy that if the designed bandpass filter is not applied at the data preprocessing stage, the mode separation for the two modes is not possible. In fact, directly applying HHT will produce only one modal frequency possessing a higher average relative error 6.25%, when compared to 1.46% and 1.47% using the proposed approach and the FFT, respectively, as listed in Table 7. Nonetheless, the modal signal reconstruction after the bandpass filtering also provides assistance in further denoising. The proposed filtering approach has improved the HHT and proven to be effective in mode separation and accurate evaluation of modal frequencies.

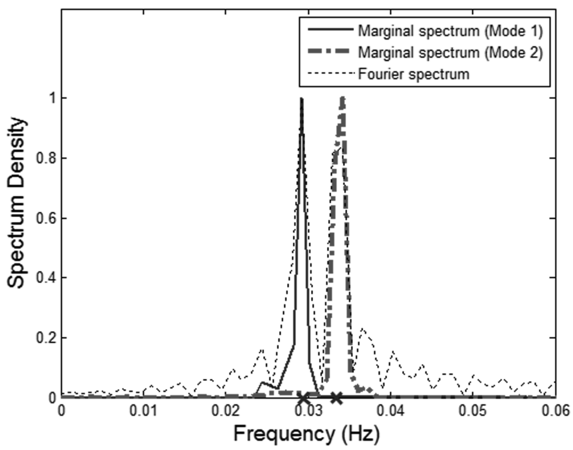

Fig. 13 Fourier spectrum vs. marginal spectrum using the proposed filtering approach (× indicates the real frequencies of the two modes)

Table 7 Evaluated modal frequencies for the linear sum of two cosine waves using the FFT, direct HHT, and the proposed HHT-based bandpass filtering approach

	Frequency (Hz)		Relative Error		Average Relative Error
	Mode 1	Mode 2	Mode 1	Mode 2	
Real	0.02941	0.03333			
FFT	0.02930	0.03419	0.37%	2.56%	1.47%
HHT	0.03125	0.03125	6.25%	6.25%	6.25%
HHT-based Bandpass Filtering	0.02930	0.03418	0.39%	2.54%	1.46%

9. Conclusions

An efficient HHT-based bandpass filtering approach for the modal evaluation of structural systems was proposed, particularly for solving the mode-mixing problem in HHT. Accounting for sampling errors, the developed identification approach is capable to provide reliable indices that are indicative of current conditions of structures for accurate safety assessment. The denoising identification approach includes 1) a designed bandpass filter will separate structural modes, thereby solving the mode-mixing problem of HHT in addition to providing noise filtering, in which a narrow bandwidth is automatically determined according to the input signal and the cutoff frequency design; 2) the modal signal reconstruction technique, using an orthogonalization coefficient to select a proper IMF from the EMD, will extract a significant mode for the system and filter out any further noisy components; 3) modal evaluation of systems using the selected IMF, obtained from the EMD of the reconstructed modal signal, will accurately estimate the modal frequency in the time-frequency diagram.

The effectiveness of the developed approach for evaluating modal frequency has been demonstrated through a mode-by-mode sequential analysis. Using the theoretical estimation of the benchmark structure via a shear-building model as a gauge, the relative estimation error of 1.40% was achieved. In addition, a simulated time-varying structural system was tested to ascertain degree of damage, which was possible to achieve a relative estimation error of 2.06%. A linear sum of two cosine waves was also tested to manifest the efficiency of the proposed approach at mode separation, which achieved a smaller estimation error of 1.46% when compared to the error of 6.25% using the pure HHT.

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References

Bao, C., Hao, H., Li, Z.X. and Zhu, X. (2009), "Time-varying system identification using a newly improved HHT algorithm", *Comput. Struct.*, **87**(23-24), 1611-1623.

Chen, C. and Ricles, J.M. (2010), "Stability analysis of direct integration algorithms applied to MDOF nonlinear structural dynamics", *J. Eng. Mech.*, **136**(4), 485-495.

Cheng, J., Yu, D. and Yang, Y. (2006), "Research on the intrinsic mode function (IMF) criterion in EMD method", *Mech. Syst. Signal Pr.*, **20**(4), 817-824.

Chopra, A.K. (1995), *Dynamics of Structures*, Prentice-Hall.

Duffy, D.G. (2004), "The application of Hilbert-Huang transforms to meteorological datasets", *J. Atmos. Ocean. Tech.*, **21**, 599-611.

Etter, D.M. (1993), *Engineering Problem Solving with MATLAB*, Prentice-Hall.

Hearn, G. and Testa, R.B. (1991), "Modal analysis for damage detection in structures", *J. Struct. Eng.,* **117**(10), 3042-3063.

Huang, N.E. and Shen, S.S.P. (2005), *Hilbert-Huang Transform and Its Applications, Interdisciplinary Mathematical Sciences*, Vol. 5, World Scientific Publishing Co., Singapore.

Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.C., Tung, C.C. and Liu, H.H. (1998), "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis", *P. Roy. Soc. London*, **454**, 903-995.

Huang, N.E., Wu, M.C., Long, S.R., Shen, S.P., Qu, W., Gloersen, P. and Fan, K.L. (2003), "A confidence limit for the empirical mode decomposition and Hilbert spectral analysis", *P. Roy. Soc. London*, **459**, 2317-2345.

Huang, N.E., Wu, M.L., Qu, W., Long, S.R. and Shen, S.S.P. (2003), "Applications of Hilbert-Huang transform to non-stationary financial time series analysis", *Appl. Stoch. Model. Bus.*, **19**(3), 245-268.

Kibboua, A., Farsi, M.N., Chatelain, J.L., Guillier, B., Bechtoula, H. and Mehani, Y. (2008), "Modal analysis and ambient vibration measurements on Mila-Algeria cable stayed bridge", *Struct. Eng. Mech.*, **29**(2), 171-186.

Lin, J.W. (2001), "Adaptive algorithms for the identification of nonlinear structural systems", Ph.D. Thesis, Columbia University, New York.

Lin, J.W. and Betti, R. (2004), "On-line identification and damage detection in non-linear structural systems using a variable forgetting factor approach", *Earthq. Eng. Struct. Dyn.*, **33**(4), 419-444.

Lin, J.W. and Chen, H.J. (2009), "Repetitive identification of structural systems using a nonlinear model parameter refinement approach", *Shock Vib.,* **16**(3), 229-240.

Lin, J.W., Chen, H.J. and Lin, J.Y. (2008), "Hilbert-Huang transform based modal analysis of structures", *Proceedings of 2008 ASME Pressure Vessels and Piping Conference- Seismic Engineering*, Chicago, July.

Messina, A.R. and Vittal, V. (2007), "Extraction of dynamic patterns from wide-area measurements using empirical orthogonal functions", *IEEE T. Power Syst.,* **22**(2), 682-692.

Ni, Y.Q., Ko, J.M., Hua, X.G. and Zhou, H.F. (2007), "Variability of measured modal frequencies of a cable-stayed bridge under different wind conditions", *Smart Struct. Syst.,* **3**(3), 341-356.

Peng, Z.K., Tse, P.W. and Chu, F.L. (2005), "An improved Hilbert-Huang transform and its application in vibration signal analysis", *J. Sound Vib.*, **286**, 187-205.

Schlurmann, T. (2002), "Spectral analysis of nonlinear water waves based on the Hilbert-Huang transformation", *J. Offshore Mech. Arct.*, **124**(1), 22-27.

Smyth, A.W., Masri, S.F., Chassiakos, A.G. and Caughey, T.K. (1999), "On-line parametric identification of MDOF nonlinear hysteretic systems", *J. Eng. Mech.*, **125**, 133-142.

Udwadia, F.E. (2005), "Structural identification and damage detection from noisy modal data", *J. Aerospace Eng.,* **18**(3), 179-187.

Xun, J. and Yan, S. (2008), "A revised Hilbert-Huang transformation based on the neural networks and its application in vibration signal analysis of a deployable structure", *Mech. Syst. Signal Pr.*, **22**(7), 1705-1723.