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Response of a frame structure on a canyon site to spatially varying ground motions

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Abstract. This paper studies the effects of spatially varying ground motions on the responses of a bridge frame located on a canyon site. Compared to the spatial ground motions on a uniform flat site, which is the usual assumptions in the analysis of spatial ground motion variation effects on structures, the spatial ground motions at different locations on surface of a canyon site have different intensities owing to local site amplifications, besides the loss of coherency and phase difference. In the proposed approach, the spatial ground motions are modelled in two steps. Firstly, the base rock motions are assumed to have the same intensity and are modelled with a filtered Tajimi-Kanai power spectral density function and an empirical spatial ground motion coherency loss function. Then, power spectral density function of ground motion on surface of the canyon site is derived by considering the site amplification effect based on the one dimensional seismic wave propagation theory. Dynamic, quasi-static and total responses of the model structure to various cases of spatially varying ground motions are estimated. For comparison, responses to uniform ground motion, to spatial ground motions without considering local site effects, to spatial ground motions without considering coherency loss or phase shift are also calculated. Discussions on the ground motion spatial variation and local soil site amplification effects on structural responses are made. In particular, the effects of neglecting the site amplifications in the analysis as adopted in most studies of spatial ground motion effect on structural responses are highlighted.

Keywords: site amplification effect; ground motion spatial variation; dynamic responses; quasi-static responses; total responses.

1. Introduction

Earthquake ground motions at multiple supports of large dimensional structures inevitably vary owing to seismic wave propagation effects. Many researchers have investigated seismic ground motion spatial variations. Most of these studies are based on processing the recorded ground motions at dense seismographic arrays, such as the SMART-1 array. Many empirical spatial ground motion coherency loss functions have been derived (Bolt *et al*. 1982, Harichandran and Vanmarck 1986, Loh and Yeah 1988, Hao *et al*. 1989, Abrahamson *et al.* 1991). In all those studies the site

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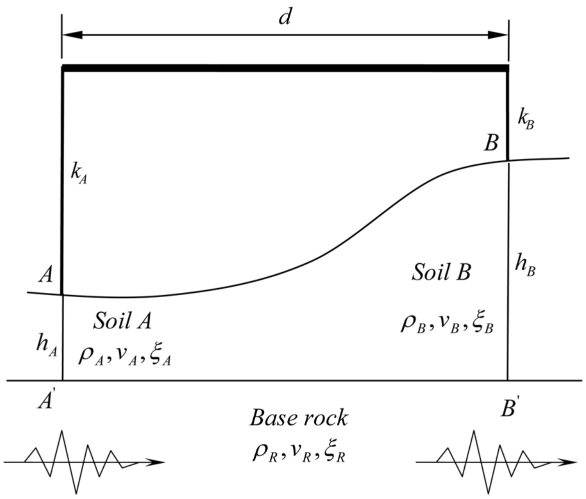

Fig. 1 Schematic view of a bridge frame crossing a canyon site

under consideration is assumed to be uniform and homogeneous. Therefore the ground motion power spectral densities at various locations of the site under consideration are assumed to be the same. In other words, the only variations in spatial ground motions are loss of coherency and a phase shift owing to seismic wave propagation. However, this assumption will lead to inaccurate ground motion representation when a site has varying conditions such as a canyon site as shown in Fig. 1. At a canyon site, the spatial ground motions at base rock can still be assumed to have the same power spectral density, but on ground surface at points A and B the ground motion power spectral densities will be very different owing to seismic wave propagation through different wave paths that cause different site amplifications. Uniform ground motion power spectral density assumption in such a situation may lead to erroneous estimation of structural responses.

Some researchers have tried to model the effect of local site conditions on earthquake ground motion spatial variations. Der Kiureghian *et al*. (1996) proposed a transfer function that implicitly modelled the site effect on seismic wave propagation. In the model, the ground motion power spectral density function was represented by a site-dependent transfer function and a white noise spectrum. Typical site-dependent parameters, i.e., the central frequency and damping ratio for three generic site conditions, namely, firm, medium and soft site were proposed. The advantage of the model is that it is straightforward to use. The drawback is it can only approximately represent the local site effects on ground motions. For example, it is well known that seismic wave will be amplified and filtered when propagating through a layered soil site. The amplifications occur at various vibration modes of the site. Therefore, the energy of surface motions will concentrate at a few frequencies. The power spectral density function of the surface motion then may have multiple peaks. This phenomenon, however, cannot be considered in Der Kiureghian's model since only one peak corresponding to the fundamental vibration mode of the site can be involved. In a recent study (Hao and Chouw 2006), derivations of earthquake ground motion spatial variation on a site with uneven surface and different geological properties were presented. In the latter study, spatial base rock motion was modelled by a Tajimi-Kanai power spectral density function (Tajimi 1960) together with an empirical coherency loss function (Hao *et al*. 1989). Power spectral density functions of the surface motions were derived based on the one dimensional seismic wave propagation theory.

Compared to the model by Der Kiureghian *et al*. (1996), the latter study by Hao and Chouw (2006) modelled the base rock motion by the Tajimi-Kanai power spectral density function instead of a white noise, and the seismic wave propagation and specific site amplification effects were explicitly represented in terms of the site conditions such as the soil depth and properties. The multiple vibration modes of local site can be easily considered. Therefore the latter model gives more realistic prediction of local site effects on seismic ground motions besides explicitly relating the site conditions to ground motion model.

Previous studies of ground motion spatial variation effects on structural responses include stochastic response analysis of a simply supported beam (Harichandran and Wang 1988), continuous beams (Harichandran and Wang 1990, Zerva 1990), an arch with multiple horizontal input (Hao 1993), an arch with multiple simultaneous horizontal and vertical excitations (Hao 1994), a symmetric building structure (Hao and Duan 1996), an asymmetric building structure (Hao and Duan 1995), and a cable-stayed bridge (Dumanoglu and Soyluk 2002). Most of these studies assumed linear elastic responses. Many researchers have also performed time history analysis of structural responses to spatially varying ground motions. In these studies, both linear elastic, nonlinear inelastic responses, pounding responses, soil-structure interaction effects were considered. The spatial ground motion time histories were obtained either by considering the wave passage effect only (Jankowski *et al*. 2000), or stochastically simulated to be compatible to a selected empirical coherency loss function (Hao 1989, Monti *et al.* 1996, Sextos *et al*. 2003a, b, Chouw and Hao 2005). In most of these studies, the site was assumed to be homogeneous and flat, local site effect was not considered.

Using the model developed by Der Kiureghian *et al*. (1996), Zembaty and Rutenburg (2002) derived the displacement and shear force response spectra with consideration of ground motion spatial variation and site effects. They concluded that site effects modified the overall behaviour of the multi-supported structure significantly. Dumanogluid and Soyluk (2003) also used this model and analysed responses of a long span structure to spatially varying ground motions with site effect. It was concluded that although it was difficult to draw general conclusions because of the limited analyses performed, it was clear that ground motion spatial variation and site effects significantly affect the structural responses; considering different site effects at multiple supports generated larger structural responses; the more significant was the difference between the site conditions at the multiple supports, the larger was the structural responses. Another study that used this model to consider the site effects and ground motion spatial variation was reported by Ates *et al*. (2005). Similar conclusions were drawn, i.e., site effects significantly affect structural responses. Sextos *et al.* (2003a, b) discussed the importance of considering ground motion spatial variations, site effect, soil-structure interaction and nonlinear inelastic responses in bridge response analysis and design. They also outlined the possible numerical approaches for bridge response analysis.

In the present study, the spatial ground motion model with site effect derived by Hao and Chouw (2006) is used to analyse the responses of a bridge frame on a canyon site. Stochastic method is used to perform parametric analysis in this study. Dynamic, quasi-static and total structural responses are calculated. The influences of site conditions and ground motion spatial variations on structural responses are highlighted. Structural responses to uniform ground motion, to spatial ground motion without considering coherency loss or phase shift and to spatial ground motion without considering the site effect are calculated and compared. Discussions on the ground motion spatial variation and site effect in terms of the site properties on structural responses are made.

2. Bridge and spatial ground motion model

2.1 Bridge model

Fig. 1 illustrates the schematic view of a model bridge frame on a canyon site, in which A and B are the two supports on ground surface, the corresponding points at base rock are A' and B'. ρ_j, v_j, ξ_j and h_j are the density, shear wave velocity, damping ratio and depth of the soil under support j, respectively, where j represents A or B. The corresponding parameters on the base rock are ρ_R, v_R and ξ_R. The deck of the bridge frame is idealized as a rigid beam supported by two piers. It should be noted that only one bridge frame is modelled in the present study, the adjacent bridge structures are neglected. This simplification implies no pounding between adjacent bridge structures is considered. This is a rational assumption since with the new development of modular expansion joint (MEJ), which allows a large joint movement and at the same time without impending the smoothness of traffic flow, completely precluding seismic pounding between adjacent bridge structures is possible (Chouw and Hao 2008). This means that each bridge frame can vibrate independently during an earthquake without pounding between adjacent structures. In the numerical analysis, without losing generality, the viscous damping ratio of the structure is assumed to be 5% in the present study.

2.2 Base rock motion

Assume the amplitudes of the power spectral densities at different locations on the base rock are the same and in the form of the filtered Tajimi-Kanai power spectral density function

$$S_g(\omega) = |H_P(\omega)|^2 S_0(\omega) = \frac{\omega^4}{(\omega_f^2-\omega^2)^2+(2\omega_f\omega\xi_f)^2}\frac{\omega_g^4+4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\xi_g^2\omega_g^2\omega^2}\Gamma \tag{1}$$

in which $|H_P(\omega)|^2$ is a high pass filter (Ruiz and Penzien 1969), $S_0(\omega)$ is the Tajimi-Kanai power spectral density function (Tajimi 1960), ω_g and ξ_g are the central frequency and damping ratio of the Tajimi-Kanai power spectral density function, Γ is a scaling factor depending on the ground motion intensity, and ω_f and ξ_f are the central frequency and damping ratio of the high pass filter. In this study, it is assumed that $f_f = \omega_f/2\pi = 0.25\text{Hz}$, $\xi_f = 0.6$, $f_g = \omega_g/2\pi = 5.0\text{Hz}$, $\xi_g = 0.6$ and $\Gamma = 0.022\ \text{m}^2/\text{s}^3$. These values correspond to a peak ground acceleration (PGA) 0.5g (Hao 1998) with duration $T = 20s$. Fig. 2 shows the power spectral density of the base rock motion.

Ground motion spatial variation at the base rock is modelled with a coherency loss function (Hao *et al*. 1989)

$$\gamma_{A'B'}(i\omega) = |\gamma_{A'B'}(i\omega)|e^{i\omega d/v_{app}} = e^{-\beta d}e^{-\alpha(\omega)\sqrt{d}(\omega/2\pi)^2}e^{i\omega d/v_{app}} \tag{2}$$

in which

$$\alpha(\omega) = \begin{cases} 2\pi a/\omega + b\omega/2\pi + c & 0.314 rad/s \le \omega \le 62.83 rad/s \\ 0.1a + 10b + c & \omega > 62.83 rad/s \end{cases} \tag{3}$$

where a, b, c and β are constants, d is the distance between the two supports, v_{app} is the apparent wave propagation velocity. The cross power spectral density function of the motion at points A' and B' on the base rock is thus

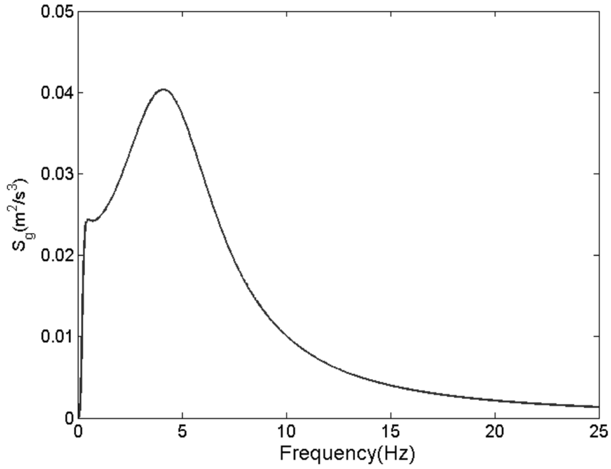

Fig. 2 Filtered ground motion power spectral density function on the base rock

$$S_{A'B'}(i\omega) = S_g(\omega)\gamma_{A'B'}(i\omega) \tag{4}$$

It should be noted that the above coherency function was obtained by processing recorded spatial ground motions on ground surface. Here it is used to model spatial variations of ground motion at base rock. This is because no information about ground motion spatial variations at the base rock is available. It is believed that seismic wave propagation through a heterogeneous soil site will change ground motion spatial variations. The present assumption may lead to some inaccurate estimation of coherency loss between spatial base rock motions. Further research into the influence of local site conditions on spatial ground motion coherency loss is deemed necessary.

2.3 Site amplification

Using seismic wave propagation theory presented by Aki (1980), Safak (1995) derived the transfer function for shear wave propagation in a horizontal layer as

$$\frac{U_j(i\omega)}{U_{j'}(i\omega)} = \frac{2(1+r_j-i\xi_j)\exp(-i\omega\tau_j(1-2i\xi_j))}{1+(r_j-i\xi_j)\exp(-2i\omega\tau_j(1-2i\xi_j))} = H_j(i\omega) \quad j = A \text{ or } B \tag{5}$$

where $U_j(i\omega)$ and $U_{j'}(i\omega)$ is the Fourier transform of the motion $u_j(t)$ and $u_{j'}(t)$ on the ground surface and at the base rock, respectively. $\xi_j = 1/4Q$ is the damping ratio accounting for energy dissipation owing to seismic wave propagation, and Q is the quality factor; $\tau_j = h_j/v_j$ is the wave propagation time from point j' to j, and r_j is the reflection coefficient for up-going waves

$$r_j = \frac{\rho_R v_R - \rho_j v_j}{\rho_R v_R + \rho_j v_j} \qquad j = A \text{ or } B \tag{6}$$

In engineering application, usually the outcrop motion on the rock surface is available, instead of the base rock motion. The parameters defined above corresponding to the Tajimi-Kanai power spectral density function in Eq. (1) also correspond to the outcrop motion on hard rock. Therefore, the constant 2 in Eq. (5), which is a measure of free surface reflection, in the transfer function is dropped. Then it has

$$H_j(i\omega) = \frac{(1+r_j-i\xi_j)\exp(-i\omega\tau_j(1-2i\xi_j))}{1+(r_j-i\xi_j)\exp(-2i\omega\tau_j(1-2i\xi_j))} \quad j = A \text{ or } B \tag{7}$$

The auto and cross power spectral density function at point j and between points A and B are

$$\begin{aligned} S_j(\omega) &= |H_j(i\omega)|^2 S_g(\omega) \quad j = A \text{ or } B \\ S_{AB}(i\omega) &= H_A(i\omega)H_B^*(i\omega)S_{A'B'}(i\omega) \end{aligned} \tag{8}$$

in which the superscript '*' represents complex conjugate.

The coherency loss between ground motions at points A and B is

$$\begin{aligned} \gamma_{AB}(i\omega) &= \frac{S_{AB}(i\omega)}{\sqrt{S_A(\omega)S_B(\omega)}} = \frac{H_A(i\omega)H_B^*(i\omega)\gamma_{A'B'}(i\omega)}{\sqrt{|H_A(i\omega)^2||H_B(i\omega)^2|}} = \exp[i(\theta_A(\omega)-\theta_B(\omega))]\gamma_{A'B'}(i\omega) \\ &= |\gamma_{A'B'}(i\omega)|\exp[i(\theta_A(\omega)-\theta_B(\omega)+\omega d/v_{app})] \end{aligned} \tag{9}$$

where $\theta_A(\omega)-\theta_B(\omega) = \tan^{-1}\dfrac{\mathrm{Im}(H_A(i\omega)H_B^*(i\omega))}{\mathrm{Re}(H_A(i\omega)H_B^*(i\omega))}$ is the phase difference of motions at points A and B owing to wave propagation at the site. This derivation indicates that the wave propagation through a homogeneous site has no effect on coherency loss $|\gamma_{A'B'}(i\omega)|$, but it changes the phase delay between the spatial ground motion at base rock and on ground surface, and changes the ground motion intensity. It should be noted that this derivation is based on assumption that site condition is homogeneous, and ground motion is stationary. In real case, a soil site will not be homogeneous. The soil properties may vary randomly in space. Moreover, ground motion is not stationary. All these will cause coherency loss in spatial ground motions. However, study of the influence of local site conditions on spatial ground motion coherency loss is beyond the scope of the present paper. It should also be noted that the transfer function expressed in Eq. (7) is derived for the case with only one soil layer. If multiple soil layers are under consideration, it can be straightforwardly extended based on the seismic wave propagation theory as discussed by Wolf (1985).

3. Structural response equation formulation

The purpose of this paper is to investigate the ground motion spatial variation and site effect on responses of multi-supported structures, the soil-structure interaction effect is thus ignored. Without losing generality, a 3-DOF mathematical model, with one for the bridge deck and two for the support movements, is used in the present study. Effectively such structural model represents only a single dynamic mode of vibration with two additional kinematic degrees of freedom representing the spatial excitations in the longitudinal direction. The dynamic equilibrium equation can be written as

$$\begin{bmatrix} m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{v}^t \\ \ddot{u}_A \\ \ddot{u}_B \end{Bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{v}^t \\ \dot{u}_A \\ \dot{u}_B \end{Bmatrix} + \begin{bmatrix} k_A+k_B & -k_A & -k_B \\ -k_A & k_A & 0 \\ -k_B & 0 & k_B \end{bmatrix}\begin{Bmatrix} v^t \\ u_A \\ u_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

where m is the lumped mass of the bridge deck, v^t is the total displacement response, and u_A and u_B are the ground displacement at support A and B respectively, k_A and k_B are the stiffness of the two columns. The total response consists of dynamic response and quasi-static response

$$v^t = v + v^{qs} \tag{11}$$

The quasi-static response can be derived as

$$v^{qs} = \frac{1}{k_A + k_B}[k_A \; k_B]\begin{bmatrix} u_A \\ u_B \end{bmatrix} = [\phi_A \quad \phi_B]\begin{bmatrix} u_A \\ u_B \end{bmatrix} = (\phi_A u_A + \phi_B u_B) \tag{12}$$

in which $\phi_A = k_A/(k_A + k_B)$ and $\varphi_B = k_B/(k_A + k_B)$. The dynamic response can be obtained by solving the dynamic equilibrium equation

$$m\ddot{v} + c\dot{v} + (k_A + k_B)v = -m\ddot{v}^{qs} = -m(\phi_A \ddot{u}_A + \phi_B \ddot{u}_B) \tag{13}$$

Transfer Eq. (13) into frequency domain, the dynamic response can be obtained by

$$\bar{v}(i\omega) = \frac{-1}{\omega_0^2 - \omega^2 + 2i\xi_0\omega_0\omega}\bar{\ddot{v}}^{qs}(i\omega) = -H_s(i\omega)\bar{\ddot{v}}^{qs}(i\omega) = -H_s(i\omega)[\phi_A\bar{\ddot{u}}_A(i\omega) + \phi_B\bar{\ddot{u}}_B(i\omega)] \tag{14}$$

in which $\omega_0 = \sqrt{(k_A + k_B)/m}$ is the circular natural vibration frequency of the structure, ξ_0 is the damping ratio, and $H_s(i\omega)$ is the transfer function of the structure.

The power spectral density function of dynamic, quasi-static and total response can then be derived as

$$S_v(\omega) = |H_s(i\omega)|^2\{\phi_A^2 S_A(\omega) + \phi_B^2 S_B(\omega) + 2\phi_A\phi_B \mathrm{Re}[S_{AB}(i\omega)]\}$$

$$S_{v^{qs}}(\omega) = \frac{1}{\omega^4}\{\phi_A^2 S_A(\omega) + \phi_B^2 S_B(\omega) + 2\phi_A\phi_B \mathrm{Re}[S_{AB}(i\omega)]\}$$

$$S_{v^t}(\omega) = S_v(\omega) + S_{v^{qs}}(\omega) - \frac{2}{\omega^2}\mathrm{Re}\{H_s(i\omega)(\phi_A^2 S_A(\omega) + \phi_B^2 S_B(\omega) + 2\phi_A\phi_B \mathrm{Re}[S_{AB}(i\omega)])\} \tag{15}$$

in which '*Re*' denotes the real part of a complex number. In this study, the uniform ground motion is assumed to be the same as u_A. Under uniform ground motion excitation, Eq. (15) reduces to

$$S_{v_u}(\omega) = |H_s(i\omega)|^2 S_A(\omega)$$

$$S_{v_u^{qs}}(\omega) = \frac{1}{\omega^4}S_A(\omega)$$

$$S_{v_u^t}(\omega) = S_{v_u}(\omega) + S_{v_u^{qs}}(\omega) - \frac{2}{\omega^2}\mathrm{Re}[H_s(i\omega)S_A(\omega)] \tag{16}$$

4. Maximum response calculation

Standard random vibration method (Der Kiureghian 1980) is used to calculate the mean peak displacement, it is briefly described in the following.

For a zero mean stationary process $x(t)$ with known power spectral density function $S(\omega)$, its mth order spectral moment is defined as

$$\lambda_m \approx \int_0^{\omega_c} \omega^m S(\omega) d\omega \tag{17}$$

where ω_c is a high cut-off frequency.

The zero mean cross rate v and shape factor of the power spectral density function δ, can be obtained by

$$v = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{18}$$

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \tag{19}$$

the mean peak response can then be calculated by

$$x_{\max} = \left(\sqrt{2\ln v_e T} + \frac{0.5772}{\sqrt{2\ln v_e T}}\right)\sigma \tag{20}$$

where T is the duration of the stationary process, $\sigma = \sqrt{\lambda_0}$ is the standard deviation of the process, and

$$v_e T = \begin{cases} \max(2.1, 2\delta T) & 0 \le \delta < 0.1 \\ (1.63\delta^{0.45} - 0.38) vT & 0.1 \le \delta < 0.69 \\ vT & \delta \ge 0.69 \end{cases} \tag{21}$$

In the present study, the high cut-off frequency is taken as 25Hz since it covers the predominant vibration modes of most engineering structures and the dominant earthquake ground motion frequencies.

5. Numerical results and discussions

The effects of ground motion spatial variations and site conditions on structural responses are investigated in detail in the present study. Dynamic, quasi-static and total responses of the structure in Fig. 1 under different ground motions and site conditions are calculated. The phase shift effect of spatial ground motion owing to seismic wave propagation depends on a dimensionless parameter $f_0 t_d$ (Bolt *et al.* 1982, Harichandran and Wang 1988, Zerva 1990, Hao 1993), in which f_0 is the structural vibration frequency, and t_d is the time lag between ground motions at two points separated by d. In the previous studies without considering the site effect and with a flat ground surface assumption, $t_d = d/v_{app}$, in which v_{app} is the apparent wave propagation velocity corresponding to the spatial motions at the site. In this study, as discussed above, vertical wave propagation is assumed in the local site, then the time lag between motions at points A and B on ground surface can be estimated as $t_d = d/v_{app} + \tau_B - \tau_A$, in which τ_B and τ_A, as defined above, are time required for wave to propagate from B' to B and A' to A, respectively. The spatial ground motion phase shift effect is investigated by varying the vibration frequency f_0 of the structure.

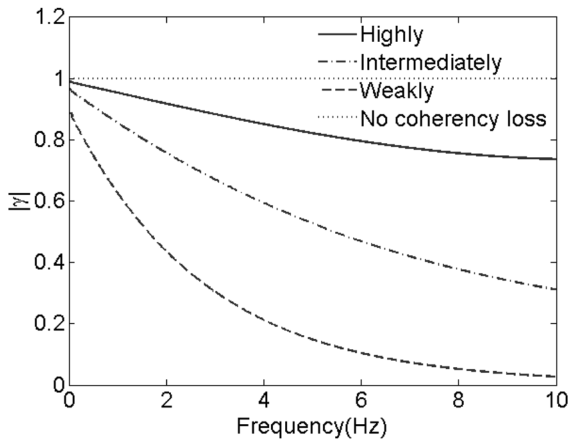

Fig. 3 Different coherency loss functions

Table 1 Parameters for coherency loss functions

Coherency loss	β	a	b	c
Highly(event 45)	1.109×10^{-4}	3.583×10^{-3}	-1.811×10^{-5}	1.177×10^{-4}
Intermediately	3.697×10^{-4}	1.194×10^{-2}	-1.811×10^{-5}	1.177×10^{-4}
Weakly	1.109×10^{-3}	3.583×10^{-2}	-1.811×10^{-5}	1.177×10^{-4}

Table 2 Parameters of base rock and different types of soil

Type	ρ (kg/m^3)	v (m/s)	ξ	$I_{R/S}$
Base rock	3000	1500	0.05	/
Firm soil	2000	450	0.05	5
Medium soil	1500	300	0.05	10
Soft soil	1500	100	0.05	30

The constants of coherency loss function in Eq. (2) are obtained by processing recorded motions during Event 45 at the SMART-1 array (Hao 1989). It should be noted that this coherency loss function represents highly correlated ground motions. For comparison, two modified coherency loss functions are also used in the study, which represent intermediately and weakly correlated ground motions, respectively. Fig. 3 shows different coherency loss functions corresponding to parameters given in Table 1. For spatial ground motion without coherency loss, $|\gamma_{A'B'}(i\omega)| = 1$ in Eq. (2).

The main parameters for base rock and soil conditions can be combined together to form a single coefficient defined as rock/soil impedance ratio (Rosset 1977)

$$I_{R/S} = \frac{\rho_R v_R}{\rho_S v_S} \tag{22}$$

This impedance coefficient reflects the differences between base rock and soil conditions. Without losing generality, three types of soils are studied in the paper. The corresponding parameters for the soil layer and the base rock are given in Table 2.

5.1 Effect of soil depth

The effects of soil depth on structural responses are investigated first. Six different soil depths are discussed, i.e., the bridge frame locates on a flat site with $h_A = h_B = 0$ m, $h_A = h_B = 30$ m, $h_A = h_B = 50$ m or locates on a canyon site with $h_A = 0$ m, $h_B = 30$ m, $h_A = 0$ m, $h_B = 50$ m and $h_A = 30$ m, $h_B = 50$ m. To preclude the influence of other parameters, the soil under both site A and B are assumed to be firm soil with $I_{R/S} = 5$, and the ground motions are assumed to be intermediately correlated.

As shown in Fig. 4, different soil depths lead to different transfer functions. The peaks occur at the corresponding vibration modes of the sites. Take $h = 50$ m as a example, the resonant frequencies of the soil layer are $f_k = v_s k/4h$, $k = 1, 3, 5, \ldots$, where v_s and h is the shear wave velocity and depth of the soil layer respectively, obvious peaks can be obtained at $f = 2.25$, 6.75 and 11.25 Hz with $v_s = 450$ m/s and $h = 50$ m. The deeper is the soil, the more flexible is the site, and the lower is the fundamental vibration frequency. The transfer function directly alters the ground motion power spectral density function on ground surface as compared to that at the base rock, as shown in Fig. 5. Motions on ground surface have a narrower band, but higher peak, as compared to that at the base rock, indicating the effect of site filtering and amplification on base rock motion. If the ground surface is flat, the time lag between motions at A and B are the same as those at the base rock ($\tau_B - \tau_A = 0$) because soil properties are assumed to be the same at the two wave paths. In this case, wave propagation through the site will not cause further phase difference. However, if a canyon site is assumed, the time for wave propagating from base rock to ground surface is different ($\tau_B - \tau_A \neq 0$), which results in an additional phase difference between motions at A and B, as compared to those at the base rock, as shown in Fig. 6.

Dynamic, quasi-static and total responses with varying structural vibration frequencies are calculated, and normalized by the corresponding responses to uniform excitation, which is defined as the motion at Point A, as discussed above. Figs. 7 and 8 show the normalized dynamic responses and total responses with respect to the dimensionless parameter, $f_0 t_d$, respectively. This parameter measures the relation between phase shift or time lag of spatial ground motions at points A and B

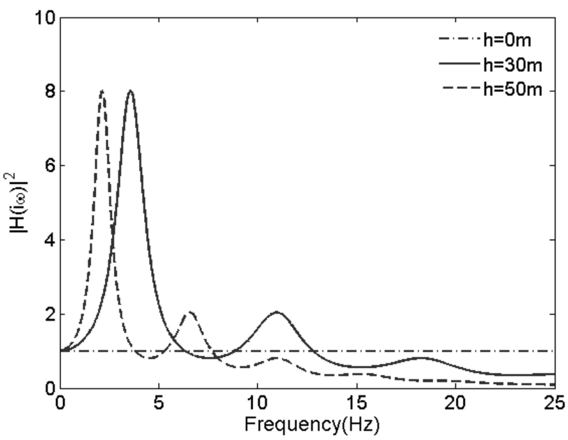

Fig. 4 Site transfer functions for different soil depths

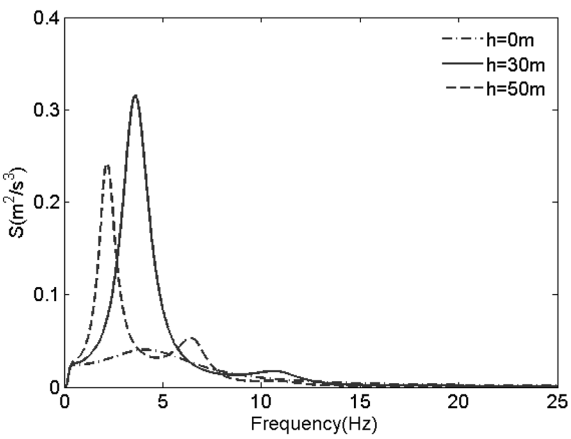

Fig. 5 Power spectral densities of ground motions on site of different depths

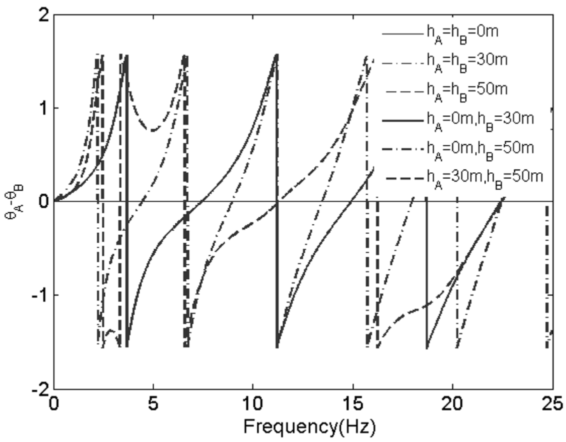

Fig. 6 Phase difference caused by seismic wave propagation through sites of different depths

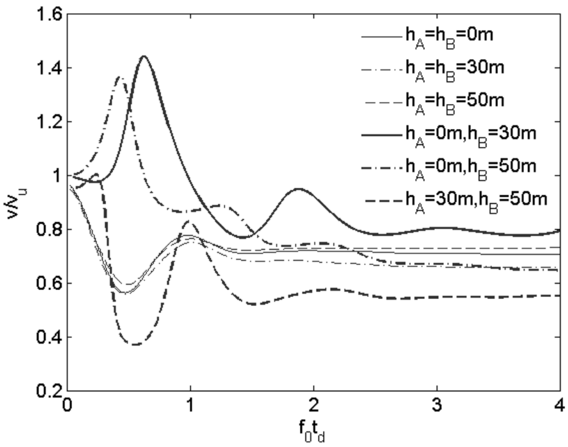

Fig. 7 Normalized dynamic responses for different soil depths

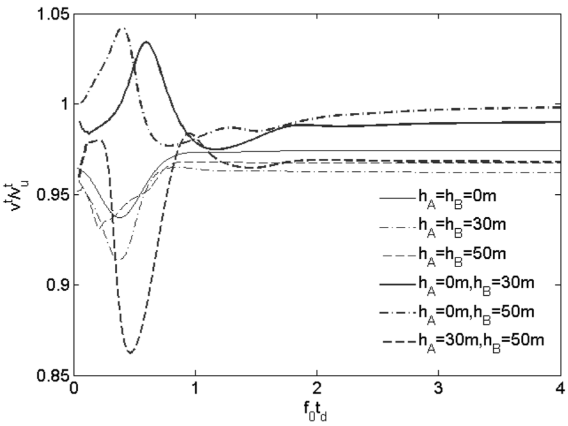

Fig. 8 Normalized total responses for different soil depths

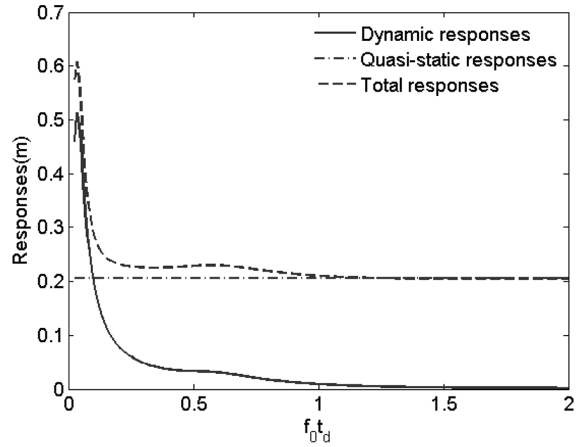

Fig. 9 Dynamic, quasi-static and total responses with $h_A = 0$ m and $h_B = 30$ m

and the fundamental vibration mode with frequency f_0. When a flat site is considered, $f_0t_d = f_0d/v_{app}$, and the multiple ground excitations and the structural vibration mode are in-phase if $f_0t_d = 1.0, 2.0, \ldots$, whereas they are out-of-phase if $f_0t_d = 0.5, 1.5, \ldots$ for the special case (Hao 1998, Hao and Zhang 1999).

As shown in Fig. 7, if the site is flat, non-uniform ground motion always reduces the dynamic responses as compared to the uniform ground motion. The normalized dynamic responses reach their minimum value at $f_0t_d = 0.5$, 1.5 and maximum value at $f_0t_d = 1.0$, 2.0 because of the out-of-phase and in-phase ground motion inputs. This observation is the same as those reported in many previous studies (Hao 1998, Hao and Zhang 1999). If a canyon site is assumed with Point A on base rock and Point B on soil surface, the maximum responses, however, do not occur at $f_0t_d = 1.0$. This is because of the dominance of site amplification effect on ground motions and resonant responses. The maximum response occurs when the structure is resonant with the soil site. For example, when $h_A = 0$ m and $h_B = 30$ m, the first peak occurs at $f_0 t_d = 0.625$, or $f_0 = 3.75$ Hz because $t_d = d/v_{app} + \tau_B = d/v_{app} + h_B/v_B = 0.16667$ sec. The second peak can be observed when $f_0 = 11.25$ Hz. As shown in Figs. 4 and 5, the resonant frequencies of the site with soil depth 30 m are $f_k = v_s k/4h = 450k/(4\times 30) = 3.75\,k$, $k = 1, 3, 5, \ldots$. If $h_A = 0$ m and $h_B = 50$ m, the first peak occurs at $f_0 t_d = 0.475$, or $f_0 = 2.25$ Hz because $t_d = 0.2111$ sec. Again as shown in Figs. 4 and 5, 2.25 Hz is the fundamental vibration frequency of the soil site with depth 50 m. The following peaks can also be observed when resonance occurs. If both point A and B locate on soil surface with $h_A = 30$ m and $h_B = 50$ m, the spatial ground motion wave passage effect dominates the site effect on dynamic structural responses, i.e., the minimum values occur around $f_0 t_d = 0.5, 1.5$, and the maximum values around $f_0 t_d = 1.0, 2.0$. This is because, although site A and B have different fundamental vibration modes and different peak values in their respective power spectral density function as shown in Fig. 5, the mean peak responses to ground motion at site A and B are similar to each other because they depend on the spectral moments as defined above. Therefore, normalization removes the site amplification effects, which leaves the wave passage effects to govern the normalized dynamic response in this case. It can also be noted that the normalized dynamic responses are always smaller than 1.0 when wave passage effect dominates, indicating the spatial ground motion phase shift always results in a reduction in dynamic structural responses. Similar observation has also been obtained in previous studies (Hao 1998, Hao and Zhang 1999). When the vibration frequency of the structure coincides with the fundamental frequency of the soil layer, however, the normalized peak dynamic responses can be larger than 1.0, indicating the significance of site amplifications on ground motions and hence on structural responses. These observations indicate the importance of considering both the site and the ground motion spatial variation effects in structural response analysis.

Quasi-static responses are independent of the fundamental vibration frequency of the structure (Eqs. (15), (16)). The normalized quasi-static responses are therefore constant for each case with respect to $f_0 t_d$. The normalized total responses are given in Fig. 8. As shown, when the dimensionless parameter $f_0 t_d$ is less than 1.5, the normalized total responses are similar to the normalized dynamic responses, indicating dynamic response dominates the total response. When $f_0 t_d$ increases, however, the normalized responses approach to a constant, equal to the quasi-static response. Neither spatial ground motion wave passage effect, nor the site amplification effect is prominent. This is because increasing $f_0 t_d$ implies the structure becomes stiffer, as f_0 is increased in this study. The dynamic response is smaller when structure is stiffer. At large $f_0 t_d$, quasi-static response dominates the total response, as shown in Fig. 9. This observation indicates the importance of quasi-static responses for stiff structures.

5.2 Effect of soil properties

To study the effect of soil properties on ground motion spatial variation and hence on structural responses, different soil types shown in Table 2 are considered. The soil under point A is assumed to be firm soil ($I_{R/S} = 5$) and unchanged in all the cases, while soil under support B varies from firm soil ($I_{R/S} = 5$) to soft soil ($I_{R/S} = 30$). The soil depths are assumed to be $h_A = 30$ m and $h_B = 50$ m, and the ground motions are intermediately correlated. Fig. 10 shows the transfer function at support B for different cases. Fig. 11 shows the corresponding power spectral density function of motion on ground surface at Point B. For comparison purpose, the power spectral density function of motion at

Point A is also shown in these two figures. Fig. 12 shows the phase differences between motions at Point A and B. The normalized dynamic responses and total responses are shown in Figs. 13 and 14, respectively.

Fig. 10 clearly shows again the site effects. As shown, peak value of the transfer function increases, while the frequency band becomes narrower with the decrease of the site stiffness. This directly affects the ground motions on ground surface, resulting in substantial spatial variations between ground motions at Points A and B. Soft soil ($I_{R/S} = 30$) and medium soil ($I_{R/S} = 10$) significantly amplifies the ground motions at its resonant frequencies, firm soil ($I_{R/S} = 5$) also amplifies ground motions, but at higher frequencies and with a less extent. As a result, the ground

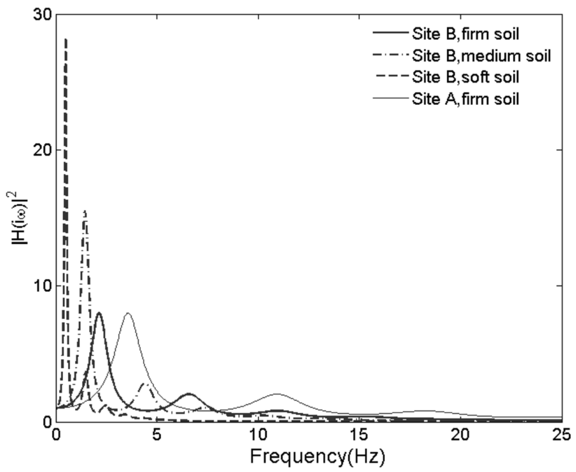

Fig. 10 Soil site transfer function for different soil properties

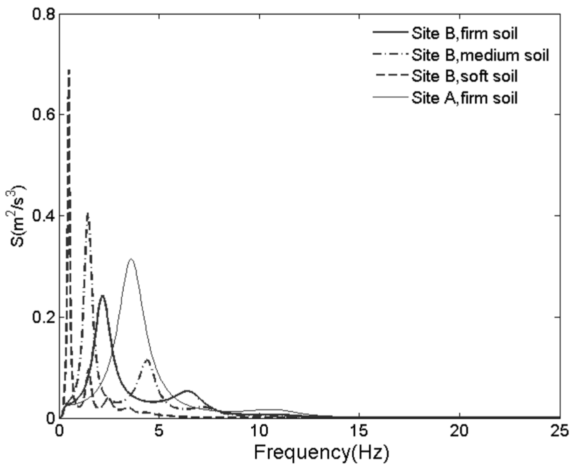

Fig. 11 Power spectral densities of ground motions at sites with different soil properties

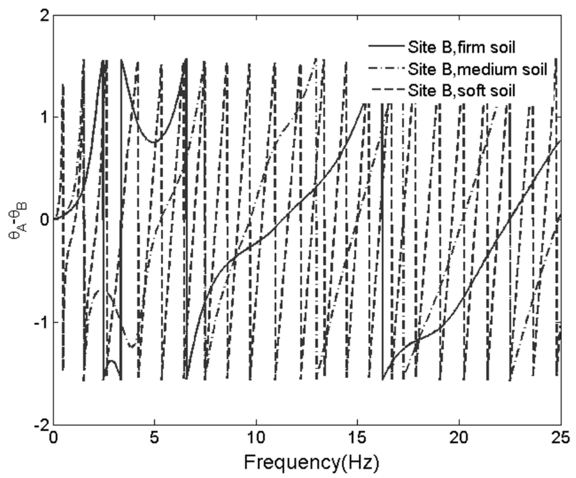

Fig. 12 Phase difference owing to seismic wave propagation through sites with different soil properties

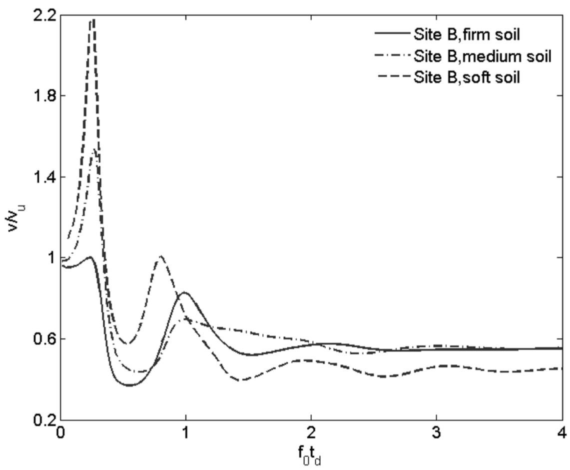

Fig. 13 Normalized dynamic responses for different soil properties

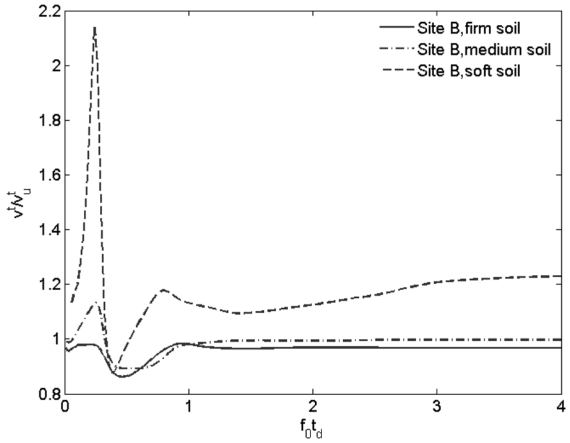

Fig. 14 Normalized total responses for different soil properties

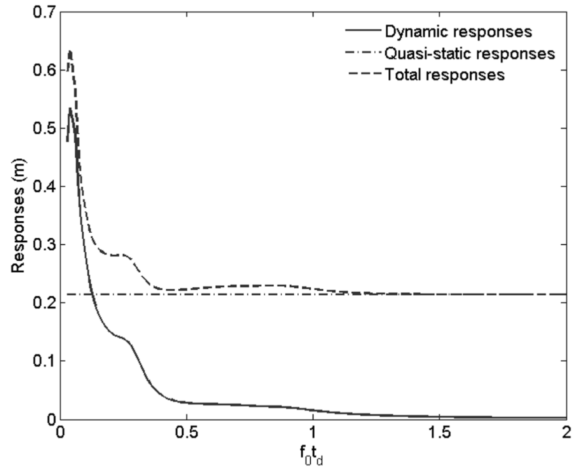

Fig. 15 Dynamic, quasi-static and total responses (medium soil at support B)

motion power spectral densities at ground surface are very different as shown in Fig. 11. Soil properties also affect the seismic wave propagation velocity and hence the phase difference between motions at Point *A* and *B*. Fig. 12 shows the phase differences between motions at *A* and *B* owing to wave propagation from base rock to ground surface. It shows that the phase differences vary rapidly with respect to frequency. The softer is the soil, the more drastic variation is the phase difference because the wave velocity is slower.

Again, as shown in Fig. 13, when the vibration frequency of the structure is low, site effect dominates the dynamic responses. When the soil properties of site *A* and *B* are different from each other significantly, i.e., the maximum responses occur when the structure resonates with the soil site. For example, when site *B* is the medium soil, the first peak occurs at $f_0t_d = 0.3$, or $f_0 = 1.5$ Hz because $t_d = 0.2$ sec. As shown in Figs. 10 and 11, the fundamental vibration frequency for the medium site is 1.5 Hz. When site *B* is a soft soil site, the first peak occurs at $f_0t_d = 0.267$, or $f_0 = 0.5$ Hz because $t_d = 0.5333$ sec, and the second peak at $f_0t_d = 0.8$, corresponding to the second mode of the site *B*. Subsequent peaks of these two cases are associated with the in-phase excitations and the minimum values are associated with the out-of-phase effect. This is because when the structure becomes stiffer, the dynamic response and hence the site resonance effect becomes less significant as compared with the ground motion spatial variation effect. As also can be seen in Fig. 13, soft soil amplification effect results in larger dynamic responses, normalized dynamic responses are usually larger than 1.0 when the responses are dominated by the site effect, and the results are always less than 1.0 when spatial ground motion phase shift effect governs the dynamic responses.

Total responses shown in Fig. 14 follow the similar pattern as that discussed above, i.e., the normalized total responses are similar to the normalized dynamic responses when f_0t_d is less than 1.5. However, if the structure is stiff, the dynamic responses are small and the total responses are dominated by the quasi-static responses, as shown in Fig. 15 (medium soil at support *B*).

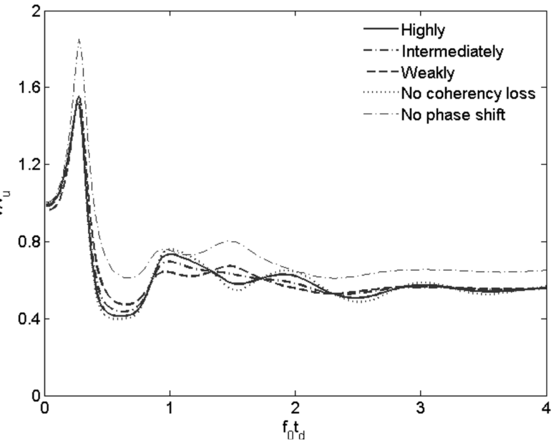

Fig. 16 Normalized dynamic responses for different coherency losses

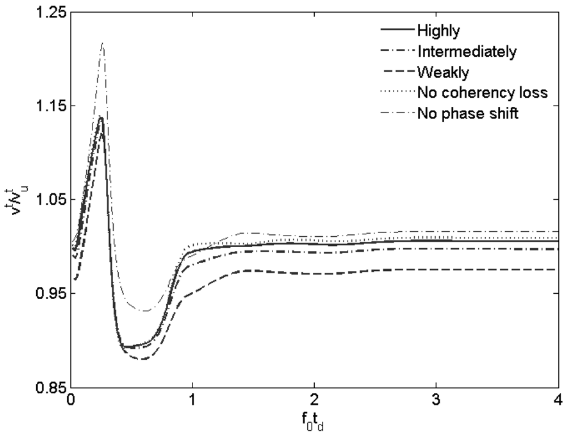

Fig. 17 Normalized total responses for different coherency losses

5.3 Effect of coherency loss

To investigate the influence of ground motion spatial variation, different coherency losses are considered in the paper as shown in Fig. 3, i.e., highly, intermediately and weakly correlated coherency loss functions. Moreover, two special cases, i.e., intermediate coherency loss without considering phase shift ($\cos(\omega t_d) = 1.0$), and no coherency loss ($|\gamma_{A'B'}(i\omega)| = 1$), are also considered. All the results are normalized by the corresponding uniform excitation. For these cases, the canyon site with $h_A = 30$ m and $h_B = 50$ m is considered, and medium soil ($I_{R/S} = 10$) are assumed at both sites A and B. Fig. 16 shows the normalized dynamic responses and Fig. 17 shows the normalized total responses.

As shown in Fig. 16, site effect governs the dynamic responses when $f_0t_d < 0.5$, i.e., peak response occurs at the resonant frequency with $f_0 = 0.2$ Hz and $f_0t_d = 0.3$. However, if $f_0t_d > 0.5$, spatial ground motion wave passage effect dominates the dynamic responses, i.e. the normalized dynamic responses reach their minimum value at $f_0t_d = 0.5, 1.5$ and their maximum value at $f_0t_d = 1.0, 2.0$ because of the out-of-phase and in-phase ground motion excitations. The more correlated are the ground motions, the more pronounced are the in-phase and out-of-phase effects. This means that the influence of site effect is more significant when the structure is relatively flexible, while spatial ground motion wave passage effect dominates the dynamic responses when the structure is stiff. If multiple ground motion phase shift is not considered, normalized peak response occurs at vibration modes of the soil site, no in-phase or out-of-phase effects are present. For total responses as shown in Fig. 17, similar observations can be drawn, i.e., dynamic response dominates total response when $f_0t_d < 1.0$, and quasi-static response is more significant when the structure is stiff.

6. Conclusions

This paper studies the combined effects of ground motion spatial variation and local site conditions on the responses of a bridge frame located on a canyon site. Dynamic, quasi-static and total responses of the model structure to various cases of spatially varying ground motions are investigated. Following conclusions can be drawn:

1. Wave propagation through multiple sites with different site conditions causes further variations of spatial ground motions. Depending on the soil conditions along each wave path, spatial ground motions at different locations on surface of a canyon site have different power spectral densities and more pronounced phase shift as compared to those on the base rock.

2. Local site conditions significantly affect spatial surface ground motions, and hence the structural responses. The dynamic peak responses occur when the structure resonates with the site, and when the spatial ground motion and structural vibration mode are in-phase. The minimum dynamic responses occur when the spatial ground motion and structural vibration mode is out-of-phase.

3. Dynamic response governs the total response when the structure is flexible, while quasi-static response dominates it when the structure is stiff.

4. Different site conditions at two structural supports causes more significant spatial variations of ground motions, and hence larger structural responses.

5. Spatial ground motion coherency loss has a relatively less significant effect on structural responses when the structure is flexible and the total response is governed by the dynamic response. However, coherency loss effect is prominent, especially when the structure is stiff.

6. Uniform site assumption leads to underestimation of spatial variations of ground motions on a canyon site, and therefore underestimation of structural responses.

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