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Topological optimized design considering dynamic problem with non-stochastic structural uncertainty

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Abstract. This study shows how uncertainties of data like material properties quantitatively have an influence on structural topology optimization results for dynamic problems, here such as both optimal topology and shape. In general, the data uncertainties may result in uncertainties of structural behaviors like deflection or stress in structural analyses. Therefore optimization solutions naturally depend on the uncertainties in structural behaviors, since structural behaviors estimated by the structural analysis method like FEM need to execute optimization procedures. In order to quantitatively estimate the effect of data uncertainties on topology optimization solutions of dynamic problems, a so-called interval analysis is utilized in this study, and it is a well-known non-stochastic approach for uncertainty estimate. Topology optimization is realized by using a typical SIMP method, and for dynamic problems the optimization seeks to maximize the first-order eigenfrequency subject to a given material limit like a volume. Numerical applications topologically optimizing dynamic wall structures with varied supports are studied to verify the non-stochastic interval analysis is also suitable to estimate topology optimization results with dynamic problems.

Keywords: topology optimization design; uncertainty; dynamic problem; eigenfrequency.

1. Introduction

In the recent years, there has been an increased interest in the simulation of structural systems with uncertainties. The interest in uncertain structural systems stems from the fact that uncertainties remain in most models of real world problems. Uncertainties arise either due to our lack of knowledge, or due to intrinsic variabilities of physical quantities. Data like domain geometry, material properties, or loads, are usually not known perfectly. Due to the uncertainties in the model, it is uncertain to what degree the prognoses of numerical simulations match reality and this fact is often ignored in traditional engineering practice. Clearly, it is desirable to quantify the uncertainties in the answer, and different approaches have been proposed for this as follows.

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A stochastic approach is described by stochastic models, and then uncertain parameters are described by random variables. Uncertain spatial properties are modeled by random fields. Alternatively, fuzzy sets may be used to describe uncertainties (Neumaier 2003, 2004). Uncertain parameters are described by possibility functions specifying their degree of belonging to a set. Maglaras *et al.* (1997) compare random and fuzzy models of uncertainty and state that uncertainty is better represented by a stochastic description if enough statistical information is available and that otherwise fuzzy theory is better suited. However, given that only small amounts of statistical information about data are available only in a few specialized cases, the fuzzy and stochastic approaches can not deliver reliable solutions without sufficient experimental data.

In contrast to fuzzy and stochastic methods, set methods are independent of a probability or possibility measure. They assume that parameters are inside given sets. Then they compute sets in that the response is guaranteed to lie. Representatives of this approach are interval analysis (Moore 1966) and its generalizations to ellipsoidal and convex modeling (Zhiping 2003).

In structural designs and analyses, what causes uncertainty of structural responses is generally divided into internal and external components of structures. The former is the uncertainty by boundary conditions, loading conditions, assumptions of modeling, and analytical assumption, the latter is the uncertainty by the workmanships and natural environmental conditions.

In particular, these uncertainties of structural behaviors may also have an effect on optimal designs of structures. Topology optimization design (Bendsøe *et al.* 1988) as a representative of structural optimization designs produces both optimal shape and topology of required structures and then provides conceptual design information to designers. Topology optimization design including the effect of uncertainty may give more reliable and practical solutions than conventional design assuming uncertainties are excluded in data and all data are nominal values.

This study presents a dynamic topology optimization design considering numerical uncertainties of structural behaviors. The uncertainties of structural behaviors result from numerical uncertainties of initial data such as structural parameters and loading conditions. Here in order to evaluate data uncertainties and structural behaviors during optimization procedures, interval arithmetic is utilized as a kind of non-stochastic methods. It is assumed in the interval arithmetic that the numerical uncertainties of initial data are defined as a tolerance error and systematic uncertainties of structures are represented by uncertainty combination of initial data. Optimal design results of structure can be easily yielded with reliable structural safety by considering critical cases of uncertainty combinations. It is a main advantage of the present method. For dynamic topology optimization design, a density distribution method (Bendsøe *et al.* 1988) is carried out since it is superior to other optimization methods with respect to reducing computational burdens.

In this study, numerical applications of varied structures like a cantilever, a clamped structure, and a structure with fixed four supports are investigated for the dynamic topology optimization design by finite element method using interval arithmetic. The optimal results with uncertainties are compared with conventional topology optimization solutions without considering the uncertainties and efficiency of the present approach is demonstrated.

The outline of this study is as follows. In Section 2, the theory of interval arithmetic and an interval change function are described. Dynamic topology optimization formulation and algorithm related to data uncertainty are presented in Section 3. Numerical applications and discussion for uncertainty estimate of dynamic topology optimization design of some structure are studied in Section 4 followed by the conclusions in Section 5.

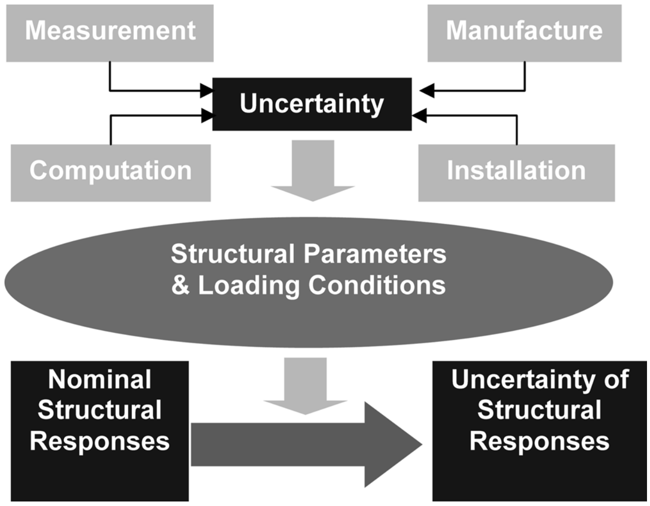


Fig. 1 Structural response uncertainty by uncertainties of initial data

2. Non-stochastic interval analysis

2.1 Interval arithmetic

In structural designs and analyses, what causes uncertainty of structural responses is generally divided into internal and external components of structures. The former is the uncertainty by boundary conditions, loading conditions, assumptions of modeling, and analytical assumption, the latter is the uncertainty by the workmanships and environmental conditions. Fig. 1 shows the uncertainty of structural response and its reason with respect to initial data of structures.

An interval Analysis is a field of mathematics that accounts for numerical imprecision and physical uncertainty with intervals using set-based operations. In the interval arithmetic, the errors or uncertainties are always denoted by intervals.

From this principle, we define intervals firstly. In general, an interval arithmetic operation $\circ$ between intervals a and b is given as

$$a \circ b = hull\{\tilde{a} \circ \tilde{b} | \tilde{a} \in a, \tilde{b} \in b; \tilde{a} \circ \tilde{b} \text{ is defined}\} \tag{1}$$

where the *hull* of a set produces the minimum and maximum bounds. *Interval vectors* and *interval matrices* are nothing more than standard vectors and matrices with intervals instead of scalar values for components and elements (Alefeld *et al.* 2000, Sunaga 1958, Dwyer 1951, Moore 1966).

Let $c = (c_1, c_2, \ldots, c_m)^T$ be a structural parameter vector with bound error or uncertainties and $c \in a \circ b$
where

$$c_i \in c_i^I = \{c_i^c - \Delta c_i, c_i^c + \Delta c_i\} \tag{2}$$

then

$$c \in c^I = \{c^c - \Delta c, c^c + \Delta c\} \tag{3}$$

where $\underline{c} = c^c - \Delta c$ is the lower bound of an interval and $\overline{c} = c^c + \Delta c$ is the upper bound of an interval. Also we define the *mid-point* of an interval c^c by

$$c^c = (c_1^c, c_2^c, \ldots, c_m^c)^T \tag{4}$$

We define the *uncertainty* of an interval Δc by

$$\Delta c = (\Delta c_1, \Delta c_2, \ldots, \Delta c_m)^T \tag{5}$$

In a similar way, expressed by $\mathbf{X}^I = [\underline{X}, \overline{X}]$, the mid-point and uncertainty of a n-dimensional interval vector $\mathbf{X}^I = (X_1^I, X_2^I, \ldots, X_n^I)^T$ can be described by

$$\mathbf{X}^c = (X_1^c, X_2^c, \ldots, X_n^c)^T \tag{6}$$

and

$$\Delta\mathbf{X} = (\Delta X_1, \Delta X_2, \ldots, \Delta X_n)^T \tag{7}$$

Commonly used notions are the *mid-point* of an interval vector $\mathbf{X}^c$

$$\mathbf{X}^c = \frac{\overline{X} + \underline{X}}{2} \tag{8}$$

and the *uncertainty* of an interval vector $\Delta\mathbf{X}$

$$\Delta\mathbf{X} = \frac{\overline{X} - \underline{X}}{2} \tag{9}$$

A matrix whose elements are the interval parameters is called an interval matrix and expressed by $\mathbf{A}^I = [\underline{A}, \overline{A}]$, in which $\underline{A}$ and $\overline{A}$ consist of each lower and upper bound. Similarly, the mid-point and uncertainty of n-dimensional interval matrix $\mathbf{A}^I = (A_1^c, A_2^c, \ldots, A_n^c)^T$ can be expressed by

$$\mathbf{A}^c = (A_1^c, A_2^c, \ldots, A_n^c)^T \tag{10}$$

and

$$\Delta\mathbf{A} = (\Delta A_1, \Delta A_2, \ldots, \Delta A_n)^T \tag{11}$$

Commonly used notions are the *mid-point* of an interval matrix $\mathbf{A}^c$ as follows.

$$\mathbf{A}^c = \frac{\overline{A} + \underline{A}}{2} \tag{12}$$

and the *uncertainty* of an interval matrix $\Delta\mathbf{A}$ as follows.

$$\Delta\mathbf{A} = \frac{\overline{A} - \underline{A}}{2} \tag{13}$$

For many operations, including standard arithmetic operations of *addition*, *subtraction*, *multiplication* and *division*, the resulting set is also an interval that can be conveniently defined in term of end-points of the argument intervals.

Let $\mathbf{X}^I = [\underline{X}, \overline{X}]$ and $\mathbf{Y}^I = [\underline{Y}, \overline{Y}]$ be the intervals, then the operations are defined by the following formulas.

$$\mathbf{X}^I + \mathbf{Y}^I = [\underline{X}, \overline{X}] + [\underline{Y}, \overline{Y}] = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}] \tag{14}$$

$$\mathbf{X}^I - \mathbf{Y}^I = [\underline{X}, \overline{X}] - [\underline{Y}, \overline{Y}] = [\underline{X} - \overline{Y}, \overline{X} - \underline{Y}] \tag{15}$$

$$\mathbf{X}^I \times \mathbf{Y}^I = [\underline{X}, \overline{X}] \times [\underline{Y}, \overline{Y}] = [\min(\underline{X} \cdot \underline{Y}, \underline{X} \cdot \overline{Y}, \overline{X} \cdot \underline{Y} \cdot \overline{X} \cdot \overline{Y}), \max(\underline{X} \cdot \underline{Y}, \underline{X} \cdot \overline{Y}, \overline{X} \cdot \underline{Y} \cdot \overline{X} \cdot \overline{Y})] \tag{16}$$

$$\mathbf{X}^I / \mathbf{Y}^I = [\underline{X}, \overline{X}] / [\underline{Y}, \overline{Y}] = \begin{cases} [\underline{X}, \overline{X}] \times \left[\frac{1}{\underline{Y}}, \frac{1}{\overline{Y}}\right], & \text{if } 0 \in [\underline{Y} \cdot \overline{Y}] \\ [-\infty, +\infty], & \text{otherwise} \end{cases} \tag{17}$$

2.2 Interval change function

An interval change function is evaluated as a mathematical formulation which is composed of the upper and lower bounds with respect to the tolerance error $\mathbf{x}$. A basic idea behind the interval change function is quantitatively to calculate the changes of the required results that take place in uncertainty problems, when a small change (i.e., uncertainty) is made by the uncertain parameters against some nominal values in the structural system.

Considering whether numerical uncertainties of the initial data exist or not, a generalized scenario function of the uncertainty $\Im_i$ may be written as follows:

$$\Im_i = \frac{b_1^c b_2^c b_3^c \ldots b_q^c}{a_1^c a_2^c a_3^c \ldots a_p^c} [\mathbf{f}(\mathbf{x})_i, \mathbf{g}(\mathbf{x})_i] \tag{18}$$

where

a^c, b^c : Mid-point of each uncertain parameter

i : 2^{p+q}, The number of uncertainty scenarios

p, q : The number of uncertain parameters

$\mathbf{f}(\mathbf{x})_i = \dfrac{(1-\mathbf{x})^q}{(1+\mathbf{x})^p}$: Lower bounds of an interval change function

$\mathbf{g}(\mathbf{x})_i = \dfrac{(1+\mathbf{x})^q}{(1-\mathbf{x})^p}$: Upper bounds of an interval change function

$\mathbf{x}$: The tolerance error, $0 \le \mathbf{x} < 1, \mathbf{x} \in \mathbf{R}$

When different parts of the structure are uncertain, it is a systematic uncertainty problem of structures. Except for members with certainty, the interval change functions determine geometric and material properties of uncertain members with respect to the structural system. The different combinations of members with and without uncertainties results in different systematic behaviors.

3. Topology optimization formulation for dynamic problems

3.1 Governing equation for dynamic problems

Governing equation for free vibration systems considered in this study can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} = \underline{0} \tag{19}$$

By using Laplace transformat*i*on Eq. (19) can be rewritten as

$$\mathbf{M}\mathbf{U}(l)l^2 + \mathbf{K}\mathbf{U}(l) = \underline{0} \tag{20}$$

By substituting ω^2 for l into Eq. (20), the final eigenvalue problem is defined as

$$[\mathbf{K} - \omega_i^2\mathbf{M}]\mathbf{u}_i = \underline{0} \tag{21}$$

where **K** and **M** are the global stiffness and mass matrix, respectively. ω_i is the *i*-th eigenfrequency and u_i denotes the corresponding eigenvector depending on ω_i. In order to numerically solve Eq. (21), **K** and **M** have to be the symmetric and positive definite (Lehoucq *et al.* 1998) stiffness and mass matrices of the finite element-based, generalized structural eigenvalue.

3.2 Topology optimization formulations for dynamic problems

Eigenvalue optimization designs are profitable for mechanical structural systems subjected to dynamic loading conditions like earthquakes and wind loads. The dynamic behaviors of structural systems can be estimated by eigenfrequency which describes structural stiffness. In general maximizing first-order eigenfrequency can be an objective for dynamic topology optimization problems since stiffness of structures also increases when eigenfrequency increases. Problems of topology optimization for maximizing natural eigenfrequencies of vibrating elastostatic structures have been considered in the studies (Pedersen 2000, Rong *et al.* 2002, Barbarosie *et al.* 2009).

Assuming that damping can be neglected, such a dynamic design problem can be formulated as follows.

$$\max_{\underline{\Phi}} : \quad \omega_1^2(\Phi) = \frac{\mathbf{u}_1^T\mathbf{K}\mathbf{u}_1}{\mathbf{u}_1^T\mathbf{M}\mathbf{u}_1} \tag{22.1}$$

$$\textit{subject to}: \quad \frac{V(\Phi)}{V_0} \le g \tag{22.2}$$

$$: [\mathbf{K} - \omega_i^2\mathbf{M}]\mathbf{u}_i = \underline{0} \tag{22.3}$$

$$: 0 < \Phi_{\min} \le \Phi \le \Phi_{\max} \tag{22.4}$$

where these discrete formulations for the dynamic problem are equal to continuous formulations for static problems except for objective and governing equation.

3.3 Constitutions of K and M by interpolation scheme of SIMP material

According to the SIMP approach (Bendsøe *et al.* 1988), the material density distribution affects element stiffness and the element stiffness-density relationship may be expressed in terms related to Young's modulus E_i^h, is associated with the updated element density Φ_i^h and it is defined as

$$E_i^h(\Phi_i^h) = E_0\left(\frac{\Phi_i^h}{\Phi_0}\right)^k, \quad k \ge 1, 0 \le \Phi_i^h \le 1, \quad i = 1 \dots N_e \tag{23}$$

where E_0 and Φ_0 denote nominal values of Young's modulus and material density of elements, respectively, and N_e is the number of elements.

According to the penalized Young's module, element stiffness matrix of four-node square elements with eight-DOF used in this study is written as

$$\mathbf{K}_e = \int_V \mathbf{B}^T \mathbf{C} \mathbf{B} dV$$

$$= \frac{E_i^h(\Phi_i^h)}{1-\nu^2}\begin{bmatrix} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\ \cdot & k_1 & k_8 & k_7 & k_6 & k_5 & k_4 & k_3 \\ \cdot & \cdot & k_1 & k_6 & k_5 & k_4 & k_3 & k_2 \\ \cdot & \cdot & \cdot & k_1 & k_8 & k_3 & k_2 & k_5 \\ \cdot & \cdot & \cdot & \cdot & k_1 & k_2 & k_3 & k_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & k_1 & k_8 & k_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & k_1 & k_6 \\ sym. & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & k_1 \end{bmatrix} \tag{24}$$

where

$$k_1 = \frac{1}{2} - \frac{\nu}{12}, \quad k_5 = -\frac{1}{4} + \frac{\nu}{12}$$

$$k_2 = \frac{1}{8} + \frac{\nu}{8}, \quad k_6 = -\frac{1}{8} - \frac{\nu}{8}$$

$$k_3 = -\frac{1}{4} - \frac{\nu}{12}, \quad k_7 = \frac{\nu}{6}$$

$$k_4 = -\frac{1}{8} + \frac{3\nu}{8}, \quad k_8 = \frac{1}{8} - \frac{3\nu}{8}$$

Please note that the stiffness formulation is used for both static and dynamic problems in this study. For example, an isotropic material model with a plane stress (such as a wall structure) is used here without loss of generality, so that

$$\mathbf{C}_i^h = \frac{E_i^h(\Phi_i^h)}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{25}$$

where $\mathbf{C}_i^h$ is a material tensor of each finite element i and includes the updated term of Young's modulus E_i^h which has been defined by the updated element density average Φ_i^h. ν is Poisson's ratio.

According to dynamic topology optimization problems using SIMP material, mass matrix also includes the same penalty formulation such as the stiffness matrix. Therefore it can be written as

$$\mathbf{M}_e = (\Phi_e)^k \mathbf{M}_0 \tag{26}$$

For the mass matrix, a lumped mass matrix $\mathbf{M}_L$, a consistent mass matrix $\mathbf{M}_C$ or a combination of those two can be used. The lumped and consistent mass matrices are written as respectively in case discretization of eight-node square elements with 8 DOFs.

$$\mathbf{M}_C^e = \int_V \Phi \mathbf{N}^T \mathbf{N} dV$$

$$= \frac{1}{4}\Phi A \begin{bmatrix} 4 & 0 & 2 & 0 & 1 & 0 & 2 & 0 \\ \cdot & 4 & 0 & 2 & 0 & 1 & 0 & 2 \\ \cdot & \cdot & 4 & 0 & 2 & 0 & 1 & 0 \\ \cdot & \cdot & \cdot & 4 & 0 & 2 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & 4 & 0 & 2 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 4 & 0 & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 4 & 0 \\ sym. & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 4 \end{bmatrix} \tag{27}$$

$$\mathbf{M}_L^e = \frac{1}{4}\Phi A \mathbf{I}_{(8\times 8)} \tag{28}$$

where Φ and A denote the material density and area of elements, respectively and $\mathbf{I}$ is the 8×8 unit matrice.

3.4 Definition of uncertainty data by using interval arithmetic

In this study it is assumed that material properties such as E and ν have data uncertainties. When uncertainties of the data are considered, stiffness of structure may be described by interval arithmetic formulations in global systems.

Through interval arithmetic, E and ν including data uncertainty described by upper and lower bounds may be written as follows, respectively.

$$E_i = \left(\frac{\Phi_i}{\Phi_o}\right)^k [E_0^C - \alpha E_0^C, E_0^C + \alpha E_0^C] \tag{29}$$

$$\nu_i = [\,\nu_0^C - \beta\nu_0^C,\ \nu_0^C + \beta\nu_0^C] \tag{30}$$

where Φ_0, E_0^C, and ν_0^C are nominal values of density, Young's modulus, and Poisson's ratio, respectively. $\alpha,\ \beta \in \mathbf{R}$ are tolerance errors of uncertainty of data.

3.5 Numerical algorithm for dynamic topology optimization design considering uncertainty

The topology optimization processes are composed of structural analysis, sensitivity analysis and optimization method in turn of procedures.

When the repetitive solution is converged during optimization procedures, all iterations are finished and optimal solutions are obtained. For structural analysis, finite element method using interval arithmetic of section 2.1 is utilized.

A variational approach with adjoint method is applied for sensitivity analysis. With respect to design variables *s* (for instance, material element densities), the total differential form (Haug *et al.* 1986) of the objective function is the combination of parts of an explicit partial derivative and an implicit partial derivative as follows.

$$\nabla_s f = \nabla_s^{ex} f + \overline{\nabla}_u f^T \nabla_s \mathbf{u} \tag{31}$$

According to dynamic topology optimization, the total derivative of objective function of Eq. (22.1) is written as a simple discrete formulation as follows.

$$\frac{\partial \omega_1^2}{\partial \Phi_e} = \frac{u_1^{e^T}[k(\Phi_0)^{k-1}\mathbf{K}_0 - \omega_1^2 k(\Phi_e)^{k-1}\mathbf{M}_0]\mathbf{u}_1^e}{\mathbf{u}_1^T \mathbf{M} \mathbf{u}_1} \tag{32}$$

An OC (Optimality Criteria) method (Sigmund 2001) of gradient-based concepts is used for the optimization method since it can reduce computational consumptions of many design variables. By considering heuristic updating scheme (Bendsøe 1995) introducing a moving limit to prevent Φ_e from changing to much in one iteration, the design variables can now be updated using

$$\frac{\max\left(0, -\dfrac{\partial \omega_1^2}{\partial \Phi_e}\right)}{\lambda V_e} = B_n^e \tag{33}$$

and

$$\Phi_e^{n+1} = B_e^n \Phi_e^n \tag{34}$$

The above-mentioned algorithm of total processes is shown in Fig. 2. The developed MATLAB code for dynamic topology optimization design is based on MATLAB code (Sigmund 2001) for static designs. In MATLAB program Eigenvalue problem can be easily solved by using internal functions for user comfort. One of internal functions is "eigs" to automatically produce Eigenfrequency and Eigenvector by inputting a global stiffness matrix, a global mass matrix, and an Eigenmode order. It is shown in Fig. 3.

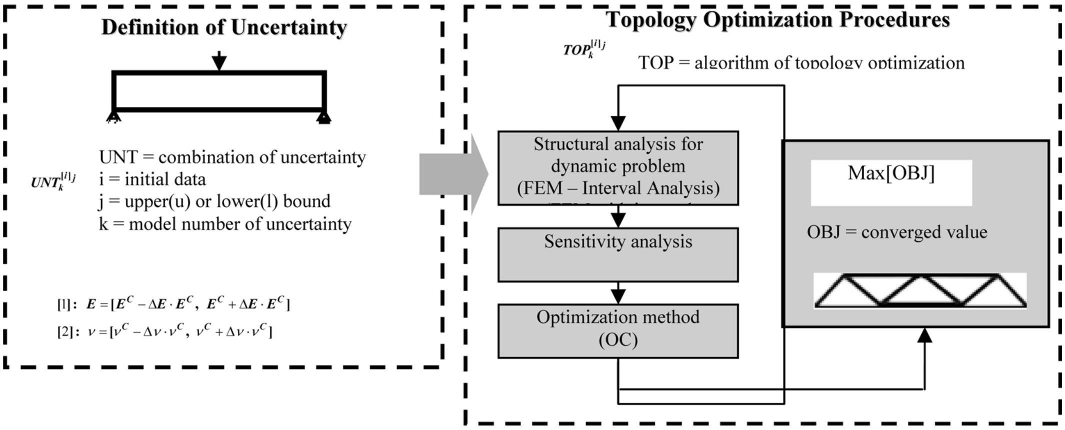


Fig. 2 Topology optimization algorithm by considering data uncertainty

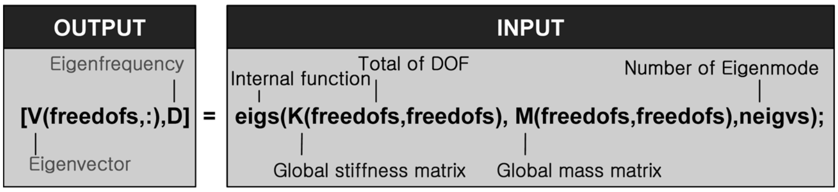
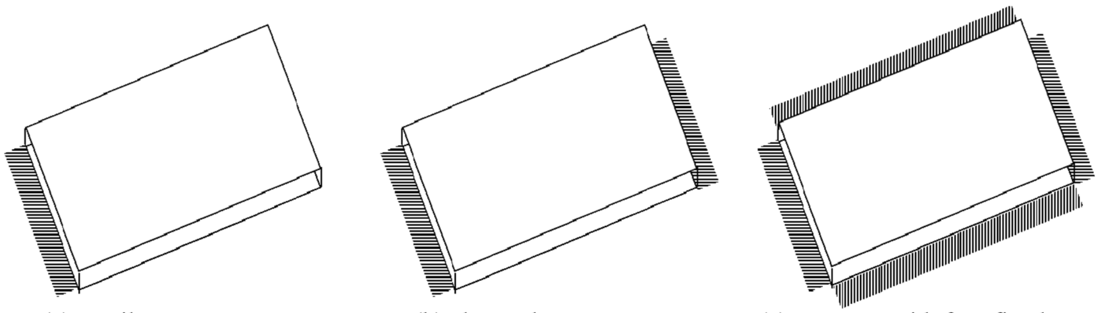


Fig. 3 Internal function “eigs” stored in MATLAB codes



(a) cantilever structure (b) clamped structure (c) structure with four fixed supports

Fig. 4 Structural models

4. Numerical applications and discussion

Structural models for the numerical examples of dynamic topology optimization design considering uncertainties are a cantilever structure, a clamped structure, a structure with four fixed supports as shown in Fig. 4. A 4 m × 2 m design domain is discretized using 40 × 20 square finite elements with four nodes. The nominal material parameters are Young’s modulus of concrete $E = 1.0$ GPa and Poisson’s ratio $\nu = 0.3$. For dynamic topology optimization problem objective function is maximal fundamental first-order eigenfrequency. Penalty parameter is $k = 3.0$ for the SIMP approach. Mass is combined with each half of consistent mass and lumped masses, and it is

Table 1 Uncertainty combination models

Data	Error of uncertainty																
	5%								10%								0%
	A5	B5	C5	D5	E5	F5	G5	H5	A10	B10	C10	D10	E10	F10	G10	H10	I0
E	l	l	o	u	u	o	l	u	l	l	o	u	u	o	l	u	o
ν	l	o	l	u	o	u	u	l	l	o	l	u	o	u	u	l	o

used for eigenvalue analyses into dynamic topology optimization design. The volumes of 30% for a cantilever structure, 40% for a clamped structure, and 50% for a structure with four fixed supports are fixed during entire optimization procedures.

Material properties of E and ν are assumed to be uncertainty data with the error of 5% and 10% in this study. Uncertainty phases on systematic structural analyses and topology optimization design results are divided by whether they are uncertain or not into dynamic structural behaviors as shown in Table 1. As can be seen, uncertainty models are A5~H5 with the error of 5%, A10~H10 with the error of 10%, and I0 without error. Here l, o, and u denote a lower bound value, a nominal value, and an upper bound value, respectively.

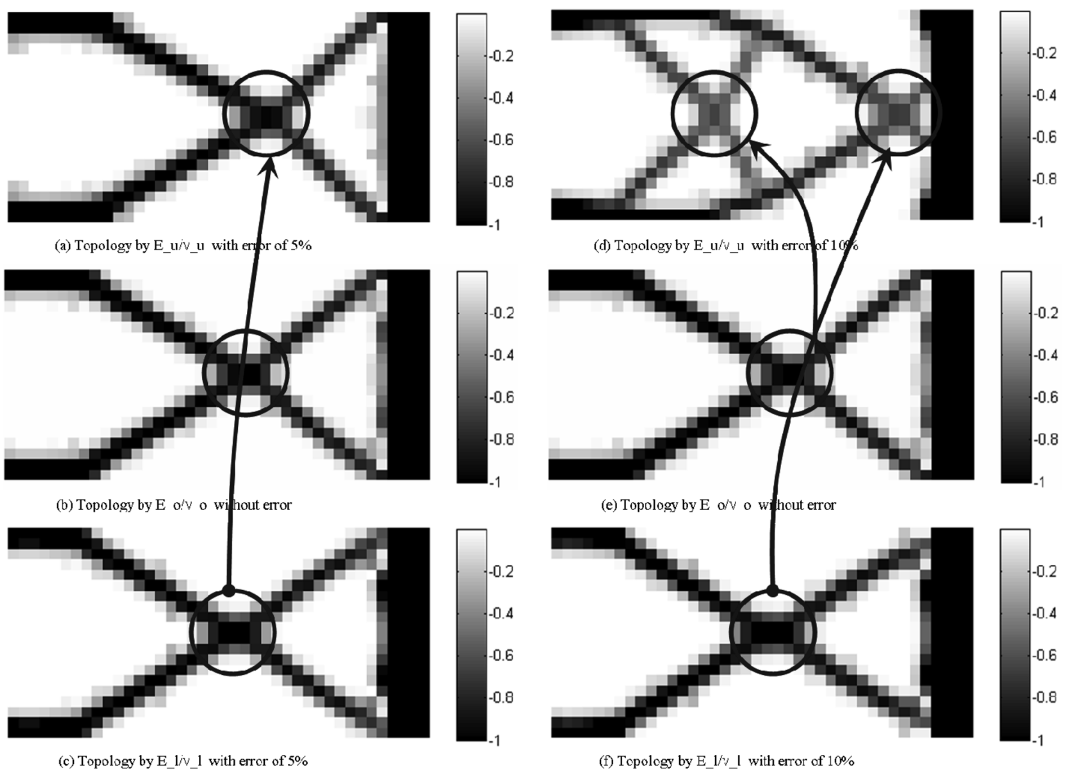


Fig. 5 Optimal topologies by considering the uncertainty error of 5% and 10% including a nominal optimal topology

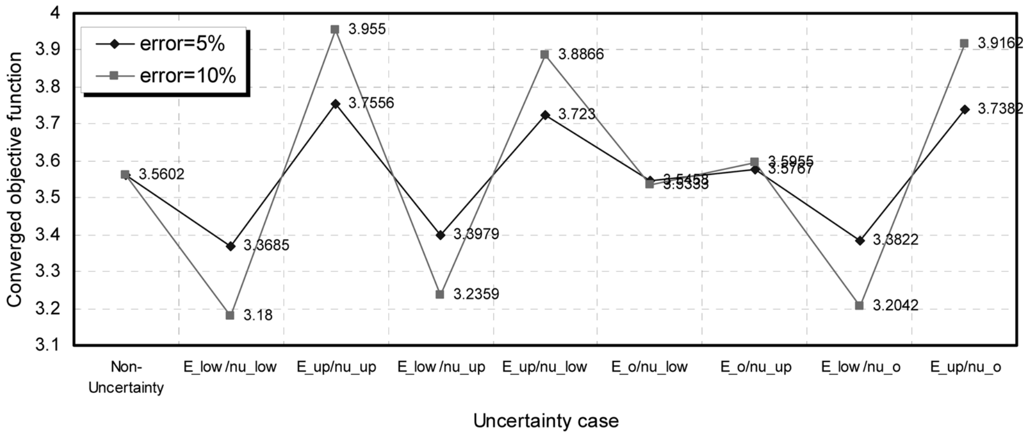


Fig. 6 Converged objective function values of each uncertainty case

4.1 Dynamic topology optimization designs of a cantilever structure

Fig. 5 shows optimal topologies of a cantilever structure as shown in Fig. 4(a) by using a jagged density distribution contour designed by error of 5% and 10% including a nominal optimal solution without considering uncertainties. Results of uncertainty cases of E and ν to take all lower bound values or all upper bound values, in which differences from a nominal optimal solution are the greatest, are shown here. As can be seen, a joint part of a blue circle connected by four structural members moves toward a maximal stiffness structure according to degree of data error. The range of the movement at error of 10% is greater than that of 5%. Two joints are produced in case E_u/ν_u with the error of 10%.

Fig. 6 shows converged objective function values of each uncertainty case as shown in Table 1. In uncertainty cases *E_up/nu_up* and *E_low/nu_low*, differences from the nominal value of the case *E_o/nu_o* are the biggest at the error of both 5% and 10%.

4.2 Dynamic topology optimization designs of a clamped structure

Fig. 7 presents optimal topologies of a clamped structure as sketched in Fig. 4(b) by using a jagged density distribution contour designed by error of 5% and 10% including a nominal optimal solution without considering uncertainties. As can be seen a hole size changes according to ultimate uncertainty cases E_u/ν_u and $E\_l/\ \nu\_l$ in comparison with a nominal case, while global topology is fixed. Moreover, the change of hole size is the greatest at the error of 10% like an example 4.1.

Fig. 8 shows converged objective function values of each uncertainty case as shown in Table 1. The graph is all the same to that of Fig. 6.

4.3 Dynamic topology optimization designs of a structure by four fixed supports

Fig. 9 presents optimal topologies of a structure with four fixed supports as sketched in Fig. 4(c) by using a jagged density distribution contour designed by error of 5% and 10% according to ultimate uncertainty cases E_u/ν_u and E_l/ν_l in comparison with a nominal case, including a

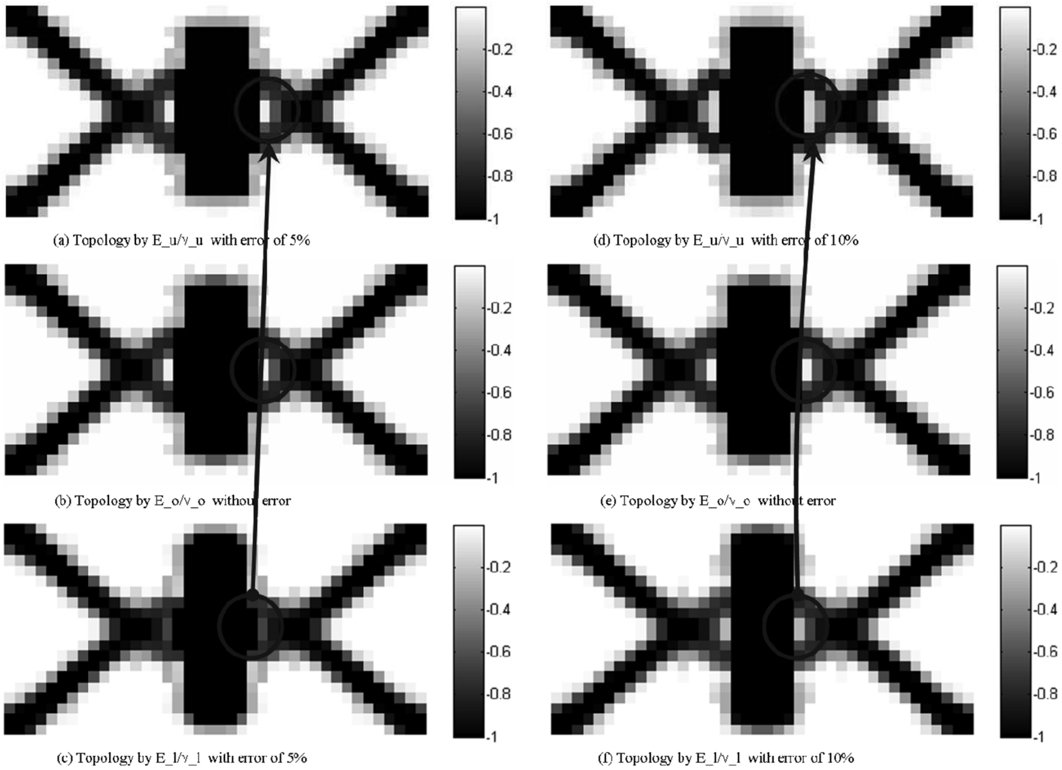


Fig. 7 Optimal topologies by considering the uncertainty error of 5% and 10% including a nominal optimal topology

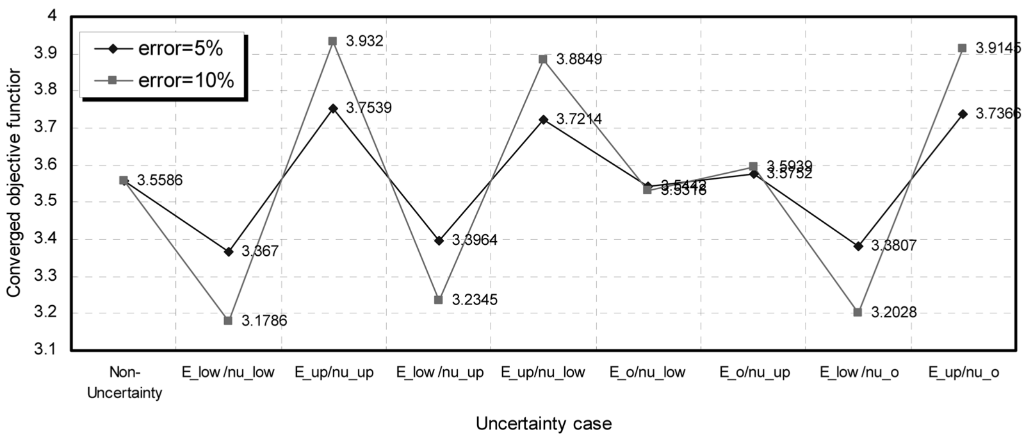


Fig. 8 Converged objective function values of each uncertainty case

nominal optimal solution without considering uncertainties. As can be seen results on the error of 5% are almost all the same, but changes of holes in small circles and horizontal members in large circles occurs at error of 10%.

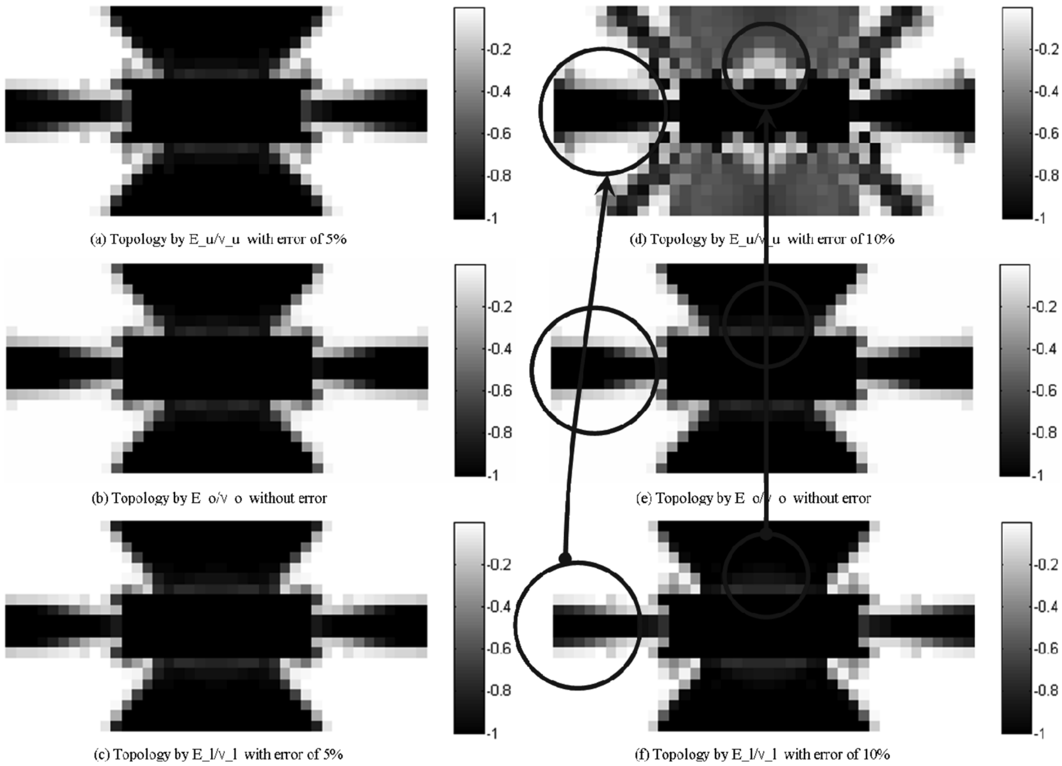


Fig. 9 Optimal topologies by considering the uncertainty error of 5% and 10% including a nominal optimal topology

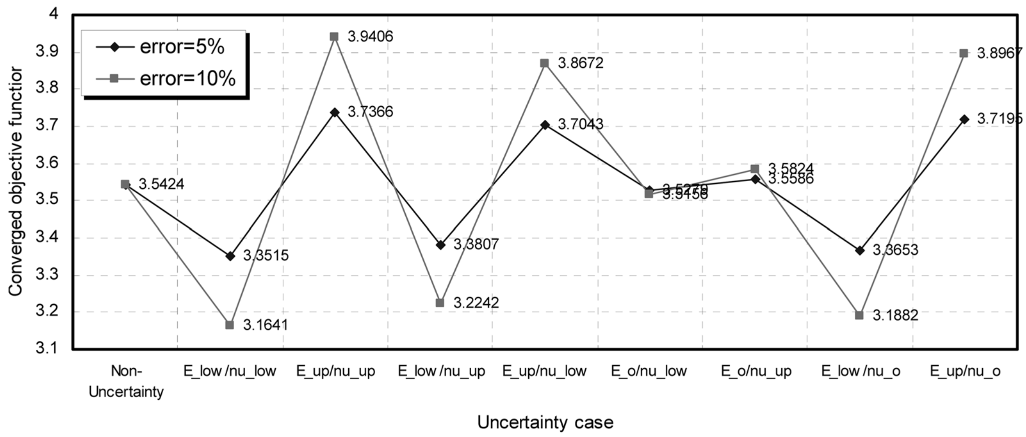


Fig. 10 Converged objective function values of each uncertainty case

Fig. 10 shows converged objective function values of each uncertainty case as shown in Table 1. The history of the curve is all the same to that of Figs. 6 and 8.

Finally, as can be seem in numerical applications of section 4.1, 4.2, and 4.3, it can be found that the effect of uncertainty cases as shown in Table 1 is *E_up* & *ν_up* > *E_low* & *ν_low* > *E_low* &

v_o E_up & *v_o* > *E_up* & *v_low* > *E_low* & *v_up* > *E_o* & *v_up* > *E_o* & *v_low*. From this fact, it can be found that the weakest effect factor of material properties on uncertainties of dynamic topology optimization is Poisson's ratio ν, and dynamic topology optimization results are the most sensitive to uncertainties of Young's modulus E.

As can be seen in final topologies in Figs. 5, 7, and 9, the use of different uncertainty values of Young's modulus and Poisson's ratio in SIMP gives slightly different optimal topologies. However the slight different results are significant, since they are linked to investigate the most appropriate location and connectivity among the so-called quasi-members described by density distributions. Although these present trivial exercises focus on macro-mechanics like civil and architectural engineering, the consideration of structural uncertainty may be more significant in the fields concentrating on micro-mechanics such as magnetics and electronics.

5. Conclusions

In this study, uncertainties of dynamic topology optimization design results are quantitatively and non-stochastically estimated by using a well-known interval analysis which is based on interval arithmetic of lower and upper bounds of given data.

According to numerical applications the following results are obtained in this study. First, when data uncertainties of material properties like Young's modulus and Poisson' ratio are assumed to carry out dynamic topology optimization design, a globally optimized topology like connectivity among members is fixed, but a locally optimal shape like material boundaries is very sensitive at like holes or joints, which extend, move, or are newly created into a given design space. Second, uncertainty error of Young's modulus is more sensitive on topology optimization results than that of Poisson's ratio. Third, like as uncertainties of static topology optimization from data uncertainties which are verified at last researches (Lee *et al.* 2006a, b), data uncertainties also affect topology optimization results of dynamic problems.

In order to escape the interval analysis from a classical mathematical tool and treat it into practical mechanical problems, in the future's work, uncertainties on varied mechanical problems like buckling or plasticity resulted from the data uncertainty need to be investigated, and finally may be linked to uncertainty estimate on structural optimization design, especially topology optimization, generally termed a synthesis of mechanical principles.

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