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Estimation of load and resistance factors based on the fourth moment method

Zhao-Hui Lu[1a], *Yan-Gang Zhao[2] and Alfredo H-S. Ang[3b]

[1]*School of Civil Engineering and Architecture, Central South University, 22 Shaoshannan Road, Changsha 410075, China*
[2]*Department of Architecture, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama 221-8686, Japan*
[3]*Department of Civil and Environmental Engineering, University of California, Irvine, E-4150 Engineering Gateway, Irvine, CA 92697-2175, USA*

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Abstract. The load and resistance factors are generally obtained using the First Order Reliability Method (FORM), in which the design point should be determined and derivative-based iterations have to be used. In this paper, a simple method for estimating the load and resistance factors using the first four moments of the basic random variables is proposed and a simple formula for the target mean resistance is also proposed to avoid iteration computation. Unlike the currently used method, the load and resistance factors can be determined using the proposed method even when the probability density functions (PDFs) of the basic random variables are not available. Moreover, the proposed method does not need either the iterative computation of derivatives or any design points. Thus, the present method provides a more convenient and effective way to estimate the load and resistance factors in practical engineering. Numerical examples are presented to demonstrate the advantages of the proposed fourth moment method for determining the load and resistance factors.

Keywords: load and resistance factors; fourth moment method; target mean resistance; simple formula.

1. Introduction

As the insurance of the performance of a structure must be accomplished under conditions of uncertainty, probabilistic analysis is generally necessary for reliability-based structural designs. However, the reliability-based structural design may also be achieved without a complete probabilistic analysis. If the required safety factors are predetermined on the basis of specified probability-based requirement, reliability-based design may be accomplished through the adoption of appropriate deterministic design criteria, e.g., the use of traditional safety factors.

For practical use, design criteria should be as simple as possible; moreover, they should be developed in a way that is familiar to the practical engineers. This can be accomplished through the

*Corresponding author, Professor, E-mail: zhao@kanagawa-u.ac.jp
[a]Associate Professor
[b]Research Professor

use of load amplification factors and resistance reduction factors, known as the LRFD format (Ang and Tang 1984, AIJ 2002). The nominal design loads are amplified by appropriate load factors and the nominal resistances are reduced by corresponding resistance factors, and safety is assured if the factored resistance is at least equal to the factored loads. The appropriate load and resistance factors must be developed in order to obtain designs that achieve a prescribed level of reliability.

The load and resistance factors are generally determined using the first order reliability method (FORM) (e.g., Melchers 1999, Nowak and Collins 2000), in which the "design point" must be determined and derivative-based iterations have to be used. Also, it is inconvenient to deal with the problem of multiple design points (Der-Kiureghian and Dakessian 1998, Barranco-Cicilia *et al.* 2009) with this procedure. At the present stage, the practicing engineers in general would not perform reliability analysis in engineering designs, and they only use the load and resistance factors recommended in design codes. However, with the trend towards the performance design, there will be a necessity for designers to determine the load and resistance factors by themselves in order to conduct structural design more flexibly. In such a case, it is required that the design code recommend not only specific values of load and resistances factors but also suitable and simple methods for determining these factors. AIJ (2002) recommendation has provided a simple method based on the proposal of Mori (2002), in which all the random variables are assumed to have known probability density functions (PDFs) and required to transfer into lognormal random variables. However, in reality, the PDFs of some of the basic random variables are often unknown due to the lack of statistical data. Therefore, it is important to find a way to obtain LRFD including random variables with unknown PDFs.

In this paper, the basic principle of the load and resistance factor format is reviewed. A simple method for estimating the load and resistance factors using the first four moments of the basic random variables is proposed and a simple formula for the target mean resistance is also proposed to avoid iterative computations. Since the proposed method is based on the first four moments of the basic random variables, the load and resistances factors can be determined even when the probability density functions of the random variables are unknown. The simplicity and efficiency of the proposed method for determining the load and resistance factors are demonstrated with several numerical examples.

2. Determination of load and resistance factors

The LRFD format may be expressed as the follows.

$$\phi R_n \geq \Sigma \gamma_i S_{ni} \tag{1}$$

where ϕ = the resistance factor; γ_i = the partial load factor to be applied to load S_i; R_n = the nominal value of the resistance; and S_{ni} = the nominal value of load S_i.

In reliability-based structural design, the resistance factor ϕ and the load factor γ_i should be determined based on a specified reliability, i.e., the design format, Eq. (1), should be equivalent to the following equations in probability terms.

$$G(X) = R - \Sigma S_i \tag{2}$$

$$P_f \leq P_{fT} \quad \text{or} \quad \beta \geq \beta_T \tag{3}$$

where R and S_i are the random variables representing the uncertainty in the resistance and load effects. P_f and β are the probability of failure and reliability index corresponding to the performance function Eq. (2). P_{fT} and β_T are the target probability of failure and target reliability index, respectively.

If R and S_i are mutually independent normal random variables, the second moment method is correct and the design formula becomes

$$\beta_{2M} \geq \beta_T \tag{4}$$

where

$$\beta_{2M} = \frac{\mu_G}{\sigma_G} \tag{5a}$$

$$\mu_G = \mu_R - \Sigma\mu_{Si} \qquad \sigma_G = \sqrt{\sigma_R^2 + \Sigma\sigma_{Si}^2} \tag{5b}$$

where β_{2M} is the second moment reliability index; μ_G and σ_G are the mean value and standard deviation of the performance function $G(\mathbf{X})$, respectively; μ_R and σ_R are the mean value and standard deviation of R, respectively; and μ_{Si} and σ_{Si} are the mean value and standard deviation of S_i, respectively.

Substituting Eq. (5) in Eq. (4) leads to

$$\mu_R(1 - \alpha_R V_R \beta_T) \geq \Sigma\mu_{Si}(1 + \alpha_{Si} V_{Si} \beta_T) \tag{6}$$

Comparing Eq. (6) with Eq. (1), the load and resistance factors can be expressed as

$$\phi = (1 - \alpha_R V_R \beta_T)\frac{\mu_R}{R_n} \tag{7a}$$

$$\gamma_i = (1 + \alpha_{Si} V_{Si} \beta_T)\frac{\mu_{Si}}{S_{ni}} \tag{7b}$$

where V_R and V_{Si} are the coefficient of variation (COV) of R and S_i, respectively; and α_R and α_{Si} are the direction cosines (also known as separating factors) of R and S_i, respectively.

$$\alpha_R = \frac{\sigma_R}{\sigma_G}, \quad \alpha_{Si} = \frac{\sigma_{Si}}{\sigma_G} \tag{8}$$

When R and S_i are non-normal random variables, the second moment reliability index expressed in Eq. (5) does not correctly reflect the real failure probability corresponding to the performance function in Eq. (2). The reliability index in this case is generally obtained by the first order reliability method (FORM), where the design format can be expressed as

$$R^* \geq \Sigma S_i^* \tag{9}$$

And the load and resistance factors can be obtained as (Ang and Tang 1984)

$$\phi = \frac{R^*}{R_n}, \quad \gamma_i = \frac{S_i^*}{S_{ni}} \tag{10}$$

where R^* and S_i^* are the values of variables R and S_i at the design point of FORM, respectively. Since R^* and S_i^* are usually obtained using derivative-based iterations, explicit expressions of R^*

and S_i^* are not available. Some simplifications have been proposed in order to avoid iterative computations (Ugata 2000, Mori 2002).

In the present paper, the reliability index in Eq. (3) is derived using the method of moments. Since the central moments of the performance function, as described in Eq. (2), can be obtained easily, the probability of failure, which is defined as $P[G(\mathbf{X}) < 0]$ can be expressed as a function of the central moments. Since no derivative-based iteration is necessary in the proposed method, the required load and resistance factors are much easier to determine.

3. Load and resistance factors by method of moments

3.1 Determination of load and resistance factors using the third-moment method

Substituting the third-moment reliability index in the design format described in Eq. (3), produces

$$\beta_{3M} \geq \beta_T \tag{11}$$

where the third-moment reliability index β_{3M} is given by Zhao *et al.* (2006)

$$\beta_{3M} = \frac{1}{\alpha_{3G}}\left(3 - \sqrt{9 + \alpha_{3G}^2 - 6\alpha_{3G}\beta_{2M}}\right) \tag{12}$$

where α_{3G} is the skewness of $G(\mathbf{X})$. The α_{3G} in Eq. (2) can be computed by

$$\alpha_{3G} = \frac{1}{\sigma_G^3}\left(\alpha_{3R}\sigma_R^3 - \Sigma\alpha_{3Si}\sigma_{Si}^3\right) \tag{13}$$

Substituting Eq. (13) in Eq. (11) leads to

$$\beta_{2M} \geq \beta_T - \frac{1}{6}\alpha_{3G}(\beta_T^2 - 1) \tag{14}$$

Denoting the right side of Eq. (14) as β_{2T}, one obtains

$$\beta_{2M} \geq \beta_{2T} \tag{15}$$

$$\beta_{2T} = \beta_T - \frac{1}{6}\alpha_{3G}(\beta_T^2 - 1) \tag{16}$$

It can be observed that Eq. (15) is the same as Eq. (3), which implies that if the second moment reliability index β_{2M} is at least equal to β_{2T}, the reliability index β will be at least equal to the target reliability index β_T, and the required reliability is satisfied. Therefore, β_{2T} can be considered to be a target value of β_{2M}, and is hereafter denoted as the target second moment reliability index.

Substituting Eq. (5) in Eq. (15) leads to

$$\mu_R(1 - \alpha_R V_R \beta_{2T}) \geq \Sigma\mu_{Si}(1 + \alpha_{Si} V_{Si} \beta_{2T}) \tag{17}$$

Comparing Eq. (17) with Eq. (1), the load and resistance factors may be expressed as

$$\phi = (1 - \alpha_R V_R \beta_{2T})\frac{\mu_R}{R_n} \tag{18a}$$

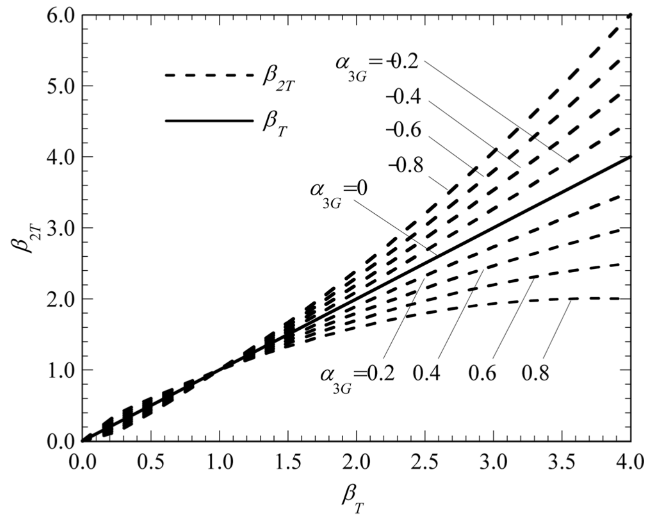


Fig. 1 The target 2M reliability index based on the third moment method

$$\gamma_i = (1+\alpha_{Si}V_{Si}\beta_{2T})\frac{\mu_{Si}}{S_{ni}} \tag{18b}$$

where V_R and V_{Si} are the coefficients of variation, and α_R and α_{Si} are the direction cosines, respectively, for R and S_i, which are the same as those in Eq. (8).

Comparing Eq. (18) with Eq. (7), it can be observed that after replacing β_T in Eq. (7) by β_{2T} in Eq. (18), the determination of the load and resistance factors using the third moment method is essentially the same as that by the second moment method.

The variations of the target second moment reliability index β_{2T} with respect to the target reliability index β_T are shown in Fig. 1. As can be seen from Fig. 1, β_{2T} is larger than β_T for negative α_{3G} and smaller than β_T for positive α_{3G}. When $\alpha_{3G} = 0$, $\beta_{2T} = \beta_T$, then, Eq. (15) becomes exactly the same as Eq. (4), and the load and resistance factors can be determined using Eq. (7) and Eq. (8).

3.2 Determination of load and resistance factors using the fourth moment method

Consider the performance function in Eq. (2). Without loss of generality, standardize the performance function $G(\mathbf{X})$ using the following standardized variable

$$z_u = \frac{G-\mu_G}{\sigma_G} \tag{19}$$

Then the probability of failure corresponding to the performance function, Eq. (2), can be expressed as the following according to its definition.

$$P_f = P[G \le 0] = P[z_u\sigma_G + \mu_G \le 0] = P\left[z_u \le -\frac{\mu_G}{\sigma_G}\right] = P[z_u \le -\beta_{2M}] \tag{20}$$

where β_{2M} is the second moment reliability index given in Eq. (5a).

The standardized variable z_u can be expressed as a third order polynomial function of the standard normal variable u (Fleishman 1978) as follows.

$$z_u = a_1 + a_2 u + a_3 u^2 + a_4 u^3 \tag{21}$$

where a_1, a_2, a_3, and a_4 are the polynomial coefficients that can be obtained by making the first four moments of the right side of Eq. (21) equal to those of the left side (Zhao and Lu 2008).

Eq. (21) is simple if the coefficients a_1, a_2, a_3, and a_4 are known. However, the determination of the four coefficients is not easy, since the solution of nonlinear equations has to be solved (Fleishman 1978). In order to avoid this difficulty, Eq. (21) can be simplified by the fourth moment standardization functions (Zhao and Lu 2007a), of which Eq. (21) can be expressed as

$$z_u = S(u) = -l_1 + k_1 u + l_1 u^2 + k_2 u^3 \tag{22}$$

where $S(u)$ denotes the third polynomial of u; the coefficients l_1, k_1, and k_2 are given as

$$l_1 = \frac{\alpha_{3G}}{6(1+6l_2)}, \quad l_2 = \frac{1}{36}\left(\sqrt{6\alpha_{4G} - 8\alpha_{3G}^2 - 14} - 2\right) \tag{23a}$$

$$k_1 = \frac{1-3l_2}{(1+l_1^2-l_2^2)}, \quad k_2 = \frac{l_2}{(1+l_1^2+12l_2^2)} \tag{23b}$$

where α_{4G} is the 4th dimensionless central moment, i.e., the kurtosis of $G(\mathbf{X})$, and is calculated from

$$\alpha_{4G} = \frac{1}{\sigma_G^4}\left(\alpha_{4R}\sigma_R^4 + 6\sigma_R^2\sum_{i=1}^{n}\sigma_{Si}^2 + \sum_{i=1}^{n}\alpha_{4Si}\sigma_{Si}^4 + 6\sum_{i=1}^{n-1}\sum_{j>i}^{n}\sigma_{Si}^2\sigma_{Sj}^2\right) \tag{24}$$

where α_{4G} and α_{4Si} are the kurtosis of R and S_i, respectively.

Substituting Eq. (22) in Eq. (20), one obtains

$$P_f = P[z_u = S(u) = -l_1 + k_1 u + l_1 u^2 + k_2 u^3 \le -\beta_{2M}] \tag{25}$$

Suppose the inverse function of S is

$$u = S^{-1}(z_u) \tag{26}$$

According to Eq. (25) and Eq. (26), it is not difficult to obtain

$$P_f = P[u \le S^{-1}(-\beta_{2M})] = \Phi[S^{-1}(-\beta_{2M})] \tag{27}$$

Therefore, the reliability index is expressed as

$$\beta = -\Phi^{-1}(P_f) = -S^{-1}(-\beta_{2M}) \tag{28}$$

Substituting Eq. (28) in the design format described in Eq. (3) leads to

$$\beta = -S^{-1}(-\beta_{2M}) \ge \beta_T \tag{29}$$

and which

$$\beta_{2M} \ge -S(-\beta_T) = l_1 + k_1\beta_T - l_1\beta_T^2 + k_2\beta_T^3 \tag{30}$$

Denoting the right side of Eq. (30) as β_{2T}, one obtains

$$\beta_{2M} \geq \beta_{2T} \tag{31a}$$

$$\beta_{2T} = l_1 + k_1\beta_T - l_1\beta_T^2 + k_2\beta_T^3 \tag{31b}$$

As described in the previous section, Eq. (31a) is the same as Eq. (3) implying that if the second moment reliability index β_{2M} is at least equal to β_{2T}, the reliability index β will be at least equal to the target reliability index β_T, and the required reliability is satisfied. Therefore, β_{2T} is denoted as the target second moment reliability index.

Since Eq. (31a) is the same as Eq. (4) except that the right side is β_{2T}, the load and resistance factors corresponding to Eq. (31a) can be easily obtained by substituting β_T in the right side of Eq. (4) with β_{2T}. The design formula is essentially the same as Eq. (17) and the load and resistance factors are essentially the same as Eq. (18).

Especially for the case of $\alpha_{4G} = 3$, it can be derived that $h_4 = 0$, $h_3 = \frac{1}{6}\alpha_{3G}$, and Eq. (31b) becomes

$$\beta_{2T} = k_1\beta_T - \frac{1}{6}\alpha_{3G}(\beta_T^2 - 1) \tag{32}$$

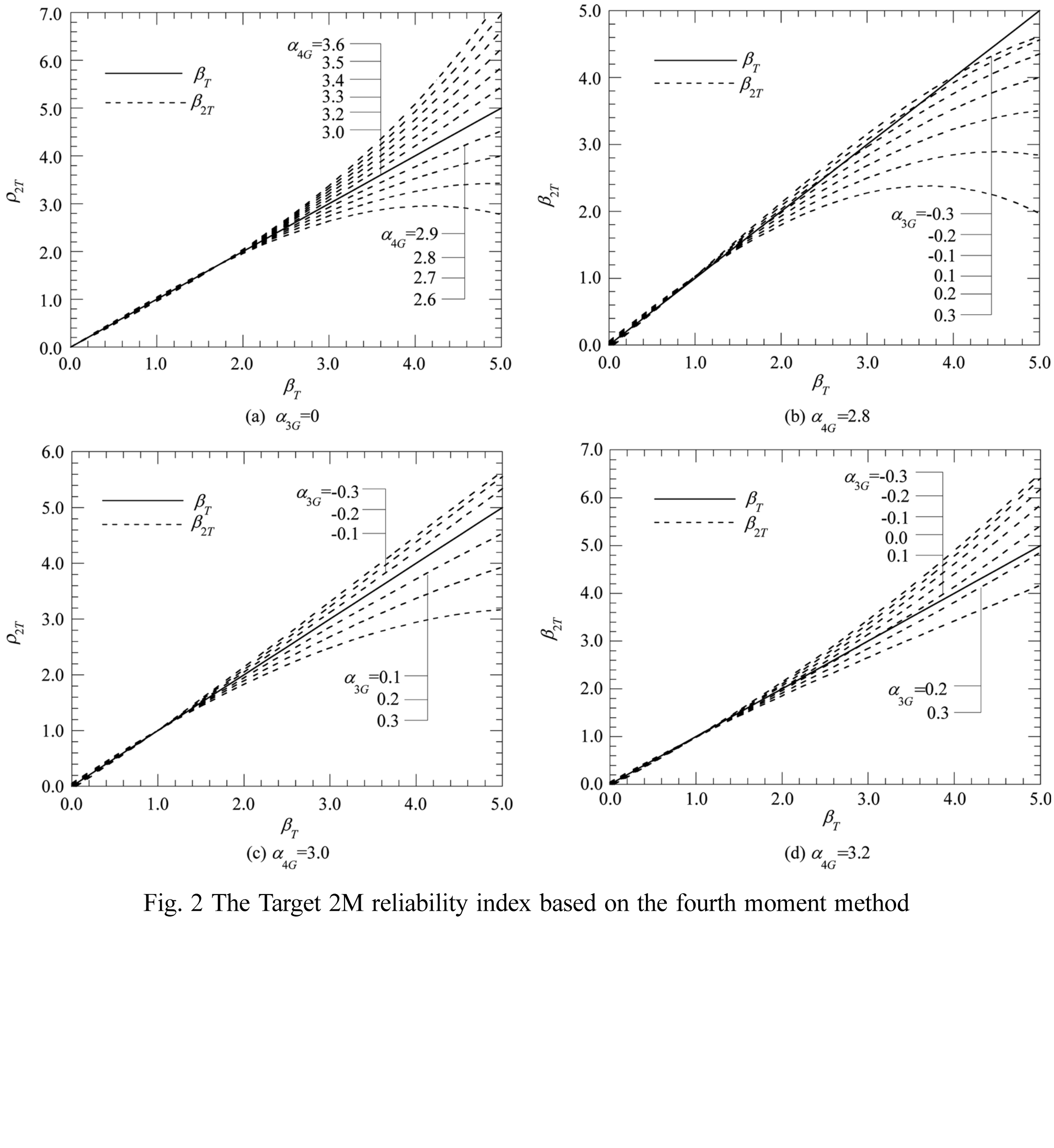


Fig. 2 The Target 2M reliability index based on the fourth moment method

When the value of α_{3G} is small enough, $k_1 = 1/(1+\alpha_{3G}^2/36) \approx 1$, then Eq. (32) becomes essentially the same as Eq. (16) based on the third moment method.

When $\alpha_{4G} = 3$ and $\alpha_{3G} = 0$, l_1, l_2, k_1, and k_2 will be obtained as $l_1 = l_2 = k_2 = 0$ and $k_1 = 1$, and thus $\beta_{2T} = \beta_T$. Then, Eq. (31a) becomes the same as Eq. (4), and the load and resistance factors can be determined using Eq. (7) and Eq. (8).

The variations of the target second moment reliability index β_{2T} with respect to the target reliability index β_T are shown in Fig. 2(a) in the case of $\alpha_{3G} = 0$, and in Figs. 2(b), (c), and (d) in the cases of $\alpha_{4G} = 2.8$, $\alpha_{4G} = 3.0$, and $\alpha_{4G} = 3.2$, respectively. From these figures, one can see that β_{2T} is generally larger than β_T for negative α_{3G} and smaller than β_T for positive α_{3G}. One can also see that β_{2T} is generally larger than β_T for $\alpha_{4G} > 3.0$ and smaller than β_T for positive $\alpha_{4G} < 3.0$.

4. Determination of the mean resistance

4.1 The iteration method

Since the load and resistance factors are determined when the reliability index is equal to the target reliability index, the mean value of the resistance should be determined under this condition (hereafter referred to as the target mean resistance). Generally, the target mean resistance is computed using the following iteration equation.

$$\mu_{Rk} = \mu_{Rk-1} + (\beta_T - \beta_{k-1})\sigma_G \tag{33}$$

where μ_{Rk} and $\mu_{Rk\text{-}1}$ are the *kth* and (k-1)*th* iteration value of the mean value of resistance; β_{k-1} is the (k-1)th iteration value of the third or fourth moment reliability index.

4.2 Simple formulas for the target mean resistance

The following simple formula is proposed to avoid the iterative computations of the target mean resistance

$$\mu_{RT} = \Sigma\mu_{Si} + \beta_{2T_0}\sigma_{G_0} \tag{34}$$

where μ_{RT} = the target mean resistance; σ_{G0} = the standard deviation of $G(\mathbf{X})$ and β_{2T0} = the target 2M reliability index, which are obtained using μ_{R0}.

μ_{R0} is given by the following equation, which is obtained from try and error

$$\mu_{R_0} = \Sigma\mu_{Si} + \sqrt{\beta_T^{3.3}\Sigma\sigma_{Si}^2} \tag{35}$$

The steps for determining the load and resistance factors using the fourth moment method are as follows:

(1) Calculate μ_{R0} using the Eq. (35).

(2) Calculate σ_{G0}, α_{3G0}, α_{4G0}, and β_{2T0} using Eq. (5), Eq. (13), Eq. (24), and Eq. (31b), respectively, and determine μ_{RT} with the aid of Eq. (34).

(3) Calculate σ_G, α_{3G}, α_{4G}, and β_{2T} using Eq. (5), Eq. (13), Eq. (24), and Eq. (31b), respectively, and calculate α_R and α_{Si} with the aid of Eq. (8).

(4) Determine the load and resistance factors using Eq. (18).

4.3 The efficiency of the simple formula

In order to investigate the efficiency of the proposed simple formula in the fourth moment method, several cases under different conditions are examined.

Case 1: Consider the following performance function

$$G(X) = R-(D+L+S) \tag{36}$$

where

R = resistance, with unknown probability density function (PDF), $\mu_R/R_n = 1.1$, $V = 0.15$, $\alpha_{3R} = 0.453$, $\alpha_{4R} = 3.368$;

D = dead load, with unknown PDF, $\mu_D/D_n = 1$, $V = 0.1$, $\alpha_{3D} = 0.0$, $\alpha_{4D} = 3.0$;

L = live load, with unknown PDF, $\mu_L/L_n = 0.45$, $V = 0.4$, $\alpha_{3L} = 1.264$, $\alpha_{4L} = 5.969$; and S = snow load, with unknown PDF, $\mu_S/S_n = 0.47$, $V = 0.25$, $\alpha_{3S} = 1.140$, $\alpha_{4S} = 5.4$.

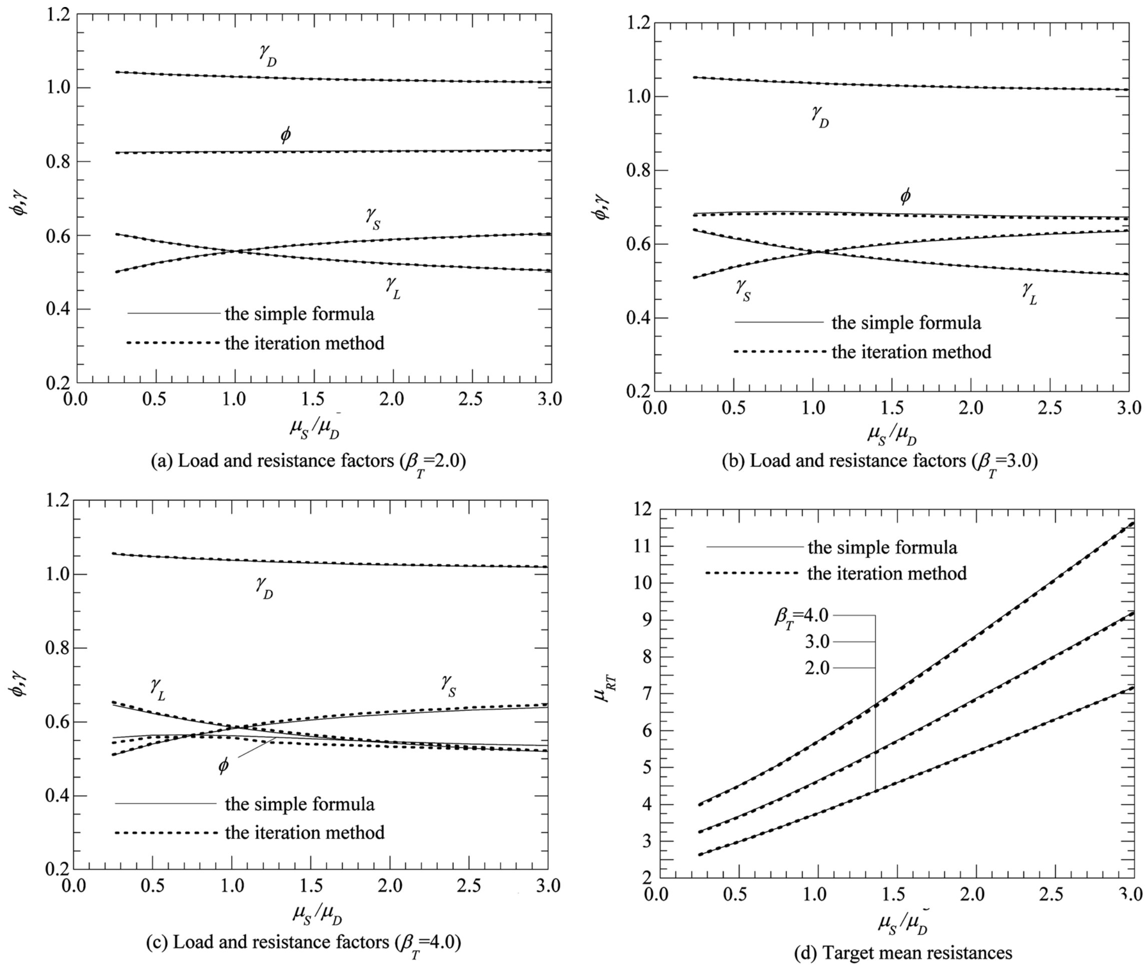


Fig. 3 Figure for case 1

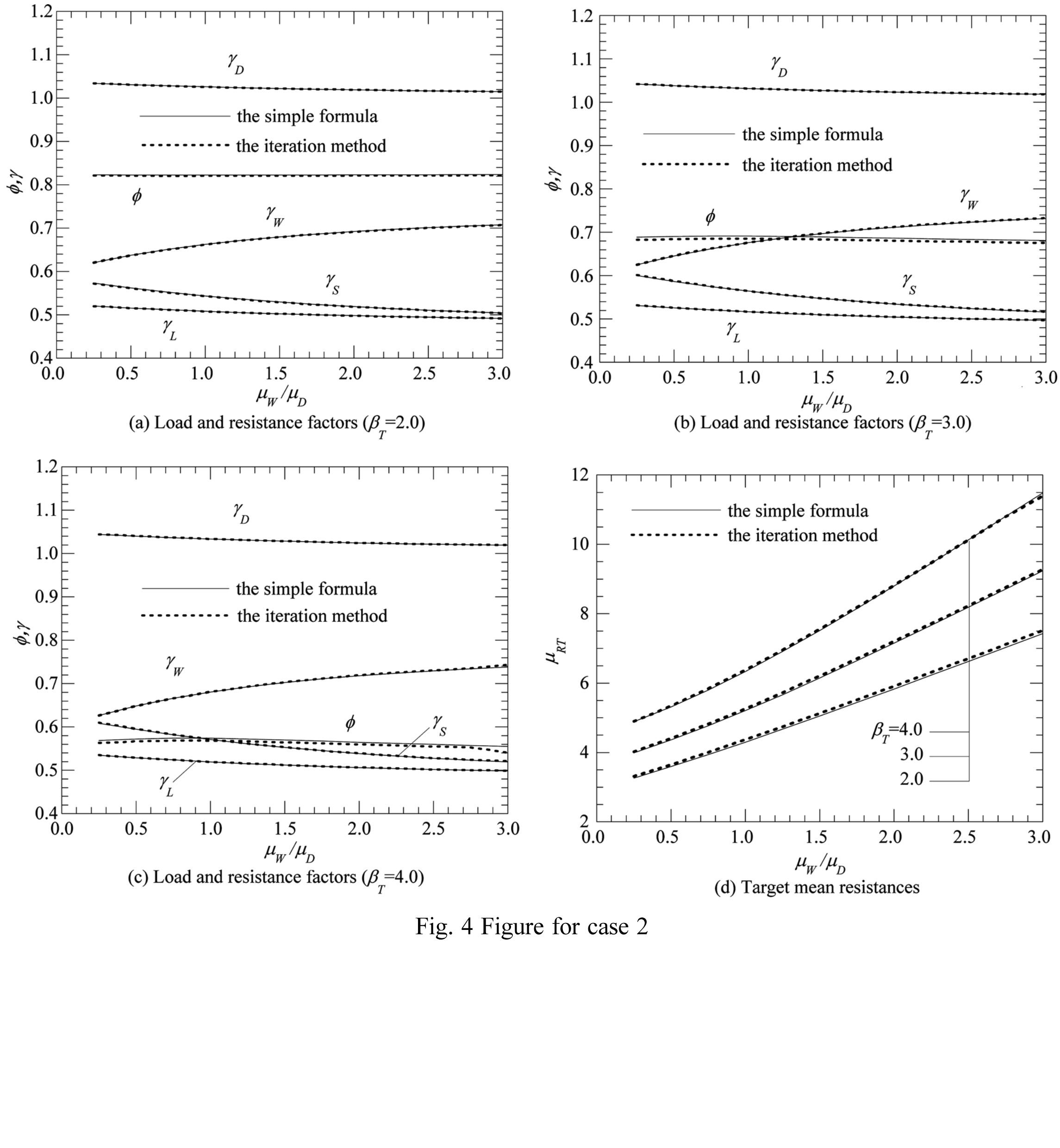
Consider the mean value of D, L with $\mu_D = 1.0$, $\mu_L/\mu_D = 0.5$, the load and resistance factors obtained using the simple formula are illustrated in Figs. 3(a)-(c), compared with the corresponding factors obtained using iterative calculations of the fourth moment for $\beta_T = 2.0$, 3.0, and 4.0. The target mean resistances obtained using the simple formula and those obtained using iterative calculations are illustrated in Fig. 3(d). It can be observed from Fig. 3 that the load and resistance factors and the target mean resistances obtained by the two methods are essentially the same for a given target reliability index.

Case 2: Consider the following performance function

$$G(X) = R-(D+L+S+W) \tag{37}$$

where

R = resistance, with unknown PDF, $\mu_R/R_n = 1.1$, $V = 0.15$, $\alpha_{3R} = 0.453$, $\alpha_{4R} = 3.368$;
D = dead load, with unknown PDF, $\mu_D/D_n = 1$, $V = 0.1$, $\alpha_{3D} = 0.0$, $\alpha_{4D} = 3.0$;



(a) Load and resistance factors (β_T=2.0)
(b) Load and resistance factors (β_T=3.0)
(c) Load and resistance factors (β_T=4.0)
(d) Target mean resistances

Fig. 4 Figure for case 2

L = live load, with unknown PDF, μ_L/L_n = 0.45, V = 0.4, α_{3L} = 1.264, α_{4L} = 5.969;
S = snow load, with unknown PDF, μ_S/S_n = 0.47, V = 0.25, α_{3L} = 1.140, α_{4S} = 5.4; and
W = wind load, with unknown PDF, with μ_W/W_n= 0.6, V = 0.2, α_{3W} = 1.140, α_{4W} = 5.4.

Consider the mean value of D, L, and S with μ_D =1.0, μ_L/μ_D = 0.5, μ_S/μ_D = 0.5, the load and resistance factors obtained using the simple formula are illustrated in Figs. 4(a)-(c), compared with the corresponding factors obtained using iterative calculations of the fourth moments for β_T = 2.0, 3.0, and 4.0. The target mean resistances obtained using the two methods are illustrated in Fig. 4(d). As can be observed from Fig. 4, the load and resistance factors and the target mean resistances obtained by the simple formula are quite satisfactory.

5. Investigations and discussion of the proposed method for LRFD

5.1 LRFD including random variables with unknown PDFs

Consider the statically indeterminate beam shown in Fig. 5, where the beam is loaded with three uniformly distributed loads, i.e., the dead load (D), live load (L), and snow load (S), in which the snow load is the dominating load and is time-dependent. The probabilistic member strength and loads are listed in Table 1. It is assumed that the design working life is 50 years.

The limit state function for the beam can be expressed as

$$G(X) = M_P - (M_D + M_L + M_S) \tag{38}$$

where M_P is the resistance; $M_D = (Dl^2)/12$, $M_L = (Ll^2)/12$, and $M_S = (Sl^2)/12$ are the load effects of D, L, and S, respectively.

The objective is to determine the load and resistance factors for the performance function of Eq. (38) in order to achieve a reliability of β_T = 3.5.

Since the PDFs of D and L are unknown, the method of FORM is not feasible. Here, the load and resistance factors are obtained using the method of moments.

Because S is a Gumbel random variable, the probability distribution of the maximum S during 50

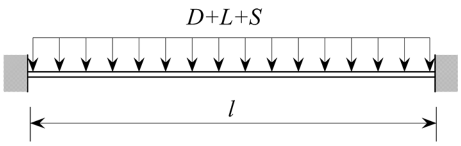


Fig. 5 A statically indeterminate beam

Table 1 Basic random variables for Ex. 1

R or S_i	PDF	Mean/Nominal	Mean	V_i	α_{3i}	α_{4i}
D	Unknown	μ_D/D_n = 1.0	μ_D	0.05	0.0	3.0
L	Unknown	μ_L/L_n = 0.45	$0.3\mu_D$	0.4	1.264	6.907
S	Gumbel	μ_S/S_n = 0.47	$1.25\mu_D$	0.35	1.140	5.4
Mp	Lognormal	μ_{Mp}/M_{pn} = 1.0	μ_{Mp}	0.1	0.301	3.162

years is also the Gumbel distribution (Melchers 1999). The values of mean, mean/nominal, coefficient of variation, skewness, and kurtosis corresponding to the maximum snow load over 50 years are readily obtained as: $\mu_{S50} = 2.585\mu_D$, $\mu_{S50}/S_n = 0.972$, $V_{S50} = 0.169$, $\alpha_{3S50} = 1.140$, and $\alpha_{4S50} = 5.4$.

According to Eq. (35), Eq.(5), and Eq. (13),

$$\mu_{M_{p0}} = \Sigma\mu_{M_{Si}} + \sqrt{\beta_T^{3.3}\Sigma\sigma_{M_{Si}}^2} = 7.556\mu_{M_D}, \quad \mu_{M_D} = (\mu_D l^2)/12$$

$$\sigma_{G0} = \sqrt{\sigma_{M_{p0}}^2 + \Sigma\sigma_{M_{Si}}^2} = 0.599\mu_{M_D}$$

$$\alpha_{3G0} = \frac{1}{\sigma_{G0}^3}(\alpha_{3M_p}\sigma_{M_{p0}}^3 - \Sigma\alpha_{3M_{Si}}\sigma_{M_{Si}}^3) = -0.417$$

The load and resistances can be determined using the third moment method:

After α_{G0} is obtained, β_{2T0} can be computed by Eq. (16)

$$\beta_{2T0} = \beta_T - \frac{1}{6}\alpha_{3G0}(\beta_T^2 - 1) = 4.283$$

The target mean resistance μ_{MpT} can be estimated with the aid of Eq. (34)

$$\mu_{MpT} = \Sigma\mu_{M_{Si}} + \beta_{2T_0}\sigma_{G_0} = 6.449\mu_{M_D}$$

Then, with the aid of Eq. (5), Eq. (13), and Eq. (16), σ_G, α_{3G}, and β_{2T} can be obtained as

$$\sigma_G = 0.565\mu_{M_D},\ \alpha_{3G} = -0.512, \quad \text{and} \quad \beta_{2T} = 4.46$$

Calculate α_{Mp} and α_{Si} with the aid of Eq. (8)

$$\alpha_{M_p} = \sigma_{M_p}/\sigma_G = 0.570, \quad \alpha_{M_D} = \sigma_{M_D}/\sigma_G = 0.177$$

$$\alpha_{M_L} = \sigma_{M_L}/\sigma_G = 0.212, \quad \alpha_{M_{S50}} = \sigma_{M_{S50}}/\sigma_G = 0.774$$

Determine the load and resistance factors using Eq. (18), obtaining

$$\phi = \mu_{M_p}(1 - \alpha_{M_p}V_{M_p}\beta_{2T})/R_n = 0.873$$

$$\gamma_{M_D} = \mu_{M_D}(1 + \alpha_{M_D}V_{M_D}\beta_{2T})/D_n = 1.079$$

$$\gamma_{ML} = \mu_{ML}(1 + \alpha_{ML}V_{ML}\beta_{2T})/L_n = 0.620$$

$$\gamma_{M_{S50}} = \mu_{M_{S50}}(1 + \alpha_{M_{S50}}V_{M_{S50}}\beta_{2T})/S_n = 1.540$$

Then, the LRFD format and the target mean resistance for this example using the third moment method are obtained as

$$0.87M_{Pn} \geq 1.08M_{Dn} + 0.62M_{Ln} + 1.54M_{Sn}$$

$$\mu_{Mp} \geq 6.42\mu_{MD}$$

where $M_{Dn} = (D_n l^2)/12$, $M_{Ln} = (L_n l^2)/12$, and $M_{Sn} = (S_n l^2)/12$.

As for the fourth moment method, the calculation process is illustrated as follows:

μ_{G0}, σ_{G0}, and α_{3G0} are determined, α_{4G0} and β_{2T0} can be obtained using Eqs. (24) and (31b), respectively,

$$\alpha_{4G0} = 3.695$$

$$l_{20} = 0.0168, \quad l_{10} = -0.0631, \quad k_{10} = 0.946, \quad k_{20} = 0.0167$$

$$\beta_{2T0} = l_{10} + k_{10}\beta_T - l_{10}\beta_T^2 + k_{20}\beta_T^3 = 4.736$$

The target mean resistance μ_{MpT} can be estimated with the aid of Eq. (34)

$$\mu_{MpT} = \Sigma\mu_{M_{Si}} + \beta_{2T_0}\sigma_{G_0} = 6.72\mu_{M_D}$$

Then, with the aid of Eq. (5), Eq. (13), Eq. (24), and Eq. (31b), σ_G, α_{3G}, α_{4G}, and β_{2T} can be obtained as

$$\sigma_G = 0.573\mu_{M_D}, \quad \alpha_{3G} = -0.488, \quad \alpha_{4G} = 3.824, \quad \beta_{2T} = 4.889$$

Calculate α_{Mp} and α_{MSi} with the aid of Eq. (8)

$$\alpha_{M_p} = 0.582, \quad \alpha_{M_D} = 0.174, \quad \alpha_{M_L} = 0.209, \quad \text{and} \quad \alpha_{M_{S50}} = 0.763$$

Determine the load and resistance factors using Eq. (18)

$$\phi = 0.86, \quad \gamma_{M_D} = 1.09, \quad \gamma_{ML} = 0.63, \quad \gamma_{M_{S50}} = 1.59$$

Finally, the LRFD format and the target mean resistance for this example using the fourth moment method are obtained as

$$0.86M_{Pn} \geq 1.09M_{Dn} + 0.63M_{Ln} + 1.59M_{Sn}$$

$$\mu_{Mp} \geq 6.67\mu_{MD}$$

The results obtained using the Monte-Carlo Simulation (MCS) method (Melchers 1999, Schueller 2009) with 1,000,000 samples (the COV of MCS estimate is 6.5%) is as follows:

$$\mu_{Mp} \geq 6.74\mu_{MD}$$

As can be seen from the design results, the target mean resistance obtained by the fourth moment method is in close agreement with that of MCS.

5.2 Sectional design for a simple non-linear case

Consider the following nonlinear performance function of the fully plastic flexural capacity of a steel beam section

$$G(X) = YZ - M \tag{39}$$

where

Y = the yield strength of steel, a lognormal variable.

Z = section modulus of the section, a lognormal variable.

M = the applied bending moment at the pertinent section, a Gumbel variable.

Determine the mean design section modulus for the performance function of Eq. (39), in order to achieve a reliability of $\beta_T = 2.5$.

The purpose of this design problem is to determine the appropriate μ_Z for any given μ_M to satisfy the required reliability. Assume a mean value of Y to be $\mu_Y = 276$ Mpa and the coefficients of variation of Y, Z, and M are $V_Y = 0.1$, $V_Z = 0.05$, and $V_M = 0.3$, respectively. We determine the required design section as follows.

First, to calculate the value of μ_{Z0}

$$\mu_G = \mu_Y\mu_{Z_0} - \mu_M = \sqrt{\beta_T^{3.3}\sigma_M^2}$$

$$\mu_{Z_0} = (\sqrt{\beta_T^{3.3}\sigma_M^2} + \mu_M)/\mu_Y = 8.555\times 10^{-3}\mu_M$$

Let $R = YZ$, then

$$\sigma_{R_0} = \sigma_{YZ_0} = \sqrt{(\mu_Y\mu_{Z_0})^2[(1+V_Y^2)(1+V_Z^2)-1]} = 0.2642\mu_M$$

Therefore

$$\sigma_{G_0} = \sqrt{\sigma_{R0}^2 + \sigma_M^2} = 0.3997\mu_M$$

The skewness of Y, Z, and M are readily obtained as

$$\alpha_{3Y} = 0.301,\quad \alpha_{3Z} = 0.150,\quad \alpha_{3M} = 1.14$$

The skewness of R can be obtained by

$$\alpha_{3R_0} = \alpha_{3YZ_0} = 0.3371$$

Thus

$$\alpha_{3G_0} = [(\alpha_{3R_0}\sigma_{R_0}^3 - \alpha_{3M}\sigma_M^3)]/\sigma_{G_0}^3 = -0.3846$$

The kurtosis of Y, Z, and M are readily obtained as

$$\alpha_{4Y} = 3.162,\quad \alpha_{4Z} = 3.04,\quad \alpha_{4M} = 5.4$$

The kurtosis of R can be obtained by

$$\alpha_{4R_0} = \alpha_{4YZ_0} = 3.203$$

therefore

$$\alpha_{4G_0} = (\alpha_{4R_0}\sigma_{R_0}^4 + \alpha_{4M}\sigma_M^4 + 6\sigma_{R_0}^2\sigma_M^2)/\sigma_{G_0}^4 = 3.800$$

and then

$$l_{20} = 0.0211,\quad l_{10} = -0.0569,\quad k_{10} = 0.9341,\quad k_{20} = 0.0209$$

$$\beta_{2T0} = l_{10} + k_{10}\beta_T - l_{10}\beta_T^2 + k_{20}\beta_T^3 = 2.961$$

Therefore, at the limit state, the appropriate μ_{ZT} obtained by using the fourth moment method is shown as follows.

$$\mu_{ZT} = \{\mu_M + \beta_{2T0}\sigma_{\sigma_0}\}/\mu_Y = 0.0079\mu_M$$

At the limit state, the design results of μ_{ZT} using the third moment method and FORM are obtained as $0.0077\mu_M$ and $0.0079\mu_M$, respectively. The result obtained using Monte-Carlo Simulation (MCS) with 1,000,000 samples (the COV of MCS estimate is 1.3%) is $0.0079\mu_M$. One can see that the same results are obtained by the present fourth moment method, FORM, and MCS method.

From the numerical examples, one can see that the present method needs neither iterative computations of derivatives nor any design points. The designers or users can easily produce a reliability-based design with the aid of the present method.

5.3 Comparison with FORM

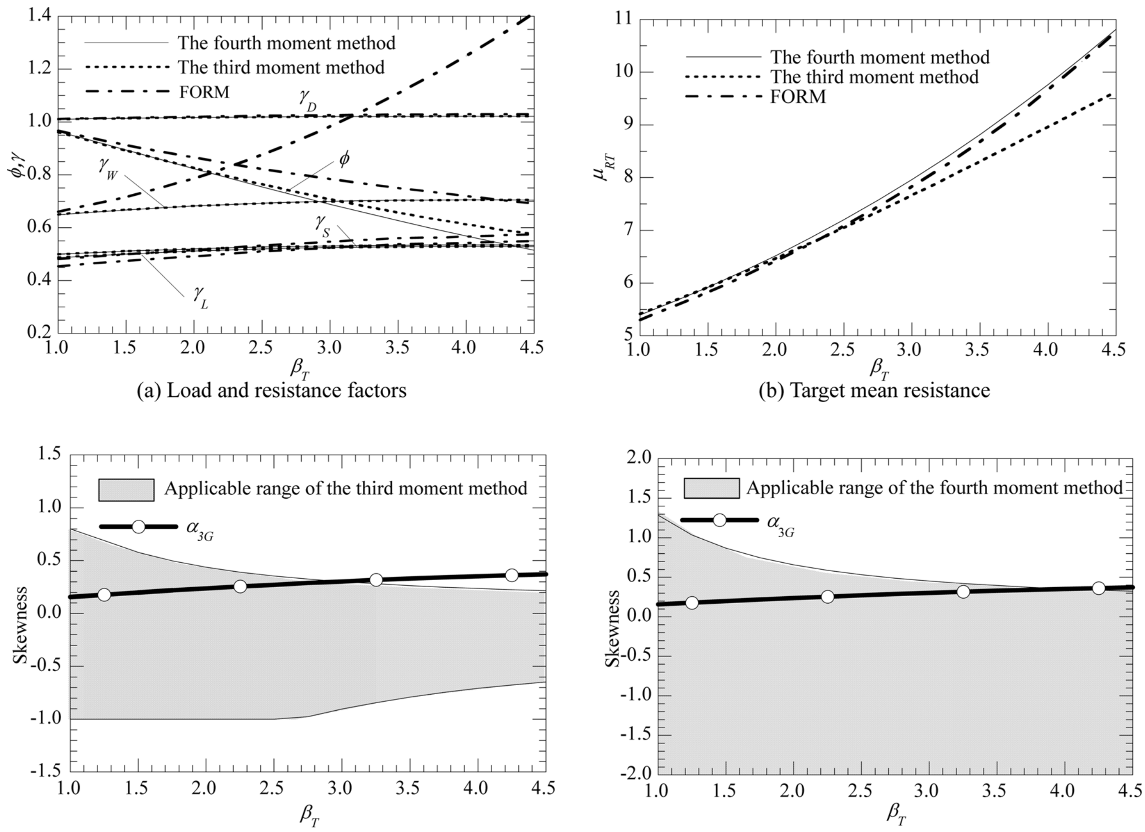
Consider the following performance function

$$G(X) = R-(D+L+S+W) \tag{40}$$

where

R = resistance, a lognormal variable with $\mu_R/R_n = 1.1$, $V = 0.15$;
D = dead load, a normal variable with, $\mu_D/D_n = 1$, $V = 0.1$;
L = live load, a lognormal variable with $\mu_L/L_n = 0.45$, $V = 0.4$;
S = snow load, a Gumbel variable with $\mu_S/S_n = 0.47$, $V = 0.25$; and
W = wind load, a Gumbel variable with $\mu_W/W_n = 0.6$, $V = 0.2$.

Consider the mean value of D, L, S, and W with $\mu_D = 1.0$, $\mu_L/\mu_D = 0.5$, $\mu_S/\mu_D = 2.0$, $\mu_W/\mu_D = 2.0$.



(a) Load and resistance factors (b) Target mean resistance

(c) The applicable range of the third moment method (d) The applicable range of the fourth moment method

Fig. 6 Comparisons of LRFD determined by FORM, the third moment method, and the fourth moment method

Table 2 Iteration numbers of FORM for LRFD

β_T	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
Iteration numbers	8	9	9	10	11	11	12	13	14	14	15	16	18	19	20

The load and resistance factors and the target mean resistances obtained using the fourth moment method and those obtained by the third moment method and FORM are illustrated in Figs. 6(a) and (b), respectively. The applicable range of the third moment method (Zhao *et al.* 2006) and the fourth moment method (Zhao and Lu 2007b) are illustrated in Figs. 6(c) and (d), respectively. The iterantion numbers of FORM for LRFD changed with the target reliability index (illustrated in the cases of β_T = 1.0, 1.25, 1.50, 1.75, 2.0, 2.25, 2.50, 2.75, 3.0, 3.25, 3.50, 3.75, 4.0, 4.25, 4.50) are given in Table 2. From Fig. 6 and Table 2, one can observe the following:

(1) Although the load and resistance factors obtained by the method of moments are different from those obtained by FORM, the target mean resistances obtained by both methods are essentially the same. This can be explained by that the different combinations of load and resistances factors can result in the same design results, while the set of factors obtained by the proposed methods may be different from the one determined by FORM since the formulae for load and resistances factors [i.e., Eq. (10) and Eqs. (18a) and (18b)] are different between the two methods. Therefore, in design practice, if the resistance factors determined by either of the method are adopted, the corresponding load factors (i.e., determined by the same method as the one used to estimate the resistance factors) should be used.

(2) When $\beta_T > 3.0$, the third moment method will give unconcervative results, this is because the skewness exceeds the range of the third moment method.

(3) The target mean resistances obtained using the fourth moment method are in close agreement with those of FORM over the entire range since the skewness is within the range of the fourth moment method.

(4) The derivative-based iteration numbers of FORM for LRFD varied from 8 to 20 as the target reliability index changed from 1.0 to 4.50. While the iteration computation is avoided in the proposed methods. For this reason, the proposed method is simpler to be used.

5.4 Two simple parabolic limit state functions

Consider the following two parabolic performance functions

$$G(X) = 10 + (X_1 - 10)^2 - X_2 \tag{41a}$$

$$G(X) = 10 - (X_1 - 10)^2 - X_2 \tag{41b}$$

in which X_1 is a normal random variable with the mean and coefficient of variation of 10 and 0.1; X_2 is also a random variable with the coefficient of variation of 0.7. For a given target reliability index of β_T = 2.0, determine the mean value of X_2.

For the performance function of Eq. (41a), using FORM, the mean value of X_2 is obtained as: μ_{X2} = 4.17 with corresponding design points of (10, 10) in **X** space; whereas using the fourth moment method and the MCS method with 100,000 samples (the COV of MCS estimate of 2.07%), the mean values of X_2 are obtained as 4.43 and 4.44, respectively. As can be observed from the results, FORM provides inaccurate result for this performance function, whereas the result of the present method is in close agreement with that of MCS.

For the performance function expressed in Eq. (41b), because this is a typical function with multiple design points when using FORM, FORM cannot solve this simple problem; whereas using the fourth moment method and the MCS method with 100,000 samples (the COV of MCS estimate is 2.07%), the mean values of X_2 are obtained as 3.16 and 3.20, respectively.

6. Conclusions

1. A method for the determination of load and resistance factors for reliability-based structural design using the fourth moment method is proposed and a simple formula for the target mean resistance is also proposed to avoid iteration computation. Derivative-based iteration, which is necessary in FORM, is demonstrated to be not necessary in the proposed method. The proposed method is therefore much easier to apply.
2. Although the load and resistance factors obtained by the proposed method may be different from those obtained by FORM, in general, the target mean resistances obtained by both methods are essentially the same. Therefore, in design practice, if the resistance factors determined by either of the method are adopted, the corresponding load factors (i.e., determined by the same method as the one used to estimate the resistance factors) should be used.
3. Since the proposed method is based on the first four moments of the basic random variables, the load and resistances factors can be determined even when the PDFs of the random variables are unknown.
4. The fourth moment method generally provides better results than the third moment method.

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