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Technical Note

Sensitivity analysis of responses for vibration control systems

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1. Introduction

In the optimization of structural analysis, the design sensitivity analysis of eigenvalues and eigenvectors plays an essential role. The designer can use this information directly in an interactive computer-aided design procedure as a valuable guide. Significant work has been done in this area (Moon and Kim 2004, Wu and Mohanty 2006, Moens and Vandepitte 2007).

In the control engineering, the eigenvalue and eigenvector sensitivity analysis inclued (Kida *et al.* 1981, Davison 1975, Schaechter 1985, Fujita *et al.* 1990, Chen and Zhang 2006, Kuchaksarai and Bargi 2006). In the sensitivity analyis, the control-force distribution vector **b**, the feedback gains $\mathbf{G}_0$ and $\mathbf{H}_0$ were assumed to be unchanged, whereas, the changes of structural parameters will cause the changes not only for the state matrix, but also for the feedback gain vector. Thus, the results of the response sensitivity obtained by Chen and Zhang (2006) are only an approximation.

This study presents the sensitivity analysis of responses by considering the effect of changes of both state matrix of the open-loop system and feedback gain on sensitivity.

2. Modal control equations in the state space and gain vector

Consider the vibration control system indicated by the following state equations

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}z(t) + \mathbf{d}(t) \tag{1}$$

where $\mathbf{A}$ is the state matrix, $\mathbf{x}(t) \in \mathbf{R}^{n \times 1}$ is the state vector, $z(t)$ is input control force, $\mathbf{b} \in \mathbf{R}^{n \times 1}$ is called actuator distribution vector.

Using the modal transformation, we obtain modal control equations

$$\dot{\xi} = diag(s_k)\xi + \mathbf{V}^H\mathbf{b}z(t) + \mathbf{V}^H\mathbf{d}(t) \tag{2}$$

where $diag(s_k)$ is the eigenvalue diagonal matrix of $\mathbf{A}$, and $\mathbf{U}$, $\mathbf{V}$ are the right and left modal matrix of $\mathbf{A}$, respectively.

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Eq. (2) can be written as

$$\dot{\xi} = \Lambda\xi + \mathbf{p}z(t) + \mathbf{V}^H\mathbf{d}(t) \tag{3}$$

If the control loop, which generates the input vector by linear feedback of the state vector of the system, is introduced, the response characteristic of the closed-loop system will be different from that of the open-loop system. Thus, it is possible to reassign a closed-loop system eigenvalues, which correspond to the controllable modes of the system, so that the closed-loop response characteristic is superior to the characteristic of the original uncontrolled system.

Since Eq. (3) is much simpler than the state Eq. (1), the gain matrix of the closed-loop system can be derived directly without the tedious mathematical manipulation.

If the direct state feedback control is used, the modal control force is given as follows

$$\mathbf{p}z(t) = \mathbf{V}^H\mathbf{b}\mathbf{g}^T\xi \qquad \mathbf{g}^T = [g_1, g_2, \ldots, g_n] \tag{4}$$

Substituting Eq. (4) into Eq. (3) yields

$$\dot{\xi} = [diag(s_k) + \mathbf{p}\mathbf{g}^T]\xi + \mathbf{V}^H\mathbf{d}(t) \tag{5}$$

To guarantee the asymptotic stability of the control structure, it is necessary to import larger negative real part for the eigenvalues of open-loop system. To this end, the eigenvalues in Eq. (5) are assigned to be $\rho_i (i = 1, 2, \ldots, n)$, the corresponding eigenvectors are $\mathbf{w}_i (i = 1, 2, \ldots, n)$, they satisfy the following eigenproblem

$$[diag(s_k) + \mathbf{p}\mathbf{g}^T - \rho_i\mathbf{I}]\mathbf{w}_i = \mathbf{0} \qquad (i = 1, 2, \ldots, n) \tag{6}$$

since $\mathbf{w}_i \neq \mathbf{0}$, we obtain

$$\det[diag(s_k) + \mathbf{p}\mathbf{g}^T - \rho_i\mathbf{I}] = \mathbf{0} \qquad (i = 1, 2, \ldots, n) \tag{7}$$

In order to obtain a convenient form, the following notations can be introduced

$$\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n]^T \qquad \mathbf{e}^T = [1, 1, \ldots, 1] \tag{8}$$

where

$$\mathbf{f}_i = \left[\frac{p_1}{\rho_i - s_1}, \frac{p_2}{\rho_i - s_2}, \ldots, \frac{p_n}{\rho_i - s_n}\right] \qquad (i = 1, 2, \ldots, n) \tag{9}$$

By using these notations, from Eq. (7) we have

$$\mathbf{F}\mathbf{g} = \mathbf{e} \tag{10}$$

It is possible to solve Eq. (10) for the modal gain vector **g**.

If the modal gain vector **g** has been solved, the control law can be obtained

$$z(t) = \mathbf{g}^T\xi(t) \qquad z(t) = \mathbf{g}^T\mathbf{V}^H\mathbf{x}(t) \tag{11}$$

3. Sensitivity of responses of closed-loop control systems

Substituting Eq. (11) into Eq. (1), the control equation of the closed-loop systems can be obtained

$$\dot{\mathbf{x}}(t) = (\mathbf{A}+\mathbf{b}\bar{\mathbf{g}})\mathbf{x}(t)+\mathbf{d}(t) = \mathbf{C}\mathbf{x}(t)+\mathbf{d}(t) \tag{12}$$

$$\bar{\mathbf{g}} = \mathbf{g}^T\mathbf{V}^H \qquad \mathbf{C} = \mathbf{A}+\mathbf{b}\bar{\mathbf{g}} \tag{13}$$

The response sensitivity of the closed-loop systems, $\partial\mathbf{x}/\partial b_j$, can be determined by differentiating Eq. (12) with respect to the structural parameter b_j

$$\dot{\mathbf{x}}_{,j} = \mathbf{C}(b)\mathbf{x}_{,j}+\mathbf{C}_{,j}\mathbf{x}(t) \tag{14}$$

where $\dot{\mathbf{x}}_{,j} = \partial\dot{\mathbf{x}}/\partial b_j$, $\mathbf{x}_{,j} = \partial\mathbf{x}/\partial b_j$, $\mathbf{C}_{,j} = \partial\mathbf{C}/\partial b_j$, $\partial\mathbf{d}(t)/\partial b_j = 0$.

From Eq. (13), we have

$$\mathbf{C}_{,j} = \frac{\partial\mathbf{C}}{\partial b_j} = \frac{\partial\mathbf{A}}{\partial b_j}+\frac{\partial\mathbf{b}}{\partial b_j}\bar{\mathbf{g}}+\mathbf{b}\frac{\partial\bar{\mathbf{g}}}{\partial b_j} \tag{15}$$

Eq. (15) indicates that the sensitivity of state matrix $\mathbf{C}$ involves computations of $\partial\mathbf{A}/\partial b_j$, $\partial\mathbf{b}/\partial b_j$, and $\partial\bar{\mathbf{g}}/\partial b_j$ it is easy to calculate $\partial\mathbf{A}/\partial b_j$ by all appearances.

For $\partial\bar{\mathbf{g}}/\partial b_j$, considering Eq. (13), we obtain

$$\frac{\partial\bar{\mathbf{g}}}{\partial b_j} = \frac{\partial\mathbf{g}^T}{\partial b_j}\mathbf{V}^H+\mathbf{g}^T\frac{\partial\mathbf{V}^H}{\partial b_j} \tag{16}$$

The computation of $\partial V/\partial b_j$ can be found in (Chen and Liu 1993).

The sensitivity of the modal gain vector, $\partial\mathbf{g}/\partial b_j$, can be obtained by differentiating Eq. (10) with respect to the structural parameter b_j

$$\mathbf{F}\frac{\partial\mathbf{g}}{\partial b_j} = -\frac{\partial\mathbf{F}}{\partial b_j}\mathbf{g} \tag{17}$$

where

$$f_{il} = \frac{p_l}{\rho_i-s_l} \qquad (l = 1, 2, \ldots, n) \tag{18}$$

$$\frac{\partial f_{il}}{\partial b_j} = \frac{\partial p_l}{\partial b_j}\frac{1}{\rho_i-s_l}-\frac{p_l}{(\rho_i-s_l)^2}\frac{\partial s_l}{\partial b_j} \qquad (l = 1, 2, \ldots, n) \tag{19}$$

Finally, the modal gain vector sensitivity can be solved from Eq. (17)

$$\frac{\partial\mathbf{g}}{\partial b_j} = -\mathbf{F}^{-1}\frac{\partial\mathbf{F}}{\partial b_j}\mathbf{g} \qquad (j = 1, 2, \ldots, m) \tag{20}$$

4. Conclusions

By considering the effect of changes of both state matrix of the open-loop system and feedback gain on sensitivities, the response sensitivity analysis was developed. Because the response sensitivity involves the gain vector sensitivity, the new method for computing modal gain vector and the corresponding sensitivity analysis were discussed. The advantage of the present method is that the computation step is more straightforward and convenient to implement on the computer.

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