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Numerical assessment of seismic safety of liquid storage tanks and performance of base isolation system

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Abstract. Seismic isolation is a well-known method to mitigate the earthquake effects on structures by increasing their fundamental natural periods at the expense of larger displacements in the structural system. In this paper, the seismic response of isolated and fixed base vertical, cylindrical, liquid storage tanks is investigated using a Finite Element Model (FEM), taking into account fluid-structure interaction effects. Three vertical, cylindrical tanks with different ratios of height to radius ($H/R = 2.6$, 1.0 and 0.3) are numerically analyzed and the results of response-history analysis, including base shear, overturning moment and free surface displacement are reported for isolated and non-isolated tanks. Isolated tanks equipped by lead rubber bearings isolators and the bearing are modeled by using a non-linear spring in FEM model. It is observed that the seismic isolation of liquid storage tanks is quite effective and the response of isolated tanks is significantly influenced by the system parameters such as their fundamental frequencies and the aspect ratio of the tanks. However, the base isolation does not significantly affect the surface wave height and even it can causes adverse effects on the free surface sloshing motion.

Keywords: liquid storage tank; seismic analysis; numerical analysis; base isolation.

1. Introduction

In past earthquakes there have been a number of reports on damage to liquid storage tanks (Kelly and Mayes 1989). The earthquake damage has been due to a number of causes with the most common being buckling of tank walls and uplift at the tank anchorages. Investigations on the seismic response of liquid storage tanks have been conducted over the past 30 years. These research activities have been devoted to improving the seismic performance of liquid storage tanks. The seismic behavior of liquid tanks, however, is quite complicated due to the liquid-structure interaction.

Most of previous investigations focus on proposing mechanical models for computing the seismic response of liquid storage tanks. Historically, mechanical models were first developed for tanks with rigid walls. Housner (1954, 1957, 1963) was perhaps the first to propose such a simplified mechanical model for circular and rectangular rigid tanks. His simplified model is a two degree-of-freedom (DOF) system for rigid tanks, one DOF accounting for the motion of the tank-liquid

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system, with a part of the liquid being rigidly attached (impulsive mode) and the other DOF for the motion of the sloshing liquid (convective mode). The mechanical model of Howsner is still widely used with certain modifications for the analysis of rectangular and cylindrical tanks. Wozniak and Mitchell (1978) have generalized Houser's model for short and slender tanks. Veletsos and Yang (1977) have used a different approach to propose a similar type of a mechanical model for circular rigid tanks. Subsequently, Haroun and Housner (1981) and Veletsos (1984) have developed mechanical models for flexible tanks. Malhotra *et al.* (2000) proposed further simplifications of the mechanical model of Veletsos (1984) for flexible tanks. Although the dynamic behavior of liquid storage tanks has been quite well understood, the protecting method to mitigate the earthquake effects on liquid storage tanks is not properly developed.

One of the modern methods to reduce the seismic response of liquid tanks is based on the base isolation technique. The base isolation is a strategy that attempts to isolate the structure at its base, moving the dominant frequency of the isolated tank away from the dominant frequency range of the earthquake acceleration spectrum. There are several studies for investigating the effectiveness of seismic isolation for buildings, but a very few studies are reported for seismic isolation of liquid storage tanks which has a vital industrial application.

Kim and Lee (1995) experimentally investigated the earthquake performance of liquid storage tanks isolated by the elastomeric bearings. Chalhoub and Kelly (1988) and Malhotra (1997) also studied the seismic response of isolated liquid storage tanks. Shenton and Hampton (1999) investigated the seismic response of isolated elevated tanks and found that seismic isolation is effective in reducing the tower drift, base shear, overturning moment, and tank wall pressure for the full range of tank capacities. Wang *et al.* (2001) investigated the response of liquid storage tanks isolated by friction pendulum system. Shrimali and Jangid (2002) investigated the seismic response of liquid storage tanks isolated by lead-rubber bearings under bi-directional earthquake excitation and observed that the seismic response of isolated tanks is insensitive to interaction effect of the bearing forces. Shrimali and Jangid (2004) presented the earthquake analysis of base-isolated liquid storage tanks using linear theory of base isolation. Jadhav and Jangid (2006) also investigated the seismic response of liquid storage tanks isolated by elastomeric bearings and sliding systems under near-fault ground motions. Regardless of the method of investigation and the type of used isolation system, all of above researchers found that base isolation technique is effective in reducing the seismic response of the tanks over traditional fixed base tank. However, most of previous researchers either used experimental measurements with restrict results or simplified the seismic behavior of isolated liquid tank using mechanical equivalent mass-spring models. In this regards, it seems that numerical methods with capability of detail modeling is not properly introduced in literature because of related computational costs.

In this research, the numerical model based on finite element method is used to precise evolution of dynamic behavior of seismically isolated liquid storage tanks. Fluid-structure interaction and variation of shell thickness is also considered. First, the FEM strategy is presented and validated by experimental data. Then, in order to quantitatively investigate the effectiveness of base isolation system, parametric study is performed on slender, medium and broad tanks. Finally, conclusions are made using the comparison between the results of isolated and corresponding fixed base considered tanks.

2. Finite element modeling and strategy

ANSYS package (2005), as a numerical based code, is used to model the tank-fluid system in three dimensional spaces under seismic load. Finite Element (FE) method is employed to numerical investigation of the behavior of seismically isolated tanks. For this purpose four-noded, 24 DOF quadrilateral elastic shell elements that have both membrane and bending capabilities are used to model the tank walls. The fluid domain is modeled with three dimensional, eight-noded, 24 DOF fluid elements. These elements have 3 DOF at each node (displacement in three directions). To use the advantages of symmetry, only one half of the tank is modeled and proper boundary conditions are imposed on the nodes lying in the plane of symmetry.

The interaction between the tank and the fluid was addressed by properly coupling the nodes that lie in the common faces of these two domains in the radial direction. The fluid applies only normal pressures on the tank wall and relative movements in the tangential and vertical directions between shell and fluid elements are allowed to occur. Fluid element nodes are also allowed to move on the surface of the tank bottom plate, at the tank base. Meshing of the fluid domain is undertaken so that the solid elements would be as close to a cubic shape as possible. The general FEM of the considered tanks is illustrated in Fig. 1.

For the particular employed contained fluid element, only the lumped mass matrix is available. Newmark's Method has been employed to simulate the time history response analysis during earthquake ground motion. A Rayleigh damping matrix has been defined in the two significant modes; the first sloshing mode and the first horizontal coupled (impulsive) mode. The damping ratio for the sloshing mode and impulsive mode is considered to be 0.5% and 2.0% respectively. This corresponds to the linear elastic range of steel cylindrical tanks. The material property of the tanks walls were considered to be $E_S = 2\text{E}+08$ (kPa), Poisson's ratio $\upsilon = 0.3$ and density $\gamma_S = 78$ (kN/m^3).

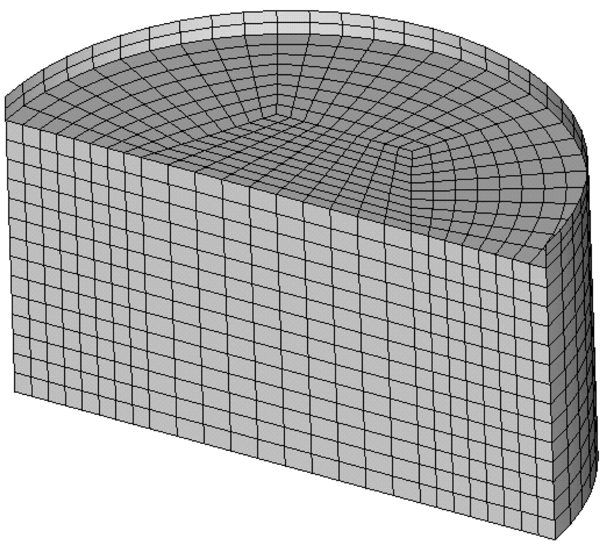

Fig. 1 Finite Element Model (FEM) which is used for numerical analysis

3. Verification of the numerical modeling strategy

Prior to use the FE model for a parametric study on storage tanks, the accuracy of the introduced modeling strategy is investigated in this section. For this purpose, FEM response under both harmonic and earthquake excitation is compared with experimental measurements.

3.1 Verification of FEM under harmonic excitation

In order to verify numerical modeling strategy presented in previous section, a series of the shaking table tests on rectangular tank conducted at the Hydraulic Institute of University of Stuttgart is used. The test tank was rectangular in shape having dimensions of $1 \times 0.96 \times 0.4\ \mathrm{m}^3$ (height × length × width) (Fig. 2). The tank was made of fiber reinforced acrylic glass with a thickness of 0.02 m and the tank was mounted on a 1×2 m shake-table. This experimental setup is used for various research purposes which are published in separate papers (Goudarzi and Sabbagh-Yazdi 2009, Goudarzi *et al.*, 2010). In this paper, in order to validate the FEM performance, only measurements of some experiments which are performed to evaluate the time history of free surface motions under harmonic sinusoidal excitation are used.

The surge motion was given as $X_{(t)} = D\sin(\omega t)$ where D = maximum horizontal amplitude (which was considered to 0.005 m) and ω = horizontal forced frequency. Experiments were carried out for partially filled tank with water depth equal to 0.624 m which are equivalent to aspect ratios (h/α) of 1.3. The test tank was excited with three horizontal forced frequencies: smaller, closed to and bigger than the first fundamental frequency of contained liquid. These frequencies correspond to different conditions such as under-resonant, near resonant and over-resonant frequencies. For mentioned forced frequencies, the FEM results and corresponding experimental measurements of free surface displacement at the left wall of the tank are shown in Fig. 3. As can be seen, for all cases, the tolerance of FEM results under harmonic loads is in an acceptable range.

Fig. 2 Test tank on the shake-table

3.2 Verification of FEM under seismic excitation

In order to verify the FEM results under seismic excitation, the results of free surface displacement obtained from the numerical model are compared with experimental results reported by Chalhoub and Kelly (1988). They used a cylindrical steel tank, with a height of 60.96 cm and a

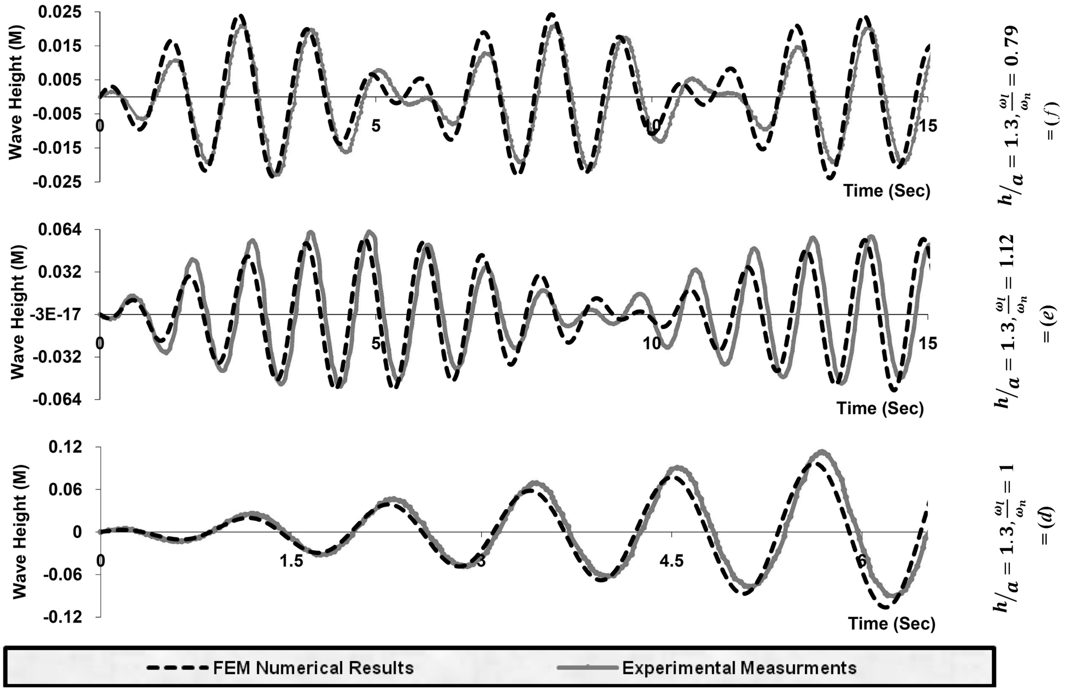

Fig. 3 Comparison between the results of experimental measurements and numerical model for sloshing wave height at the left wall of tanks (Harmonic Excitation)

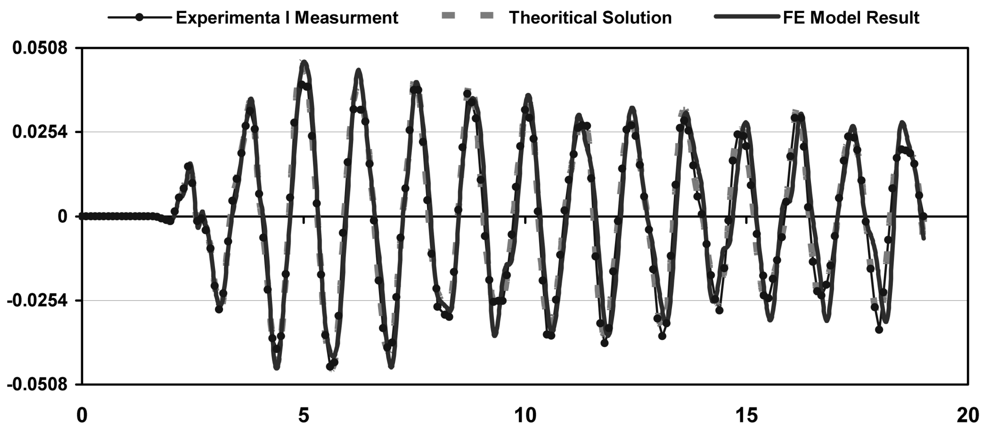

Fig. 4 Comparing between FE model results of sloshing wave height, experimental measurements and analytical solution (Meter) (Seismic Excitation)

wall thickness of 1 mm and a diameter of 121.92 cm. The El Centro earthquake record scaled by peak acceleration 0.114 g which was applied in the experiment is considered as an input base excitation for the FE analysis.

In Fig. 4, the results of free surface displacements at the shell wall computed by FEM are compared with measured free surface elevation as well as theoretical solution given by Chalhoub and Kelly (1988). The first three modes of sloshing are used to calculate free surface theoretical solution. As can be seen in Fig. 4, there are good agreement between FEM results, the theoretical solution and the experimental measurements. The FEM results present maximum differences of 15% and 14%, respectively, with measured and analytical peak free surface (MSWH) elevation. These levels of difference are inevitable because of the errors in measurements and the assumptions are made in the theoretical solution.

4. Introudcing of isolation system

The properties of the isolation system utilized in this study were selected to represent Lead-Rubber elastomeric bearings which can be used also for Friction Pendulum™ sliding bearings.

Fig. 5 presents the assumed hysteretic loop shape of the isolator, namely, bilinear response with

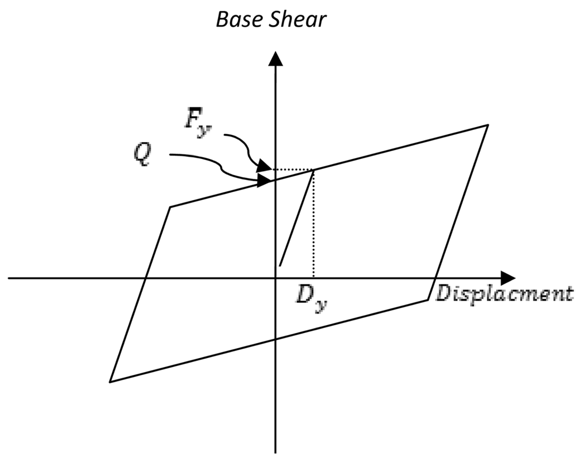

Fig. 5 Energy dissipation loop for the set of lead core rubber bearings

Table 1 Isolation system parameters and corresponding relations

$E_p = M_{imp}(2\pi/T_{iso})^2$	E_p = Post-elastic isolation stiffness T_{iso} = Isolation period M_{imp} = Impulsive Mass
$E_e = (D_y \cdot K_p + Q)/T_{iso}^2$	D_y = Yield displacement which was set equal to 12 mm E_e = Global Elastic Isolation stiffness Q = Characterized strength
$\alpha = K_p/K_e$	α = Post-yield stiffness ratio
$F_y = K_e \cdot D_y$	F_y = Global yield force of the isolation system

kinematic hardening. For isolated structures a fundamental period, i.e., the natural period of the structure (T_{iso}) moving as an almost rigid body on the isolators, is usually selected in the range of 1.5 to 3 sec based on the post-elastic stiffness. For this study, a fundamental period of 2.5 sec is chosen for the analysis of this study. Due to the fact that only impulsive mass participates in the isolated mode, the characteristic strength, Q, was set to 8% of this part of liquid mass.

In order to consider the seismic isolators at the base of the structure, the seismic isolation system are modeled as horizontal nonlinear springs with material nonlinearity which introduced to connect the tank base to the ground.

5. Numerical investigation

5.1 Specifications of utilized tanks

In present study, three tanks of different aspect ratios including broad tank ($H/R = 0.3$), medium tank ($H/R = 1$) and tall tank ($H/R = 2.6$) are utilized. Each tank was designed based on API code of practice (American Petroleum Institute API 1998). Dimensions and other geometry characterizes of these tanks are listed in Table 2.

5.2 Utilized earthquake excitations

Five earthquake base excitations are utilized as the record excitations of the system in which the peak ground accelerations are considered between 2.9 to 8.3 m/sec^2. Specifications of the earthquakes used for the time history analysis are listed in Table 3.

Table 2 Geometry dimensions of the tanks

	Radius (m)	Tank height (m)	Liquid height (m)	Lower shell thickness (m)	Upper shell thickness (m)	Liquid density (Kg/m^2)	Type of liquid	Bulk modulus of elasticity (N/m^2)
Tank1	54.5	17.5	15.85	0.03	0.03	885	Crude oil	1.65E+09
Tank2	37	40.6	37.4	0.033	0.033	480	LNG	2.00E+09
Tank3	2.5	8	6.5	0.006	0.006	1000	Water	2.10E+09

Table 3 Earthquake specifications used for the time history analysis

Earthquake	Year	Peak Ground Acceleration (g)	Predominant Period (sec)	Abbreviation
Elsentro	1940	4.1	0.5	EL
Imperial Vallay	1979	2.9	0.24	IV
Chichi	1999	3.03	1.14	CHI
Tabas	1978	8.3	0.2	TAB
Northriche	1994	5.9	0.54	NOR

6. Numerical results and discusion

6.1 FEM results for designing parameters

In this section, FEM results which is obtained from time history analyses of isolated and non-isolated tanks are presented. These results, including base shear and overturning moment, are tabulated in Tables 4 and 5, respectively. The overturning moment is computed with respect to the center of the cylindrical tank, neglecting the contribution of hydrodynamic pressures exerted on the base plate. “BI” and “NONBI” labels respectively refer to the results obtained from time history analysis of isolated and non-isolated tanks. In these tables, the deviation percentage of isolated tanks results from the results of corresponding non-isolated tanks is also presented.

In Fig. 6, the time history of global overturning moment below the foundation slab versus the corresponding base displacement is plotted for all fixed base and isolated cases. For comparison purposes, the overturning moment time history for corresponding non-isolated tanks is also shown.

In view of the results, a series of conclusions can be made concerning the most prominent design

Table 4 Maximum base shear force (kN)
Error (%) = 100 × (NONBI − BI)/NONBI

		EL	IV	CHI	TAB	NOR
Tank1	NONBI Tank	221551	179155	193340	532728	331382
	BI Tank	23808	29118	71722	34921	40136
	Error (%)	89%	84%	63%	93%	88%
Tank2	NONBI Tank	415386	412824	348148	984647	684681
	BI Tank	44008	62774	133732	64927	76936
	Error (%)	89%	85%	62%	93%	89%
Tank3	NONBI Tank	515	635	467	1862	722
	BI Tank	112	175	255	177	154
	Error (%)	78%	72%	45%	90%	79%

Table 5 Maximum overturning moment (kN-m)
Error (%) = 100 × (NONBI − BI)/NONBI

		EL	IV	CHI	TAB	NOR
Tank1	NONBI Tank	1496733	1186920	1286895	3875070	2217541
	BI Tank	304099	353480	895820	498130	501234
	Error (%)	80%	70%	30%	87%	77%
Tank2	NONBI Tank	6571204	7021780	5731903	3875070	11158885
	BI Tank	1363830	1805626	4162671	2029499	2308497
	Error (%)	79%	74%	27%	48%	79%
Tank3	NONBI Tank	1439	1807	1282	5375	1774
	BI Tank	736	1218	1622	1161	1102
	Error (%)	49%	33%	-26%	78%	38%

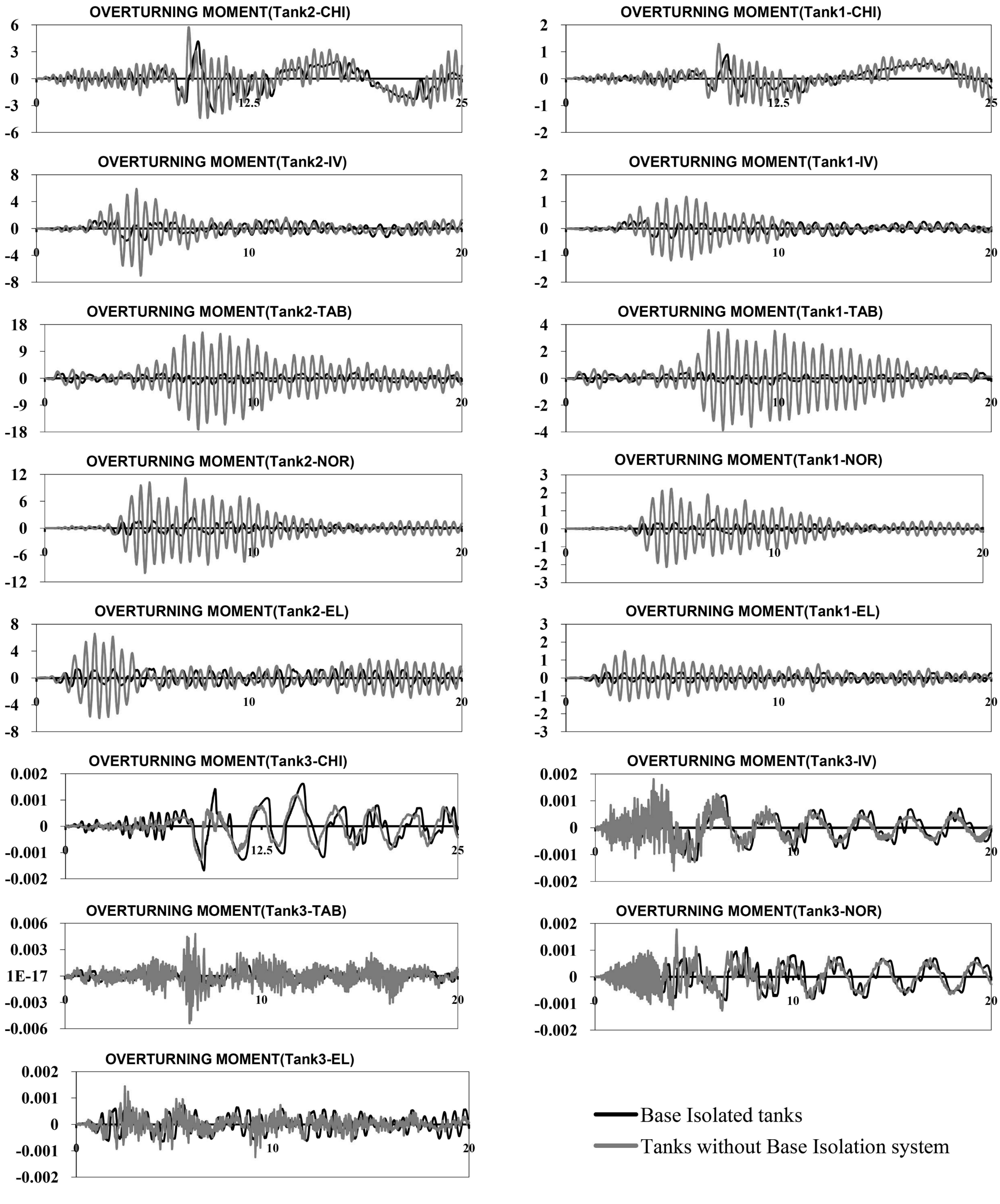

Fig. 6 Time history response of overturning moment for considered tanks with and without base isolation system (*E+9 *N-m*)

parameters. The average reductions of base shear forces for isolated tanks are 85%, 84% and 75% for three considered tanks. In terms of overturning moment, average reductions of the order of 69%, 61% and 32% for broad, medium and slender isolated tanks with respect to corresponding fixed base tanks are respectively calculated. It can be seen that base isolation method is quite efficient to suppressing of main design parameters. It seems the effectiveness of base isolation system to mitigating of base shear force is not significantly affected by changing of tank aspect ratio. On the other hands, overturning moment is considerably reduced for slender tank. It may be due to the facts that the fundamental convective period of slender tank (2.34 sec) is very close to the effective isolation period (2.5 sec). But, for the natural frequencies of sloshing liquid mass of the broad and medium tanks (9.37 sec and 15.4 sec) are well apart from the dominant natural frequencies of used isolation system. Moreover, the natural period of impulsive mode which has more important rules to compute the design parameter is slightly reduced by decreasing of aspect ratio. Therefore, for slender tank, the contribution of convective mode in final response of the tanks is considerably increased.

For specific tank aspect ratio, the minimum mitigation of designing parameters is computed under CHI-CHI excitation. Among the applied earthquake excitations, the CHI earthquake with larger predominant period includes the long period motion which is important to exciting of the primary sloshing mode. As can be seen in Table 5 and Fig. 6, for slender tank under CHI earthquake, the maximum overturning moment of isolated tank is even larger than non-isolated tank. In other words, for this case, using the isolation system adversely increases the main designing parameter i.e., overturning moment. This is due to the fact that both fundamental period of isolation system and predominate period of exerted earthquake be relatively close to the fundamental period of convective mode of liquid motion. It can be conclude that the effectiveness of base isolation method is very sensitive to the physical and geometrical parameters of considered tanks. It should be note that the contained liquid usually has a significantly longer fundamental natural period than most of the buildings.

6.2 FEM results of free surface motion

Maximum Sloshing Wave Height (MSWH) is one of the major considerations in the design of liquid storage tanks. MSWH is used to provide sufficient freeboard between the liquid surface and the tank roof to prevent sloshing waves from impacting the roof. If the sufficient free board is not provided, the impaction of liquid to the roof should be considered (Malholtra 2005, 2006, Goudarzi *et al.* 2010). The results of FEM for the isolated and non-isolated tanks are compared in Table 6.

Table 6 Results of MSWH computed by FEM for isolated and non isolated tanks

		EL	IV	CHI	TAB	NOR
		Liquid Sloshing Height				
Tank1	Base Isolated Tank	0.35	1.45	3.39	1.73	0.71
	No Base Isolated Tanks	0.36	1.51	3.27	1.73	0.77
	Error (%)	-2.86	-4.14	3.59	0.00	-8.45
Tank2	Base Isolated Tank	0.56	1.97	8.06	3.09	0.71
	No Base Isolated Tanks	0.57	2.11	8.02	3.14	1.03
	Error (%)	-2.25	-7.09	0.51	-1.65	-45.73
Tank3	Base Isolated Tank	0.46	1.10	1.82	1.26	1.36
	No Base Isolated Tanks	0.48	1.10	2.03	0.95	1.20
	Error (%)	-3.75	-0.01	-11.34	24.55	11.85

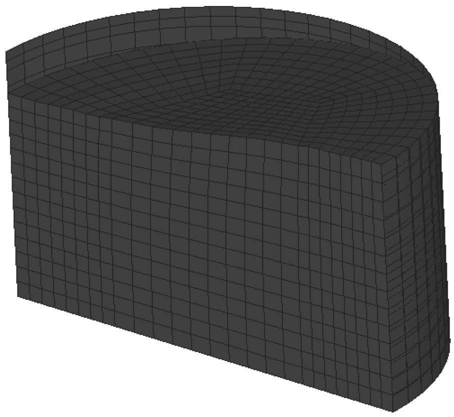

Fig. 7 Medium tank under TAB Earthquake (MSWH occurs at the tank wall)

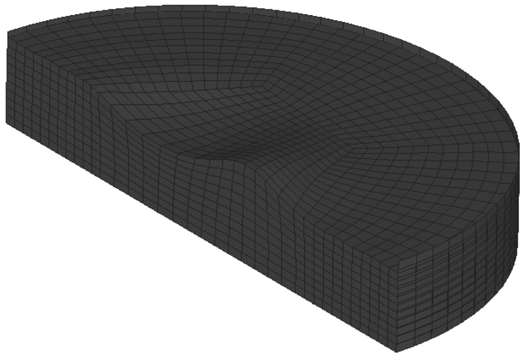

Fig. 8 Broad tank under CHI Earthquake (MSWH occurs in the middle of tank)

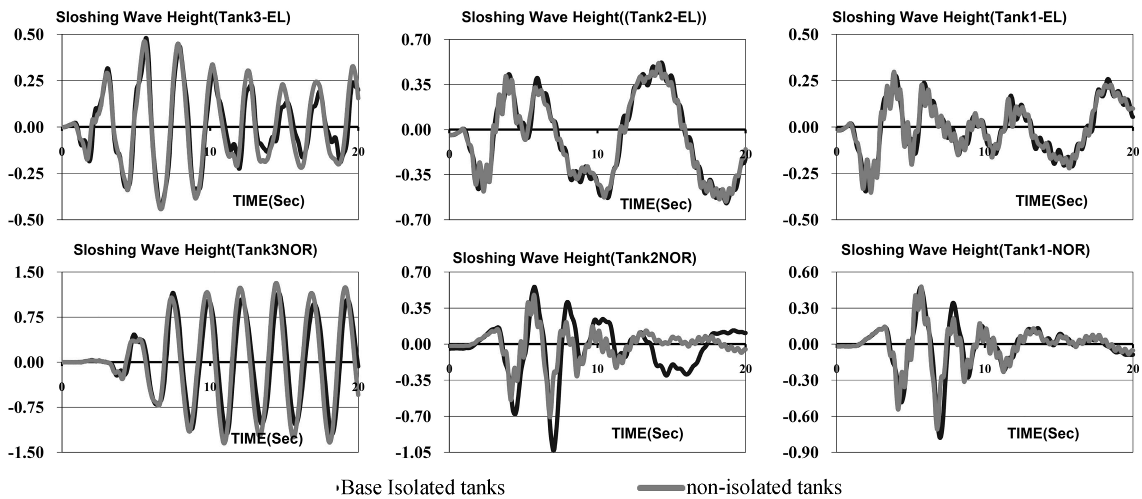

Fig. 9 Time history results of sloshing wave height (Meter)

The MSWH which is obtained by FEM takes place either at the wall sides of the tanks (Fig. 7) or in the middle of free surface (Fig. 8). In Table 6, only the maximum sloshing wave height occurring in the middle of free surface and is more important to provide the proper freeboard are reported. Time history of MSWH obtained by both isolated and fixed base tank is also plotted in Fig. 9 for some cases. According to the result of Table 6, the errors are less than 8% for most of considered cases. Also, for specific aspect ratio, MSWH occurs for the tanks subjected to CHI earthquake due to the special nature of excitation (including long period motion).

Free surface sloshing motions are shown in Fig. 9 are quite similar in both fixed base and isolated cases, but the component of sloshing motion associated with the wall vibration is not clearly visible in the base-isolated cases due to fluid-structure interaction. Even, the sloshing height is slightly amplified in some cases. Since, the convective effect is almost uncoupled from shell flexibility, impulsive response, and shell motion; the average surface motion period is close to the first convective mode, implying that the significant hydrodynamic sloshing motion is dominated by the fundamental convective modes.

Considering the maximum value of vertical surface displacement obtained from FEM results, available freeboard, estimated by design code for selected tanks under applied excitation, is not generally adequate to avoid damage to the roof structure. This means that the analytical relations suggested by codes could not properly predict the sloshing motion. Hence exact numerical modeling should be applied (Goudarzi and Sabbagh-Yazdi 2009).

7. Conclusions

A numerical method based on Finite Element Model (FEM) is conducted to evaluate dynamic response of seismically isolated tanks. FEM is generated using ANSYS package and is validated by comparison of it results with experimental measurements of small scale tank under harmonic and seismic excitations. Using the validated FEM, parametric study is conducted on slender, medium and broad tanks. In order to evaluate the effectiveness of base isolation system, the isolated and corresponding fixed base tanks are analyzed. The comparisons are made between the FEM analysis results of isolated and non-isolated tanks for main designing parameters in terms of base shear, overturning moment and free surface displacement. From the results of these comparisons, the following conclusions may be drawn:

1. The comparison with experimental measurements reveals that the use of the considered FEM provides enough accuracy for evaluating seismic behavior of nonlinear isolated and non-isolated tanks.
2. MSWH obtained from FE analysis occurs in the side wall of slender tanks. In media or broad tanks, however, it may occur in the middle of the free surface.
3. Long period ground motion is the main parameter which can significantly affects the seismic response of isolated tank.
4. It was shown that seismic isolation is very effective in mitigating dynamic responses. The efficiency becomes more pronounced as the effective frequency becomes lower and the intensity of ground motion become larger.
5. The average reductions of base shear forces of isolated tanks are 85%, 84% and 75% for broad, medium and slender isolated tanks. It seems the effectiveness of base isolation system to mitigating of base shear force is not significantly affected by changing of tank aspect ratio.
6. In terms of overturning moment, the average reductions of the order of 69%, 61% and 32% for broad, medium and slender tanks is obtained due to applying of isolation system. Therefore, overturning moment is considerably mitigated by reduction of tank aspect ratio.
7. The effectiveness of base isolation considerably reduces for exerted earthquake records including long period motion. Especially for slender tanks, base isolation may even increase the overturning moment.
8. Results of free surface displacement for both isolated and non-isolated tanks have quite similar trends for considered tanks. The errors between the maximum sloshing wave height of fixed base and isolated tank are less than 8% for most of considered cases. Even, the sloshing height is slightly amplified in some cases. Therefore, the base isolation system can cause adverse effects on the free surface sloshing motion.

9. The component of sloshing motion associated with the wall vibration due to fluid-structure interaction is not clearly visible in the base-isolated model.

It can be conclude that the effectiveness of base isolation method is very sensitive to the physical and geometrical parameters of considered tanks. This suggests that a careful selection of isolators with a certain limit on the mechanical properties of isolators is required for the optimal seismic isolation design of liquid storage tanks.

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