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Differential transform method for free vibration analysis of a moving beam

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Abstract. In this study, the Differential Transform Method (DTM) is employed in order to solve the governing differential equation of a moving Bernoulli-Euler beam with axial force effect and investigate its free flexural vibration characteristics. The free vibration analysis of a moving Bernoulli-Euler beam using DTM has not been investigated by any of the studies in open literature so far. At first, the terms are found directly from the analytical solution of the differential equation that describes the deformations of the cross-section according to Bernoulli-Euler beam theory. After the analytical solution, an efficient and easy mathematical technique called DTM is used to solve the differential equation of the motion. The calculated natural frequencies of the moving beams with various combinations of boundary conditions using DTM are tabulated in several tables and are compared with the results of the analytical solution where a very good agreement is observed.

Keywords: Differential Transform Method; free vibration; moving beam; natural frequencies.

1. Introduction

Axially moving beams can represent many engineering devices, such as power transmission belt and chain drives, high-speed magnetic tapes, band saws, pipe-conveying fluids and aerial cable tramways. The calculation of free vibration has a great importance in the dynamic analysis of these moving beams.

Previously, numerous researchers studied on this subject. Tabarrok *et al.* (1974), Buffinton and Kane (1985) and Wickert and Mote (1990) developed the governing differential equations of motion more accurately and sought the solutions for the problem. Hwang and Perkins investigated the stability and free vibration of moving beams by using geometric nonlinearity resulting from the large deformations (Hwang and Perkins 1992a, b). In the other study, a finite element based solution for free and forced vibration of a moving beam using the Lagrangian Multiplier Method was investigated by Sreeram and Sivaneri (1998). Öz and Pakdemirli (1999) and Öz (2001) applied the method of multiple scales to calculate analytically the stability boundaries of an axially accelerating beam under pinned-pinned and clamped-clamped conditions, respectively. An artificial neural network algorithm to determine stability boundary of an axially accelerating beam was used by Özkaya and Öz (2002). In the other study, natural frequencies of axially travelling tensioned beam

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in contact with a stationary mass were obtained by Öz (2003). Pellicano investigated the dynamic properties of axially moving systems (Pellicano 2005). On the other hand, nonlinear free transverse vibration of axially moving strips was studied by Chen and Yang (2007). Banerjee and Gunawardana developed the dynamic stiffness matrix of a moving Bernoulli-Euler beam and investigated its free flexural vibration characteristics (Banerjee and Gunawardana 2007). Lee and Jang investigated the effects of the continuously incoming and outgoing semi-infinite beam parts on the dynamic characteristics and stability of an axially moving beam by using the spectral element method (Lee and Jang 2007). In the other study, an approximate Galerkin finite-element method was applied to solve the initial boundary-value problem of a viscously damped axially moving pre-tensioned beam including arbitrary support excitations by Cepon and Boltezar (2007). On the other hand, the multidimensional Lindstedt-Poincare method was extended to the nonlinear vibration of axially moving beams by Chen *et al*. (2007). Tang *et al*. analyzed the natural frequencies, modes and critical speeds of axially moving beams based on Timoshenko model (Tang *et al*. 2008). Chen and Wang investigated the stability of an axially accelerating viscoelastic beam by using an asymptotic perturbation method and differential quadrature validation (Chen and Wang 2009).

DTM was applied to solve linear and non-linear initial value problems and partial differential equations by many researches. The concept of DTM was first introduced by Zhou and he used DTM to solve both linear and non-linear initial value problems in electric circuit analysis (Zhou 1986). Chen and Ho solved eigenvalue problems for the free and transverse vibration problems of a rotating twisted Timoshenko beam under axial loading by using DTM (Chen and Ho 1996, 1999). DTM was applied to solve a second order non-linear differential equation that describes the under damped and over damped motion of a system subject to external excitation by Jang and Chen (1997). Malik and Dang applied DTM to the free vibration of Bernoulli-Euler beams (Malik and Dang 1998). Bert and Zeng used DTM to investigate analysis of axial vibration of compound bars (Bert and Zeng 2004). Özdemir and Kaya investigated flapwise bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by DTM (Özdemir and Kaya 2006). In the other study, the out-of-plane free vibration analysis of a double tapered Bernoulli-Euler beam, mounted on the periphery of a rotating rigid hub is performed using DTM by Ozgumus and Kaya (2006). Çatal suggested DTM for the free vibration analysis of both ends simply supported and one end fixed, the other end simply supported Timoshenko beams resting on elastic soil foundation (Çatal 2006, 2008). Çatal and Çatal calculated the critical buckling loads of partially embedded Timoshenko pile in elastic soil by DTM (Çatal and Çatal 2006). Ho and Chen investigated the vibration problems of an axially loaded non-uniform spinning twisted Timoshenko beam by using DTM (Ho and Chen 2006). Free vibration analysis of a rotating, double tapered Timoshenko beam featuring coupling between flapwise bending and torsional vibrations is performed using DTM by Ozgumus and Kaya (2007). In the other study, Kaya and Ozgumus introduced DTM to analyze the free vibration response of an axially loaded, closed-section composite Timoshenko beam which features material coupling between flapwise bending and torsional vibrations due to ply orientation (Kaya and Ozgumus 2007). Numerical solution to buckling analysis of Bernoulli-Euler beams and columns were obtained using DTM and harmonic differential quadrature for various support conditions considering the variation of flexural rigidity by Rajasekaran (2008). In this study, solution technique is applied to find the buckling load of fully or partially embedded columns such as piles. For the first time, Yesilce and Catal investigated the free vibration analysis of one fixed, the other end simply supported Reddy-Bickford beam by using DTM in the other study (Yesilce and Catal 2009). Since previous studies have shown DTM to be an efficient tool and it has been applied to solve boundary value problems for many linear, non-linear integrodifferential and differential-difference equations that are very important in fluid mechanics, viscoelasticity, control theory, acoustics, etc. Besides the variety of the problems to that DTM may be applied, its accuracy and simplicity in calculating the natural frequencies and plotting the mode shapes makes this method outstanding among many other methods.

In this study, the free vibration analysis of simply supported, fixed-fixed supported and one end fixed, the other end simply supported and moving Bernoulli-Euler beams is performed. The equation of motion, including the parameters for nondimensionalized multiplication factors for the constant velocity and nondimensionalized multiplication factor for the axial tensile force, are solved using an efficient mathematical technique, called DTM. The natural frequencies of the moving beams are calculated by using the computer package, Matlab.

2. The mathematical model and formulation

A moving beam, its notation and coordinate system are presented in Fig. 1. The equation of motion for a moving Bernoulli-Euler beam can be written as (Banerjee and Gunawardana 2007)

$$EI_x \cdot \frac{\partial^4 w(x,t)}{\partial x^4} - N \cdot \frac{\partial^2 w(x,t)}{\partial x^2} + m \cdot \left[\frac{\partial^2 w(x,t)}{\partial t^2} + 2 \cdot v \cdot \frac{\partial^2 w(x,t)}{\partial x \cdot \partial t} + v^2 \cdot \frac{\partial^2 w(x,t)}{\partial x^2}\right] = 0 \quad (1)$$

where $w(x,t)$ represents transverse displacement function of the beam, m is mass per unit length of the beam, N is the axial tensile force, v is the axial velocity of the beam, I_x is moment of inertia, E is Young's modulus of the beam, x is spatial coordinate and t is time variable.

Assuming that the motion is harmonic we substitute for $w(x, t)$ the following

$$w(x,t) = w(x) \cdot e^{i\omega t} \quad (2)$$

where $w(x)$ is the amplitude of the bending displacement, ω is the circular frequency and $i = \sqrt{-1}$. Eq. (1) can be converted into an ordinary differential equation by using Eq. (2) as

$$\frac{EI_x}{L^4} \cdot \frac{d^4 w(z)}{dz^4} + \left(\frac{m \cdot v^2 - N}{L^2}\right) \cdot \frac{d^2 w(z)}{dz^2} + \left(\frac{2 \cdot m \cdot \omega \cdot v \cdot i}{L}\right) \cdot \frac{dw(z)}{dz} - m \cdot \omega^2 \cdot w(z) = 0 \quad (3)$$

where $z = x/L$ and L is length of the beam.

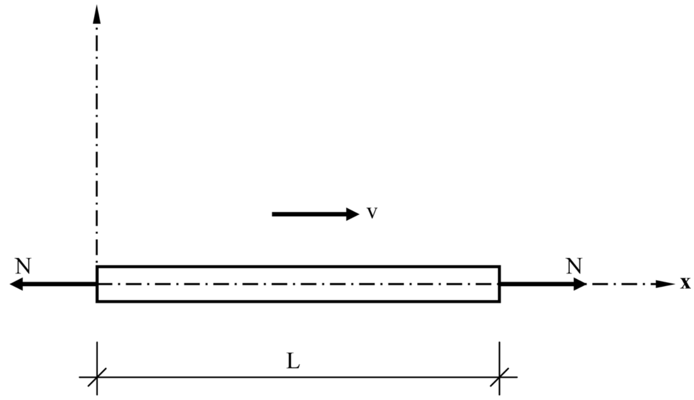

Fig. 1 Notation and coordinate system of a moving beam

The expression for bending rotation $\theta(z,t)$ is given by

$$\theta(z,t) = \frac{1}{L}\cdot\frac{dw(z)}{dz}\cdot e^{i\omega t} \tag{4}$$

The shear force function $Q(z, t)$ can be obtained as

$$Q(z,t) = \left[\frac{EI_x}{L^3}\cdot\frac{d^3w(z)}{dz^3}+\left(\frac{m\cdot v^2-N}{L}\right)\cdot\frac{dw(z)}{dz}+m\cdot\omega\cdot v\cdot i\cdot w(z)\right]\cdot e^{i\omega t} \tag{5}$$

Similarly, the bending moment function $M(z, t)$ can be obtained as

$$M(z,t) = -\frac{EI_x}{L^2}\cdot\frac{d^2w(z)}{dz^2}\cdot e^{i\omega t} \tag{6}$$

3. The differential transform method (DTM)

Partial differential equations are often used to describe engineering problems whose closed form solutions are very difficult to establish in many cases. Therefore, approximate numerical methods are often preferred. However, in spite of the advantages of these on hand methods and the computer codes that are based on them, closed form solutions are more attractive due to their implementation of the physics of the problem and their convenience for parametric studies. Moreover, closed form solutions have the capability and facility to solve inverse problem of determining and designing the geometry and characteristics of an engineering system and to achieve a prescribed behavior of the system. Considering the advantages of the closed form solutions mentioned above, DTM is introduced in this study as the solution method. DTM is a semi-analytic transformation technique based on Taylor series expansion and is a useful tool to obtain analytical solutions of the differential equations. Certain transformation rules are applied and the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions in DTM. The solution of these algebraic equations gives the desired solution of the problem. The different from high-order Taylor series method is; Taylor series method requires symbolic computation of the necessary derivatives of the data functions and is expensive for large orders. DTM is an iterative procedure to obtain analytic Taylor series solutions of differential equations (Yesilce and Catal 2009).

A function $w(z)$, which is analytic in a domain D, can be represented by a power series with a center at $z = z_0$, any point in D. The differential transform of the function $w(z)$ is given by

$$W(k) = \frac{1}{k!}\cdot\left(\frac{d^kw(z)}{dz^k}\right)_{z=z_0} \tag{7}$$

where $w(z)$ is the original function and $W(k)$ is the transformed function. The inverse transformation is defined as

$$w(z) = \sum_{k=0}^{\infty}(z-z_0)^k\cdot W(k) \tag{8}$$

From Eqs. (7) and (8) we get

$$w(z) = \sum_{k=0}^{\infty} \frac{(z-z_0)^k}{k!} \cdot \left(\frac{d^k w(z)}{dz^k}\right)_{z=z_0} \tag{9}$$

Eq. (9) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivative are calculated by iterative procedure that are described by the transformed equations of the original functions. In real applications, the function $w(z)$ in Eq. (8) is expressed by a finite series and can be written as

$$w(z) = \sum_{k=0}^{\bar{N}} (z-z_0)^k \cdot W(k) \tag{10}$$

Eq. (10) implies that $\sum_{k=\bar{N}+1}^{\infty} (z-z_0)^k W(k)$ is negligibly small. Where $\bar{N}$ is series size and the value of $\bar{N}$ depends on the convergence of the eigenvalues.

Table 1 DTM theorems used for equations of motion

Original Function	Transformed Function
$w(z) = u(z) \pm v(z)$	$W(k) = U(k) \pm V(k)$
$w(z) = a \cdot u(z)$	$W(k) = a \cdot U(k)$
$w(z) = \frac{d^m u(z)}{dz^m}$	$W(k) = \frac{(k+m)!}{k!} \cdot U(k+m)$
$w(z) = u(z) \cdot v(z)$	$W(k) = \sum_{r=0}^{k} U(r) \cdot V(k-r)$
$w(z) = z^m$	$W(k) = \delta(k-m) = \begin{cases} 0 \text{ if } k \neq m \\ 0 \text{ if } k = m \end{cases}$

Table 2 DTM theorems used for boundary conditions

$z = 0$		$z = 1$	
Original Boundary Conditions	Transformed Boundary Conditions	Original Boundary Conditions	Transformed Boundary Conditions
$w(0) = 0$	$W(0) = 0$	$w(1) = 0$	$\sum_{k=0}^{\infty} W(k) = 0$
$\frac{dw}{dz}(0) = 0$	$W(1) = 0$	$\frac{dw}{dz}(1) = 0$	$\sum_{k=0}^{\infty} k \cdot W(k) = 0$
$\frac{d^2 w}{dz^2}(0) = 0$	$W(2) = 0$	$\frac{d^2 w}{dz^2}(1) = 0$	$\sum_{k=0}^{\infty} k \cdot (k-1) \cdot W(k) = 0$
$\frac{d^3 w}{dz^3}(0) = 0$	$W(3) = 0$	$\frac{d^3 w}{dz^3}(1) = 0$	$\sum_{k=0}^{\infty} k \cdot (k-1) \cdot (k-2) \cdot W(k) = 0$

Theorems that are frequently used in differential transformation of the differential equations and the boundary conditions are introduced in Table 1 and Table 2, respectively.

3.1 Using differential transformation to solve motion equations

Eq. (3) can be rewritten as follows

$$\frac{d^4w(z)}{dz^4} = (N_r - \alpha)\cdot\frac{d^2w(z)}{dz^2} - (2\cdot\beta\cdot i)\cdot\frac{dw(z)}{dz} + (\lambda^4)\cdot w(z) \tag{11}$$

where

$$\lambda = \sqrt[4]{\frac{m\cdot\omega^2\cdot L^4}{EI_x}} \quad \text{(Frequency factor)} \tag{12}$$

$$\alpha = \frac{m\cdot v^2\cdot L^2}{EI_x} \quad \text{(The first nondimensionalized multiplication factor for the velocity)} \tag{13}$$

$$\beta = \frac{m\cdot\omega\cdot v\cdot L^3}{EI_x} \quad \text{(The second nondimensionalized multiplication factor for the velocity)} \tag{14}$$

$$N_r = \frac{N\cdot L^2}{EI_x} \quad \text{(Nondimensionalized multiplication factor for the axial tensile force)} \tag{15}$$

DTM is applied to Eq. (11) by using the theorems introduced in Table 1 and the following expression is obtained

$$W(k+4) = (N_r - \alpha)\cdot\frac{W(k+2)}{(k+3)\cdot(k+4)} - (2\cdot\beta\cdot i)\cdot\frac{W(k+1)}{(k+2)\cdot(k+3)\cdot(k+4)} + (\lambda^4)\cdot\frac{W(k)}{(k+1)\cdot(k+2)\cdot(k+3)\cdot(k+4)} \tag{16}$$

where $W(k)$ is the transformed function of $w(z)$.

The boundary conditions of a simply supported Bernoulli-Euler beam are given below

$$w(z=0)=0 \tag{17}$$

$$M(z=0)=0 \tag{18}$$

$$w(z=1)=0 \tag{19}$$

$$M(z=1)=0 \tag{20}$$

Applying DTM to Eqs. (17)-(20) and using the theorems introduced in Table 2, the transformed boundary conditions of a simply supported beam are obtained as

for $z = 0$; $$W(0) = W(2) = 0 \tag{21}$$

for $z = 1$; $$\sum_{k=0}^{\bar{N}} W(k) = \sum_{k=0}^{\bar{N}} \bar{M}(k) = 0 \tag{22}$$

where $\bar{M}(k)$ is the transformed function of $M(z)$.

The boundary conditions of a fixed-fixed beam are given below

$$w(z=0) = 0 \tag{23}$$

$$\theta(z=0) = 0 \tag{24}$$

$$w(z=1) = 0 \tag{25}$$

$$\theta(z=1) = 0 \tag{26}$$

Applying DTM to Eqs. (23)-(26), the transformed boundary conditions of a fixed-fixed beam are obtained as

for $z = 0$; $$W(0) = W(1) = 0 \tag{27}$$

for $z = 1$; $$\sum_{k=0}^{\bar{N}} W(k) = \sum_{k=0}^{\bar{N}} \Phi(k) = 0 \tag{28}$$

where $\Phi(k)$ is the transformed function of $\theta(z)$.

The boundary conditions of one end ($z = 0$) fixed and the other end ($z = 1$) simply supported Bernoulli-Euler beam are given below

$$w(z=0) = 0 \tag{29}$$

$$\theta(z=0) = 0 \tag{30}$$

$$w(z=1) = 0 \tag{31}$$

$$M(z=1) = 0 \tag{32}$$

Applying DTM to Eqs. (29)-(32), the transformed boundary conditions of one end fixed and the other end simply supported beam are obtained as

for $z = 0$; $$W(0) = W(1) = 0 \tag{33}$$

for $z = 1$; $$\sum_{k=0}^{\bar{N}} W(k) = \sum_{k=0}^{\bar{N}} \overline{M}(k) = 0 \tag{34}$$

For simply supported beam, substituting the boundary conditions expressed in Eqs. (21) and (22) into Eq. (16) and taking $W(1) = c_1$, $W(3) = c_2$; for fixed-fixed supported beam, substituting the boundary conditions expressed in Eqs. (27) and (28) into Eq. (16) and taking $W(2) = c_1$, $W(3) = c_2$; for one end fixed and the other end simply supported beam, substituting the boundary conditions expressed in Eqs. (33) and (34) into Eq. (16) and taking $W(2) = c_1$, $W(3) = c_2$; the following matrix expression is obtained

$$\begin{bmatrix} A_{11}^{(\bar{N})}(\omega) & A_{12}^{(\bar{N})}(\omega) \\ A_{21}^{(\bar{N})}(\omega) & A_{22}^{(\bar{N})}(\omega) \end{bmatrix} \cdot \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{35}$$

where c_1 and c_2 are constants and $A_{j1}^{(\bar{N})}(\omega), A_{j2}^{(\bar{N})}(\omega)$ ($j = 1, 2$) are polynomials of ω corresponding $\bar{N}$.

In the last step, for non-trivial solution, equating the coefficient matrix that is given in Eq. (35) to zero one determines the natural frequencies of the vibrating system as is given in Eq. (36).

$$\begin{vmatrix} A_{11}^{(\overline{N})}(\omega) & A_{12}^{(\overline{N})}(\omega) \\ A_{21}^{(\overline{N})}(\omega) & A_{22}^{(\overline{N})}(\omega) \end{vmatrix} = 0 \tag{36}$$

The jth estimated eigenvalue, $\omega_j^{(\overline{N})}$ corresponds to $\overline{N}$ and the value of $\overline{N}$ is determined as

$$\left|\omega_j^{(\overline{N})} - \omega_j^{(\overline{N}-1)}\right| \leq \varepsilon \tag{37}$$

where $\omega_j^{(\overline{N}-1)}$ is the jth estimated eigenvalue corresponding to $(\overline{N}-1)$ and ε is the small tolerance parameter. If Eq. (37) is satisfied, the jth estimated eigenvalue, $\omega_j^{(\overline{N})}$ is obtained.

The procedure that is explained below can be used to plot the mode shapes of the moving Bernoulli-Euler beam. The following equalities can be written by using Eq. (35)

$$A_{11}(\omega) \cdot c_1 + A_{12}(\omega) \cdot c_2 = 0 \tag{38}$$

Using Eq. (38), the constant c_2 can be obtained in terms of c_1 as follows

$$c_2 = -\frac{A_{11}(\omega)}{A_{12}(\omega)} \cdot c_1 \tag{39}$$

All transformed functions can be expressed in terms of ω, c_1 and c_2. Since c_2 has been written in terms of c_1 above, $W(k), \Phi(k)$ and $\overline{M}(k)$ can be expressed in terms c_1 as follows

$$W(k) = W(\omega, c_1) \tag{40}$$

$$\Phi(k) = \Phi(\omega, c_1) \tag{41}$$

$$\overline{M}(k) = \overline{M}(\omega, c_1) \tag{42}$$

The mode shapes can be plotted for several values of ω by using Eq. (40).

4. Numerical analysis and discussions

For numerical analysis, the simply supported, the fixed-fixed supported and one end fixed, the other end simply supported beams are considered in the paper. Natural frequencies of the beams, ω_i (i = 1, 2, 3) are calculated by using computer programs prepared in Matlab by the author. Natural frequencies are found by determining values for which the determinant of the coefficient matrix is equal to zero.

The numerical results of this paper are obtained based on uniform, rectangular Bernoulli-Euler beams with the following data as:

m = 0.31855 kN.sec^2/m ; $EI_x = 7.421875 \times 10^4$ kN.m^2 ; L = 3.0 m ; N_r = 0.00, 0.50 and 1.00; α = 0.50, 1.00 and 2.00

Using DTM, the frequency values of the moving simply supported beam for the first three modes are presented in Table 3, the first three frequency values of the moving fixed-fixed beam are presented in Table 4 and one end fixed, the other end simply supported beam's the first three frequency values are presented in Table 5 being compared with the frequency values obtained by using analytical method for the different values of the first nondimensionalized multiplication factor for the velocity (α) and nondimensionalized multiplication factor for the axial tensile force (N_r).

Table 3 The first three natural frequencies of a moving beam with simply supported boundary condition for a range of axial load (N_r) and moving speed (α) parameters

METHOD	$\overline{N}$	$N_r = 0.00$, $\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	18	513.2040	2095.1676	---	496.8372	2080.0642	---	463.2732	2049.6414	---
	28	513.2040	2106.3344	4753.9788	496.8372	2095.3074	4746.4218	463.2732	2073.1086	4733.2551
	38	513.2040	2106.3344	4754.4013	496.8372	2095.3074	4744.8359	463.2732	2073.1086	4725.6734
Analytic Method		513.2040	2106.3344	4754.4013	496.8372	2095.3074	4744.8359	463.2732	2073.1086	4725.6734

METHOD	$\overline{N}$	$N_r = 0.50$, $\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	18	526.7454	2108.5621	---	510.7003	2093.4862	---	477.8615	2063.1166	---
	28	526.7454	2119.7267	4767.3768	510.7003	2108.7272	4759.8285	477.8615	2086.5870	4746.6791
	38	526.7454	2119.7267	4767.7985	510.7003	2108.7272	4758.2412	477.8615	2086.5870	4739.0951
Analytic Method		526.7454	2119.7267	4767.7985	510.7003	2108.7272	4758.2412	477.8615	2086.5870	4739.0951

METHOD	$\overline{N}$	$N_r = 1.00$, $\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	18	539.9494	2121.8727	---	524.2011	2106.8239	---	492.0269	2076.5062	---
	28	539.9494	2133.0349	4780.7374	524.2011	2122.0622	4773.1976	492.0269	2099.9789	4760.0649
	38	539.9494	2133.0349	4781.1581	524.2011	2122.0622	4771.6087	492.0269	2099.9789	4752.4787
Analytic Method		539.9494	2133.0349	4781.1581	524.2011	2122.0622	4771.6087	492.0269	2099.9789	4752.4787

Table 4 The first three natural frequencies of a moving beam with fixed-fixed supported boundary condition for a range of axial load (N_r) and moving speed (α) parameters

METHOD	$\bar{N}$	$N_r = 0.00$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	24	1188.4691	3295.5910	---	1177.0048	3284.1362	---	1154.0594	3261.1949	---
	34	1188.4691	3299.3809	6474.3052	1177.0048	3291.0973	6465.8910	1154.0594	3274.4817	6448.8437
	44	1188.4691	3299.3809	6476.3793	1177.0048	3291.0973	6468.4431	1154.0594	3274.4817	6452.5635
Analytic Method		1188.4691	3299.3809	6476.3793	1177.0048	3291.0973	6468.4431	1154.0594	3274.4817	6452.5635
METHOD	$\bar{N}$	$N_r = 0.50$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	24	1195.8666	3305.8525	---	1184.4521	3294.3504	---	1161.6072	3271.3360	---
	34	1195.8666	3309.3697	6485.2355	1184.4521	3301.0798	6476.8349	1161.6072	3284.4522	6459.8112
	44	1195.8666	3309.3697	6487.3378	1184.4521	3301.0798	6479.4010	1161.6072	3284.4522	6463.5203
Analytic Method		1195.8666	3309.3697	6487.3378	1184.4521	3301.0798	6479.4010	1161.6072	3284.4522	6463.5203
METHOD	$\bar{N}$	$N_r = 1.00$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	24	1203.2144	3316.0988	---	1191.8490	3304.5448	---	1169.1031	3281.4498	---
	34	1203.2144	3319.3265	6496.1445	1191.8490	3311.0302	6487.7581	1169.1031	3294.3908	6470.7589
	44	1203.2144	3319.3265	6498.2773	1191.8490	3311.0302	6490.3399	1169.1031	3294.3908	6474.4582
Analytic Method		1203.2144	3319.3265	6498.2773	1191.8490	3311.0302	6490.3399	1169.1031	3294.3908	6474.4582

Table 5 The first three natural frequencies of a moving beam with one end fixed and the other end simply supported boundary condition for a range of axial load (N_r) and moving speed (α) parameters

METHOD	$\overline{N}$	$N_r = 0.00$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	20	813.5463	2645.9725	---	800.1197	2623.6030	---	773.0699	2577.4379	---
	30	813.5463	2670.2269	5574.8766	800.1197	2660.7023	5561.4429	773.0699	2641.5731	5533.0130
	40	813.5463	2670.2269	5582.3526	800.1197	2660.7023	5573.6679	773.0699	2641.5731	5556.2819
Analytic Method		813.5463	2670.2269	5582.3526	800.1197	2660.7023	5573.6679	773.0699	2641.5731	5556.2819
METHOD	$\overline{N}$	$N_r = 0.50$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	20	823.6082	2657.7423	---	810.2993	2635.4946	---	783.4961	2589.5062	---
	30	823.6082	2681.7175	5586.9824	810.2993	2672.1978	5573.5576	783.4961	2653.0797	5545.1534
	40	823.6082	2681.7175	5594.4368	810.2993	2672.1978	5585.7546	783.4961	2653.0797	5568.3737
Analytic Method		823.6082	2681.7175	5594.4368	810.2993	2672.1978	5585.7546	783.4961	2653.0797	5568.3737
METHOD	$\overline{N}$	$N_r = 1.00$								
		$\alpha = 0.50$			$\alpha = 1.00$			$\alpha = 2.00$		
		ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)	ω_1 (rad/sec)	ω_2 (rad/sec)	ω_3 (rad/sec)
DTM	20	833.5408	2669.4651	---	820.3456	2647.3428	---	793.7797	2601.5355	---
	30	833.5408	2693.1586	5599.0610	820.3456	2683.6437	5585.6444	793.7797	2664.5364	5557.2650
	40	833.5408	2693.1586	5606.4948	820.3456	2683.6437	5597.8151	793.7797	2664.5364	5580.4392
Analytic Method		833.5408	2693.1586	5606.4948	820.3456	2683.6437	5597.8151	793.7797	2664.5364	5580.4392

As the axial tensile force acting to the beams is increased for the other variable (α) is being constant, the natural frequency values of all moving beams increased. This result indicates that, the increasing for the axial tensile force leads to augmentation in natural frequency values for all boundary conditions. This result is very important for the effect of axial tensile force.

A decrease is observed in natural frequency values of the first three modes of the moving beams for the condition of N_r ratio being constant and the values of the first nondimensionalized multiplication factor for the velocity (α) is increased. This result indicates that, the first nondimensionalized multiplication factor for the velocity leads to reduction in natural frequency values for all boundary conditions.

In application of DTM, the natural frequency values of the moving beams are calculated by increasing series size $\bar{N}$. In Table 3-Table 5, convergences of the first three natural frequencies are introduced. Here, it is seen that; for simply supported beam, when the series size is taken 38; for fixed-fixed beam, when the series size is taken 44 and for one end fixed, the other end simply supported beam, when the series size is taken 40, the natural frequency values of the third mode can be appeared. Additionally, here it is seen that higher modes appear when more terms are taken into account in DTM applications. Thus, depending on the order of the required mode, one must try a few values for the term number at the beginning of the calculations in order to find the adequate number of terms.

5. Conclusions

In this study, starting from the governing differential equation of motion in free vibration of the moving beam, DTM algorithms are developed by using Bernoulli-Euler beam theory and the iterative-based computer programs are developed for the solution of linear-homogeneous frequency equation set relating to free vibration of the moving beams with simply supported, fixed-fixed and one end fixed, the other end simply supported boundary conditions. Variation in free vibration natural frequencies for the first three modes of the beams is investigated for the different values of the first nondimensionalized multiplication factor for the velocity and nondimensionalized multiplication factor for the axial tensile force. The calculated natural frequencies of the moving beams by using DTM are compared with the results of the analytical solution.

The essential steps of the DTM application includes transforming the governing equation of motion into algebraic equations, solving the transformed equations and then applying a process of inverse transformation to obtain any desired natural frequency. All the steps of the DTM are very straightforward and the application of the DTM to both the equation of motion and the boundary conditions seem to be very involved computationally. However, all the algebraic calculations are finished quickly using symbolic computational software. Besides all these, the analysis of the convergence of the results show that DTM solutions converge fast. When the results of the DTM are compared with the results of analytical method, very good agreement is observed.

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