DOI: http://dx.doi.org/10.12989/sem.2010.35.2.257
Structural Engineering and Mechanics, Vol. 35, No. 2 (2010) 257-260

Technical Note

A new structural damage detection index based on analyzing vibration modes by the wavelet transform

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(Received October 1, 2008, Accepted September 29, 2009)

1. Introduction

The wavelet transform (WT) is a signal-processing tool that is sensitive to sudden local variations in data. This feature of the WT has enabled to determine structural damage locations by processing dynamic shapes such as vibration modes. When such data is transformed by the WT, the wavelet coefficients appearing as sudden peaks indicate the possible defect locations, since crack or notch type defect induces local singularity to mode shape. One can obtain a suitable damage index by subtracting the WT coefficients of the healthy response from the damaged ones (Castro *et al.* 2006). The first advantage of such index is to reduce boundary distortions substantially. These distortions arise because of the discontinuity of modal shape at end points, and make difficult to perceive damage-induced peaks. Second, this index is especially suitable for the cases of smaller-extent damage and largely sampled structural response (Zhong and Oyadiji 2007). However, healthy structural response may not be available in general. In this note, a new damage detection index that does not require such healthy structural vibration mode but is as efficient as the approach set forth above is presented. The performance of the method is demonstrated by the first three vibration modes of a simply supported (SS) beam.

2. Theoretical background

The continuous WT (CWT) of a vibration mode $f(x)$ is defined as $W(a,b)=a^{-1/2}\int_{-\infty}^{\infty}f(x)\,\psi_{a,b}^{*}(x)dx$, where $\psi_{a,b}^{*}(x)$ is derived by $\psi_{a,b}(x) = \psi((x-b)/a)$ from a mother wavelet $\psi(x)$ satisfying some mathematical requirements. The a $(a>0)$ and b are real numbers called as scale and shifting parameters, respectively. The overstar indicates complex conjugate, since a wavelet can, in general, be complex (Addison 2002). At a damage location $d \in [0, L]$ $W(a,d)$ forms a high-magnitude peak due to the singularity at d, whereas other wavelet coefficients away from the point d are close to zero, if a wavelet with enough vanishing moments is used. The number of vanishing moments (NVM) of a wavelet is said to be N if $\int_{-\infty}^{\infty}x^{k}\psi(x)dx = 0$ holds, where $k=0, 1,.., (N-1)$ (Debnath

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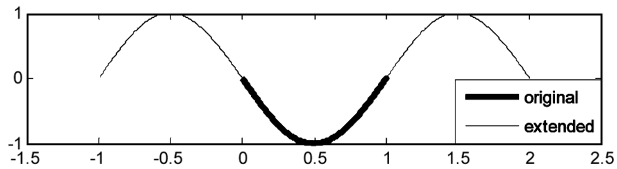

Fig. 1 Extending a mode shape

2002). The redundancy of the CWT can be removed by sampling scale and shifting parameters as $a = 2^m$ and $b = n2^m$, where n and m are integers. The transform is then called as the discrete WT (DWT). If an orthonormal wavelet family is used, $f(x)$ can be decomposed by the DWT to the sum of approximation function (AF) $A_m(x)$ at the decomposition level (DL) m, and the detail functions $D_i(x)$ up to that level as $f(x) = A_m(x) + \sum_{i=-\infty}^{m} D_i(x)$ (Addison 2002).

The damaged vibration mode (DM) may be assumed to be composed of undamaged mode (UM) and other components such as damage-induced variations and noise (Zhong and Oyadiji 2007). Hence, the AF at a suitable DL is expected to be well-correlated with the UM, since both are free of damage-induced components, and contain mainly the lower frequency parts. Therefore, this AF can be used instead of UM to obtain the CWT-based damage index. The following steps are proposed in this note to obtain such an AF:

1) The DM is extended by its asymetry as shown in Fig. 1 to reduce border distortion. This can be achieved by "wextend" command in MATLAB environment. 2) The extended signal is upsampled to the nearest power of 2 by cubic spline interpolation, since the DWT is defined for a sequence with length of some power of 2 (Misiti *et al*. 2007). 3) A suitable AF is extracted from this extended and upsampled DM by symN wavelet (N: NVM). It is important to choose the proper NVM and DL for this purpose. Here, it is proposed to process DM by a wavelet with high NVM, since the frequency spectrum of the scale function of the wavelet narrows about zero frequency and becomes flat with rising NVM (Debnath 2002). Hence, the AF obtained by such a wavelet will be freer of high frequency components. The agreement of the UM and the extracted AF is shown by the well-known modal assurance criterion (MAC). If A and U represent, respectively, the AF and UM of length K, then $MAC_1(A, U) = (\sum_{k=1}^{K} A(k)U(k))^2/((\sum_{k=1}^{K} A(k)A(k))(\sum_{k=1}^{K} U(k)U(k))$. Since the WT is sensitive to derivative discontinuities, the closeness of the derivatives of A and U should be ensured, so that $MAC_2(A', U')$ and $MAC_3(A'', U'')$ are introduced, where $' = d/dx$. However, higher-order derivatives are ignored since they are sensitive to noise. If UM is not known, these MACs can not be computed. In this case, another index is needed to make sure that these MACs are comparatively large and the correct DL is chosen. For this purpose, it is proposed to compute the approximation energy ratio (E_A) defined as $E_A(\%) = 100\sum_{k=0}^{K-1} A_m(k)^2/\sum_{k=0}^{K-1} f(k)^2$ at each DL, m, for a discrete mode function f with length K. It was seen that, when the first notable decrease in E_A occurs at a DL, the AF at the preceding DL is compatible with the UM.

After the suitable AF is obtained, the following damage indexes are computed

$$W_1 = W_D(a, b) - W_U(a, b), \quad W_2 = W_D(a, b) - W_A(a, b) \tag{1}$$

where W_D, W_U and W_A denote, respectively, the CWT coefficients of the DM, UM, and the AF. W_1 represents the classical approach while W_2 denotes the new damage index proposed in this note.

3. Application to a SS beam

The method is applied to the first three vibration modes of a beam with length $L = 1$ m and section sizes 40(width) × 30(height)mm^2. Three notches are located at $x_1 = 0.3$ m, $x_2 = 0.5$ m and $x_3 = 0.9$ m, respectively. A notched section height is 26 mm, and its width along the beam axis is 1 mm. Variation of the MACs wrt the NVM and DL are shown in Fig. 2, considering the second mode shape, obtained in ANSYS environment with BEAM188 element, as an example. According to the Figure, the fourth DL seems unsuitable since all MACs are relatively lower at this level.

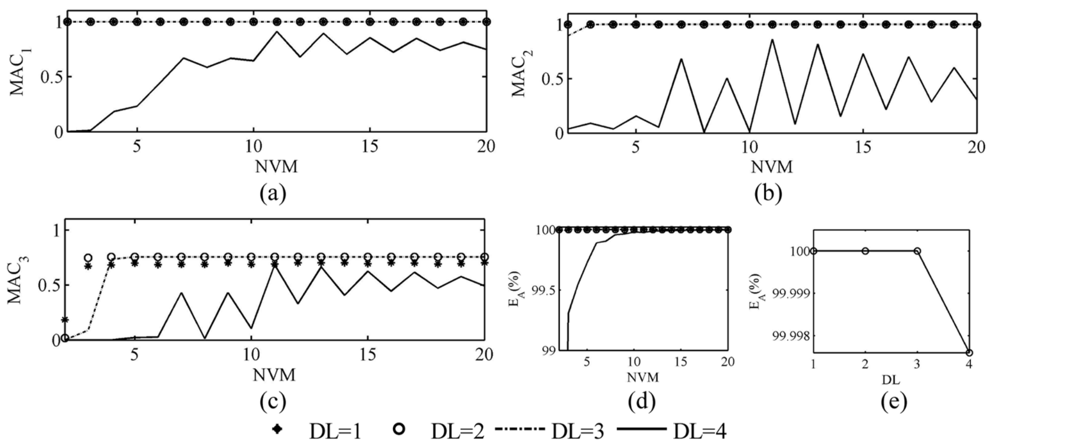

Fig. 2 Variation of MACs and E_A wrt NVM and DL. In (e) the trend of E_A is illustrated for sym20 wavelet

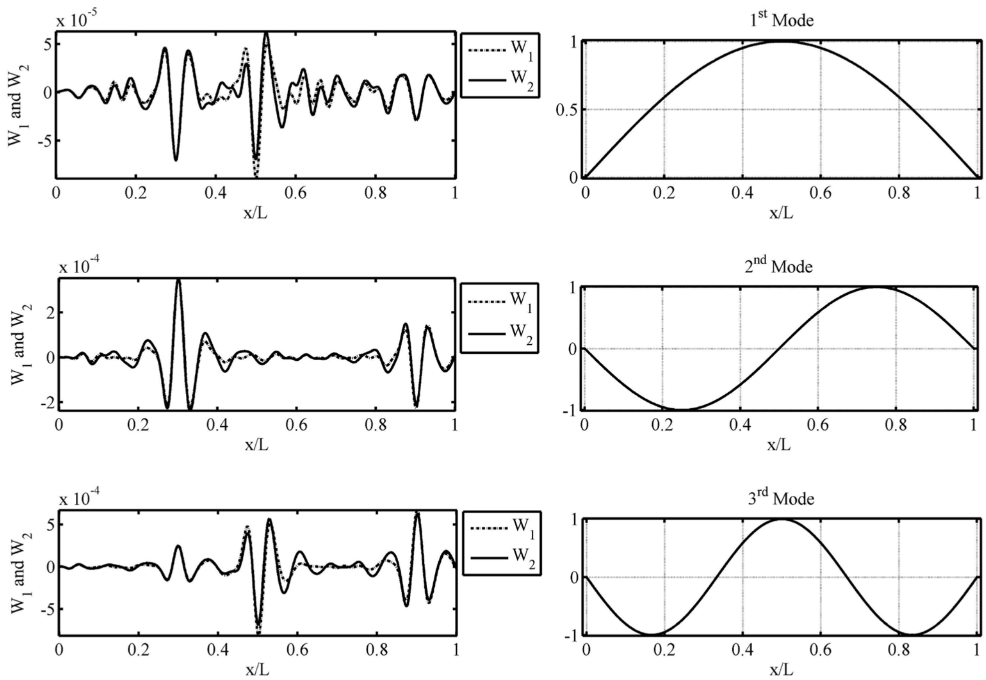

Fig. 3 Comparison of the damage indexes W_1 and W_2 using the first three modes of the SS beam

Thus, the most suitable AF should be searched at the first three levels, where MAC_1 is equal to 1 for all NVMs, and MAC_2 is equal to 1 for NVM > 3. Besides, MAC_3 has its greatest values at the second and third levels when NVM > 4. Figs. 2(d),(e) show that E_A drops significantly at the fourth DL. Hence, as previously stated, the preceding DL, i.e., the third level, is suitable if large NVMs are considered, because all three MACs are extremely large, implying that AF at this level is relatively close to UM. These trends of MACs and E_A were also observed for the other modes, and similar results were obtained. Therefore, using sym20 and regarding the third DL, the AFs for the first three modes were extracted. Later, the graphs in Fig. 3 were plotted, where comparison of the damage indexes given in Eq. (1) are illustrated. Douka *et al.* (2003) suggest that wavelets with high NVM provide more stable performance. Thus, sym10 is employed to compute W_1 and W_2, and the graphs are presented for the scale $a = 40$. From the figure, it is obvious that the three damage locations ($x/L = 0.3$, 0.5, and 0.9) can be determined by the first and third modes, while the second damage is undetectable with the second vibration mode, since it is at the nodal point of the mode. However, the closeness between W_1 and W_2, particularly at damage locations, are apparent at each mode. Thus, the present method can be concluded to be reliable and effective.

4. Conclusions

In this note, a new wavelet-based damage detection method that needs only damaged vibration modes has been proposed. The method is based on extracting a suitable AF by the DWT decomposition of DM, and employing it as reference data. It was shown that the AF extracted at suitable DL using wavelet with high NVM may be well-correlated with UM. Therefore, it can be employed as reference. Numerical applications illustrated that this approach produces reliable results.

Acknowledgement

This work was made during the project 107M371. TÜBİTAK is appreciated for valuable supports to the project.

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