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Evolutionary topology optimization of geometrically and materially nonlinear structures under prescribed design load

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Abstract. This paper presents topology optimization of geometrically and materially nonlinear structures using a bi-directional evolutionary optimization (BESO) method. To maximum the stiffness of nonlinear structures under prescribed design load, the complementary work is selected as the objective function of the optimization. An optimal design can be obtained by gradually removing inefficient material and adding efficient ones. The proposed method can be applied to a series of geometrically and/or materially nonlinear structures. The results show considerable differences in topologies and stiffness of the optimal designs for linear and nonlinear structures. It is found that the optimal designs for nonlinear structures are much stiffer than those for linear structures when large design loads (which result in significantly nonlinear deformations) are applied.

Keywords: bi-directional evolutionary structural optimization (BESO); topology optimization; complementary work.

1. Introduction

Topology optimization is to find the best distribution of material in the design domain and many methods have been developed in last several decades (Bendsøe and Kikuchi 1988, Bendsøe and Sigmund 2003, Xie and Steven 1993, 1997). Among them, the evolutionary structural optimization (ESO) method has been under continuous development since it was first proposed by Xie and Steven in the early 1990s. The basic concept is that by slowly removing inefficient materials, the structure may evolve towards an optimum. Bi-directional evolutionary structural optimization (BESO) is an extension of ESO which allows for materials to be added to the structure at the same time as the inefficient ones to be removed (Yang *et al*. 1999, Huang and Xie 2007).

Most works dealing with the optimization methods have been concerned with the optimization of structures with linear material and small deformation behaviour. However, the linear assumptions are not always valid for applications involving nonlinear material and large deformation. Using the sensitivity/gradient based optimization methods, a number of papers has considered topology

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optimization of geometrically nonlinear structures (Buhl *et al*. 2000, Bruns and Tortorelli 2003, Pedersen *et al*. 2001). Topology optimization of materially nonlinear structures has also been conducted by several researchers (Yuge and Kikuchi 1995, Bendsøe *et al*. 1996, Pedersen 1998, Maute *et al*. 1998). However, reports on topology optimization with both geometrical and material nonlinearities are still very limited.

In this paper, the topology optimization of geometrically and materially nonlinear structures will be studied using the BESO method (Huang and Xie 2007). The paper is organized as follows: Section 2 formulates the basic problem of optimization. In Section 3, sensitivity analysis has been conducted using an adjoint method proposed by Buhl *et al.* (2000) and sensitivity number which denotes the variation of objection function due to removing an element is derived. Section 4 explains the procedure for removing and adding material. Section 5 presents and discusses the numerical results for linear and nonlinear structures. Concluding remarks are made in Section 6.

2. Problem statement and nonlinear analysis

In many industrial applications, the maximum stiffness of a structure is pursued. Consider a nonlinear structure subjected to a applied load which increases monotonously with displacement up to a maximum, **F**, the corresponding nonlinear force-displacement curve is depicted in Fig. 1. To maximize the structural stiffness, the natural choice of the objective function is the minimization of the displacement, $\mathbf{U}^*$ or end compliance $\mathbf{F}^T\mathbf{U}^*$, in the deflected configuration (Buhl *et al.* 2000). However, minimization of the end-compliance may result in degenerated structures which can only support the maximum load they are designed for. To prevent this problem and make sure that the structure is stable for any load up to the maximum design load, one may minimize the complementary work. Thus, the optimization problem for maximizing stiffness can be formulated with the volume constraint using the element as the design variable

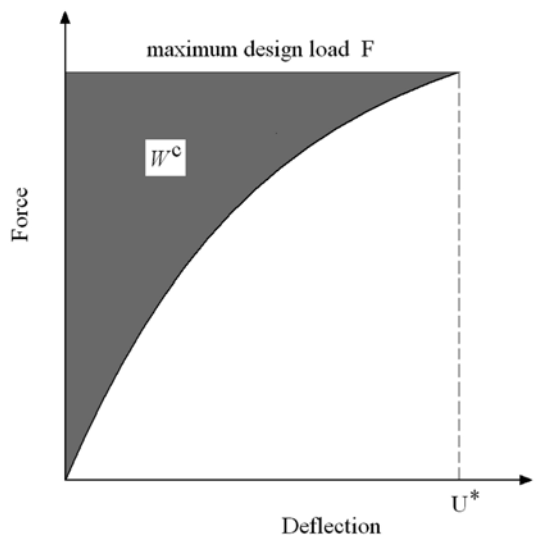


Fig. 1 Typical load-deflection curve in nonlinear finite element analysis with force control

$$\text{Minimize} \quad f(x) = W^C = \lim_{n\to\infty}\left[\frac{1}{2}\sum_{i=1}^{n}\Delta\mathbf{F}^T(\mathbf{U}_i+\mathbf{U}_{i-1})\right] \tag{1a}$$

$$\text{Subject to} \quad g = V^* - \sum_{j=1}^{M} V_j x_j = 0 \tag{1b}$$

$$x_j \in \{0, 1\} \tag{1c}$$

where $\mathbf{U}$ is the displacement vector. n is the number of increments in the load vector. The size of the increment is determined by $\Delta\mathbf{F} = \mathbf{F}/n$. V_j is the volume of an individual element and V^* the prescribed total structural volume. The binary design variable x_j declares the absence (0) or presence (1) of an element. M is the total element number in the system.

For nonlinear problems, the nonlinear equilibrium $\mathbf{R} = 0$ must be found using an iterative procedure whereas the equilibrium of a linear problem is found from the solution of a linear system of equations. The residual force, $\mathbf{R}$, is defined as the error between the internal force vector and external force vector as

$$\mathbf{R} = \mathbf{F} - \mathbf{F}^{int} = 0 \tag{2}$$

The internal force vector can be expressed by

$$\mathbf{F}^{int} = \sum^{M}\int_{e}\mathbf{C}^{e^T}\mathbf{B}\sigma dv = \sum^{M}\mathbf{C}^{e^T}\mathbf{F}^e \tag{3}$$

Where $\mathbf{C}^e$ is a matrix which transforms the nodal force vector of element to the globally nodal force vector and $\mathbf{F}^e$ the nodal force vector of element.

Normally, the equilibrium (2) is solved incrementally and iteratively using the Newton-Raphson method, which requires the determination of the tangent stiffness matrix in each step as

$$\mathbf{K}^t = -\frac{\partial\mathbf{R}}{\partial\mathbf{U}} \tag{4}$$

The details on nonlinear analysis may be consulted to nonlinear finite element analysis book, e.g., Crisfield (1991). The objective of this paper is to present optimization formulation and sensitivity numbers for nonlinear structures.

3. Sensitivity number

Consider a nonlinear structural system corresponding to design independent loads. If we assume the design variable continuously varies from 1 to 0, the sensitivity of the complementary work with respect to a change in design variable x is

$$\frac{df(x)}{dx} = \lim_{n\to\infty}\left[\frac{1}{2}\sum_{i=1}^{n}(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)\left(\frac{d\mathbf{U}_i}{dx} + \frac{d\mathbf{U}_{i-1}}{dx}\right)\right] \tag{5}$$

Following the work of Buhl *et al.* (2000), an adjoint equation is introduced by adding a series of vectors of Lagrangian multipliers λ_i into the objective function as

$$f(x) = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}[(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)(\mathbf{U}_i + \mathbf{U}_{i-1}) + \boldsymbol{\lambda}_i^T\mathbf{R}_i] \tag{6}$$

where $\mathbf{R}_i$ is the average residual force in ith incremental step approximately by

$$\mathbf{R}_i = \mathbf{F}_i + \mathbf{F}_{i-1} - \mathbf{F}_i^{int} - \mathbf{F}_{i-1}^{int} = 0 \tag{7}$$

Because $\mathbf{R}_i$ is equal to zero, the modified objective function (6) is same as the original objective function (1a). Thus, the sensitivity of the modified objective function is

$$\frac{df(x)}{dx} = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\left[(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T)\left(\frac{d\mathbf{U}_i}{dx} + \frac{d\mathbf{U}_{i-1}}{dx}\right) + \boldsymbol{\lambda}_i^T\left(\frac{\partial\mathbf{R}_i}{\partial\mathbf{U}_i}\frac{d\mathbf{U}_i}{dx} + \frac{\partial\mathbf{R}_i}{\partial\mathbf{U}_{i-1}}\frac{d\mathbf{U}_{i-1}}{dx} + \frac{\partial\mathbf{R}_i}{\partial x}\right)\right] \tag{8}$$

It is assumed a linear force-displacement relationship in a small increment, that is $\partial\mathbf{R}_i/\partial\mathbf{U}_i = \partial\mathbf{R}_i/\partial\mathbf{U}_{i-1} = -\mathbf{K}_i^t$. Substituting it into (8), the sensitivity of the modified objective function can be re-written as

$$\frac{df(x)}{dx} = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}\left[(\mathbf{F}_i^T - \mathbf{F}_{i-1}^T - \boldsymbol{\lambda}_i^T\mathbf{K}_i^t)\left(\frac{d\mathbf{U}_i}{dx} + \frac{d\mathbf{U}_{i-1}}{dx}\right) + \boldsymbol{\lambda}_i^T\left(\frac{\partial\mathbf{R}_i}{\partial x}\right)\right] \tag{9}$$

In order to eliminate the unknowns $d\mathbf{U}_i/dx + d\mathbf{U}_{i-1}/dx$, $\boldsymbol{\lambda}_i$ is chosen as

$$\mathbf{K}_i^t\boldsymbol{\lambda}_i = \mathbf{F}_i - \mathbf{F}_{i-1} \tag{10}$$

This equation defines the adjoint structures. From the linear assumption of force-displacement relationship in a small increment, the increment of the force can be approximately expressed by

$$\mathbf{K}_i^t(\mathbf{U}_i - \mathbf{U}_{i-1}) = \mathbf{F}_i - \mathbf{F}_{i-1} \tag{11}$$

Comparing Eq. (10) with Eq. (11), $\boldsymbol{\lambda}_i$ can be obtained with

$$\boldsymbol{\lambda}_i = \mathbf{U}_i - \mathbf{U}_{i-1} \tag{12}$$

Substituting $\boldsymbol{\lambda}_i$ into Eq. (9) and utilizing Eq. (7), the sensitivity of the objective function is expressed by

$$\frac{df(x)}{dx} = -\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{U}_i^T - \mathbf{U}_{i-1}^T)\left(\frac{\partial\mathbf{F}_i^{int}}{\partial x} + \frac{\partial\mathbf{F}_{i-1}^{int}}{\partial x}\right) \tag{13}$$

In the evolutionary structural optimization method, a structure can be optimized by removing and adding elements. That is to say that, the element itself, rather than its associated physical parameters, is treated as the design variable. Thus, when one element is totally removed from the system, the variation of the objective function is approximately

$$\Delta f(x) = -\lim_{n\to\infty}\left[\frac{1}{2}\sum_{i=1}^{n}(\mathbf{U}_i^T - \mathbf{U}_{i-1}^T)(\Delta\mathbf{F}_i^{int} + \Delta\mathbf{F}_{i-1}^{int})\right] \tag{14}$$

From Eq. (3), the variation of internal force is

$$\Delta\mathbf{F}_i^{int} = -\mathbf{C}^{e^T}\mathbf{F}_i^e \tag{15}$$

where the negative sign means removing elements, Substituting Eq. (15) into Eq. (14), the sensitivity number is calculated as

$$\alpha^e = \Delta f(x) = \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{U}_i^T - \mathbf{U}_{i-1}^T)(\mathbf{C}^{eT}\mathbf{F}_i^e + \mathbf{C}^{eT}\mathbf{F}_{i-1}^e) = \lim_{n\to\infty}\sum_{i=1}^{n}(E_i^e - E_{i-1}^e) = E_n^e \tag{16}$$

where E_n^e is the final elemental elastic and plastic strain energy. The above equation indicates that the complementary work increases with the elemental strain energy as the element is removed from the structure. Therefore, in linear cases, the above equation can also be stated that the external work or total strain energy increases with the elemental strain energy as the element is removed from the structure (Chu *et al.* 1996).

However, the sensitivity number for adding material cannot be obtained directly from the FEA results because the candidate elements for addition are not included in the analysis. Here, a mesh-independent Gaussian-weighted filter (Bruns and Tortorelli 2003, Murio 1993) is used to evaluate sensitivity numbers for candidate elements for addition, although other filters may be implemented, e.g., linearly weighted kernel (Bendsøe and Sigmund 2003, Sigmund and Peterson 1998) and checkerboard suppression filter (Li *et al.* 2001). Thus, the sensitivity number for element i is given by

$$\alpha_i = \frac{\sum_{j=1}^{N}\omega(r_{ij})\alpha_i}{\sum_{j=1}^{N}\omega(r_{ij})} \tag{17}$$

where N is the total number of elements in the mesh and $\omega(r_{ij})$ is the weight factor given as

$$\omega(r_{ij}) = \begin{cases} \dfrac{\exp\left(-\left(r_{ij}^2/2\left(\frac{r}{3}\right)^2\right)\right)}{2\pi(r/3)} & \text{if} \quad r_{ij} \le r \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

where r_{ij} is defined as the distance between the centers of the elements i and j and r is the filter radius specified by the user.

To enhance the convergence of the BESO method, it is proposed to further improve the accuracy of the current sensitivity numbers by considering the deformation history of each element. A simple way to achieve this is to average the current sensitivity number with that of the previous iteration (Huang and Xie 2007) as

$$\alpha_i = \frac{\alpha_i^n + \alpha_i^{n-1}}{2} \tag{19}$$

where n is the current iteration number. Then let $\alpha_i^n = \alpha_i$ which will be used for next iteration. Thus, the updated sensitivity number includes all sensitivity information in the previous iterations.

4. Bi-direction Evolutionary Structural Optimization (BESO) method

The detail procedure of the BESO method has been presented by Huang and Xie (2007). To minimize the complementary work of nonlinear structures, the evolutionary process will be conducted by removing elements with smallest sensitivity numbers and adding elements with highest ones. The volume of the structure may gradually decrease or increase until the objective volume is reached as

$$V_{i+1} = V_i(1 \pm ER) \qquad (i = 1, 2, 3, \ldots) \tag{20}$$

where ER is called the evolutionary volume ratio.

Then the elements are sorted according to their values of the sensitivity number (from the highest to the lowest). For solid element (1), it will be removed (switched to 0) if

$$\alpha_i \leq \alpha_{del}^{th} \tag{21a}$$

For void elements (0), it will be added (switched to 1) if

$$\alpha_i > \alpha_{add}^{th} \tag{21b}$$

where α_{del}^{th} and α_{add}^{th} are the threshold sensitivity numbers for removing and adding elements and $\alpha_{del}^{th} \leq \alpha_{add}^{th}$. α_{del}^{th} and α_{add}^{th} are determined by the following three simple steps:

1. Let $\alpha_{add}^{th} = \alpha_{del}^{th} = \alpha_{th}$, thus α_{th} can be easily determined by V_{i+1}. For example, if there are 1000 elements in design domain and $\alpha_1 > \alpha_2 \ldots > \alpha_{1000}$ and V_{i+1} corresponds to a design with 725 elements then $\alpha_{th} = \alpha_{725}$.
2. Calculate the admission volume ratio (AR), which is defined the number of added elements divided by the total number of elements in the design domain. If $AR \leq AR_{\max}$ where $AR_{\max}$ is a prescribed maximum volume addition ratio, skip step 3. Otherwise recalculate α_{del}^{th} and α_{add}^{th} as in step 3.
3. Calculate α_{add}^{th} by first sorting the sensitivity number of void elements (0). The number of elements to be switched from 0 to 1 will be equal to $AR_{\max}$ multiplied by the total number of elements in the design domain. α_{add}^{th} is the sensitivity number of the element ranked just below the last added element. α_{del}^{th} is then determined so that the removed volume is equal to ($V_{i+1} - V_i$ + the volume of the added elements).

It is noted that $AR_{\max}$ is introduced to ensure that not too many elements are added in a single iteration. Normally $AR_{\max}$ is greater than 1% so that it does not suppress the capability and advantages of adding elements.

Once the objective volume is reached, the volume of the structure keeps to be constant in thereafter iterations. Then, the following convergence criterion is applied to stop the whole optimization process

$$\frac{\left|\sum_{j=1}^{N}(W_{i-j+1}^{C} - W_{i-N-j+1}^{C})\right|}{\sum_{j=1}^{N} W_{i-j+1}^{C}} \leq \tau \tag{22}$$

where W_i^C is the complementary work for the structure in ith iteration, N is integral number and τ

is a allowable convergence error normally selected from 0.1% to 0.01%. N is selected to be 5 through this paper which means the there is no signification improvement in objective function (less than τ) over last 10 iterations.

5. Examples and discussion

5.1 Geometrically nonlinear structure

The first example considers the stiffness optimization design of a slender cantilever (Buhl *et al.* 2000) under a concentrated loading as shown in Fig. 2. The cantilever is fixed at one end and has length 1m, width 0.25 m and thickness 0.1 m, the force is applied downward. It is assumed that the available material can only cover 50% volume of the design domain, and material has Young's modulus 3GPa and Poisson's ratio 0.4. BESO parameters in the following examples are ER = 1%, AR_{max} = 1%, filter radius r = 0.02 m and τ = 1%.

If the optimization problems are solved using linear finite element analysis, the optimal topology would not depend on the magnitude of the load. The topology optimization using linear finite element analysis was first carried to find the linear design and the topology is symmetric as shown in Fig. 3(a). The nonlinear designs obtained from the proposed method using the geometrically nonlinear finite element analysis are shown in Figs. 3(b)-(c) for F = 60 kN and 144 kN respectively. It shows that the topologies using linear and nonlinear finite element analysis are different. Also, the topologies using nonlinear finite element analysis depends on the magnitude of the maximum design load.

In order to sort out which design is better, the complementary works are calculated using the nonlinear finite element analysis and compared in Table 1. It can be seen that designs using geometrically nonlinear finite element analysis are always stiffer than that using linear finite element analysis. However, these improvements without involving snap-through effects are marginal as discussed by Buhl *et al.* (2000). It is shown in Table 1 that compared with nonlinear designs using SIMP method (Buhl *et al.* 2000), the present designs have similar but lower complementary works, W^C. The topologies from BESO and SIMP methods are very similar too. The difference in the values of the complementary works can be attributed to the effect of grey areas (intermediate density elements) in the SIMP topologies where the strain energy of intermediate density elements depends on the assumed power-law relationship.

The numerical experiment revealed that the current BESO method could not find an optimal solution for the above nonlinear structure with a large applied force, such as 240kN. This was because the design process was interrupted by a convergence problem of nonlinear finite element

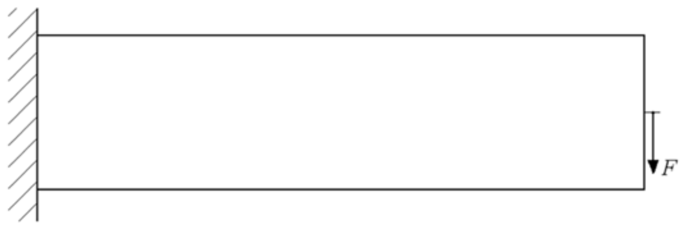


Fig. 2 Design domain and support conditions for a cantilever

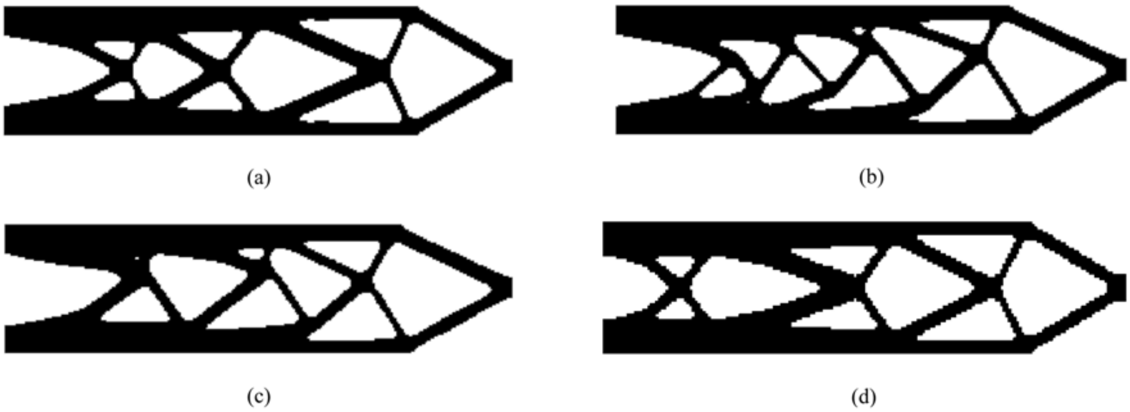


Fig. 3 Optimized topologies for the optimization problem sketched in Fig. 2 (a) linear optimal design, (b) Geometrically nonlinear design with F = 60 kN, (c) Geometrically nonlinear design with F = 144 kN. (d) Materially nonlinear design with F = 144 kN

Table 1 Comparison of complimentary work, W^C, between linear and various nonlinear designs

Maximum design load	W^C (kJ)	
	60kN	144kN
Linear design	2.183	12.53
Nonlinear designs	2.171	12.38
Nonlinear designs in (Buhl *et al.* 2000)	2.331	13.29

analysis because the applied force was beyond the load-carrying capability of an intermediate design.

5.2 Materially nonlinear structure

Consider the above cantilever under an applied load $F = 144$ kN. The material is assumed as an elastic, linear hardening plastic model with Young's modulus $E = 3$ GPa, Poisson's ratio $v = 0.4$, yield stress $\sigma_y = 100$ MPa, hardening modulus $E_p = 0.1E$. The nonlinear design obtained from the proposed method using the materially nonlinear finite element analysis is shown in Fig. 3(d) which is similar to the linear design. It indicates that the material nonlinearity has insignificant effect on the optimal topology in this case.

To show the effect of the material nonlinearity, we consider the design problem as sketched in Fig. 4. The dimension of the plate is defined as 2 m×2 m×0.01 m. The maximum design load is 20 kN. The structure is made with a frictional material such as soils or rock which exhibits pressure-dependent yield (the material becomes stronger as the pressure increases). Thus a linear Drucker-Prager elastic-perfectly-plastic model with friction angle $\beta = 40^{\circ}$ and dilation angle $\psi = 40^{\circ}$ was employed. The material has yield stress in uniaxial compression $\sigma_y = 40$ MPa, Young's modulus $E = 20$ GPa and Poisson's ratio $\nu = 0.3$. Suppose only 20% of design domain volume material is available for constructing the final structure. The used BESO parameters are

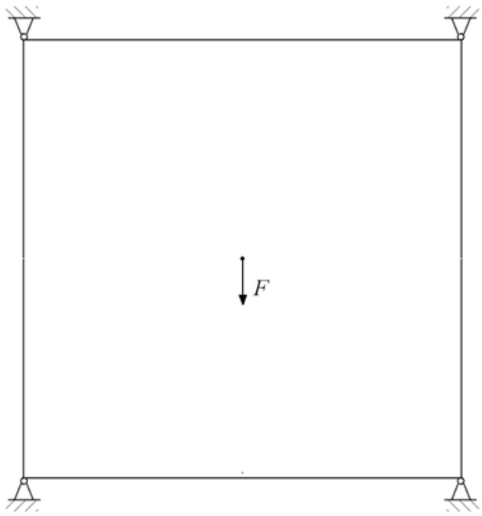


Fig. 4 Design domain and support conditions for a materially nonlinear structure

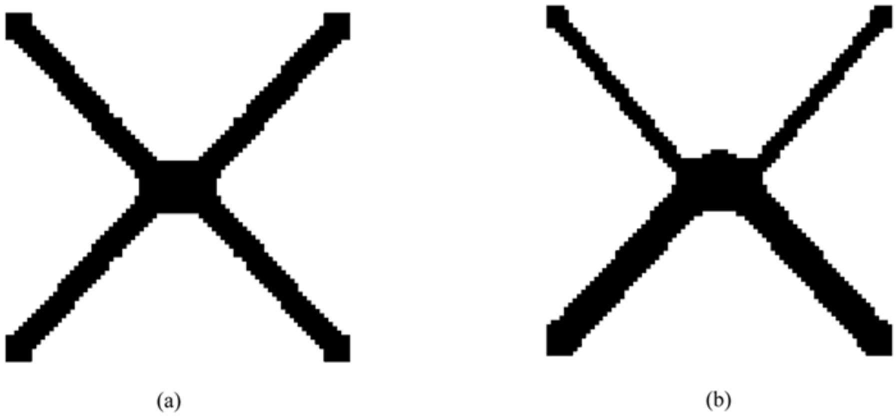


Fig. 5 Optimized topologies for the optimization problem sketched in Fig. 4 (a) linear optimal design, (b) materially nonlinear optimal design

$ER = 2\%$, $AR_{\max} = 2\%$, filter radius $r = 0.1$ m and $\tau = 0.1\%$.

To show difference between the linear design and elastoplastic design, the problem is solved first using linear finite element analysis and the resulted optimal design is shown in Fig. 5(a). Since the material features symmetrical tension and compression behaviours, the linear design shows symmetric tension and compression supports. If the problem is solved using nonlinear finite element analysis, the optimal design has different topology shown in Fig. 5(b). Comparing with the linear design, the nonlinear design has taken detailed account of material constitutive behaviour, and presents with compression-dominated structure.

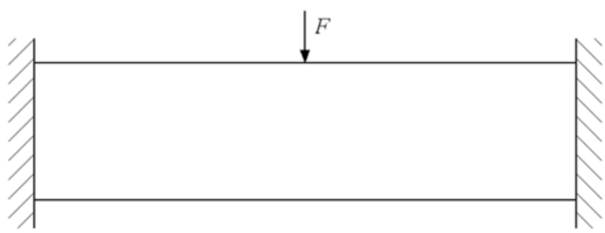
When the nonlinear finite element analysis is applied to both designs, the results shows the complementary work and final deflection are 11.38J and 0.94 mm for the linear design and 6.93J and 0.78 mm for the nonlinear design. It can be concluded that the nonlinear design is much stiffer than the linear design.

5.3 Geometrically and materially nonlinear structure

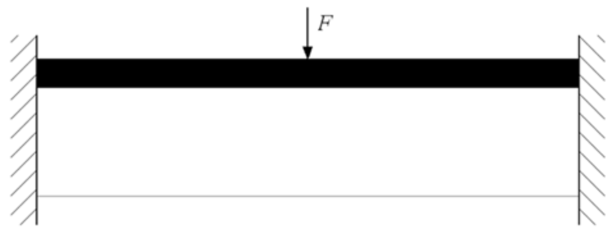
The proposed BESO method starts from the full design can apply directly to both geometrically and materially nonlinear structures and somewhat save the computation time. To further improve the computational efficiency, BESO may start from an initial guess design which is only with objective material volume. The optimal design is achieved by shifting the position of elements. Because only small portion of elements in the design domain is calculated in nonlinear finite element analyses, the computation time would be saved significantly.

A beam shown in Fig. 6(a) is clamped on both sides and a concentrated force, 10 kN, is acted downward at the top edge. It is 4 m long, 1m wide, and 0.01 m thick. The nonlinear material is approximated with the well-known Ramberg-Osgood plasticity model with Young's modulus $E = 500$ MPa, Poisson's ratio $v = 0.3$, the yield stress $\sigma_y = 1$ MPa, the yield offset $\alpha = 0.002$ and the hardening exponent $n = 3$. The objective volume is only 20% of design domain. The initial guess design is shown in Fig. 6(b) and BESO parameters are $ER = 0$, $AR_{\max} = 2\%$, $r = 0.05$ m and $\tau = 0.01\%$. The convergence criterion used here is very strict, ensuring that the BESO algorithm really has converged to an (at least) local optimum.

The optimal design using the linear finite element analysis is shown in Fig. 7(a) and design using the nonlinear finite element analysis is given in Fig. 7(b). Once again, these two designs exhibit disparities to a great extent. Using nonlinear finite element analysis, the corresponding complementary work and deflection are 633.6J, 0.11m for the linear design and 456.9J, 0.09m for the nonlinear design respectively. It also shows that the nonlinear design is much stiffer than the linear design. Fig. 8 shows the evolutionary histories of the complementary work and structural topology while the material volume keeps constant. After 230 iterations, the solution is stably



(a)



(b)

Fig. 6 Design domain and initial guess design (a) design domain, (b) initial guess design

(a)

(b)

Fig. 7 Optimized topologies for the optimization problem sketched in Fig. 6 (a) linear optimal design, (b) geometrically and materially nonlinear optimal design

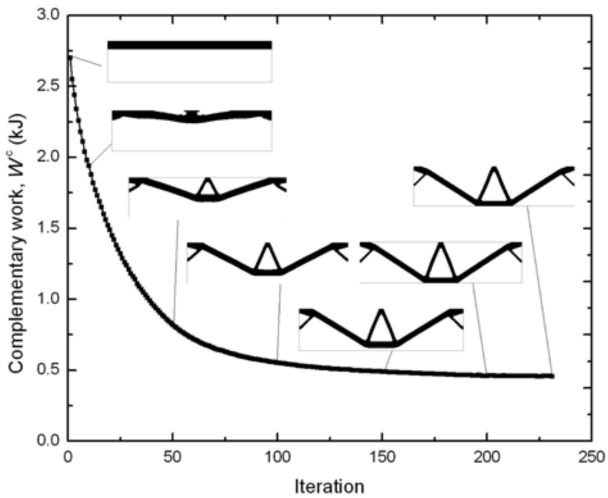


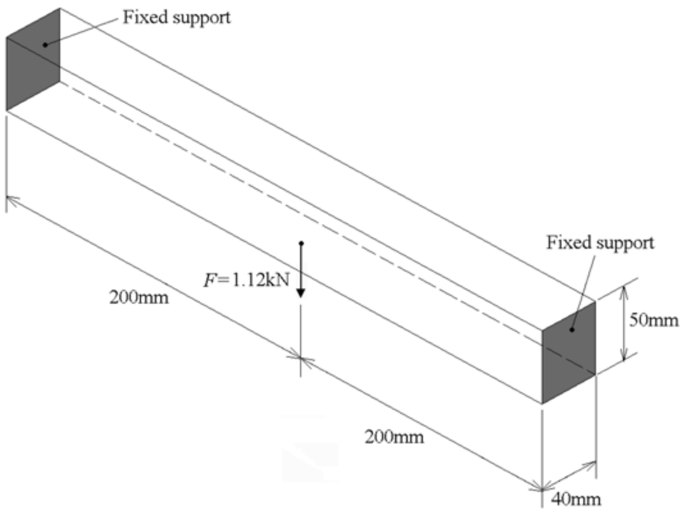
Fig. 8 Evolutionary histories of complementary work and topology for the optimization problem using nonlinear finite element analysis

convergent to an optimal solution. However, designs with the complementary work just a few percent above the "optimal" one may be obtained using a large τ such as 1% in approximately 130 iterations. It should be noted that the material nonlinearity has less effect on the final design in this case. In other words, the difference between the linear design and the nonlinear design mainly comes from the geometrical nonlinearity.

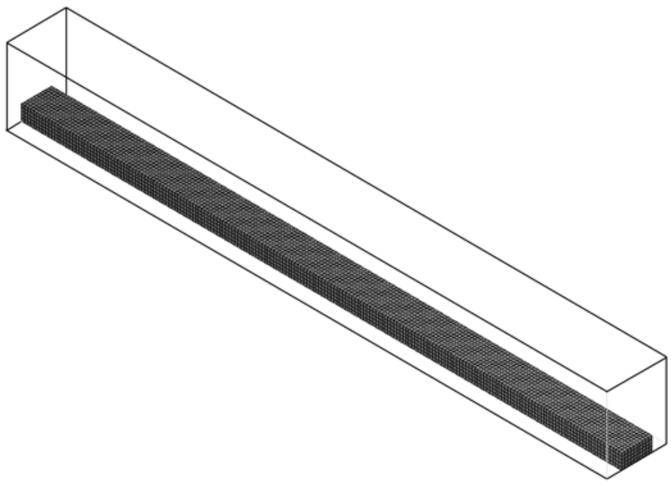
5.4 Three-dimensional structure

The above BESO method is easily extended to the topology optimization problem for a 3D structure. As an example, Fig. 9(a) shows the maximum design domain, load and supports conditions. The material is assumed as a elastic, linear hardening plastic model with Young's modulus $E = 1$ GPa, Poisson's ratio $v = 0.3$, yield stress $\sigma_y = 10$ MPa, hardening modulus $E_p = 0.3E$. The design domain is meshed with 100,000 hexahedral elements. However, the objective volume is only 5% of design domain. The initial guess design is shown in Fig. 9(b) with only 5,000 hexahedral elements. BESO parameters are $ER = 0$, $AR_{\max} = 2\%$, $r = 4$ mm and $\tau = 0.01\%$.

Fig. 10(a) shows the optimal design using linear finite element analysis and Fig. 10(b) is the optimal design using geometrically and materially nonlinear finite element analysis. There is a significant difference between two topologies. Using geometrically and materially nonlinear finite element analysis, a force-displacement diagram for linear and nonlinear designs is compared in



(a)



(b)

Fig. 9 Design domain and initial guess design for a 3D structure (a) design domain, (b) initial guess design

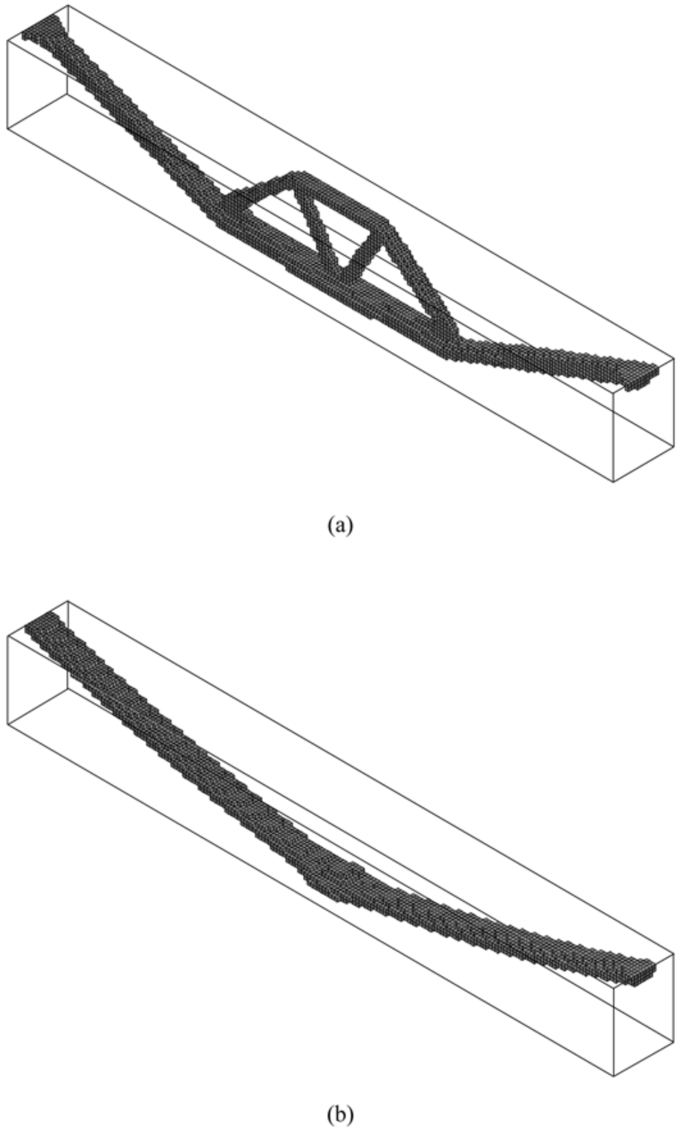


Fig. 10 Optimized topologies for the optimization problem sketched in Fig. 9 (a) optimal design using linear finite element analysis, (b) optimal design using geometrically and materially nonlinear finite element analysis

Fig. 11. Notice that the topology optimized using nonlinear finite element analysis is stiffer than that using linear finite element analysis at the maximum design load. However, it has larger deflection for smaller loads. It attributes to the fact that the present optimization procedure considers the overall performance of the design at loads up to the maximum design load. The complementary works at the maximum design load are 6.7J and 5.1J for linear and nonlinear designs respectively. So we also conclude that the nonlinear design is stiffer than the linear design.

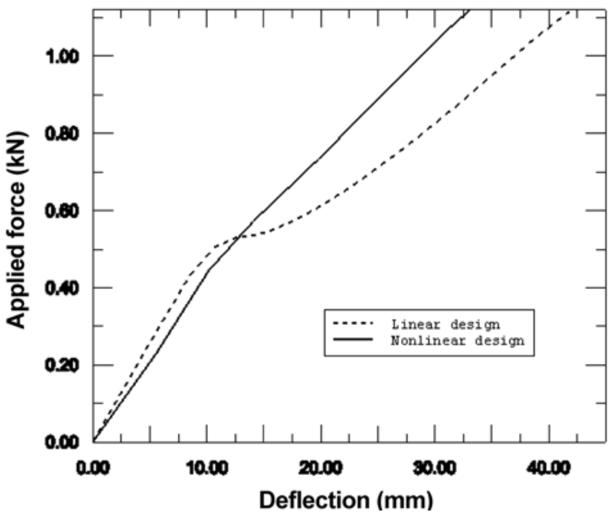


Fig. 11 Comparison of the force-displacement relationships between the optimal design using linear and nonlinear finite element analysis

6. Conclusions

Topology optimization of nonlinear structures under prescribed loading conditions using the BESO method has been proposed. To minimize the complementary work of nonlinear structure, sensitivity numbers are determined by elemental strain energies at the final equilibrium. The optimal design is obtained by gradually removing elements with the lowest sensitivity numbers and adding elements with highest sensitivity numbers. This procedure has been applied to a number of design problems involving geometrical and/or material nonlinearities. According to comparison, it is found that the optimal designs for nonlinear structures are stiffer than those for linear structures, especially when large design loads are applied.

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