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Free transverse vibrations of an elastically connected simply supported twin pipe system

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Abstract. In this paper, free vibration analyses of a parallel placed twin pipe system simulated by simply supported-simply supported and fixed-fixed Euler-Bernoulli beams resting on Winkler elastic soil are presented. The motion of the system is described by a homogenous set of two partial differential equations, which is solved by a simulation method called the Differential Transform Method (DTM). Free vibrations of an elastically connected twin pipe system are realized by synchronous and asynchronous deflections. The results of the presented theoretical analyses for simply supported Euler-Bernoulli beams are compared with existing ones in open literature and very good agreement is demonstrated.

Keywords: Differential Transform Method; DTM; elastic soil; vibration; twin beam; twin pipeline.

1. Introduction

Beam-type structures are widely used in many branches of modern aerospace, mechanical and civil engineering. In civil engineering, there are numerous studies dealing with problems related to soil-structure interaction; such as railroad tracks, highway pavements, strip foundations and continuously supported pipelines, in which the structure is modeled by means of a beam on an elastic foundation.

There are different types of beam models. One of the well known models is the Euler-Bernoulli beam theory that works well for slender beams. According to the Euler-Bernoulli beam theory, the length of each beam section is much greater than the height of each section and the shear and rotary inertia effects are ignored.

Besides beam models, there are also various types of foundation models such as Winkler, Pasternak, Vlasov, etc. A well known and widely used mechanical model is the one devised by Winkler. According to the Winkler model, the beam-supporting soil is modeled as a series of closely spaced, mutually independent, linear elastic vertical springs which provide resistance in direct proportion to the deflection of the beam. In the Winkler model, the properties of the soil are described only by the parameter k, which represents the stiffness of the vertical springs (Winkler 1867, Avramidis and Morfidis 2006).

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Free vibrations of single beams on continuous elastic foundations have been analyzed by a number of investigators. However, there are only few contributions dealing with the vibrations of double-beam systems. An elastically connected double-beam system is a typical model of a complex continuous system of two one-dimensional solids attached together by a Winkler elastic soil. But, the general vibration analyses of an elastically connected double-beam system are complicated and laborious in view of a large variety of possible combinations of boundary conditions, and thus, the solution of the governing coupled partial differential equations is difficult (Abu-Hilal 2006, Oniszczuk 2000, 2002, 2003).

Seelig and Hoppman's studies appear to be the first on the vibration of elastically connected double beam systems (Seelig and Hoppmann 1964a, b). Then, different aspects of dynamics of an elastically connected double-beam system have been treated by many authors: Kessel (1966), Saito and Chonan (1969), Kessel and Raske (1971), Hamada *et al.* (1983), Kukla and Skalmierski (1994), Kukla (1994), Oniszczuk (1999), Vu *et al.* (2000), Oniszczuk (2000, 2003), Erol and Gurgoze (2004) and Abu-Hilal (2006).

In this paper, the free vibration analyses of a parallel placed twin pipe system are studied. The system consists of two identical, parallel placed, elastic, homogeneous, isotropic Euler-Bernoulli beams connected continuously by Winkler elastic soil. In the first part of the analyses, the beams are supposed to be simply supported; and in the second part they are supposed to be fixed at both ends. The motion of the system is described by a homogenous set of two partial differential equations, which are solved by a simulation method called the Differential Transform Method (DTM); and the natural frequencies of the system are determined. In general, an elastically connected simply supported double-beam system executes two fundamental kinds of vibrations, synchronous and asynchronous (Oniszczuk 2000, 2002). The presented theoretical analyses for simply supported Euler-Bernoulli beams are compared with a numerical example solved by Oniszczuk (2000), and very good agreement has been achieved.

The Differential Transform Method (DTM) used in the analyses is a semi analytical-numerical technique based on the Taylor series expansion method for solving differential equations. It is different from the traditional high order Taylor series method. The Taylor series method computationally takes long time for large orders. However, with DTM, doing some simple mathematical operations on differential equations, a closed form series solution or an approximate solution can be obtained quickly. This method was first proposed by Zhou in 1986 for solving both linear and nonlinear initial-value problems of electrical circuits (Zhou 1986). Later, Chen and Ho developed this method for partial differential equations (Chen and Ho 1999) and Ayaz studied two and three dimensional differential transform method of solution of the initial value problem for partial differential equations (Ayaz 2003, 2004). Arikoglu and Ozkol extended the differential transform method to solve the integro-differential equations (Arikoglu and Ozkol 2005). Catal used DTM for free vibration analyses of both ends simply supported beam resting on elastic foundation (Catal 2006). Recently, the second author used the DTM method successfully to handle various kinds of rotating beam problems (Kaya 2006, Ozdemir and Kaya 2006a, b).

2. The equations of motion and boundary conditions

In this study, a vibrating twin pipe system which is represented by two parallel, slender, prismatic and homogenous Euler-Bernoulli beams resting on a Winkler elastic soil is investigated. In the

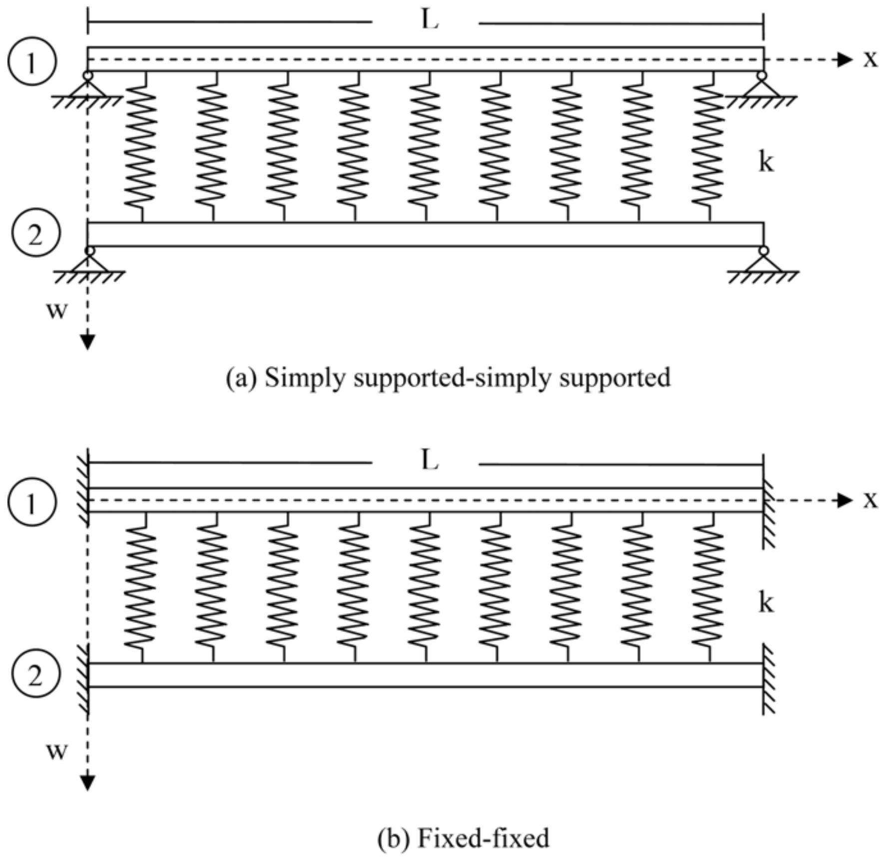

Fig. 1 Physical models of elastically connected twin pipe systems

analyses, it is assumed that both beams have the same length; and two kinds of boundary conditions are studied: simply supported (Fig. 1(a)) and fixed at both ends (Fig. 1(b)).

Free transverse vibrations of the system under consideration are described by the following differential Eq. (1)

$$\begin{aligned} &K_1\frac{\partial^4 w_1}{\partial x^4}+m_1\frac{\partial^2 w_1}{\partial t^2}+k(w_1-w_2) = 0 \\ &K_2\frac{\partial^4 w_2}{\partial x^4}+m_2\frac{\partial^2 w_2}{\partial t^2}+k(w_2-w_1) = 0 \\ &K_i = E_iI_i, \quad m_i = \rho_iA_i, \quad i = 1,2 \end{aligned} \tag{1}$$

where $w_i = w_i(x, t)$ is the transverse beam deflection; x, t are the spatial co-ordinate and the time; E_i is the Young modulus of elasticity; A_i is the cross-sectional area of the beam; I_i is the moment of inertia of the beam cross-section; K_i is the flexural rigidity of the beam; k is the stiffness modulus of a Winkler elastic layer; L is the length of the beam, and ρ_i is the mass density.

The associated boundary conditions considered in this paper are given as follows:
Simply supported-simply supported

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{at} \quad x = 0, L \tag{2}$$

Fixed-fixed

$$w = \frac{\partial w}{\partial x} = 0 \quad \text{at} \quad x = 0, L \tag{3}$$

In order to make free vibration analyses of the Euler-Bernoulli beam on the Winkler foundation, let us assume the solution is in the form of a sinusoidal variation of $w = w(x, t)$ with circular frequency ω

$$w_1(x, t) = W_1(x)e^{i\omega t}, \quad w_2(x, t) = W_2(x)e^{i\omega t} \tag{4}$$

Substituting Eq. (4) into Eq. (1), equations of motion are expressed as follows

$$\begin{aligned} K_1\frac{d^4 W_1}{dx^4} - m_1\omega^2 W_1 + k(W_1 - W_2) = 0 \\ K_1\frac{d^4 W_2}{dx^4} - m_2\omega^2 W_2 + k(W_2 - W_1) = 0 \end{aligned} \tag{5}$$

3. Nondimensionalization

The non-dimensional parameters are defined as

$$\xi = \frac{x}{L}, \quad \overline{W} = \frac{W}{L} \tag{6}$$

Using these parameters, the nondimensional form of Eq. (5) can be written as

$$\begin{aligned} \frac{d^4\overline{W}}{d\xi^4} - A_1\omega^2\overline{W}_1 + B_1(\overline{W}_1 - \overline{W}_2) = 0 \\ \frac{d^4\overline{W}}{d\xi^4} - A_2\omega^2\overline{W}_2 + B_2(\overline{W}_2 - \overline{W}_1) = 0 \end{aligned} \tag{7}$$

where, $A_1 = \frac{m_1 L^4}{K_1}$, $A_2 = \frac{m_2 L^4}{K_2}$, $B_1 = \frac{kL^4}{K_1}$ and $B_2 = \frac{kL^4}{K_2}$, respectively.

Nondimensional boundary conditions are as follows:
Simply supported-simply supported

$$\overline{W} = \frac{d^2\overline{W}}{d\xi^2} = 0 \quad \text{at} \quad \xi = 0, 1 \tag{8}$$

Fixed-fixed

$$\overline{W} = \frac{d\overline{W}}{d\xi} = 0 \quad \text{at} \quad \xi = 0, 1 \tag{9}$$

4. Differential transform method

The differential transform method (DTM) is a transformation technique based on the Taylor series expansion and is a useful tool to obtain analytical solutions of the differential equations. In this method, certain transformation rules are applied and the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions. The solution of these algebraic equations gives the desired solution of the problem. It is different from high-order Taylor series method because Taylor series method requires symbolic computation, and is laborious for large orders.

Consider a function $f(x)$ which is analytic in a domain D and let $x = x_0$ represent any point in D. The function $f(x)$ is then represented by a power series whose center is located at x_0. The differential transform of the function $f(x)$ is given by

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{10}$$

where $f(x)$ is the original function and $F(k)$ is the transformed function.

The inverse transformation is defined as

$$f(x) = \sum_{k=0}^{\infty} (x-x_0)^k F(k) \tag{11}$$

Combining Eqs. (10) and (11) gives

$$f(x) = \sum_{k=0}^{\infty} \frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{12}$$

Considering Eq. (12), once more it is noticed that the concept of differential transform is derived from Taylor series expansion. However, the method does not evaluate the derivatives symbolically.

In actual applications, the function $f(x)$ is expressed by a finite series and Eq. (12) can be written as follows

$$f(x) = \sum_{k=0}^{m} \frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{13}$$

which means that $f(x) = \sum_{k=m+1}^{\infty} \frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0}$ is negligibly small. Here, the value of m depends on the convergence rate of the natural frequencies.

Theorems that are frequently used in the transformation of the differential equations and the boundary conditions are introduced in Table 1 and Table 2, respectively.

Table 1 DTM theorems used for equations of motion

Original function	Transformed function
$f(x) = g(x) \pm h(x)$	$F(k) = G(k) \pm H(k)$
$f(x) = \lambda g(x)$	$F(k) = \lambda G(k)$
$f(x) = \dfrac{d^n g(x)}{dx^n}$	$F(k) = \dfrac{(k+n)!}{k!} G(k+n)$

Table 2 DTM theorems used for boundary conditions

$x = 0$		$x = 1$	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$f(0) = 0$	$F(0) = 0$	$f(1) = 0$	$\sum_{k=0}^{\infty} F(k) = 0$
$\dfrac{df}{dx}(0) = 0$	$F(1) = 0$	$\dfrac{df}{dx}(1) = 0$	$\sum_{k=0}^{\infty} kF(k) = 0$
$\dfrac{d^2f}{dx^2}(0) = 0$	$F(2) = 0$	$\dfrac{d^2f}{dx^2}(1) = 0$	$\sum_{k=0}^{\infty} k(k-1)F(k) = 0$
$\dfrac{d^3f}{dx^3}(0) = 0$	$F(3) = 0$	$\dfrac{d^3f}{dx^3}(1) = 0$	$\sum_{k=0}^{\infty} k(k-1)(k-2)F(k) = 0$

5. DTM formulation and solution procedure

In order to derive DTM form of Eq. (7), we will quit using the bar symbol on $\overline{W}$ and instead, W will be employed. If Table 1 is referred the following expression can be written easily.

$$
\begin{aligned}
&(k+1)(k+2)(k+3)(k+4)W_1(k+4) - A_1\omega^2 W_1(k) + B_1[W_1(k) - W_2(k)] = 0 \\
&(k+1)(k+2)(k+3)(k+4)W_2(k+4) - A_2\omega^2 W_2(k) + B_2[W_2(k) - W_1(k)] = 0
\end{aligned}
\tag{14}
$$

If Eq. (14) is arranged, a simple recurrence relation can be obtained as follows

$$
\begin{aligned}
W_1(k+4) &= \frac{(A_1\omega^2 - B_1)W_1(k) + B_1 W_2(k)}{(k+1)(k+2)(k+3)(k+4)} \\
W_2(k+4) &= \frac{(A_2\omega^2 - B_2)W_2(k) + B_2 W_1(k)}{(k+1)(k+2)(k+3)(k+4)}
\end{aligned}
\tag{15}
$$

The boundary conditions can be written from Table 2 as follows:
Simply supported-simply supported

$$W_1(0) = W_1(2) = 0, \quad W_2(0) = W_2(2) = 0$$

$$\sum_{k=0}^{\infty} W_1(k) = 0, \quad \sum_{k=0}^{\infty} W_2(k) = 0 \tag{16}$$

$$\sum_{k=0}^{\infty} k(k-1)W_1(k) = 0, \quad \sum_{k=0}^{\infty} k(k-1)W_2(k) = 0 \tag{17}$$

Fixed-fixed

$$W_1(0) = W_1(1) = 0, \quad W_2(0) = W_2(1) = 0$$

$$\sum_{k=0}^{\infty} W_1(k) = 0, \quad \sum_{k=0}^{\infty} W_2(k) = 0$$

$$\sum_{k=0}^{\infty} kW_1(k) = 0, \quad \sum_{k=0}^{\infty} kW_2(k) = 0 \tag{18}$$

The solution procedure of DTM will be shown for simply-supported-simply supported conditions and the values are set to $A_1 = 0.25$, $B_1 = 250$, $A_2 = 0.25$, $B_2 = 500$.

$$W_1(0) = 0, \quad W_1(2) = 0 \tag{19}$$

$$W_2(0) = 0, \quad W_2(2) = 0 \tag{20}$$

Eq. (19) and Eq. (20) represent the left end boundary condition. The $W_1(1)$, $W_1(3)$, $W_2(1)$ and $W_2(3)$ values are set as unknowns such as

$$\begin{aligned} W_1(1) &= c_1 \\ W_2(1) &= c_2 \\ W_1(3) &= c_3 \\ W_2(3) &= c_4 \end{aligned} \tag{21}$$

$W_1(k)$ and $W_2(k)$ values for $k = 4, 5, \ldots$ can now be evaluated in terms of constants c_1, c_2, c_3 and c_4, respectively. Substituting all $W_1(i)$ and $W_2(i)$ terms into boundary condition expressions, the following equation is obtained.

$$A_{j1}^{(n)}(\omega)c_1 + A_{j2}^{(n)}(\omega)c_2 + A_{j3}^{(n)}(\omega)c_3 + A_{j4}^{(n)}(\omega)c_4 = 0, \quad j = 1, 2, 3, \ldots, n \tag{22}$$

where $A_{j1}^{(n)}(\omega), A_{j2}^{(n)}(\omega), A_{j3}^{(n)}(\omega), A_{j4}^{(n)}(\omega)$ are polynomials of ω corresponding to n^{th} term.

When Eq. (22) is written in matrix form, we get

$$\begin{bmatrix} A_{11}^{n}(\omega) & A_{12}^{n}(\omega) & A_{13}^{n}(\omega) & A_{14}^{n}(\omega) \\ A_{21}^{n}(\omega) & A_{22}^{n}(\omega) & A_{23}^{n}(\omega) & A_{24}^{n}(\omega) \\ A_{31}^{n}(\omega) & A_{32}^{n}(\omega) & A_{33}^{n}(\omega) & A_{34}^{n}(\omega) \\ A_{41}^{n}(\omega) & A_{42}^{n}(\omega) & A_{43}^{n}(\omega) & A_{44}^{n}(\omega) \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

The eigenvalue equation is obtained from Eq. (23) as follows

$$\begin{bmatrix} A_{11}^n(\omega) & A_{12}^n(\omega) & A_{13}^n(\omega) & A_{14}^n(\omega) \\ A_{21}^n(\omega) & A_{22}^n(\omega) & A_{23}^n(\omega) & A_{24}^n(\omega) \\ A_{31}^n(\omega) & A_{32}^n(\omega) & A_{33}^n(\omega) & A_{34}^n(\omega) \\ A_{41}^n(\omega) & A_{42}^n(\omega) & A_{43}^n(\omega) & A_{44}^n(\omega) \end{bmatrix} = 0 \tag{24}$$

Solving Eq. (24), we get $\omega = \omega_j^{(n)}$ where $j = 1, 2, 3, \ldots, n$. Here, $\omega_j^{(n)}$ is the j^{th} estimated eigenvalue corresponding to n. The value of n is obtained by the following equation

$$\left|\omega_j^{(n)} - \omega_j^{(n-1)}\right| \le \varepsilon \tag{25}$$

where ε is the tolerance parameter.

If Eq. (25) is satisfied, then we have j^{th} eigenvalue $\omega_j^{(n)}$. In general, $\omega_j^{(n)}$ are conjugated complex values, and can be written as $\omega_j^{(n)} = a_j + ib_j$. Neglecting the small imaginary part b_j, we have the have j^{th} natural frequency. In this study the value of $n = 50$ was enough.

6. Numerical examples

The computer package Mathematica is used to solve recurrence relations with associated boundary conditions. The free transverse vibrations of two simply supported-simply supported and fixed-fixed beams are considered to establish the effect of physical parameters characterizing the vibrating system on the natural frequencies. Results of the simply supported double-beam system are compared with Oniszczuk's study which is solved by using the classical Bernoulli-Fourier method (Oniszczuk 2000). The following values of the parameters are used in the numerical calculations:

$E = 1 \times 10^{10}$ N.m^{-2}, $A = 5 \times 10^{-2}$m^2, $I = 4 \times 10^{-4}$m^4

$K = EI = 4 \times 10^6$N.m^{-2}, $k = (0-5) \times 10^5$ N.m^{-2}, $L = 10$m

$c = 0.5; 1.0; 2.0$, $m = \rho.A = 1 \times 10^2$kg.m^{-1}, $\rho = 2 \times 10^3$kg.m^{-3}

The problem is solved for one variant of the system which is denoted as:

$K_1 = K,\ K_2 = cK,\ m_1 = m,\ m_2 = cm$

where c is a positive constant parameter.

The calculations of ω_{in} are carried out for three values of a parameters c ($c = 0.5$; 1.0; 2.0) as a function of stiffness modulus k, which is changed in a certain interval $k = (0–5) \times 10^5$ N.m^2. The results of the calculations for simply supported beams at both ends are compared with Oniszczuk (2000); and very good agreement is observed. The results for the simply supported-simply supported beams are presented in Tables 3-5 and in Fig. 2; and the results for fixed-fixed beams are given in Tables 6-8 and in Fig. 3. Following Oniszczuk (2000), the natural frequencies ω_{in} in the Tables are additionally denoted by subscripts 1, 2, 3, to distinguish the corresponding frequencies computed for $c = 0.5$; 1; 2 respectively. If the natural frequency is independent of constant c, then this subindex is not applied.

According to the results of the analyses, it can be seen that the synchronous natural frequencies

Table 3 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 0.5$, (Simply supported)

Method	$k \times 10^{-5}$	n	1	2	3	4	5
DTM	0	ω_{in}	19.739	78.957	177.653	315.827	493.478
Bernoulli-Fourier			19.7	79.0	177.7	315.8	493.5
DTM	1	ω_{2n}	58.221	96.095	185.905	320.542	496.509
Bernoulli-Fourier			58.2	96.1	185.9	320.5	496.5
DTM	2	ω_{2n}	79.935	110.608	193.805	325.187	499.521
Bernoulli-Fourier			79.9	110.6	193.8	325.2	499.5
DTM	3	ω_{2n}	96.900	123.427	201.396	329.768	502.515
Bernoulli-Fourier			96.9	123.4	201.4	329.8	502.5
DTM	4	ω_{2n}	111.309	135.034	208.712	334.286	505.491
Bernoulli-Fourier			111.3	135.0	208.7	334.3	505.5
DTM	5	ω_{2n}	124.055	145.720	215.779	338.743	508.450
Bernoulli-Fourier			124.1	145.7	215.8	338.7	508.5

Table 4 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 1.0$, (Simply supported)

Method	$k \times 10^{-5}$	n	1	2	3	4	5
DTM	0	ω_{in}	19.739	78.957	177.653	315.827	493.478
Bernoulli-Fourier			19.7	79.0	177.7	315.8	493.5
DTM	1	ω_{2n}	48.884	90.742	183.195	318.978	495.500
Bernoulli-Fourier			48.9	90.7	183.2	319.0	495.5
DTM	2	ω_{2n}	66.254	101.164	188.575	322.098	497.515
Bernoulli-Fourier			66.3	101.2	188.6	322.1	497.5
DTM	3	ω_{2n}	79.935	110.608	193.805	325.187	499.521
Bernoulli-Fourier			79.9	110.6	193.8	325.2	499.5
DTM	4	ω_{2n}	91.595	119.307	198.898	328.248	501.519
Bernoulli-Fourier			91.6	119.3	198.9	328.2	501.5
DTM	5	ω_{2n}	101.930	127.413	203.864	331.281	503.509
Bernoulli-Fourier			101.9	127.4	203.9	331.3	503.5

Table 5 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 2.0$, (Simply supported)

Method	$k \times 10^{-5}$	n	1	2	3	4	5
DTM	0	ω_{in}	19.739	78.957	177.653	315.827	493.478
Bernoulli-Fourier			19.7	79.0	177.7	315.8	493.5
DTM	1	ω_{2n}	43.470	87.944	181.826	318.193	494.996
Bernoulli-Fourier			43.5	87.9	181.8	318.2	495.0
DTM	2	ω_{2n}	58.220	96.095	185.905	320.542	496.509
Bernoulli-Fourier			58.2	96.1	185.9	320.5	496.5
DTM	3	ω_{2n}	69.926	103.606	189.896	322.873	498.017
Bernoulli-Fourier			69.9	103.6	189.9	322.9	498.0
DTM	4	ω_{2n}	79.935	110.608	193.805	325.187	499.521
Bernoulli-Fourier			79.9	110.6	193.8	325.2	499.5
DTM	5	ω_{2n}	88.824	117.193	197.637	327.486	501.020
Bernoulli-Fourier			88.8	117.2	197.6	327.5	501.0

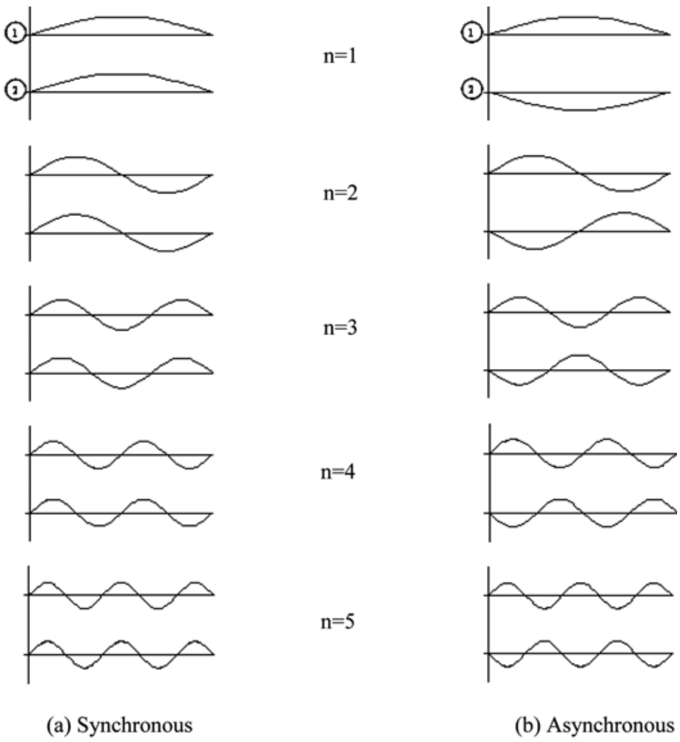

Fig. 2 General mode shapes of vibration of an elastically connected simply supported-simply supported double beam complex system corresponding to the first five pairs of the natural frequencies

Table 6 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 0.5$, (Fixed-Fixed, (DTM))

$k \times 10^{-5}$	n	1	2	3	4	5
0	ω_{in}	44.747	123.346	241.807	399.71	597.00
1	ω_{2n}	70.727	134.960	247.932	403.454	599.509
2	ω_{2n}	89.455	145.651	253.910	407.155	602.006
3	ω_{2n}	104.891	155.609	259.751	410.823	604.492
4	ω_{2n}	118.331	164.967	265.463	414.458	606.969
5	ω_{2n}	130.393	173.822	271.054	418.061	609.435

Table 7 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 1.0$, (Fixed-Fixed, (DTM))

$k \times 10^{-5}$	n	1	2	3	4	5
0	ω_{in}	44.747	123.346	241.807	399.719	597.002
1	ω_{2n}	63.263	131.203	245.908	402.213	598.674
2	ω_{2n}	77.474	138.615	249.941	404.691	600.342
3	ω_{2n}	89.455	145.651	253.910	407.155	602.006
4	ω_{2n}	100.011	152.362	257.819	409.604	603.665
5	ω_{2n}	109.555	158.790	261.669	412.038	605.319

Table 8 Natural frequencies of double-beam system ω_{in} (s^{-1}); $c = 2.0$, (Fixed-Fixed, (DTM))

$k \times 10^{-5}$	n	1	2	3	4	5
0	ω_{in}	44.747	123.346	241.807	399.719	597.002
1	ω_{2n}	59.180	129.283	244.889	401.591	598.257
2	ω_{2n}	70.727	134.960	247.932	403.454	599.509
3	ω_{2n}	80.637	140.407	250.939	405.309	600.759
4	ω_{2n}	89.455	145.651	253.910	407.155	602.006
5	ω_{2n}	97.480	150.712	256.847	408.993	603.250

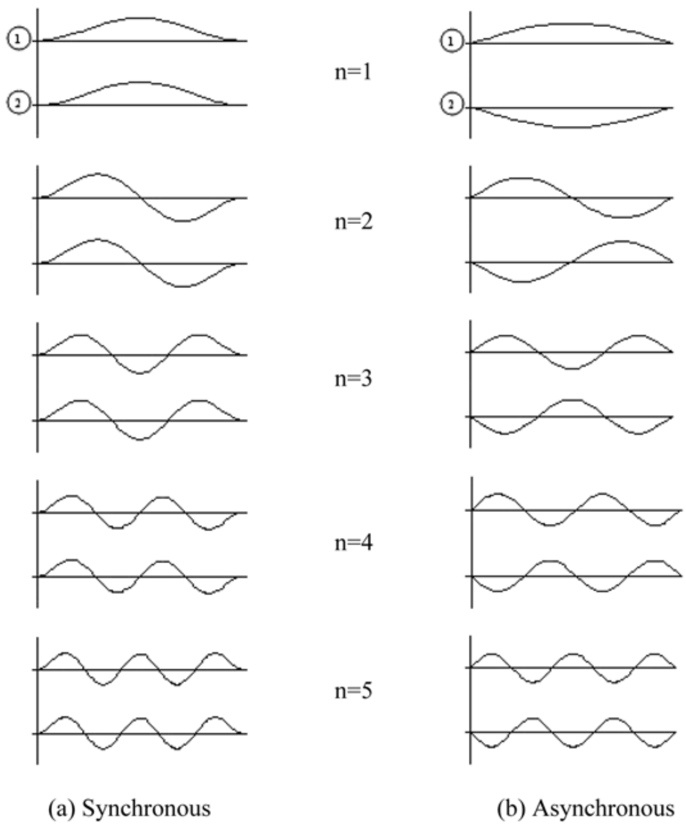

Fig. 3 General mode shapes of vibration of an elastically connected fixed-fixed double beam complex system corresponding to the first five pairs of the natural frequencies

ω_{1n} are not dependent on the stiffness modulus k unlike ω_{2n}; and there is a general tendency to increase the natural frequencies ω_{in} in the case of increasing the layer stiffness modulus k. The results also show that, the simultaneous proportional variation of flexural rigidity and mass of the second beam implies that the synchronous quantity ω_{1n} is not dependent on an assumed constant c and layer stiffness modulus k unlike the asynchronous quantity ω_{2n}. Their values diminish when parameter c grows.

7. Conclusions

In this study, the free transverse vibration analyses of elastically connected simply supported-simply supported and fixed-fixed double-beam complex systems are studied by a new and semi-analytical technique called the Differential Transform Method (DTM) in a simple and accurate way. The essential steps of the DTM application includes transforming the governing equations of motion into algebraic equations, solving the transformed equations and then applying a process of inverse transformation to obtain any j^{th} natural frequency. All steps are very straightforward, and the application of DTM to both the governing equations of motion and the boundary conditions are very easy. The simplicity of the solutions of the algebraic equations are remarkable because equations can be solved very quickly using the symbolic computational software, Mathematica. In this study, using DTM, the natural frequencies of the double beam complex systems are calculated, and the related graphics are plotted. The calculated results of simply supported-simply supported beam analyses are compared with Oniszczuk (2000), in which the differential equations of motion are formulated by classical Bernoulli-Fourier method. When the comparisons are made with the studies in the literature, a very good agreement is observed. Thus, it can be seen that, the solutions obtained for a double beam system can be helpful in the investigations of more complicated multi-beam systems.

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