

**Technical Note**

# Curvature effect on guided wave propagation in curved steel plates and hollow steel cylinders

Juwon Lee<sup>†</sup>, Won-Bae Na<sup>‡</sup> and Jeong-Tae Kim<sup>‡†</sup>*Department of Ocean Engineering, Pukyong National University, Busan 608-737, Korea**(Received December 18, 2008, Accepted January 7, 2010)***Keywords:** curvature effect; plate waves; curved steel plate; guided waves; dispersion curves.

## 1. Introduction

We examine the curvature effect on guided wave propagation in curved steel plates and hollow steel cylinders. To investigate the curvature effect, the guided wave characteristics of curved plates and hollow cylinders with radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1 were studied, respectively. For the investigation, firstly, continuity of the displacements and stresses was not assumed around the circumference of curved plates, but was assumed for hollow cylinders; hence, their formulations are different. Secondly, the guided wave characteristics (phase and group velocities) and their dispersion curves were obtained using the software package DISPERSE. This package utilizes the global matrix method, which is one of the most powerful methods for studying wave propagation in multi-layered structures such as flat plates, curved plates, and cylinders. Finally, from the dispersion curves, the detailed phase and group velocities at certain frequencies were obtained for each case and their curvature effects were examined.

This work sought to complete previously undertaken studies (Wilcox 1998). Other relevant studies were performed by Metsaveer and Klauson (1996) and Towfighi *et al.* (2002, 2003). However, the previous studies have not fully quantified the curvature effect nor detailed the curvature effects of curved steel plates and hollow steel cylinders in a vacuum. Therefore, we investigated the curvature effect of not only curved steel plates but also hollow steel cylinders in vacuum. The results, which included the dispersion curves of phase, group velocities, and detailed quantitative values at certain frequencies, provided insight into the curvature effect by showing how curvature affects phase and group velocities, especially for fundamental wave modes. Because fundamental wave modes are the most widely used modes in guided wave-based nondestructive evaluation, the results can be useful for the inspection of curved steel plates and pressure vessels.

<sup>†</sup> Graduate Student, E-mail: [i0m0free@pknu.ac.kr](mailto:i0m0free@pknu.ac.kr)<sup>‡</sup> Associate Professor, Corresponding author, E-mail: [wna@pknu.ac.kr](mailto:wna@pknu.ac.kr)<sup>‡†</sup> Professor, E-mail: [idis@pknu.ac.kr](mailto:idis@pknu.ac.kr)

## 2. Curvature effects

To represent dispersion curves in 2-D, the horizontal (frequency) and vertical (phase velocity or group velocity) axes can be dimensionless. To obtain a dimensionless frequency, we divided the frequency-thickness by the secondary bulk wave velocity (3.26 km/s for steel). To obtain the dimensionless phase or group velocity, we divided the phase or group velocity by the secondary bulk wave velocity. Consequently, the shape of the dimensionless phase velocity dispersion curves for a flat plate in a vacuum is dependent only on Poisson's ratio of the plate material (Freedman 1990). In addition to the secondary bulk wave velocity, the primary bulk wave velocity (5.96 km/s) and mass density ( $7932 \text{ k/m}^3$ ) of steel are required to calculate the phase and group velocity dispersion curves. By analyzing the guided wave characteristics (phase and group velocities) for radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1, the following results can be summarized. (1) By comparing the dimensionless dispersion curves of a steel plate in a vacuum with those of a steel plate in water, the dimensionless phase and group velocity dispersion curves were almost identical except that a Scholte wave was observed for the steel plate immersed in water. (2) For curved steel plates when a flat steel plate is used a reference, as the curvature increased, the curvature effect on the phase and group velocity dispersion curves also increased. (3) For hollow steel cylinders, the curvature effect on the phase and group velocities did not cause a significant difference except for the fundamental wave modes,  $L_1$  and  $F_1$ . As the curvature increased, the fundamental modes moved to the right. The second longitudinal wave mode,  $L_2$ , was similar to the first symmetric mode  $S_0$  of the flat steel plate.

To pinpoint the detailed changes of dimensionless phase and group velocities of the two fundamental modes, i.e., the first symmetric mode ( $S_0$ ) or the first longitudinal mode ( $L_1$ ), and the first anti-symmetric mode ( $A_0$ ) or the first flexural mode ( $F_1$ ), two specific dimensionless frequencies 0.31 and 1.0 were selected, corresponding to 1.0 and 3.26 MHz, respectively. At these frequencies, the differences (%) of the phase and group velocity were calculated with respect to the reference values of the flat steel plate.

Fig. 1 shows the differences of the phase velocity when the curved steel plates were considered. At a dimensionless frequency of 1.0, the phase velocity of  $S_0$  increased as the curvature increased (or radius-to-thickness ratio decreased), while the phase velocity of  $A_0$  decreased as the curvature increased. The same trend occurred when the dimensionless frequency was 0.31, although there was a difference in the magnitude. As shown in Fig. 2, the group velocities of  $S_0$  increased as the curvature increased, while the group velocities of  $A_0$  decreased as the curvature increased at a

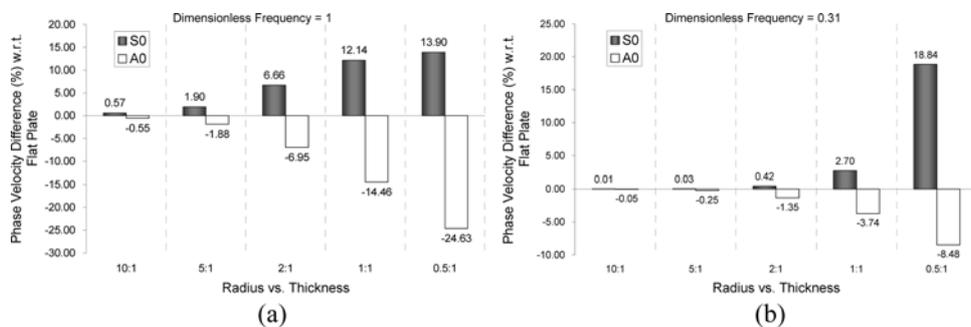


Fig. 1 Phase velocity differences (%) of circumference guided waves propagating in a steel plate with radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1. The dimensionless frequencies are (a) 1.0 and (b) 0.31, and the reference is based on the phase velocity of the flat plate

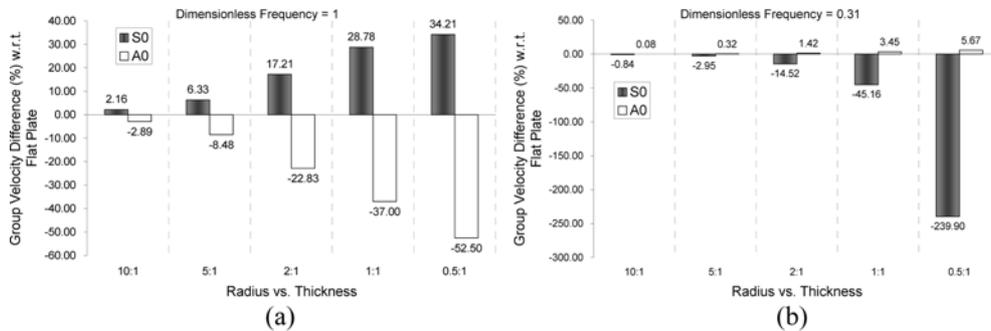


Fig. 2 Group velocity differences (%) of circumference guided waves propagating in a steel plate with radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1. The dimensionless frequencies are (a) 1.0 and (b) 0.31, and the reference is based on the phase velocity of the flat plate

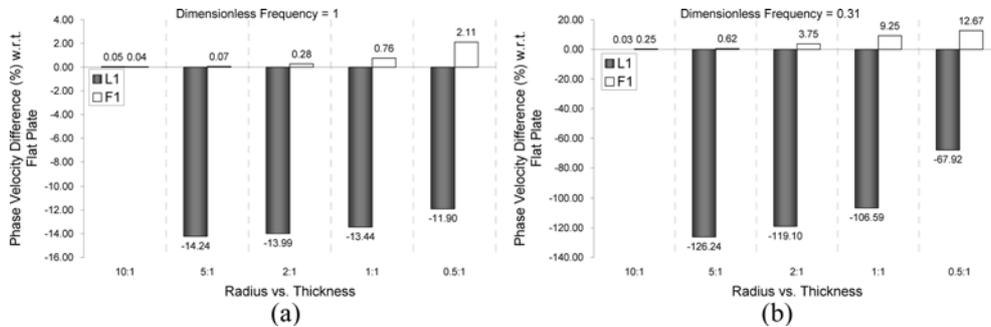


Fig. 3 Phase velocity differences (%) of cylindrical guided waves propagating in a steel plate with radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1. The dimensionless frequencies are (a) 1.0 and (b) 0.31, and the reference is based on the phase velocity of the flat plate

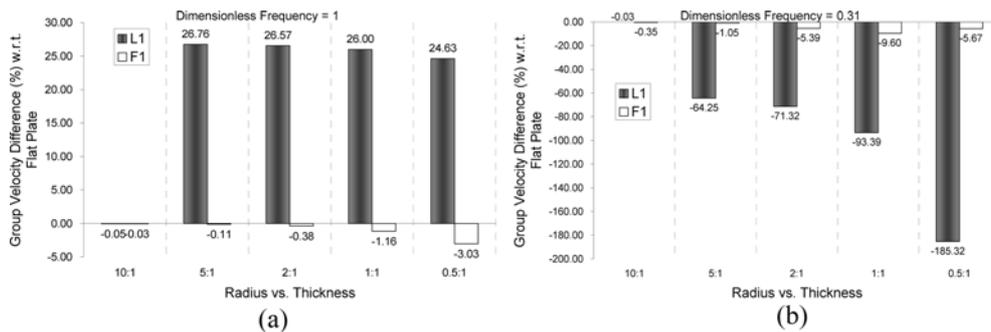


Fig. 4 Group velocity differences (%) of cylindrical guided waves propagating in a steel plate with radius-to-thickness ratios of 10:1, 5:1, 2:1, 1:1, and 0.5:1. The dimensionless frequencies are (a) 1.0 and (b) 0.31, and the reference is based on the phase velocity of the flat plate

dimensionless frequency of 1.0. However, the opposite trend was observed at a dimensionless frequency of 0.31. The group velocity differences of  $A_0$  at a dimensionless frequency of 0.31 were not as significant as the curvature increased.

Figs. 3 and 4 compare the differences in the phase and group velocities, respectively, between hollow steel cylinders and the reference values of the flat steel plate. In Fig. 3, at dimensionless

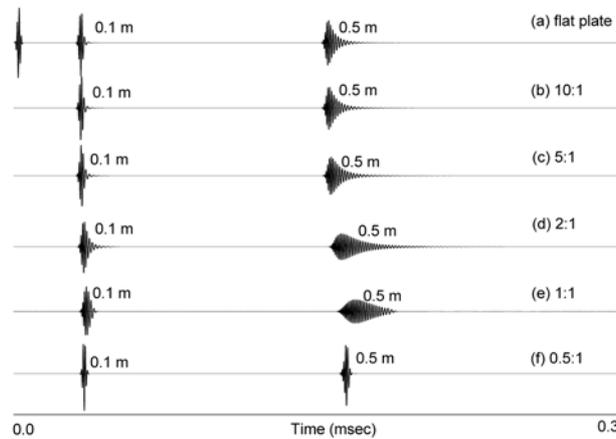


Fig. 5 (a)  $A_0$  plate mode and (b)-(f)  $F_1$  cylindrical guided wave modes of (b) 10:1, (c) 5:1, (d) 2:1, (e) 1:1, and (f) 0.5:1 curved hollow cylinders at 0.1 and 0.5 m from the excitation point. The dimensionless frequency is 0.31

frequency values of 1.0 and 0.31, the phase velocities of  $L_1$  and  $F_1$  increased as the curvature increased. The group velocities (Fig. 4) of the  $L_1$  and  $F_1$  modes decreased as the curvature increased. However, when the radius-to-thickness ratio was 0.5:1, the group velocity of the  $F_1$  mode at a dimensionless frequency of 0.31 was greater than when the radius-to-thickness ratio was 1:1. This observation can be also simulated by DISPERSE as shown in Fig. 5, which shows the curvature effect on the dispersion of  $A_0$  plate mode and  $F_1$  cylindrical wave mode. In the figure, to determine the extent of the dispersion behavior, we characterized the fundamental wave modes at 0.1 and 0.5 m from the excitation point, which correspond to a Gaussian wave with 5 cycles. As shown in Fig. 5(f), only one exception was observed for cylindrical guided wave propagation when the radius-to-thickness ratio was 0.5:1. We believe this phenomenon occurs because the characteristics of cylindrical guided waves are different from those of plate waves, especially when the radius (0.5) is smaller than the thickness (1).

## References

- Freedman, A. (1990a), "The variation with the Poisson ratio of Lamb modes in a free plate. 1. General spectra", *J. Sound Vib.*, **137**(2), 209-230.
- Freedman, A. (1990b), "The variation with the Poisson ratio of Lamb modes in a free plate. 2. At transitions and coincidence values", *J. Sound Vib.*, **137**(2), 231-247.
- Freedman, A. (1990c), "The variation with the Poisson ratio of Lamb modes in a free plate. 3. Behavior of individual modes", *J. Sound Vib.*, **137**(2), 249-266.
- Metsaveer, J. and Klauson, A. (1996), "Influence of the curvature of dispersion curves of a submerged cylindrical shells", *J. Acoust. Soc. Am.*, **100**(3), 1551-1560.
- Towfighi, S., Kundu, T. and Ehsani, M. (2002), "Elastic wave propagation in circumferential direction in anisotropic cylindrical curved plates", *J. Appl. Mech.*, **69**(3), 283-291.
- Towfighi, S. and Kundu, T. (2003), "Elastic wave propagation in anisotropic spherical curved plates", *Int. J. Solids Struct.*, **40**, 5495-5510.
- Wilcox, P.D. (1998), *Lamb Wave Inspection of Large Structures using Permanently Attached Transducers*, Ph.D. Thesis, Imperial College, U.K.