Structural Engineering and Mechanics, Vol. 34, No. 3 (2010) 285-300
DOI: http://dx.doi.org/10.12989/sem.2010.34.3.285

Thermomechanical deformation in porous generalized thermoelastic body with variable material properties

Rajneesh Kumar†

Department of Mathematics, Kurukshetra University, Kurukshetra-136119, India

Savita Devi‡

Department of Mathematics, D.N. College, Hisar-125001, India

(Received June 19, 2008, Accepted September 10, 2009)

Abstract. The two-dimensional deformation of a homogeneous, isotropic thermoelastic half-space with voids with variable modulus of elasticity and thermal conductivity subjected to thermomechanical boundary conditions has been investigated. The formulation is applied to the coupled theory(CT) as well as generalized theories: Lord and Shulman theory with one relaxation time(LS), Green and Lindsay theory with two relaxation times(GL) Chandrasekharaiah and Tzou theory with dual phase lag(C-T) of thermoelasticity. The Laplace and Fourier transforms techniques are used to solve the problem. As an application, concentrated/uniformly distributed mechanical or thermal sources have been considered to illustrate the utility of the approach. The integral transforms have been inverted by using a numerical inversion technique to obtain the components of displacement, stress, changes in volume fraction field and temperature distribution in the physical domain. The effect of dependence of modulus of elasticity on the components of stress, changes in volume fraction field and temperature distribution are illustrated graphically for a specific model. Different special cases are also deduced.

Keywords: thermoelasticity; generalized thermoelasticity; modulus of elasticity; thermal conductivity; thermal relaxation parameters; concentrated/uniformly distributed source; integral transforms.

1. Introduction

The generalized theories of thermoelasticity have been developed to overcome the physically unrealistic prediction of the coupled dynamical theory (Biot 1956) of thermoelasticity that thermal signal propagates with infinite speed. Lord and Shulman (1967)(LS) theory and Green and Lindsay (1972) temperature rate dependent (GL) theory of thermoelasticity are two well established theories of thermoelasticity, which introduce the thermal relaxation parameters in the basic equations of the coupled dynamical thermoelasticity theory and admit the finite value of heat propagation speed. The finiteness of speed of thermal signal has been found to have experimental evidence too. The generalized thermoelasticity theories are therefore, more realistic and have aroused much interst in

† Ph.D., Corresponding author, E-mail: rajneesh_kuk@rediffmail.com
‡ E-mail: savita.007@rediffmail.com

recent research. The dual-phase-lag of thermoelasticity is proposed by Chandrasekhariah (1998), Tzou (1995) (C-T model), in which the Fourier law is replaced by an approximation to a modification of the Fourier law with two different translations for the heat flux and the temperature gradient.

Each of these models has been introduced in the literature in an attempt to eliminate shortcomings of the classical dynamical coupled thermoelasticity such as: (1) infinite velocity of thermoelastic disturbances, (2) unsatisfactory thermoelastic response of a solid to short laser pulses, and (3) poor description of thermoelastic behavior at low temperatures.

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical use for investigating various types of geological and biological materials for which elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume is included among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A non-linear theory of elastic materials with voids was developed by Nunziato and Cowin (1979). Later, Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Considerable amount of work has been done in the linear theory of elastic materials with voids (Rusu 1987, Saccomandi 1992, Scarpetta 1995, Marin 1997a, b, Sharma 2001).

Cowin (1985) also extended the theory to show that the linear elastic material with voids behave like viscoelastic material.

Iesan (1986) developed the theory of the thermoelastic material with voids. Recently Kumar and Ailawalia (2009), Kumar and Rani 2005a, b, Kumar *et al.* (2009) investigated various problems in the linear theory of thermoelastic and micropolar themoelastic materials with voids and also study the effects of stiffness on reflection and transmission of micropolar thermoelastic waves at the interface between an elastic and micropolar generalized thermoelastic solid.

A comprehensive work has been done in theory of thermoelasticity with dependence of modulus of elasticity on reference temperature (Ezzat *et al.* 2001, Othman 2002, Ezzat *et al.* 2004, Youssef 2005a, b, Othman and Song 2008).

In the present investigation the equations of thermoelasticity with void, with the dependence of modulus of elasticity and thermal conductivity on the reference temperature are used to obtain the components of displacement, stress, change in volume fraction field and temperature distribution due to thermomechanical sources.

2. Basic equations

Following Cowin and Nunziato (1983) and El-Karamany (2004), the field equations and constitutive relations in thermoelastic body with voids without body forces, heat sources and extrinsic equilibrated body force can be written as

$$(\lambda+2\mu)\nabla(\nabla.\vec{u})-\mu(\nabla\times\nabla\times\vec{u})+b\nabla\phi-\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

$$K\left(n^* + t_1 \frac{\partial}{\partial t}\right)\nabla^2 T - \beta T_0\left(n_1 \frac{\partial}{\partial t} + n_0\tau_0 \frac{\partial^2}{\partial t^2} + t_2^2 \frac{\partial^3}{\partial t^3}\right)\nabla.\vec{u}$$

$$- mT_0\left(n_1 \frac{\partial}{\partial t} + n_0\tau_0 \frac{\partial^2}{\partial t^2} + t_2^2 \frac{\partial^3}{\partial t^3}\right)\phi = \rho C_e\left(n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} + t_2^2 \frac{\partial^3}{\partial t^3}\right)T \tag{2}$$

$$\alpha\nabla^2\phi - b\nabla.\vec{u} - \xi_1\phi - \omega_0 \frac{\partial\phi}{\partial t} + m\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = \rho\psi \frac{\partial^2\phi}{\partial t^2} \tag{3}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + b\phi\delta_{ij} - \beta\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T\delta_{ij} \tag{4}$$

where λ, μ - Lame's constant, $\alpha, b, \xi_1, m, \omega_0, \psi$ - material constants due to presence of voids, T - the temperature distribution, $\vec{u}$ -displacement vector, ρ, C_e - density and specific heat respectively, K - thermal conductivity, ϕ - change in volume fraction field, T_0 -uniform temperature distribution, t_{ij} - components of stress tensor, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial y}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, τ_0, τ_1 - the relaxation times, δ_{ij} - Kronecker delta, n^*, n_0, n_1, t_1, t_2 - constants $\beta = (3\lambda + 2\mu)\alpha_t$, α_t - coefficient of linear thermal expansion.

3. Formulation and solution of the problem

We consider a homogeneous, isotropic, generalized thermoelastic half space with voids in the undeformed temperature T_0. The rectangular cartesian co-ordinate system (x, y, z) having origin on the surface $z = 0$ with z-axis pointing normally in to the medium is introduced.

For two dimensional problems, we assume displacement vector $\vec{u}$ as

$$\vec{u} = (u, 0, w) \tag{5}$$

Our aim is to investigate the effect of temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume

$$\begin{aligned} &\lambda = \lambda_0(1-\alpha^*T_0),\ \ \mu = \mu_0(1-\alpha^*T_0),\ \ \beta = \beta_0(1-\alpha^*T_0),\ \ \xi_1 = \xi_{10}(1-\alpha^*T_0) \\ &m = m_0(1-\alpha^*T_0),\ \ b = b_0(1-\alpha^*T_0),\ \ \alpha = \alpha_0(1-\alpha^*T_0),\ \ \psi = \psi_0(1-\alpha^*T_0) \\ &K = K_0(1-\alpha^*T_0),\ \ \omega_0 = \omega_{10}(1-\alpha^*T_0) \end{aligned} \tag{6}$$

where $\lambda_0, \mu_0, \beta_0, \xi_{10}, m_0, b_0, \alpha_0, \psi_0, \omega_{10}$ are considered constants, α^* is called empirical material constant, incase of the reference temperature independent of modulus of elasticity, then $\alpha^* = 0$.

To facilitate the solution, following dimensionless quantities are introduced

$$x' = \frac{\omega_1^* x}{c_2},\ z' = \frac{\omega_1^* z}{c_2},\ u' = \frac{\omega_1^* u}{c_2},\ w' = \frac{\omega_1^* w}{c_2},\ t'_{33} = \frac{t_{33}}{\mu_o},\ t'_{31} = \frac{t_{31}}{\mu_o},\ \phi' = \frac{\omega_1^{*2}\psi_o}{c_2^{\,2}}\phi$$

$$t' = \omega_1^* t,\ \tau'_0 = \omega_1^*\tau_0,\ \tau'_1 = \omega_1^*\tau_1,\ T' = \frac{T}{T_0},\ P'_1 = \frac{P_1}{\mu_o},\ P'_2 = \frac{P_2}{\mu_0},\ P'_3 = \frac{P_3}{T_0} \tag{7}$$

where

$$c_2 = \left(\frac{\mu_0}{\rho}\right)^{1/2} \text{ and } \omega_1^* = \frac{c_2^2}{\kappa}$$

Using the expression relating displacement components $u(x,z,t)$ and $w(x,z,t)$ to the scalar potential functions $\psi_1(x,z,t)$ and $\psi_2(x,z,t)$ in dimensionless form

$$u = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, \quad w = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x} \tag{8}$$

Applying the Laplace and Fourier transforms defined by

$$\hat{f}(x,z,s) = \int_0^\infty f(x,z,t)e^{-st}dt$$

$$\tilde{f}(\xi,z,s) = \int_{-\infty}^\infty \hat{f}(x,z,s)e^{i\xi x}dx \tag{9}$$

on Eqs. (1)-(4) and with the help of Eqs. (5)-(9), eliminating $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}$ and $\tilde{\psi}_2$ and after some simplification (dimensionless form after avoiding the use of symbol primes) we obtain

$$\left(\frac{d^6}{dz^6} + A\frac{d^4}{dz^4} + B\frac{d^2}{dz^2} + C\right)(\tilde{\psi}_1, \tilde{\phi}, \tilde{T}) = 0 \tag{10}$$

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\tilde{\psi}_2 = 0 \tag{11}$$

where

$$A = \frac{1}{b_1 f_1}\{-b_1(f_1 f_8 + f_7) - f_1 f_6 + a_2 a_5 f_1 - a_3 \varepsilon_1 f_3 f_4\}$$

$$B = \frac{1}{b_1 f_1}\{b_1(f_7 f_8 + a_8 a_{10} f_3 f_4) - a_2(f_5 \xi^2 + a_8 \varepsilon_1 f_3 f_4 + a_5 a_7) + f_6(f_1 f_8 + f_7) + a_8 \varepsilon_1 f_3 f_4 (f_8 + \xi^2) - a_3 a_5 a_{10} f_3 f_4\}$$

$$C = \frac{1}{b_1 f_1}(-f_6 f_7 f_8 - a_8 a_{10} f_3 f_4 f_6 + \xi^2(a_2 a_8 \varepsilon_1 f_3 f_4 + a_2 a_5 a_7 + a_3 a_5 a_{10} f_3 f_4 - \varepsilon_1 f_3 f_4 f_8))$$

with

$$b_1 = 1 + a_1, \quad f_1 = n^* + t_1 s, \quad f_2 = n_1 s + \tau_0 s^2 + t_2^2 s^3, \quad f_3 = 1 + \tau_1 s, \quad f_4 = n_1 s + n_0 \tau_0 s^2 + t_2^2 s^3$$

$$f_5 = a_6 + a_7 s + a_9 s^2, \quad f_6 = b_1 \xi^2 + a_4 s^2, \quad f_7 = f_2 + f_1 \xi^2, \quad f_8 = f_5 + \xi^2, \quad \lambda_4^2 = \xi^2 + a_4 s^2$$

$$a_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad a_2 = \frac{b_0 \kappa}{\psi_0 \mu_0 \omega_1^*}, \quad a_3 = \frac{\beta_0 T_0}{\mu_0}, \quad a_4 = \frac{\rho c_2^2 a^*}{\mu_0}, \quad a_5 = \frac{b_0 \psi_0}{\alpha_0}, \quad a_6 = \frac{\xi_{10} \kappa}{\omega_1^* \alpha_0}$$

$$a_7 = \frac{\omega_{10} \kappa}{\alpha_0}, \quad a_8 = \frac{m_0 T_0 \psi_0}{\alpha_0}, \quad a_9 = \frac{\rho \psi_0 c_2^2}{\alpha_0}, \quad a_{10} = \frac{m_0 \kappa^2}{K_0 \psi_0 \omega_1^*}, \quad \varepsilon_1 = \frac{\beta_0 \kappa}{K_0}, \quad a^* = \frac{1}{(1 - \alpha^* T_0)}$$

$\rho C_e = \dfrac{K}{\kappa}$ and κ is the diffusivity.

The roots of the Eqs. (10) and (11) are $\pm\lambda_\ell(\ell = 1,2,3,4)$.

Assuming the regularity conditions, the solutions of Eqs. (10) and (11) may be written as

$$(\tilde{\psi}_1, \tilde{\phi}, \tilde{T}) = \left(\sum_{\ell=1}^{3} A_\ell e^{-\lambda_\ell z}, \sum_{\ell=1}^{3} s_\ell A_\ell e^{-\lambda_\ell z}, \sum_{\ell=1}^{3} e_\ell A_\ell e^{-\lambda_\ell z}\right) \tag{12}$$

$$\tilde{\psi}_2 = A_4 e^{-\lambda_4 z} \tag{13}$$

where

$$s_\ell = \frac{-(h_1\lambda_i^4 + h_2\lambda_i^2 + h_3)}{g_1\lambda_i^4 + g_2\lambda_i^2 + g_3}, \quad e_\ell = \frac{(k_1\lambda_i^4 + k_2\lambda_i^2 + k_3)}{g_1\lambda_i^4 + g_2\lambda_i^2 + g_3}; \quad \text{with}$$

$$g_1 = f_1, \; g_2 = -(f_7 + f_1 f_8), g_3 = f_7 f_8 + a_8 a_{10} f_3 f_4, \quad k_1 = \varepsilon_1 f_4$$

$$k_2 = a_5 a_{10} f_4 - \varepsilon_1 f_4 (f_8 + \xi^2), \; k_3 = \xi^2(-a_5 a_{10} f_4 + f_4 f_8 \varepsilon_1), \; h_1 = -a_5 f_1$$

$$h_2 = a_5(f_7 + f_1\xi^2), \; h_3 = \xi^2(-a_5 f_7 + a_8 f_3 f_4 \varepsilon_1)$$

4. Boundary conditions

4.1 Mechanical sources on the surface of half-space

The boundary conditions in this case are

$$t_{33} = -P_1 F(x,t), \quad t_{31} = -P_2 F(x,t), \quad \frac{\partial\phi}{\partial z} = 0, \quad T = P_3 F(x,t), \quad \text{at} \quad z = 0 \tag{14}$$

where P_1, P_2 are the magnitude of the forces, P_3 is the constant temperature applied on the boundary, $F(x,t)$ is the known function. Making use of Eqs. (4)-(8) and applying the Laplace and Fourier transforms defined by (9) and substituting the value of $\tilde{\psi}_1, \tilde{\phi}, \tilde{T}$ and $\tilde{\psi}_2$ from Eqs. (12) and (13) in the boundary condition (14), we obtain the components of displacement, stress, change in volume fraction field and temperature distribution as

$$\tilde{u} = \frac{\tilde{F}(\xi,s)}{\Delta}\{\sum_{\ell=1}^{3}(-i\xi)\Delta_\ell E_\ell + \Delta_4\lambda_4 E_4\}, \qquad \tilde{w} = \frac{\tilde{F}(\xi,s)}{\Delta}\{\sum_{\ell=1}^{3}(-\lambda_\ell\Delta_\ell E_\ell) + (-i\xi)\Delta_4 E_4\}$$

$$\tilde{t}_{33} = \frac{\tilde{F}(\xi,s)}{\Delta}\sum_{\ell=1}^{4} R_\ell \Delta_\ell E_\ell, \qquad \tilde{t}_{31} = \frac{\tilde{F}(\xi,s)}{\Delta}\sum_{\ell=1}^{4} q_\ell \Delta_\ell E_\ell$$

$$\tilde{\phi} = \frac{\tilde{F}(\xi,s)}{\Delta}\sum_{\ell=1}^{3} s_\ell \Delta_\ell E_\ell, \qquad \tilde{T} = \frac{\tilde{F}(\xi,s)}{\Delta}\sum_{\ell=1}^{3} e_\ell \Delta_\ell E_\ell \tag{15}$$

where

$$\Delta = \begin{vmatrix} R_1 & R_2 & R_3 & R_4 \\ q_1 & q_2 & q_3 & q_4 \\ s_1 & s_2 & s_3 & 0 \\ e_1 & e_2 & e_3 & 0 \end{vmatrix}$$

and $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are obtained by replacing first, second third and forth column of Δ by

$$[-P_1\tilde{F}(\xi,s) \ \ -P_2\tilde{F}(\xi,s) \ \ 0 \ \ 0]^T, \quad R_\ell = ((a_{11}+2)\lambda_\ell^2 - \xi^2 + a_2 d_\ell - a_3 f_3 e_\ell)/a^*, \quad a_{11} = \frac{\lambda_0}{\mu_0}$$

$$q_\ell = 2(i\xi)\lambda_\ell/a^*(\ell = 1,2,3), \quad E_\ell = e^{-\lambda_\ell x}; \ (\ell = 1,2,3,4), \quad R_4 = 2i\xi\lambda_4/a^*, \quad q_4 = -(\xi^2 + \lambda_4^2)/a^*$$

Case I Mechanical force The corresponding expressions are obtained for mechanical force in normal and tangential direction by taking $P_2 = P_3 = 0$ and $P_1 = P_3 = 0$ in Eq. (15), respectively.

Case II Thermal source The corresponding expressions are obtained for thermal source by taking $P_1 = 0, P_2 = 0$ in Eq. (15).

5. Applications

Case 5.1 Concentrated source

In this case $F(x,t)$ as

$$F(x,t) = \delta(x)\delta(t) \tag{16}$$

where $\delta(\ )$ is the Dirac delta function. Applying the Laplace and Fourier transforms defined by Eq. (9) on Eq. (16) we obtain

$$\tilde{F}(\xi,s) = 1 \tag{17}$$

Case 5.2 Uniformly distributed source

A special surface source distribution is considered: uniform distribution over the width $2a$. We take $F(x,t)$ as

$$F(x,t) = \{G_1(x)\delta(t) \tag{18}$$

The Laplace and Fourier transforms of Eq. (18) yield

$$\tilde{F}(\xi,s) = \tilde{G}_1(\xi) \tag{19}$$

The solution due to uniformly distributed source applied on the half-space is obtained by setting

$$G_1(x) = \begin{cases} 1 & \text{if} \ \ |x| \le a \\ 0 & \text{if} \ \ |x| > a \end{cases} \tag{20}$$

in Eq. (18). Applying the Fourier transforms defined by Eq. (9) on Eq. (20) and put the resulting equation in (19), we obtain

$$\tilde{F}(\xi,s) = \begin{cases} 2\sin\dfrac{\xi a}{\xi}, & \xi \neq 0 \end{cases} \tag{21}$$

The expressions for components of displacement, stress, and temperature distribution can be obtained for concentrated or uniformly distributed source by replacing $\tilde{F}(\xi,s)$ from Eqs. (17) and (21), respectively in Eq. (15).

6. Particular cases

6.1. In case of independence of modulus of elasticity we obtain the corresponding expressions in Eq. (15) with $a^* = 1$.

6.2. If we neglect the voids effect $(\alpha = b = \xi_1 = m = \psi = \omega_0 = 0)$ in our fundamental system of Eqs. (1)-(4), we obtained the corresponding expressions for thermoelastic half-space due to concentrated or distributed source with the help of Eqs. (17) and (21) with dependence of modulus of elasticity as

$$\tilde{u} = \frac{\tilde{F}(\xi,s)}{\Delta^*}\{\sum_{\ell=1}^{2}(-i\xi)\Delta_\ell^* E_\ell^* + \lambda_4\Delta_4 E_4\}, \qquad \tilde{w} = \frac{\tilde{F}(\xi,s)}{\Delta^*}\{\sum_{\ell=1}^{2}(-\lambda_\ell^*\Delta_\ell^* E_\ell^*) + (-i\xi)\Delta_4 E_4\}$$

$$\tilde{t}_{33} = \frac{\tilde{F}(\xi,s)}{\Delta^*}\{\sum_{\ell=1}^{2} R_\ell^*\Delta_\ell^* E_\ell^* + q_4\Delta_4 E_4\}, \qquad \tilde{t}_{31} = \frac{\tilde{F}(\xi,s)}{\Delta^*}\{\sum_{\ell=1}^{2} q_\ell^*\Delta_\ell^* E_\ell^* + q_4\Delta_4 E_4\}$$

$$\tilde{T} = \frac{\tilde{F}(q,p)}{\Delta^*}\sum_{i=1}^{2} e_\ell^*\Delta_\ell^* E_\ell^* \tag{22}$$

where

$$\Delta^* = \begin{vmatrix} R_1^* & R_2^* & R_4 \\ q_1^* & q_2^* & 0 \\ e_1^* & e_2^* & 0 \end{vmatrix}$$

and $\Delta_1^*, \Delta_2^*, \Delta_4$ are obtained by replacing first, second and third column of Δ^* by

$$[-P_1\tilde{F}(\xi,s) \;\; -P_2\tilde{F}(\xi,s) \;\; P_3\tilde{F}(\xi,s)]^T, \quad R_\ell^* = ((a_{11}+2)\lambda_\ell^{*2} - \xi^2 - a_3 f_3 e_\ell^*)/a^*$$

$$q_\ell^* = 2(i\xi)\lambda_\ell^*/a^*, \quad e_\ell^* = \frac{\varepsilon_1 f_4(\lambda_\ell^{*2} - \xi^2)}{(f_1\lambda_\ell^{*2} - f_7)}, \quad \lambda_\ell^{*2} = \frac{-A^* + (-1)^{i+1}\sqrt{A^{*2} - 4B^*}}{2}$$

$$E_\ell^* = e^{-\lambda_\ell^* z}; \; (\ell = 1,2) \quad \text{with}$$

$$A^* = \frac{(-1)}{b_1 f_1}\{b_1 f_7 + f_1 f_6 + \varepsilon_1 a_3 f_3 f_4\}, \quad B^* = \frac{1}{b_1 f_1}(f_6 f_7 + \xi^2 a_3 \varepsilon_1 f_3 f_4)$$

7. Special cases

We obtain the corresponding expressions for components of displacement, stress, change in volume fraction field and temperature distribution as given in Eqs. (15) and (22) in all the theories of thermoelasticity with changed values of A, B, C by considering the following:

Sub-Cases:

I-Coupled theory (CT-Theory)

$$n^* = n_1 = 1\,, \quad t_1 = t_2 = \tau_0 = \tau_1 = 0\,, \quad n_0\tau_0 = 0$$

with

$$f_1 = 1, \quad f_2 = s, \quad f_3 = 1, \quad f_4 = s$$

II-Lord and Shulman theory (LS Theory)

$$n^* = n_1 = 1 = n_0\,, \quad \tau_0 > 0\,, \quad t_1 = t_2 = \tau_1 = 0$$

$$f_1 = 1, \quad f_2 = s + \tau_0 s^2, \quad f_3 = 1, \quad f_4 = s + \tau_0 s^2$$

III-Green and Lindsay theory (GL Theory)

$$n^* = n_1 = 1\,, \quad n_0 = t_1 = t_2 = 0\,, \quad \tau_1 \geq \tau_0$$

with

$$f_1 = 1, \quad f_2 = s + \tau_0 s^2, \quad f_3 = 1 + \tau_1 s, \quad f_4 = s$$

VI-Chandrasekhariah and Tzou theory (C-T model)

$$n^* = n_1 = n_0 = 1\,, \quad t_1 = \tau_\theta > 0\,, \quad \tau_0 = \tau_q > 0\,, \quad t_2^2 = \frac{1}{2}\tau_q^2\,, \quad \tau_1 = 0\,, \quad \tau_q \geq \tau_\theta > 0$$

with

$$f_1 = 1 + \tau_\theta s, \quad f_2 = s + \tau_q s^2 + \frac{1}{2}\tau_q^2 s^3, \quad f_3 = 1, \quad f_4 = s + \tau_q s^2 + \frac{1}{2}\tau_q^2 s^3$$

8. Numerical inversion

The transformed expressions given by (15) and (22) are inverted by using the numerical inversion technique given by Kumar and Ailawalia (2003).

9. Numerical results and discussion

The numerical- discussion for both the cases (with and without dependence of modulus of elasticity) is reported. Following Dhaliwal and Singh (1980), we take the following values of relevant parameters for the magnesium crystal-like material as

$$\lambda_0 = 2.17 \times 10^{10}\ Nm^{-2}, \quad \mu_0 = 3.278 \times 10^{10}\ Nm^{-2}, \quad \rho_0 = 1.74 \times 10^3\ Kgm^{-3}, \quad T_0 = 298^{\circ}K$$

$$C_e = 1.04 \times 10^3\ Jkg^{-1}deg^{-1}, \quad K = 1.7 \times 10^2\ wm^{-1}deg^{-1}, \quad \beta_0 = 2.68 \times 10^6\ Nm^{-2}deg^{-1}$$

and the void parameters are

$$\psi_0 = 1.753\times10^{-15}\ m^2,\quad \alpha_0 = 3.688\times10^{-5}\ N,\quad \xi_{10} = 1.475\times10^{10}\ Nm^{-2}$$

$$b_0 = 1.13849\times10^{-10}\ Nm^{-2},\quad m_0 = 2\times10^{6}\ Nm^{-2}deg^{-1},\quad \omega_{10} = .0787\times10^{-3}Nm^{-2}s^{-1}$$

The comparison were carried out for

$$\alpha^* = .0051\ K^{-1},\ P_1 = P_2 = P_3 = 1,\ \tau_0 = 0.2,\ \tau_1 = 0.5,\ t_1 = \tau_\theta = 0.17,\ t_1 = \frac{\tau_q}{\sqrt{2}} = \frac{0.2}{1.414}$$

Fig. 1-24 shows the variations of normal stress t_{33}, tangential stress t_{31}, change in volume fraction field ϕ and temperature distribution T with distance x for LS with dependence of modulus of elasticity(LS-D), GL with dependence of modulus of elasticity(GL-D), C-T with dependence of modulus of elasticity(C-T-D) and LS with independence of modulus of elasticity(LS-I), GL with independence of modulus of elasticity(GL-I), C-T with independence of modulus of elasticity(C-T-I) due to concentrated or uniformly distributed thermomechanical sources, respectively. The solid line, small dashed lines, long dashed lines without center symbols denote for LS-D, GL-D, C-T-D and with center symbols denote for LS-I, GL-I, C-T-I respectively. The computations are carried in the range $0 \le x \le 10$ with non-dimensional time $t = .25$ and width $a = 1$.

10. Discussion for various cases

10.1 Concentrated force (normal direction)

Fig. 1-4 show the variations of t_{33}, t_{31}, ϕ and T with distance x. The trend of variations of t_{33}, t_{31} and T for dependence(LS-D, GL-D, C-T-D) and independence(LS-I, GL-I, C-T-I) of modulus of elasticity is similar whereas the corresponding values are different in magnitude, respectively. But it is noticed that, in the range $0 \le x \le 2$, the trend of variations of ϕ is opposite to t_{33}, t_{31}, T and far

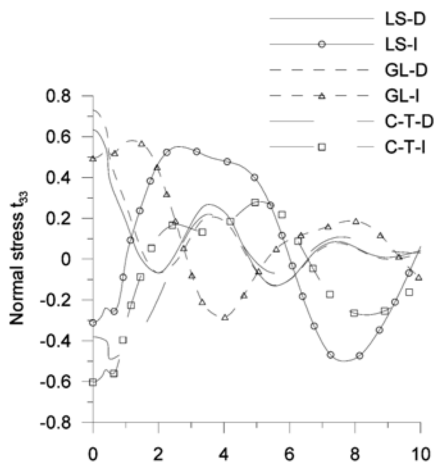

Fig. 1 Variations of normal stress t_{33} with distance x

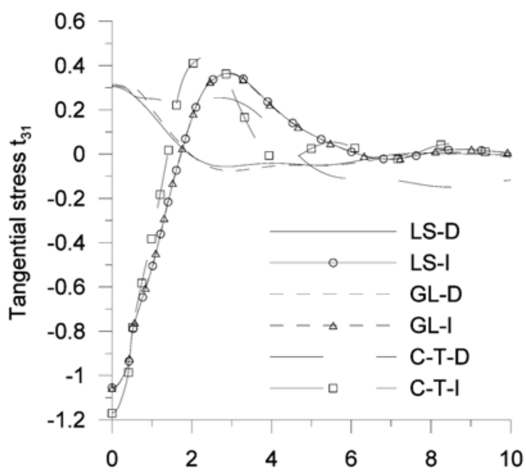

Fig. 2 Variations of tangential stress t_{31} with distance x

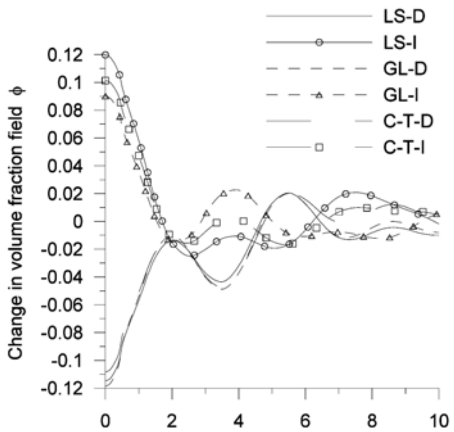

Fig. 3 Variations of change in volume fraction field ϕ with distance x

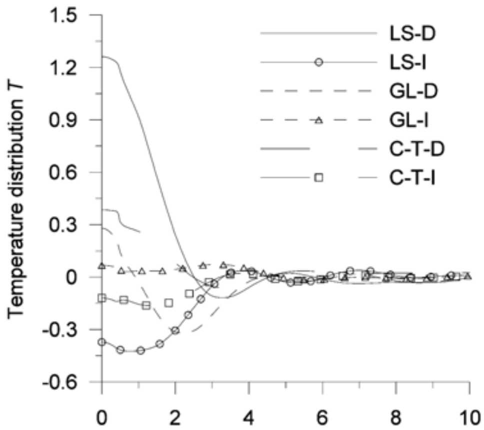

Fig. 4 Variations of temperature distribution T with distance x

from the source is similar for the same. Also the t_{33}, t_{31}, ϕ and T follow oscillatory pattern in the whole range of x. But the magnitude of oscillation is different. To compare the variations the values of t_{31} for (C-T-D) are multiplied by 10^2 times to its original values.

10.2 Uniformly distributed force (normal direction)

The variations of all the quantities t_{33}, t_{31}, ϕ and T are similar in nature to the variations obtained in case of concentrated normal force with difference in their magnitude. These variations are shown in Figs. 5-8. Also to compare the variations the values of t_{31} for (C-T-D) are multiplied by 10 times to its original values.

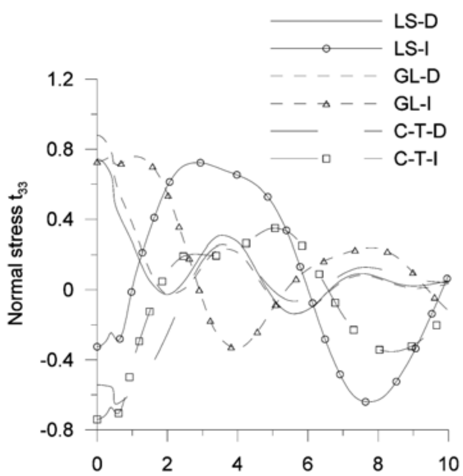

Fig. 5 Variations of normal stress t_{33} with distance x

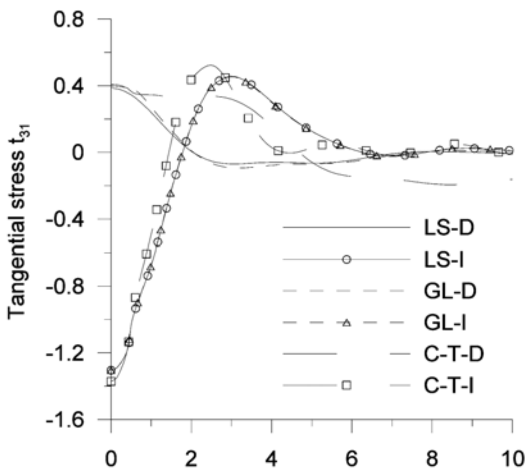

Fig. 6 Variations of tangential stress t_{31} with distance x

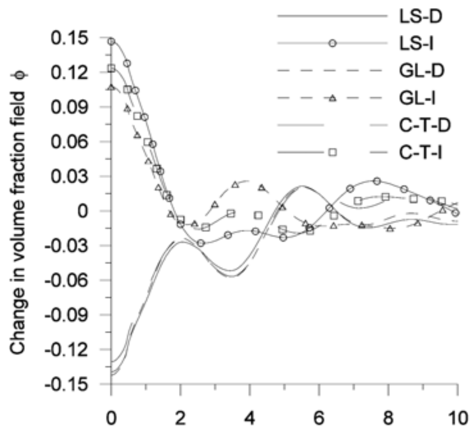

Fig. 7 Variations of change in volume fraction field ϕ with distance x

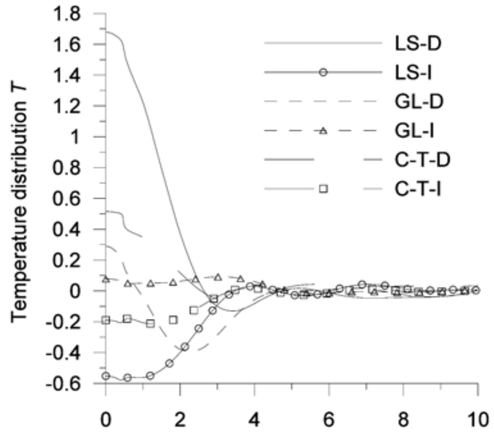

Fig. 8 Variations of temperature distribution T with distance x

10.3 Concentrated force (tangential direction)

The behavior of variations of t_{33}, t_{31}, ϕ and T for (LS-D, GL-D, C-T-D) oscillatory in the whole range of x, but the corresponding values are different in magnitude, respectively, as shown in Figs. 9-12, respectively. Also the trend of variations of t_{31}, ϕ is opposite to t_{33}, T for (LS-I, GL-I, C-T-I). The magnitude of oscillation for (LS-D, GL-D, C-T-D) is large in comparison to (LS-I, GL-I, C-T-I). To compare the variations the values of ϕ for (LS-I, GL-I, C-T-I) is multiplied by 10^2 times to its original values.

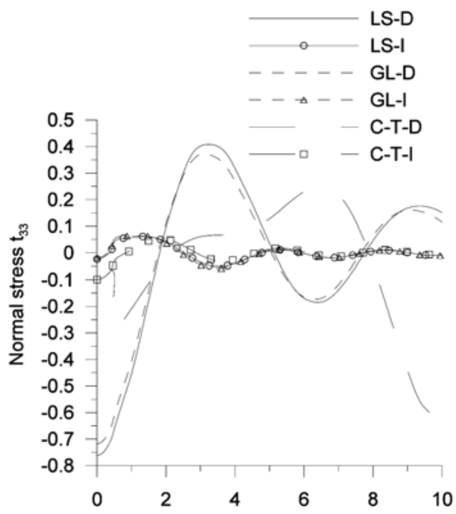

Fig. 9 Variations of normal stress t_{33} with distance x

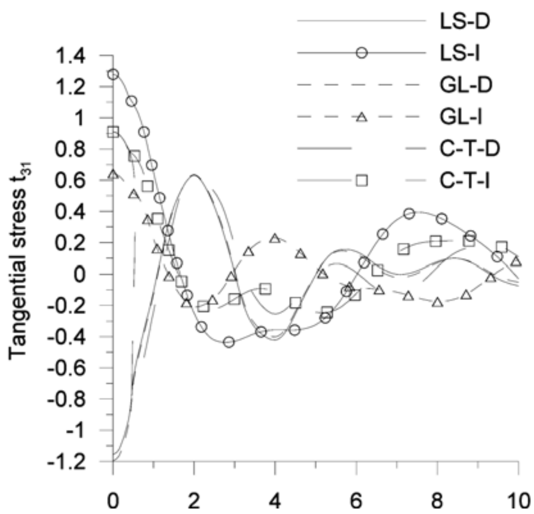

Fig. 10 Variations of tangential stress t_{31} with distance x

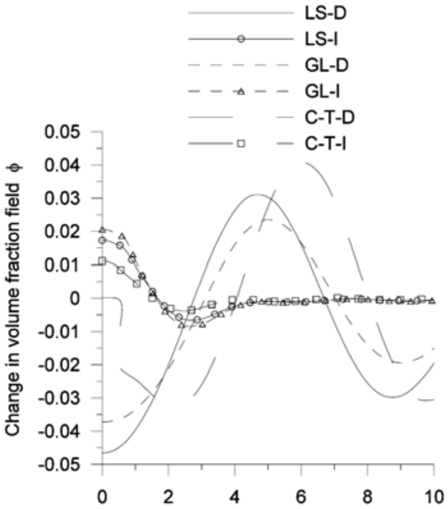

Fig. 11 Variations of change in volume fraction field ϕ with distance x

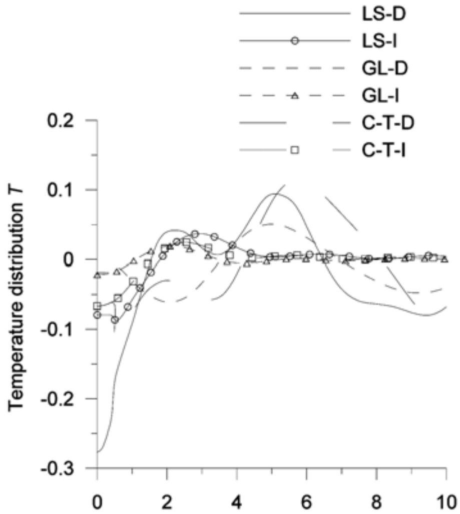

Fig. 12 Variations of temperature distribution T with distance x

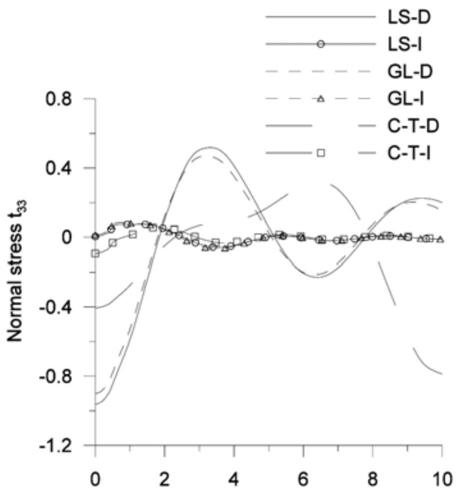

Fig. 13 Variations of normal stress t_{33} with distance x

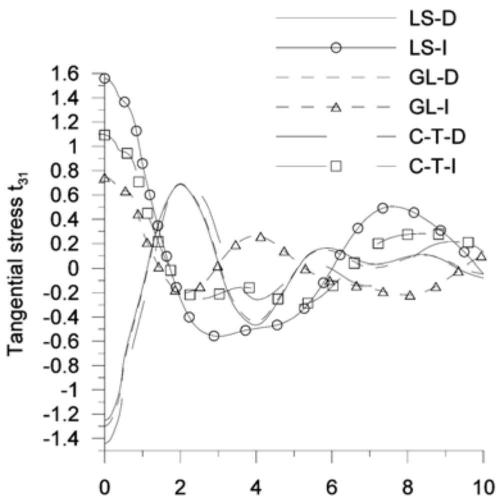

Fig. 14 Variations of tangential stress t_{31} with distance x

10.4 Uniformly distributed Force (tangential direction)

The variations of all the quantities t_{33}, t_{31}, ϕ and T are similar in nature to the variations obtained in Figs. 9-12 with difference in their magnitude. These variations are shown in Figs. 13-16. Also the variations of ϕ obtained for (LS-I, GL-I, C-T-I) is multiplied by 10^2 times to their original values.

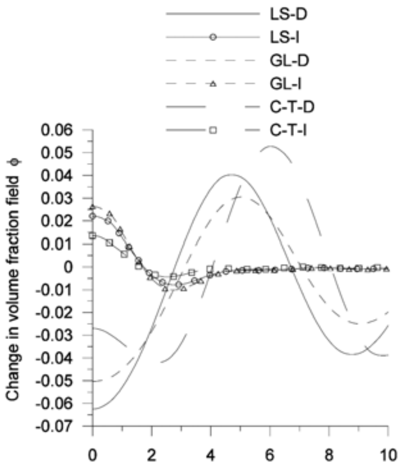

Fig. 15 Variations of change in volume fraction field ϕ with distance x

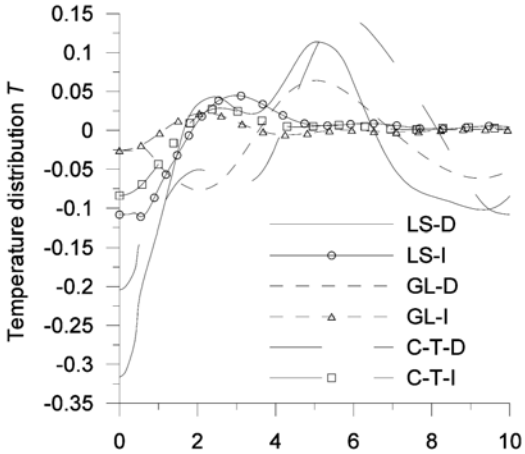

Fig. 16 Variations of temperature distribution T with distance x

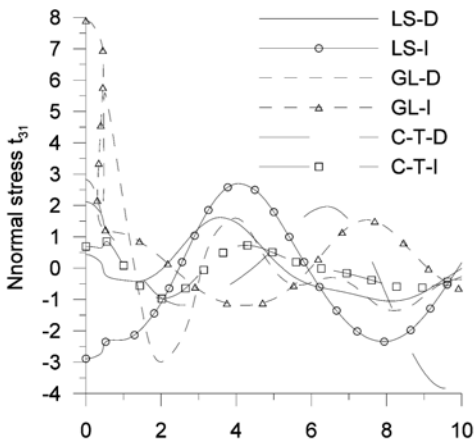

Fig. 17 Variations of normal stress t_{31} with distance x

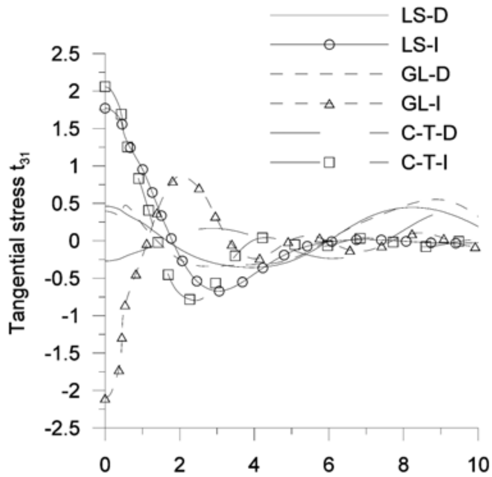

Fig. 18 Variations of tangential stress t_{31} with distance x

10.5 Concentrated thermal source

Fig. 17 shows the variation of t_{33} with distance x. The trend of variations of t_{33} for (LS-D and LS-I, GL-D and (GL-I, C-T-D and C-T-I) is similar i.e., follow oscillatory pattern whereas the corresponding values are different in their magnitude.

Fig. 18 shows the variations of t_{31} with distance x. The behavior of variations of t_{31} for (GL-D, GL-I) is opposite oscillatory in the whole range of x. The trend of variations of t_{31} for (LS-D and LS-I, C-T-D and C-T-I) is similar whereas the corresponding values are different in magnitude. To compare the variations of t_{31} for (GL-I) is demagnified by 10^2. Fig. 19 shows the variations of ϕ with distance x. Near the point of application of source the values of ϕ for (LS-D, GL-D) are more in comparison to (LS-I, GL-I), respectively. The values of ϕ for (C-T-D, C-T-I) are similar in the

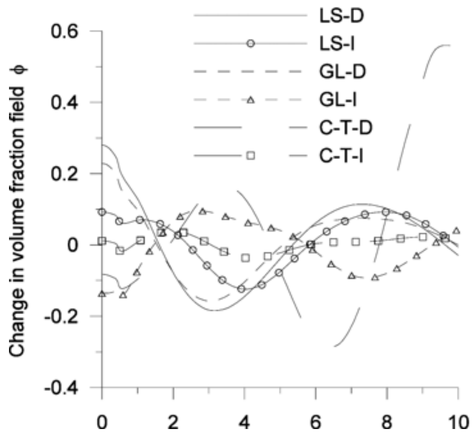

Fig. 19 Variations of change in volume fraction field ϕ with distance x

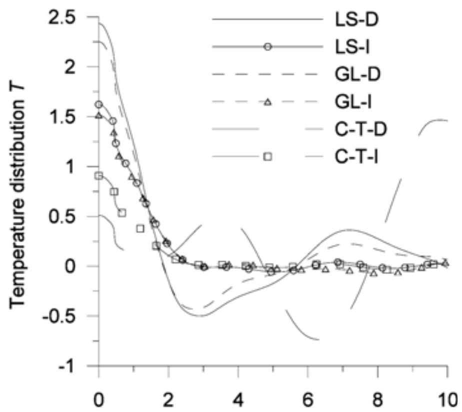

Fig. 20 Variations of temperature distribution T with distance x

range $0 \le x \le 1.8$ and contrary in the remaining range of x. As away from the source the distributions curves are all approaching toward zero except for (C-T-D). Fig. 20 shows the variations of T with distance x. The values of T for (LS-D, GL-D, C-T-D, LS-I, GL-I, C-T-I) decrease in the range $0 \le x \le 2.5$ and its behavior is oscillatory in the remaining range of x. But the magnitude of oscillation is very small except (LS-D, GL-D, C-T-D).

10.6 Uniformly distributed source

The variations of all the quantities t_{33}, t_{31}, ϕ and T are similar in nature to the variations obtained in case of concentrated thermal source with difference in their magnitude. These variations are shown in Figs. 21-24. Also the variations of t_{31} obtained for (GL-I) is demagnified by 10^2.

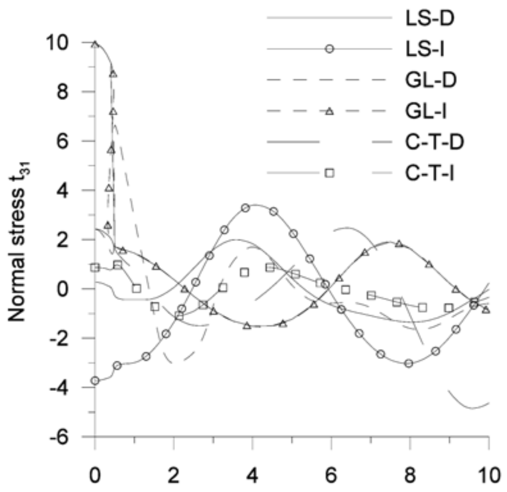

Fig. 21 Variations of normal stress t_{31} with distance x

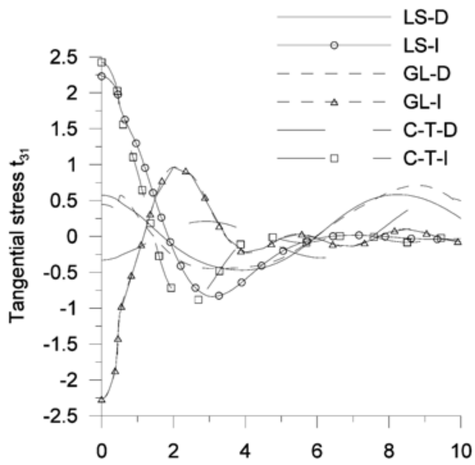

Fig. 22 Variations of tangential stress t_{31} with distance x

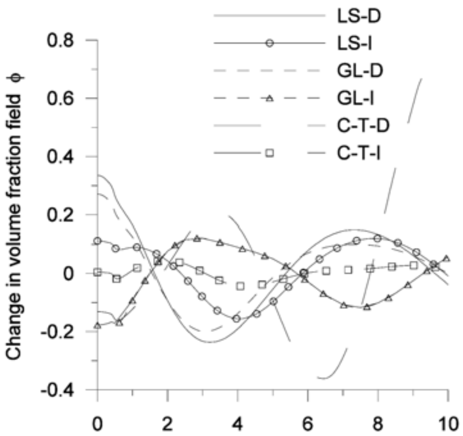

Fig. 23 Variations of change in volume fraction field ϕ with distance x

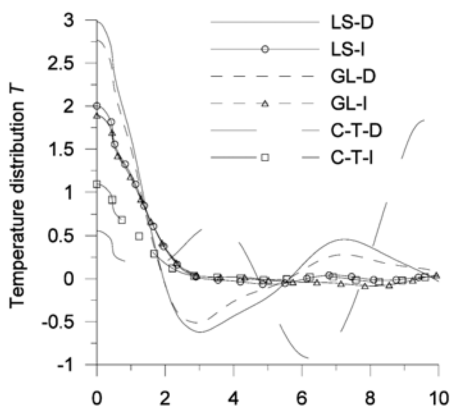

Fig. 24 Variations of temperature distribution T with distance x

11. Conclusions

Two dimensional deformation problem of a thermoelastic material with voids under the dependence of modulus of elasticity and thermal conductivity on reference temperature due to concentrated or uniformly distributed source is analyzed. It is observed that in each case, the impact (absolute) of t_{33}, t_{31}, ϕ and T is maximum near the point of application of source. As x diverges from the point of application of source these quantities are observed to follow oscillatory pattern. Also it is noticed that, near the point of application of source the values of t_{33}, t_{31} and T for (LS-D, GL-D, C-T-D) are more in comparison to (LS-I, GL-I, C-T-I) whereas when we observe in the same case the values of ϕ are quite less as compared to t_{33}, t_{31} and T.

References

Biot, M. (1956), "Thermoelasticity and irreversible thermodynamics", *J. Appl. Phys.*, **27**, 240-253.

Chandaraskharaiah, D.S. (1998), "Hyperbolic thermoelasticity: A review of recent literature", *Appl. Mech. Rev.*, **51**, 705-729.

Cowin, S.C. (1985), "The viscoelastic behavior of linear elastic materials with voids", *J. Elasticity*, **15**, 185-191.

Cowin, S.C. and Nunziato, J.W. (1983), "Linear elastic materials with voids", *J. Elasticity,* **13**, 125-147.

Dhaliwal, R.S. and Singh, A. (1980), *Dynamic Coupled Thermoelasticit*, Hindustan Publishing Co., New Delhi, India.

El-Karamany, A.S. (2004), "Boundary integral equation formulation for the generalized micro-polar thermoviscoelasticity", *Int. J. Eng. Sci.*, **42**, 157-186.

Ezzat, M.A., El-Karamany, A.S. and Samaan, A.A. (2004), "The dependence of the modulus of elasticity on reference temperature in generalized thermoelasticity with thermal relaxation", *Appl. Math. Comput.*, **147**, 169-189.

Ezzat, M.A., Othman, M.I. and El-Karamany, A.S. (2001), "The dependence of modulus of elasticity of reference temperature in generalized thermoelasticity", *J. Therm. Stresses,* **24**, 1159-1176.

Green, A.E. and Lindsay, K.A. (1972), "Thermoelasticity", *J. Elasticity,* **2**, 1-7.

Iesan, D. (1986), "A theory of thermoelastic materials with voids", *Acta Mech.*, **60**, 67-89.

Kumar, R. and Ailawalia, P. (2003), “Moving load response at thermal conducting fluid and micropolar solid surface”, *Int. J. Appl. Mech. Eng.*, **8**, 621-636.

Kumar, R. and Ailawalia, P. (2009), “Influence of various sources in micropolar thermoelastic medium with voids”, *Int. J. Struct. Eng. Mech.*, **31**(6), 715-735.

Kumar, R. and Rani, L. (2005a), “Deformation due to inclined load in thermoelastic half space with voids”, *Arch Mech.*, **57**, 7-24.

Kumar, R. and Rani, L. (2005b), “Interaction due to Mechanical and Thermal Sources in thermoelastic half-space with voids”, *J. Vib. Control*, **11**, 499-517.

Kumar, R., Sharma, N. and Ram, P. (2009), “Effects of stiffness on reflection and transmission of micropolar thermoelastic waves at the interface between an elastic and microplar generalized thermoelastic solid”, *Int. J. Struct. Eng. Mech.*, **31**(2), 117-135.

Lord, H. and Shulman, Y.A. (1967), “Generalized dynamical theory of thermoelasticity”, *J. Mech. Phys. Solid*, **15**, 299-309.

Marin, M. (1997), “On the domain of influence in thermoelasticity of bodies with voids”, *Arch Math.,* **33**(4), 301-308.

Marin, M. (1998), “Contributions on uniqueness in thermoelastodynamics on bodies with voids”, *Cienc. Mat(Havana),* **16**(2), 101-109.

Nunziato, J.W. and Cowin, S.C. (1979), “A non-linear theory of elastic materials with voids”, *Arch. Ration. Mech. An.,* **72**, 175-201.

Othman, I.A. (2002), “Lord-Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity”, *J. Therm. Stresses*, **25**(11), 1027-1045.

Othman, I.A. and Song, Y. (2008), “Reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli”, *Appl. Math. Model.*, **32**, 483-500.

Rusu, G. (1987), “On existence and uniqueness in thermoelasticity of materials with voids”, *Bull. Polish Acad. Sci. Tech. Sci.,* **35**(7-8), 339-346.

Saccomandi, G. (1992), “Some remarks about the thermoelastic theory of materials with voids”, *Rend. Mat. Appl.*, **12**, 45-58.

Scarpetta, E. (1995), “Well-posedness theorems for linear elastic materials with voids”, *Int. J. Eng. Sci.*, **33**(2), 151-161.

Sharma, J.N. and Sharma, P.K. (2001), “On the transient waves in a thermoelastic half space”, *Int. J. Appl. Mech. Eng.*, **6**, 923-946.

Tzou, D.Y. (1995), “A unified approach for heat conduction from macro to micro scales”, *J. Heat. Trans-T.* ASME, **117**, 8-16.

Youssef, H.M. (2005a), “Dependence of modulus of elasticity and thermal conductivity on reference temperature in generalized thermoelasticity for an infinite material with a spherical cavity”, *Appl. Math. Mech.*, **26**, 470-475.

Youssef, H.M. (2005b), “Generalized thermoelasticity of an infinite body with a cylindrical cavity and variable material properties”, *J. Therm. Stresses*, **28**, 521-532.