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Technical Note

Comparison of several displacement-based theories by predicting thermal response of laminated beam

Ren Xiaohui and Chen Wanji†

State Key Laboratory for Structural Analysis of Industrial Equipment, Dalian University of Technology, Dalian 116023, China
Institute for Structural Analysis of Aerocraft, Shenyang Institute of Aeronautical Engineering, Shenyang 110136, China

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1. Introduction

The typical feature of laminated structures is that the variation of in-plane displacements through the thickness shows kink at interfaces, which results in discontinuous transverse shear strain at interfaces. However, first-order theory (Whitney 1973), Reddy's theory (1984) and other global higher- order theories (Kant and Swaminathan 2002, Matsunaga 2002, Murakami 1986) are unable to satisfy this condition. Due to material constants are also different at each ply, these theories will violate transverse shear stresses continuity conditions at interfaces.

In view of this situation, Di Sciuva (1986) proposed a zig-zag model which can guarantee the continuity of transverse shear stresses. Moreover, the number of unknown variables is independent of number of layers. Cho and Oh (2004) also proposed a higher-order zig-zag model. However, the zig-zag models (Di Sciuva 1986, Cho and Oh 2004), are still unable to accurately calculate transverse shear stresses directly from constitutive equations. The 1,2-3 global-local higher-order theory (Li and Liu 1997) is very accurate for the bending problems of laminates whereas it is less accurate for the thermal expansion problems of laminated plates. To extend the capability of 1,2-3 global-local higher-order theory, Wu and Chen (2007) proposed an improved global-local higher-order theory (GLHT-32) which considering transverse normal strain. Main aim of present work is to study capabilities of several displacement-based lamianted beam theories to capture response details of laminated beams under temperature load.

2. Displacement-based theories

2.1 Global-local higher-order theory considering transverse normal strain (GLHT-32)

Displacement fields of the GLHT-32 (Wu and Chen 2007) are simply given by

† Corresponding author, E-mail: chenwj@dlut.edu.cn

$$u^k = u_0 + \Phi_1^k(z)u_1^1 + \Phi_2^k u_1 + \Phi_3^k u_2 + \Phi_4^k u_3 + \Phi_5^k \frac{\partial w_0}{\partial x} + \Phi_6^k \frac{\partial w_1}{\partial x} + \Phi_7^k \frac{\partial w_2}{\partial x}$$

$$w^k = w_0 + z w_1 + z^2 w_2 \tag{1}$$

where expression of Φ_i^k can be found in reference (Wu and Chen 2007).

2.2 Global-local higher-order theory discarding transverse normal strain (GLHT-30)

Displacement fields of the global-local higher-order theory (Li and Liu 1997) are simply given by

$$u^k = u_0 + \Phi_1^k(z)u_1^1 + \Phi_2^k(z)u_1 + \Phi_3^k(z)u_2 + \Phi_4^k(z)u_3 + \Phi_5^k w_{0,x}$$

$$w^k = w_0 \tag{2}$$

where expression of Φ_i^k can be found in references (Li and Liu 1997).

2.3 Zig-zag theory (ZZTC) satisfying continuity of transverse shear stresses at interfaces

Displacement field of the zig-zag theory (Cho and Oh 2004) can be rewritten as follows

$$u^k = u_0 + \Psi_1^k(x)u_3 + \Psi_2^k(x)\frac{\partial w_0}{\partial x} + \Psi_3^k(x)\frac{\partial w_1}{\partial x} + \Psi_3^k(x)\frac{\partial w_2}{\partial x}$$

$$w^k = w_0 + z w_1 + z^2 w_2 \tag{3}$$

where, expression of Ψ_i^k can refer to reference (Cho and Oh 2004).

2.4 Improved global displacement theory proposed by Murakami (1986)

The improved global theory MZZF obtained by adding a Murakami's zig-zag function to global displacement fields can be written as follows

$$u(x,y,z) = \sum_{i=0}^{3} z^i u_i(x,y) + M(z)u_M(x,y)$$

$$w(x,y,z) = \sum_{i=0}^{2} z^i w_i(x,y) \tag{4}$$

where $M(z)$ denotes the Murakami's zig-zag function (Murakami 1986).

2.5 Global higher order Shear Deformation Theories (HSDT)

Displacement fields of HSDT-98 (Matsunaga 2002) can be detailedly given by

$$u = \sum_{i=0}^{9} u_i z^i, \quad w = \sum_{i=0}^{8} w_i z^i \tag{5}$$

The following higher order shear deformation theories already published in previous literature are also used for comparison. They are

HSDT-54 (Matsunaga 2002)

$$u = \sum_{i=0}^{5} u_i z^i, \quad w = \sum_{i=0}^{4} w_i z^i \tag{6}$$

HSDT-33 (Kant and Swaminathan 2002)

$$u = \sum_{i=0}^{3} u_i z^i, \quad w = \sum_{i=0}^{3} w_i z^i \tag{7}$$

HSDT-30 (Kant and Swaminathan 2002)

$$u = \sum_{i=0}^{3} u_i z^i, \quad w = w_0 \tag{8}$$

HSDT-R (Reddy 1984)
Reddy's theory (1984) which is named as HSDT-R in present work.

$$u = u_0 - z\frac{\partial w}{\partial x} + \left(z - \frac{4z^3}{3h^2}\right)\gamma_x, \quad w = w_0 \tag{9}$$

2.6 First-order Shear Deformation Theories (FSDT)

First-order theory (Whitney 1973) can be written as follows

$$u = u_0 + z u_1, \quad w = w_0 \tag{10}$$

A shear correction factor of 5/6 is adopted in computed results using the first-order theory.

3. Numerical example

Example Simply-supported beam (0º/90º/0º) subjected to temperature field $T(z)\sin\pi x/L$ (Kapuria *et al.* 2003). Temperature profile $T(z)$ can be obtained by solving thermal conduction equation

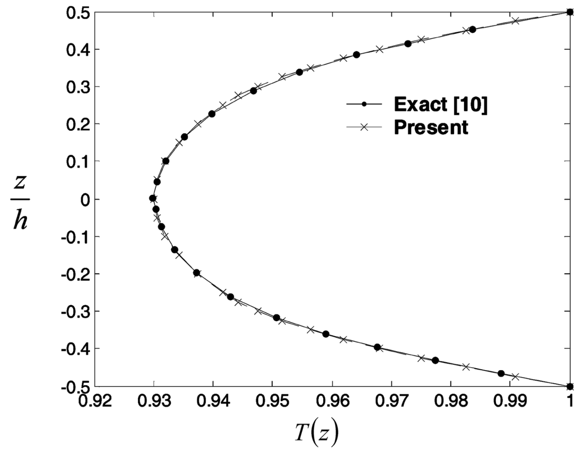

Fig. 1 Temperature distribution through thickness of laminates ($a/h = 5$)

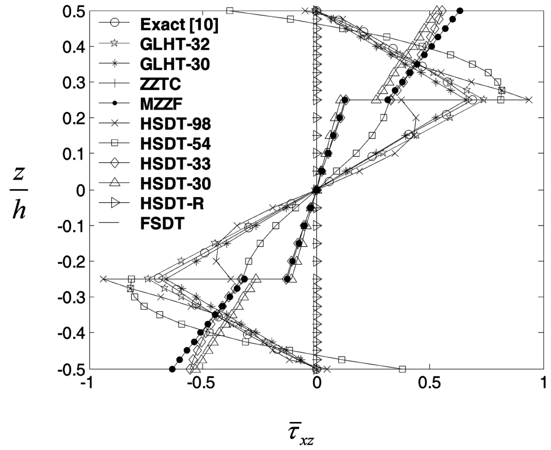

Fig. 2 Comparison of transverse shear stress from different models ($a/h = 5$)

(Tungikar and Rao 1994).

The displacements and stresses are normalized by

$$\bar{u}(0,z) = \frac{100u(0,z)}{L\alpha_T T_0}, \quad \bar{\tau}_{xz}(0,z) = \frac{\tau_{xz}(0,z)L}{\alpha_T E_T T_0 h}, \quad T_0 = 1$$

Firstly, distribution of actual temperature fields $T(z)$ through the thickness of beam can be found in Fig. 1. In Fig. 2, transverse shear stresses computed directly from constitutive equations are presented. Thereinto, transverse shear stresses calculated from ZZTC, HSDT-R and FSDT are zero along the thickness direction. The possible reason for zero transverse shear stresses is that the displacement fields of ZZTC, HSDT-R and FSDT lack the second component u_2.

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