Structural Engineering and Mechanics, Vol. 32, No. 5 (2009) 667-683

DOI: http://dx.doi.org/10.12989/sem.2009.32.5.667

FE model updating based on hybrid genetic algorithm and its verification on numerical bridge model

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(Received July 24, 2007, Accepted June 24, 2009)

Abstract. FE model-based dynamic analysis has been widely used to predict the dynamic characteristics of civil structures. In a physical point of view, an FE model is unavoidably different from the actual structure as being formulated based on extremely idealized engineering drawings and design data. The conventional model updating methods such as direct method and sensitivity-based parameter estimation are not flexible for model updating of complex and large structures. Thus, it is needed to develop a model updating method applicable to complex structures without restriction. The main objective of this paper is to present the model updating method based on the hybrid genetic algorithm (HGA) by combining the genetic algorithm as global optimization method and modified Nelder-Mead's Simplex method as local optimization method. This FE model updating method using HGA does not need the derivation of derivative function related to parameters and without application of complicated inverse analysis methods. In order to allow its application on diversified and complex structures, a commercial FEA tool is adopted to exploit previously developed element library and analysis algorithms. Moreover, an output-level objective function making use of measurement and analytical results is also presented to update simultaneously the stiffness and mass of the analysis model. The numerical examples demonstrated that the proposed method based on HGA is effective for the updating of the FE model of bridge structures.

Keywords: hybrid genetic algorithm; finite element model updating; genetic algorithm; simplex method; modal properties.

1. Introduction

Finite element (FE) model-based dynamic analysis has been widely used to predict the dynamic characteristics of civil structures. However, the results obtained from FE analysis often differ from the experimental results. This disparity is due to both modeling errors and measurement errors. Modeling errors are caused by uncertainties in geometry, boundary conditions, variation of material properties, ignorance of nonlinear effect, discretization, and other simplifications. If measurement errors can be controlled by applying high-accuracy sensors, reliable DAQ systems, and well-developed signal processing tools, the difference should be generally reduced through FE model updating procedure. In civil engineering, the derivation of an accurate FE model to predict the

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dynamic behavior, to identify the structural damage and to perform the assessment of the structure is essential for structural dynamic modification (SDM) as well as for damage detection, system identification (SI), and structural health monitoring (SHM).

Several approaches on structural model updating have been proposed and developed in the past 30 years. Conventional model updating methods can be generally classified into two categories: direct method such as optimal matrix updating and iterative method such as modal sensitivity-based parameter estimation. These methods are well reviewed by Imregun and Visser (1991) and Mottershead and Friswell (1993). Direct methods directly update the elements of stiffness and mass matrices at one-step procedure. Direct method proposed by Baruch and Bar-Itzhack (1978) assumes that the mass matrix is correct while the measured eigenvectors are updated by minimizing the weighted Euclidean norm of the difference between the measured and the analytical eigenvectors subjected to the orthogonality constraints. The error matrix method proposed by Sidhu and Ewins (1984) is another direct technique that aims at estimating the error in both mass and stiffness matrices. The updated eigienvectors are then used to update the stiffness matrix. The directly updated elements of the mass and stiffness matrices have no physical meaning, although the resulting updated matrices can exactly reproduce the measured modal data. And the sparseness, positive-definiteness and symmetry of the updated stiffness and mass matrices cannot be guaranteed. Therefore, it has been shown that direct methods are not appropriate for model updating.

Iterative parametric updating method typically involves the use of the sensitivity of the parameters to find structural changes such as stiffness or masses. For the eigensensitivity-based FE model updating method, the relationship between the perturbation in the updating parameters and the difference between the measured data and analytical results from the FE model can be represented by a sensitivity matrix (Friswell and Mottershead 1995). Iterative updating methods have been studied by many researchers such as Farhat and Hemez (1993), Friswell and Mottershead (1995), Maia and Silva (1997), Levin and Lieven (1998a), Fritzen *et al*. (1998). Eigenvalues and eigenvectors as updating parameters are generally used to construct an objective function. The use of eigendata sensitivity for analytical model updating was first proposed by Collins *et al*. (1974). Chen and Garba (1980) used matrix perturbation technique. The research on the effect of the second order sensitivities was investigated by Kim *et al*. (1983). Lin *et al*. (1995) proposed to employ both the analytical and the experimental modal data for evaluating sensitivity coefficients with the objective of improving convergence and widening the applicability of the method. Imregun *et al*. (1995a, 1995b) conducted several studies using analytical and experimental data to evaluate the effectiveness on the frequency response function data. Recently, Wu and Li (2006) proposed a two-stage eigensensitivity-based FE model updating procedure for structural parameter identification and damage detection for Phase II of the IASC-ASCE benchmark steel frame structure. Unlike the direct method, the sensitivity-based parameter updating method offers the advantage to identify the parameters that might influence directly the dynamic characteristics of the structure such as material and geometrical properties and boundary conditions. Moreover, this method secures the sparseness and positive-definiteness of the stiffness and mass matrices together with the preservation of their symmetry. However, it is required to derive the sensitivity matrix related to all the updating parameters. Generally, the sensitivity matrix is computed assuming that the change of each dynamic property is linear for infinitesimal variations of the parameters. The problem is that, in reality, responses like the mode shapes are unavoidably extremely nonlinear and non-continuous. Since previous model updating methods needed inverse analysis, their applicability remained limited to a very few types of structures. Thus, it is needed to develop a model updating method applicable to complex structures without restriction.

Recently, the improved FE model updating methods using parameter optimizing method such as genetic algorithm (GA) and non-parametric updating method such as neural network were suggested by Box (1965), Mares and Surace (1996), etc.. Levin and Lieven (1998b) employed the GA and simulated annealing (SA) algorithm independently in model updating for a beam and a flat plate wing structure. Zimmerman *et al*. (1999) investigated the GA-based approach for FE model topology and parameter adjustment. Modak and Kundra (2000) proposed a model updating method to solve a constrained nonlinear optimization problem. A simulated example of the simply supported beam and the laboratory-tested reinforced concrete beam were applied. Kim and Park (2004) introduced a multi-objective optimization technique, *Parato* GA, to model updating. The emphasis of this technique was on the selection of updating parameters. Jaishi and Ren (2006) proposed the use of modal flexibility residual for damage detection by finite element model updating which the Trust Region Newton method was used as a sensitivity-based iterative method. Research on structural model updating method using hybrid optimization method considered two optimization methods was studied by Rafiq *et al*. (2005). They utilized a genetic algorithm and regression analysis in order to predict the behavior of masonry panels. Also, merits and defects of the traditional sensitivity method, neural network method and genetic algorithm through comparison of numerical results of a real 5-story steel frame from limited modal test data were investigated by Zhu and Hao (2006). Raphael and Smith (2003) suggested Probabilistic Global Search Lausanne (PGSL) and have been applied to practical engineering tasks such as design, diagnosis and control. Saitta *et al.* (2005) introduced a system identification methodology that made use of data mining techniques to improve the reliability of identification. They used the generation of a population of candidate models as an important aspect of a suggested methodology. Also, Ian Smith's group in EPFL has been working on this subject for many years. Especially, Kripakaran and Smith (2008) introduced model filtering method using measurement-interpretation cycles as a cost effective way to determine the correct behavioral model for a structure. And they investigated the damage identification of railway truss bridge using the multiple model system identification. Although there are many papers on model updating methods based on optimization methods, they are mainly focused on simple structures, and natural frequencies and mode shapes were generally used to derive an optimal objective function.

The main objective of this paper is to present the model updating method based on the hybrid genetic algorithm (HGA) by combining the genetic algorithm as global optimization method and modified Nelder-Mead's simplex (NMS) method as local optimization method. Boundary constraints and stopping condition were included in the modified NMS method in order to guarantee a solution of problem in the variable range. This FE model updating method using hybrid optimization method does not need the derivation of derivative function related to parameters and without application of complicated inverse analysis methods. In order to allow its application on diversified and complex structures, a commercial finite element analysis tool is adopted to exploit previously developed element library and analysis algorithms. Moreover, an output-level objective function making use of measurement and analytical results is presented to update simultaneously the stiffness and mass of the analysis model. Several examples are introduced to verify numerically the proposed method and demonstrate its applicability in bridge models. The effect of measuring error is also considered as random noise and its effect is investigated.

2. Theoretical background

2.1 Methodology of FE model updating based on hybrid genetic algorithm

In this paper, the HGA for FE model updating of bridge model consists of the genetic algorithm and modified NMS method. The GA was first developed by Holland (1975) and the NMS was presented by Nelder and Mead (1965). The GA, which is a global optimization method, is a stochastic method searching randomly for the solution in the whole given domain. The modified NMS method, which is a local optimization method, is a deterministic method searching directly for

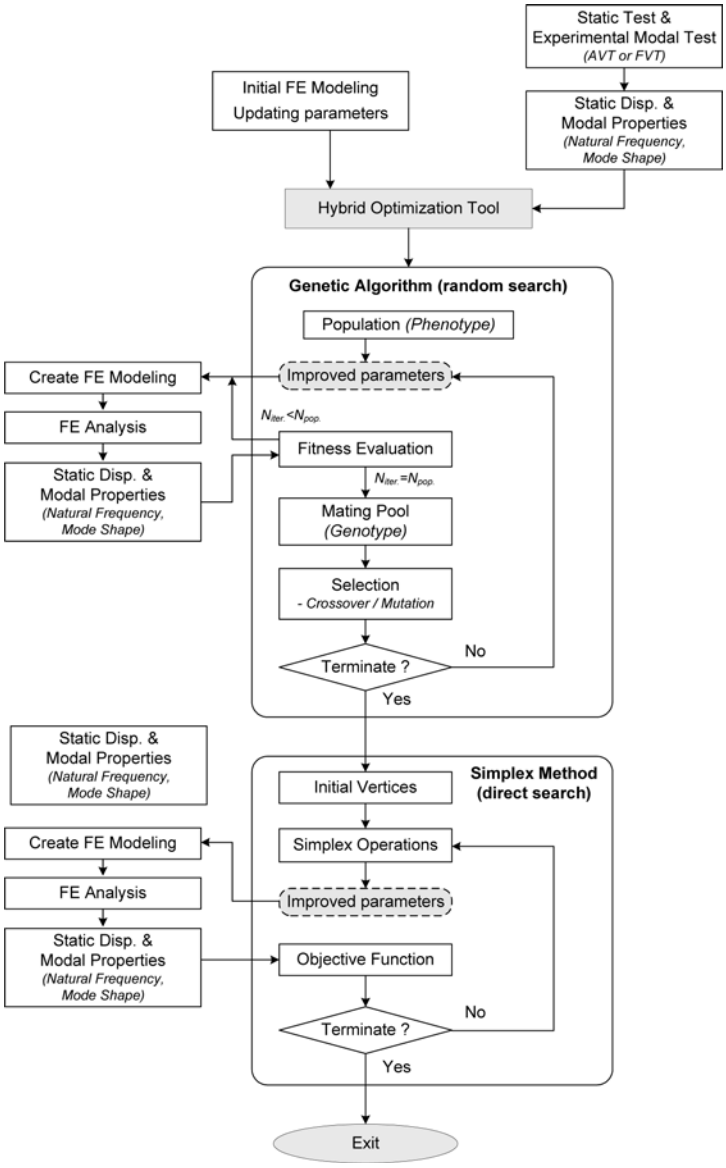


Fig. 1 Flowchart of the HGA based FE model updating method

the solution through comparison of the objective function at location nearby the given starting point. Even if both methods present different search patterns, these methods do not require the use of derivative functions and can be applied for multiple updating variables. In HGA, the GA process first determines the starting point to search optimum solution. In the next step, the modified NMS method as the local search method is used. Only a starting point obtained in the GA is used in the modified NMS method, and it has its own search pattern. Finally, the final optimum solution is determined in the modified NMS method. The HGA can find much more accurate solutions within given boundaries and decrease the search time.

In the HGA based FE model updating method, the optimization algorithm illustrated in Fig. 1 exploits the *gads* toolbox of MatLAB (2006), and the adopted finite element analysis program is the commercially available ABAQUS (2007).

2.2 Genetic algorithm

The GA, which is a stochastic search technique, is based on principles of evolutionary theory such as natural selection and evolution. This algorithm has more flexible and faster convergence characteristics than gradient-based methods with a single-point search because the objective function can be defined with a multitude of design variables. In GA the term *chromosome* typically refers to a candidate solution to a defined problem, and *fitness* is the objective function value of the candidate solution. Most GA start with an initial randomly generated population of n chromosomes. The size of the population is generally related to the problem under consideration. The length of each chromosome is the dimension of the solution space, where the component of a chromosome is the *gene*. Then the calculation of the fitness of each chromosome in the population, which is called evaluation, is performed. The Darwinian principles of reproduction, survival of the fittest, crossover, and mutation are used to create a new offspring population from a parent population (Hao and Xia 2002).

Generally, the process of genetic algorithm has four stages such as reproduction, selection, crossover, and mutation. Through this process, the next generation is generated. The process is repeated until a convergence result can be obtained. More information on GA can be found in research papers (Mares and Surace 1996, Holland 1975, Godberg 1989, Friswell and Mottershead 1998).

2.3 The modified Nelder-Mead's simplex method

Since its publication in 1965, the Nelder-Mead "simplex" algorithm has been one of the most widely used methods for nonlinear unconstrained optimization. In this paper, a simplex method was used at secondary search step. The simplex method, thus falls in the general class of direct search methods, is a local search technique that uses the evaluation of the current set of data to determine the promising search direction. A simplex (n+1 vertices) is defined by a number of points equal to one more than the number of dimensions (n variables) of the search space. The next simplex is replaced by simplex operators (reflection, contraction, expansion, and shrink) until the best vertex is satisfied with a specific convergence or stopping condition. The optimization or iteration is terminated when the condition of optimum is achieved.

However, the general objective function contains the constraint conditions and the design variables are constrained to a limited range ($x_{\min} \le x \le x_{\max}$), and it is not easy for the general simplex algorithm to achieve this. Therefore, in order to satisfy the above restriction, the modified Nelder-Mead's simplex method by Yang *et al.* (2005) as follows was used in this paper:

- If $x_{\max} \le x_r$ or $x_{\min} \ge x_r$, then the value of x_r is replaced by x_{min} or x_{max}. This process will guarantee that x_r can not go out of the boundaries.
- If x_r cannot satisfy the constraint conditions, then x_w is replaced by the second-worst point in the original simplex. This process is similar to the contraction because they are almost the same in the resulting simplex, where x_w and x_r indicate the worst point in the simplex and a new point generated by reflecting, expanding and contracting, respectively.

In addition, the weighting factors of simplex operators proposed by Barton and Ivey (1996) were considered in this paper. The value of weighting factor on reflection, contraction, expansion, and shrink operator are 1.0, 0.5, 2.0, and 0.5, respectively. The stopping condition on iterative process was also used as follows which have been proposed by Dennis and Woods (1987).

$$\Psi_k = \phi_k - \phi_{k-1} < \varepsilon_s \tag{1}$$

where,

$$\phi_k = \frac{1}{\Delta_k} \max_{\substack{i=1,\ldots,n+1 \\ x_i^k \neq x_w^k}} \left\| x_i^k - x_w^k \right\| \tag{2a}$$

$$\Delta_k = \max(1, \left\| x_w^k \right\|) \tag{2b}$$

where Ψ_k is relative difference between the evaluation value of the kth simplex, ϕ_k and k-1th simplex, ϕ_{k-1}, and ε_s is the convergence tolerance. $\left\| x_w^k \right\|$ indicates the Euclidian norm of the kth worst point on the simplex x. Δ_k is the maximum value of the Euclidian norm on the simplex x.

2.4 Formulation of the objective function

In FE model updating method, the optimization objective function enables accurate expression of the relationship between the actual structure and the analytical model. This function shall be expressed in terms of measurable physical quantities describing exactly the behavioral characteristics of the structure. In order to allow the application of a commercial finite element analysis tool, an output-level objective function using measured data and analytical results is presented. As shown in Eq. (3), the objective function suggested for the FE model updating method using HGA is formulated as a linear combination of fitness functions related to static deflections, natural frequencies and mode shapes in Eqs. (4a) to (4c). Particularly, the fitness function related to mode shapes adopts the values of the normalized modal difference (NMD) proposed by Waters (1995).

$$f_{\min} = fitness_1(f) + fitness_2(\phi) + fitness_3(u) \tag{3}$$

where,

$$fitness_1(f) = \frac{1}{m}\sum_{i=1}^{m} \alpha_i \cdot \left| \frac{f_i^e - f_i^a}{f_i^e} \right|, \ i = 1, \ldots, m \ \text{(number of modes)} \tag{4a}$$

$$fitness_2(\phi) = \frac{1}{m}\sum_{i=1}^{m} \beta_i \cdot NMD_i, \ i = 1, \ldots, m \ \text{(number of modes)} \tag{4b}$$

$$fitness_3(u) = \frac{1}{n}\sum_{j=1}^{n} \gamma_j \cdot \left| \frac{u_j^e - u_j^a}{u_j^e} \right|, \ j = 1, \ldots, n \text{ (number of measuring points)} \tag{4c}$$

In Eqs. (4a) to (4c), $fitness_1(f)$, $fitness_2(\phi)$ and $fitness_3(u)$ stand respectively for the fitness functions related to natural frequencies, mode shapes and static deflections, respectively. The superscripts e and a are corresponding to the experimental results and analytic results of the updating model, respectively. u_j is the static displacement at jth measuring point, and f_i is the natural frequency of ith mode. In addition, α_i, β_i and γ_j are the weighting factors of fitness functions related to the ith mode and jth measuring point. A value of 1.0 is adopted for all weighting factors in this paper. The normalized modal difference (NMD) can be expressed as follows, where, the modal assurance criterion (MAC) is defined by Allemang and Brown (1982).

$$NMD_i = \sqrt{\frac{1 - MAC_i}{MAC_i}} \tag{5}$$

$$MAC_i = \frac{\left| \{\phi_i^m\}^T \{\phi_i^a\} \right|^2}{(\{\phi_i^m\}^T \{\phi_i^m\})(\{\phi_i^a\}^T \{\phi_i^a\})} \tag{6}$$

where $\{\phi_i^m\}$ is the experimental mode shape that is paired with the ith analytical mode shape $\{\phi_i^a\}$. The proposed objective function does not exploit the sensitivity function relative to the updating parameters or system matrices as was done in previous model updating methods.

3. Numerical verification by 10 DOFs spring-mass model

3.1 10 DOFs spring-mass model

A 10 DOFs spring-mass model, originally used by Rad (1997) as shown in Fig. 2, is employed to investigate the use of the HGA method for FE model updating. The best combination of the fitness functions is evaluated by case studies and an analytical investigation is made to study convergence criteria and the effect of noise.

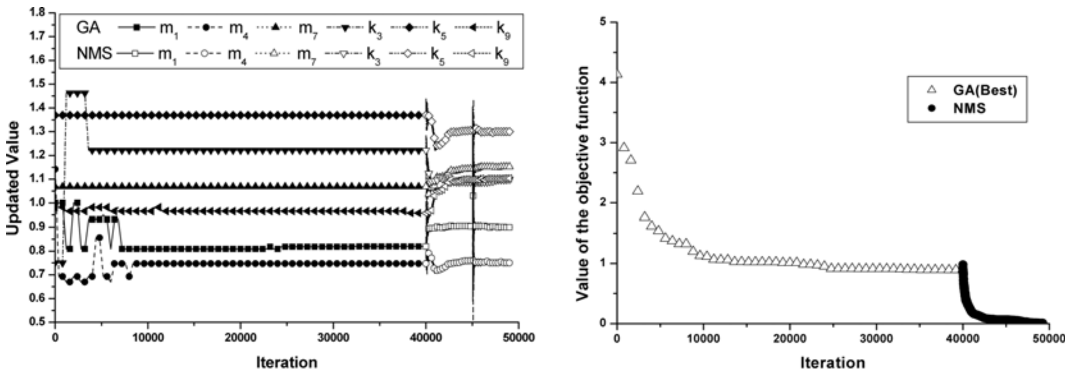
It is assumed, as was used by Rad (1997) to investigate his system identification algorithm, that for the reference model m_1 is decreased by 10% of the value in Fig. 2 and m_4 by 20%, while m_7, k_3, k_5 and k_9 are increased by 15%, 10%, 30% and 10%, respectively. For the static displacement, the model is subjected to a load 10 kN at m_{10}. Parameters of the GA used in the analysis are shown in

m1=1.0kg	m6=5.0kg	k1=0.40MN/m	k6=1.60MN/m
m2=1.0kg	m7=4.0kg	k2=0.48MN/m	k7=1.32MN/m
m3=2.0kg	m8=2.0kg	k3=0.60MN/m	k8=1.00MN/m
m4=3.0kg	m9=1.0kg	k4=1.20MN/m	k9=0.80MN/m
m5=6.0kg	m10=1.0kg	k5=2.20MN/m	k10=0.68MN/m

Fig. 2 The 10-DOF spring-mass system

Table 1 The parameters of the GA

Parameters	Function	Value
Population size	-	Updating variables×20
Generation	-	100
Elite count	-	2
Initial population range	-	[0.5, 1.5]
Selection method	Roulette wheel	-
Crossover probability	Scattered	0.8
Mutation probability	Uniform	0.1



(a) The histories of updating variables (b) The history of objective function

Fig. 3 Convergence histories for the HGA

Table 1. For easy comparison of updating results, all updating variables are normalized as a ratio to initial value as in Fig. 3, Fig. 4 and Table 5.

3.2 Selection of optimal objective function

The objective functions are evaluated to update mass as well as stiffness at the same time as shown in Eqs. (7a) to (7d). The weighting factors of each fitness functions in Eq. (4) are assumed as 1.0.

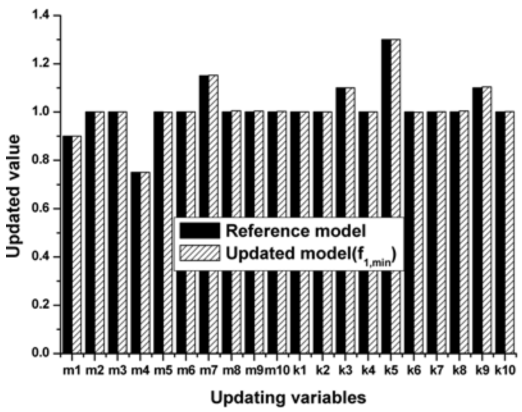
$$f_{1,\min} = fitness_1(f) + fitness_2(\phi) + fitness_3(u) \tag{7a}$$

$$f_{2,\min} = fitness_1(f) + fitness_3(u) \tag{7b}$$

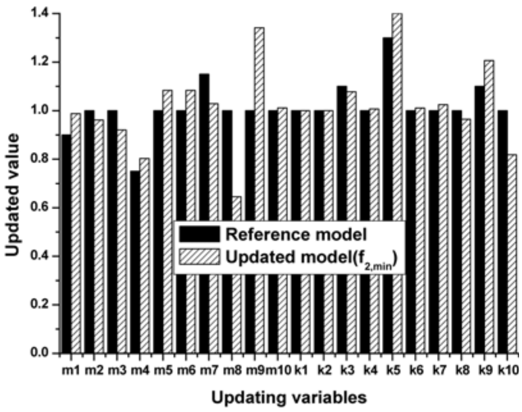
$$f_{3,\min} = fitness_1(f) + fitness_2(\phi) \tag{7c}$$

$$f_{4,\min} = fitness_2(\phi) + fitness_3(u) \tag{7d}$$

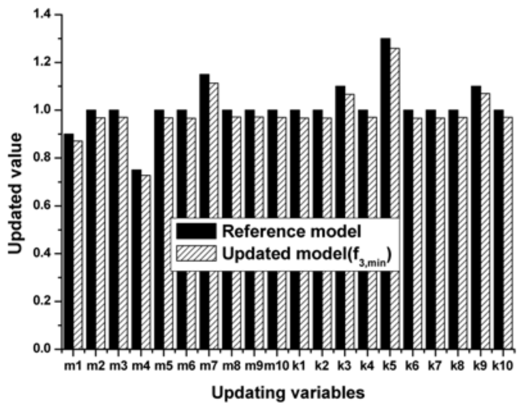
Fig. 3 shows the convergence histories of the updating variables and the objective function value. In Fig. 3(b), GA(Best) are the best individuals from each generation.



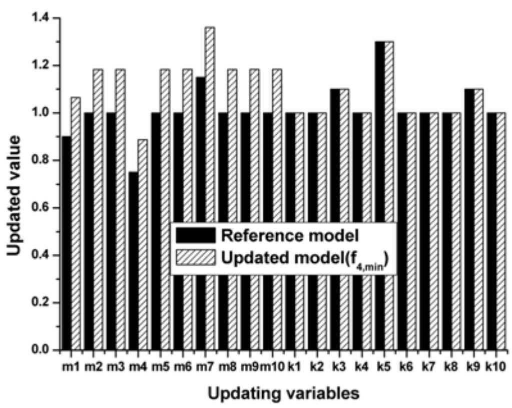
(a) Objective function, $f_{1,\min}$



(b) Objective function, $f_{2,\min}$



(c) Objective function, $f_{3,\min}$



(d) Objective function, $f_{4,\min}$

Fig. 4 Comparison of updating variables according to the type of objective function

Table 2 Comparison of objective function value

Objective function	$f_{1,\min}$	$f_{2,\min}$	$f_{3,\min}$	$f_{4,\min}$
Function value	0.011	1.905	0.339	0.808

Values of objective functions with respect to their type are shown in Table 2. It is shown that the model updating using $f_{1,\min}$ as the objective function yields better result in accuracy. Updated values of updating variables with respect to the objective function are compared in Fig. 4.

In Fig. 4(a), all variables of stiffness and mass are correctly converged to their reference values. Moreover, the updating error in case of the objective function $f_{1,\min}$ considering the static displacement decreases compared with that in the case of $f_{3,\min}$ considering the modal data only, which is commonly used in the conventional model updating methods. Consequently, it is found that the best type of objective function to update stiffness and mass simultaneously is $f_{1,\min}$ considering all the fitness functions with respect to the natural frequencies, mode shapes and static displacements. Table 3 shows the correlations of MAC, CoMAC and NMD for the final updated FE model.

Table 3 Comparison of MAC, CoMAC and NMD values ($f_{1,\min}$)

Mode	MAC		CoMAC		NMD	
	Initial	Updated	Initial	Updated	Initial	Updated
1	1.0000	1.000	0.9223	1.000	0.0062	0.000
2	0.9989	1.000	0.8397	1.000	0.0328	0.002
3	0.9956	1.000	0.7793	1.000	0.0666	0.001
4	0.9932	1.000	0.6787	1.000	0.0829	0.001
5	0.9821	1.000	0.6816	1.000	0.1351	0.001
6	0.9645	1.000	0.6961	1.000	0.1917	0.001
7	0.9693	1.000	0.7122	1.000	0.1779	0.001
8	0.9073	1.000	0.7077	1.000	0.3197	0.000
9	0.0004	1.000	0.551	1.000	51.1564	0.000
10	0.0001	1.000	0.5795	1.000	83.0597	0.000

In Table 3, MAC and CoMAC are correctly updated as 1.0 on all modes, and maximum value of NMD is 0.002 at mode 2. This means that the normalized modal difference between FE and experimental mode shape is reduced to 0.2% or smaller.

3.3 Necessary condition for the generation and population in the GA

The convergence according to the number of updating variables for the HGA method is investigated, as shown in Table 4. The error on each updating variable is calculated by the root mean square error (RMSE), as shown in Eq. (8).

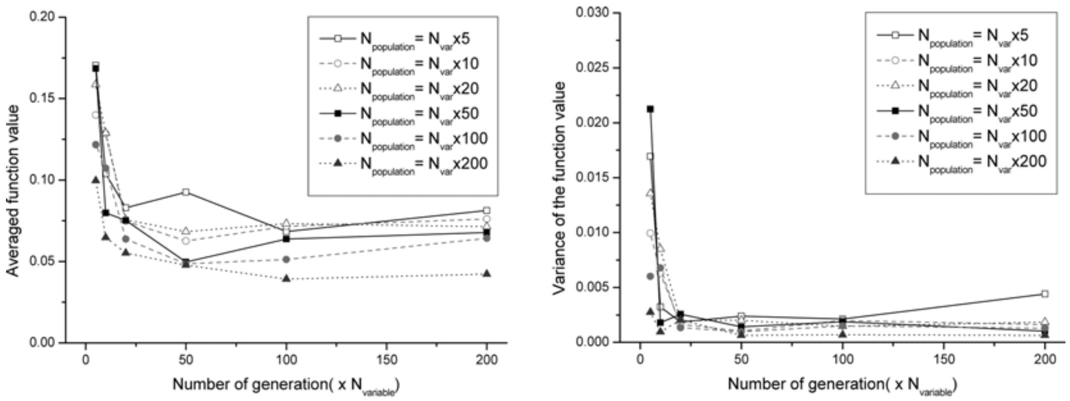
$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i^{exp} - x_i^{up}}{x_i^{exp}}\right)^2} \tag{8}$$

where x_i^{exp} and x_i^{up} indicate the value of variable, x_i, on the reference model and updated model, respectively, and N is the total number of updating variables. For the effect of the number of updating variables, the updating variables are arranged in Table 4. Only the objective function $f_{1,\min}$ considering the fitness function on natural frequencies, mode shapes and static displacements is used.

In the GA, the accuracy of solution usually increases according to the number of generation and population. But, it takes a long time to optimize due to large number of iterations. In order to get

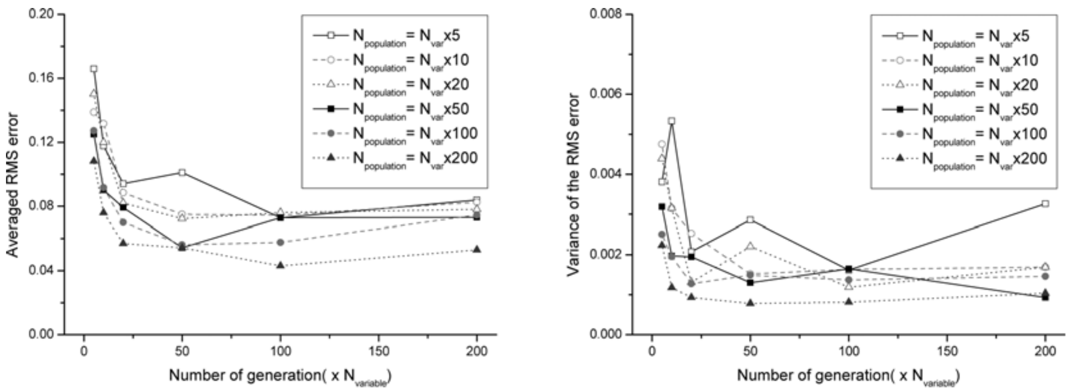
Table 4 Updating variables

Number of updating variables	Updating variables
1	k_3
3	k_3, k_5, and k_9
6	m_1, m_4, m_7, k_3, k_5, and k_9
10	k_1~k_{10}
15	m_1~m_5 & k_1~k_{10}
20	m_1~m_{10} & k_1~k_{10}



(a) Averaged function value (b) Variance of the function value

Fig. 5 Comparison of objective function value according to the number of generation



(a) Averaged RMSE (b) Variance of the RMSE

Fig. 6 Comparison of RMSE of updating variables according to the number of generation

reliable results by the proposed method, a necessary condition for the number of generation and population on the GA according to the number of updating variable is investigated. In Fig. 5 and Fig. 6, it is shown that the averaged values and variances of RMSE with respect to improved updating variables become relatively very small if the number of generation is larger than certain number. Also, it is found that the number of generation and population size on the GA should necessarily be larger than approximately twenty times the number of updating variables for the reliable application of the proposed method.

3.4 Effect of the combinatorial problem of updating variables

In model updating problems, it is very important to assure that the acquired result is well optimized so that it always gives reliable results with acceptable differences less than certain amount. Results of 10 individual model updating trial, where the same updating parameters are used while the values of population are randomly assigned in each trial, are shown in Table 5. As shown in Table 5, each model updating shows similar results with acceptable differences. In other words,

Table 5 Results of updating variables from 10 trials

Updating variables	Ref. value	Trial number										Max. diff.	Max. per.
		1	2	3	4	5	6	7	8	9	10		
m1	0.90	0.902	0.898	0.902	0.898	0.901	0.906	0.900	0.897	0.905	0.912	0.012	1.3%
m2	1.00	0.985	1.001	1.004	1.006	0.999	1.015	0.996	1.018	1.007	1.034	0.034	3.4%
m3	1.00	0.968	1.005	1.007	0.997	1.000	1.016	0.990	1.045	1.009	1.040	0.045	4.5%
m4	0.75	0.720	0.759	0.763	0.725	0.755	0.755	0.737	0.790	0.764	0.778	0.040	5.4%
m5	1.00	0.996	1.014	1.004	0.994	1.003	1.003	0.994	1.036	1.021	1.025	0.036	3.6%
m6	1.00	1.002	1.013	1.004	0.991	1.005	1.011	1.002	1.014	1.032	1.024	0.032	3.2%
m7	1.15	1.138	1.150	1.147	1.170	1.150	1.183	1.149	1.134	1.188	1.171	0.038	3.3%
m8	1.00	0.987	0.976	0.990	1.028	1.000	1.037	0.999	0.970	1.036	1.020	0.037	3.7%
m9	1.00	1.023	0.961	1.000	1.014	1.002	1.033	1.006	0.949	1.049	1.011	0.051	5.1%
m10	1.00	1.012	0.963	0.994	1.010	1.004	1.026	1.013	0.968	1.042	1.008	0.042	4.2%
k1	1.00	1.000	0.999	0.997	1.000	1.001	1.000	1.005	0.986	1.000	1.000	0.014	1.4%
k2	1.00	1.003	0.997	1.003	1.001	1.000	1.012	0.998	1.001	1.009	1.020	0.020	2.0%
k3	1.10	1.070	1.106	1.105	1.112	1.099	1.118	1.094	1.139	1.106	1.149	0.049	4.5%
k4	1.00	0.992	1.006	1.004	1.009	0.999	1.016	0.990	1.045	1.012	1.034	0.045	4.5%
k5	1.30	1.238	1.318	1.326	1.240	1.311	1.303	1.277	1.369	1.328	1.346	0.069	5.3%
k6	1.00	0.988	1.011	1.003	0.982	1.006	1.007	0.998	1.037	1.025	1.030	0.037	3.7%
k7	1.00	1.004	1.011	1.003	1.004	1.003	1.020	1.003	0.994	1.036	1.019	0.036	3.6%
k8	1.00	0.980	0.988	0.992	1.031	0.999	1.037	0.997	0.976	1.032	1.023	0.037	3.7%
k9	1.10	1.123	1.064	1.103	1.127	1.101	1.141	1.100	1.039	1.152	1.119	0.061	5.5%
k10	1.00	1.012	0.957	0.991	1.005	1.004	1.026	1.012	0.966	1.045	1.005	0.045	4.5%

even though it cannot be guaranteed that the proposed solution is the best, all the results of updating variables are converged less than about 5% in all trials. Since these differences are very small and ignorable in the civil engineering problems it can be said that the proposed solution is reliable.

4. Application on numerical bridge model

4.1 Two span continuous grid model

To verify the applicability of the proposed method to bridge structures, an FE model updating is also performed on a symmetric two-span multi-girder bridge idealized as a grid model, as shown in Fig. 7 (Kwon 2006). The analytic model is composed of a total of 58 beam elements and 2-node linear beam elements are used between the girders.

Damaged structure with reduced sectional properties as in Table 6, which Kwon (2006) used for his damage detection study, are considered as reference model. Only mode shape vectors in the z-direction are considered. Static deflections corresponding to a load of 20 kN moving from node 1 of the members' group G1 to node 13 of the members' group G5 are used in fitness function. For the static displacement, only the responses at six midspan nodes are considered.

Fig. 8 depicts the deformed shape under static load applied at node 4. Fig. 9 shows the first six

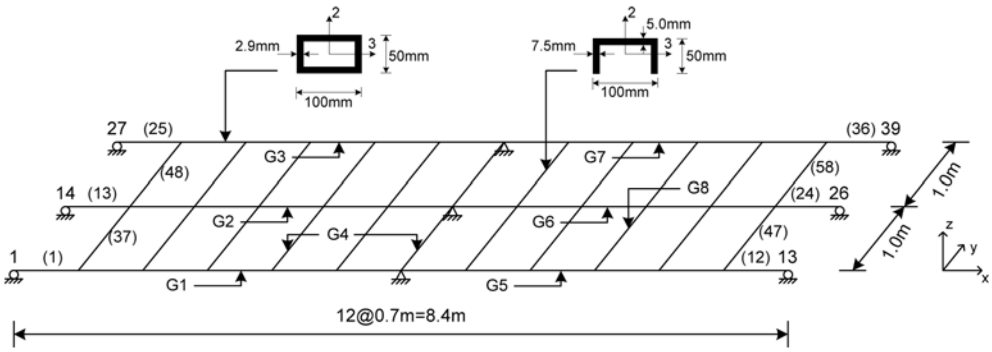


Fig. 7 Two span continuous grid model

Table 6 Cross-sectional and material properties of the members' groups

Group	Area (mm^2)			I_{33} (mm^4) $\times 10^5$			I_{22} (mm^4) $\times 10^6$	J (mm^4) $\times 10^4$	E (GPa)	Mass (kg/m^3)
	Reference	Initial	Updated	Reference	Initial	Updated				
1, 5	836.4	760.36	986.80	3.638	3.3073	3.637	1.052	80.80	210	7850
2, 6	836.4	1045.5	1140.54	3.638	2.9104	3.635	1.052	80.80		
3, 7	836.4	760.36	992.50	3.638	3.3073	3.649	1.052	80.80		
4, 8	1175.0	1566.7	963.18	2.945	3.9267	2.971	1.864	1.721		

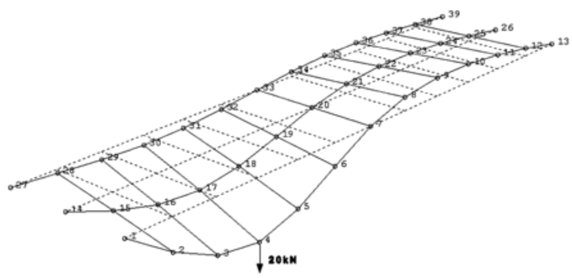
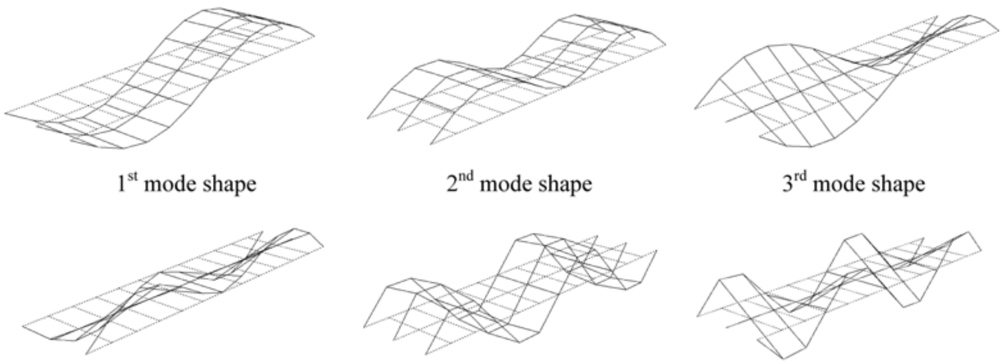


Fig. 8 Deformed shape under static load



1st mode shape 2nd mode shape 3rd mode shape

4th mode shape 5th mode shape 6th mode shape

Fig. 9 Analytical mode shapes of the reference FE model mode shapes of the reference model considered during model updating.

4.2 Model updating

The analysis conditions applied for the GA are the same as conditions on the spring-mass model, except that the number of generation is twenty times the number of updating variable. The mesh size of each updating variable applied for the NMS method is 5% of starting value. Natural frequencies and various correlation coefficients such as MAC, NMD and CoMAC are compared in Tables 7 and 8, respectively.

As it is shown in Table 7, natural frequencies of the updated model and the reference model are almost identical with difference ratio smaller than 0.02%. Also MAC and CoMAC values of updated model are 1.000 and NMD are very small in Table 8. However, even though the value of the updating variables I_{33} in Table 6 converges to the reference value very closely, the mass or area exhibits large difference compared with the reference value. This can be explained by the fact that, during the establishment of the grid model, the connecting cross-beams between the girders are modeled as single beam element, which attributes values only at each node of the girder in the global mass matrix. This implies that even though the values of the updating variables of the final improved model are different, the global mass matrix of the FE model remains identical to that of the reference model, which results in identical analytic results for both updated model and reference model. Thus, if the HGA method and objective function proposed in this paper are used for FE model updating, the application to actual structures appears to be sufficiently reliable.

Table 7 Comparison of natural frequencies according to FE model

Mode No.	Natural frequency (Hz)			Difference ratio (%)	
	Reference[a]	Initial[b]	Updated[c]	Initial[d]	Updated[e]
1	6.3006	5.3701	6.2999	-14.77	-0.01
2	9.9001	8.4331	9.9001	-14.82	0.00
3	10.439	9.6986	10.437	-7.09	-0.02
4	13.079	12.028	13.080	-8.04	0.01
5	25.342	21.493	25.344	-15.19	0.01
6	31.835	27.338	31.840	-14.13	0.02

d = (b-a)/a×100, e = (c-a)/a×100.

Table 8 Comparison of the correlation between initial and updated model

Mode No.	MAC		NMD		Node No.	CoMAC	
	Initial	Updated	Initial	Updated		Initial	Updated
1	1.000	1.000	7.0E-4	2.0E-4	2	0.9997	1.000
2	1.000	1.000	1.7E-3	3.0E-4	3	0.9997	1.000
3	1.000	1.000	5.9E-3	1.1E-3	4	0.9998	1.000
4	1.000	1.000	3.3E-3	1.0E-3	5	0.9998	1.000
5	0.9999	1.000	1.1E-2	4.0E-4	6	0.9998	1.000
6	0.5148	1.000	9.7E-1	8.0E-4	10	0.9998	1.000

Table 9 Comparison of the effect on measuring error

Noise		Area			Inertia Moment			Function value
		G1	G2	G3	G1	G2	G3	
1%	AE (%)	3.156	5.633	3.741	0.249	0.511	0.435	0.023
	SD (%)	1.754	3.801	2.045	0.143	0.314	0.382	0.003
5%	AE (%)	7.179	12.079	7.016	2.534	3.554	2.893	0.108
	SD (%)	6.952	5.137	6.379	2.388	2.672	2.494	0.029
10%	AE (%)	13.231	40.610	15.584	2.981	1.507	3.650	0.244
	SD (%)	6.059	20.029	12.756	2.073	1.275	1.656	0.029

AE = Average Error, SD = Standard Deviation.

4.3 Effect of measurement error considering random noise

To investigate the effect of measurement error, random noise was added to the results of the reference model. In real situations, experimental results such as modal data are not random since these are usually averaged and normalized, and the magnitudes of measuring errors are relatively small. In this paper, the measuring errors of natural frequencies, mode shapes, and static displacements are considered as random noise of 1%, 5%, and 10% with respect to the analytical results of reference model. FE model updating is carried out to investigate the effect of measuring error with respect to five different reference values and the results are show in Table 9.

It can be found in Table 9 that maximum model updating errors for the mass or area are 40.6%, 12.1% and 5.6% in case of 10%, 5% and 1% random noise level, respectively. On the other hand, those for the stiffness (or inertia moment) are 3.7%, 3.6% and 0.5%, respectively. This means that model updating efficiency for the stiffness may be still satisfactory with increasing noise level, while that for the mass is very vulnerable to noise. Since pure and high quality data can be achieved due to the rapid development of measurement technology and mass is rather deterministic compared with stiffness, it can be said that the proposed method may be applicable to real structures where a certain level of measurement noise is expected.

5. Conclusions

A hybrid genetic algorithm based on a genetic algorithm and modified simplex method has been addressed as global-local optimization method and a corresponding FE model updating method using a commercial finite element analysis tool has been presented. The proposed method can be applied to any FEA tool if the interface of FEA tool is prepared. Therefore, FE model updating by HGA does not require a condensation or expansion technique of system matrices to match the corresponding DOFs between measurement points and analytical model. Objective function has been formulated as a linear combination of fitness functions related to static deflections, natural frequencies and mode shapes in order to improve both the stiffness and mass simultaneously. The applicability of the proposed method to bridge structures has been investigated through numerical examples.

The proposed method has been applied to a spring-mass model with 10 DOFs and a 2-span continuous girder bridge model. The results demonstrate that the proposed method is very efficient for the model improvement. From the numerical examples, it is found that the number of generation and population on the GA should be larger than twenty times the number of updating variables for the proposed method to give reliable result. Also, random noise was added to the result of numerical analysis in order to investigate the effect of noise on the efficiency of model updating and the applicability to real structures. From the result, it can be said that the proposed method is very robust for updating of the stiffness and it may be applied to real structures where the level of measurement noise is relatively small. This fact will be proved by further applications to various real bridges.

Acknowledgements

This study has been a part of a research project supported by Korea Institute of Construction & Transportation Technology Evaluation and Planning (KICTEP) via the Infra-Structures Assessment Research Center. The authors wish to express their gratitude for the financial support.

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