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Vibration mitigation of guyed masts via tuned pendulum dampers

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Abstract. A passive vibration mitigation architecture is proposed to damp transverse vibrations of guyed masts. The scheme is based on a number of pendula attached to the mast and tuned to the vibration modes to be controlled. This scheme differs from the well-known autoparametric pendulum absorber system. The equations of motion of the guyed mast with an arbitrary number of pendula are obtained. The leading bending behaviour of a typical truss mast is described by an equivalent beam model whereas the guys are conveniently modeled as equivalent transverse springs whose stiffness comprises the elastic and geometric stiffness. By assuming a mast with an inertially and elastically isotropic cross-section, a planar model of the guyed mast is investigated. The linearization of the equations of motion of the mast subject to a harmonic distributed force leads to the transfer functions of the structure without the dampers and with the dampers. The transfer functions allow to investigate the mitigation effects of the pendula. By employing one pendulum only, tuned to the frequency of the lowest mode, the effectiveness of the passive vibration potential in reducing the motion and acceleration of the top section of the mast is demonstrated.

Keywords: guyed mast; tuned pendulum dampers; vibration absorbers; vibration mitigation; truss structure.

1. Introduction

A variety of passive tuned mass dampers (TMD) has been employed in vibration reduction of flexible structures subject to long-duration narrow-band excitations. While a TMD does not necessarily reduce the peak deformation demand in an inelastic structure subject to ground motion or wind pressure, it generally reduces the corresponding level of damage. On the other hand, in flexible structures as guyed masts and towers, TMDs are effective in the reduction of the level of vibration and acceleration. Generally, the major challenge is the practical realization of these secondary systems and their incorporation into a real structure via suitable visco-elastic components.

An interesting variant of the classical TMD is the autoparametric vibration absorber, originally proposed by Haxton and Barr (1972), realized as a pendulum attached to the main mass moving in the direction of gravity. The autoparametric coupling between the two systems allows a transfer of

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energy from the main structure to the secondary pendulum structure. Bajaj *et al*. (1994) used the method of averaging to study forced, weakly non-linear oscillations of a two-degree-of-freedom autoparametric vibration absorber system in resonant excitation. They reported a complete bifurcation analysis of the averaged equations for the subharmonic case of both internal and external resonances. Hatwal *et al*. (1983a,b) employed the harmonic balance method and direct numerical integration to study a variant of the same system at moderately higher levels of excitation, and observed that over some ranges of force, frequency, and amplitude, the system response presented an amplitude- and phase-modulated harmonic motion. For higher excitation levels, the response was found to be chaotic. Furthermore, Yabuno *et al*. (1999) investigated the stability of 1/3 order subharmonic resonance of the system. Recently, Song *et al*. (2003) analyzed the characteristics of responses of the primary system and pendulum using the harmonic balance method and the third-order approximation of the equations of motion and found the stable and unstable response regions. The main limitation of the autoparametric vibration absorber is clearly that the autoparametric transfer of energy associated with the inherent nonlinear character of the pendulum makes the system response complex, sometimes modulated or even chaotic.

A variant of the classical TMD was recently investigated by Gerges and Vickery (2003) who proposed to attach a pendulum-type TMD to the main mass of a single-degree-of-freedom structure using wire rope springs that provide both the elastic and the damping forces. They experimentally verified that the system offers good vibration absorption properties.

In this paper, we propose a practical variant of the classical TMD architecture using an array of pendula simply attached to a guyed mast to damp the transverse vibrations excited by ground motions or wind forces. The pendula are not, at low order, autoparametrically coupled to the equations of motion of the guyed mast, however, when tuned to the frequency of one of the mast vibration modes, and provided that they are properly collocated, they can exert beneficial mitigating effects on the mast vibratory response as shown in the present analyses.

2. The inertial pendulum absorbers

We first investigate the pendulum absorber attached to a single-degree-of-freedom (single-dof) structure modeled as a visco-elastic mass M whose position is described by x and is attached to a linear spring and dashpot in parallel whose constants are K and C, respectively. By denoting θ the angle that the pendulum of mass m makes with the vertical line, by assuming a dissipative mechanism in the pendulum as a linear viscous couple, $c_p\dot{\theta}$ and by letting l indicate the length of the pendulum arm, the nonlinear equations of motion of the two-dof system, subject to a harmonic force applied directly to the main mass, are

$$\begin{aligned} &M\ddot{x} + m(\ddot{x} + l\ddot{\theta}\cos\theta) - ml\dot{\theta}^2\sin\theta + C\dot{x} + Kx = Fe^{i\Omega t} \\ &ml\ddot{x}\cos\theta + ml^2\ddot{\theta} - ml\dot{x}\dot{\theta}\sin\theta + c_p\dot{\theta} + mgl\sin\theta = 0 \end{aligned} \tag{1}$$

where Ω is the circular frequency of the force, F is its magnitude and i is the imaginary unit.

Contrary to the autoparametric vibration absorber where the main mass moves in the direction of gravity, in the proposed system there are no low-order autoparametric coupling terms through which energy can be transferred from the main mass to the pendulum motion. Conversely, this occurs in the autoparametric vibration absorber when there is a 1:2 ratio between the frequency of the pendulum and the frequency of the structure.

The linearization of the equations of motion for small-amplitude vibrations leads to

$$\begin{aligned} M\ddot{x} + m(\ddot{x} + l\ddot{\theta}) + C\dot{x} + Kx &= Fe^{i\Omega t} \\ ml\ddot{x} + ml^2\ddot{\theta} + c_p\dot{\theta} + mgl\theta &= 0 \end{aligned} \tag{2}$$

Clearly, the pendulum acts as a TMD with the difference being in that the secondary mass of a standard TMD is attached to the main mass via a visco-elastic device that delivers stiffness and provides dissipation in the relative motion whereas here the pendulum possesses an inherent geometric stiffness and exerts a force in the direction of the motion of the main mass through the reactive force at the pivot. We let $\beta = \Omega/\omega_0$ be the ratio between the frequency of the external excitation and the natural frequency of the structure ($\omega_0^2 = K/M$), $r = m/M$ be the ratio between the pendulum mass and the main mass, $\xi = C/(2M\omega_0)$ and $\zeta = c_p/(2ml^2\omega_p)$ be the damping ratios relating to the main mass and to the pendulum, respectively, and $\omega_p^2 = g/l$ be the frequency of the pendulum.

In Fig. 1, the frequency-response functions, portraying variation of the dynamic amplification factor of the structure with β, are shown for several values of r. The pendulum has been tuned so as to have its frequency match the frequency of the main structure which, in turn, requires the pendulum arm to have a prescribed length, that is, $l = gM/K$. The pendulum absorber behaves as a classical TMD. The two-dof structure exhibits reduced peaks corresponding to the frequencies of the two modes, one with the mass and pendulum in phase (left) and the other with the mass and the pendulum out of phase (right). In the region between the two frequencies, the minimum amplitude is attained, corresponding to the phenomenon known as anti-resonance. The peaks at the resonances of the two modes reduce sensibly with the mass ratio, as expected.

Moreover, in Fig. 2, the frequency-response functions are shown for a fixed r, namely r = 5%,

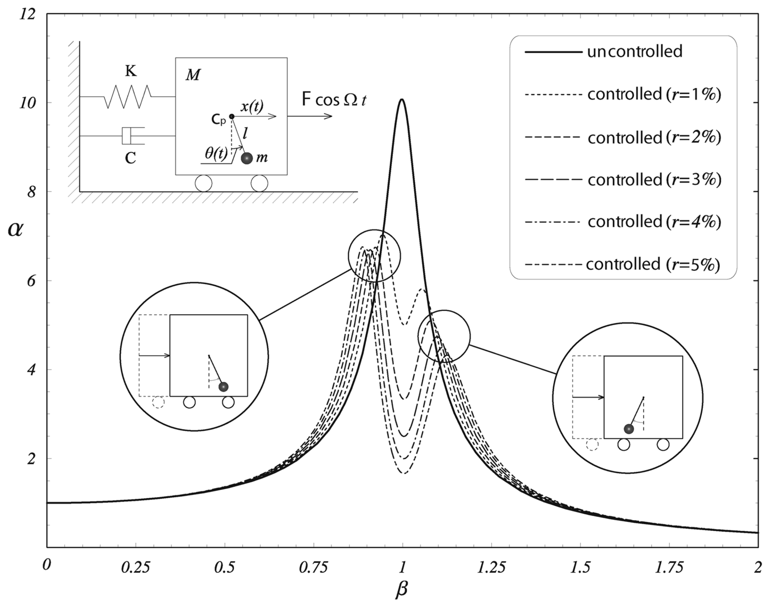

Fig. 1 Frequency-response curves of the two-dof system when ξ = 0.05, ζ = 0.0 and for several values of r

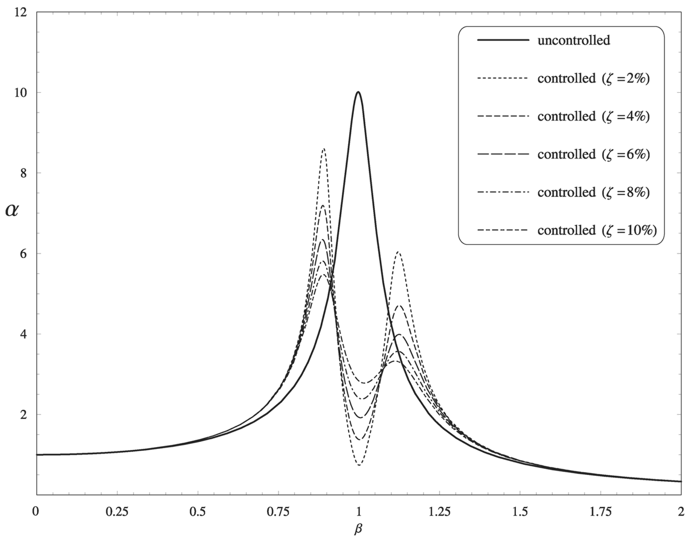

Fig. 2 Frequency-response curves of the two-dof system when $\xi = 0.05$, $r = 0.05$ and for several values of ζ

and varying the pendulum damping ratio. As in a classical TMD, the resonance peaks suffer a serious reduction with increasing damping although there occurs a simultaneous degradation of the anti-resonance.

3. Model of the guyed mast with the pendulum absorbers

A guyed mast with the pendulum absorbers comprises three elements: the mast, the cable guys, the pendula. Fig. 3 shows the structural arrangement of a typical guyed mast. The mast here considered as illustrative example (He *et al*. 2003) is supported at the base with a spherical hinge and is 150 m high. It is a triangular truss beam made of three main rods connected by three bars, 1 m long, placed at a distance of 1.25 m resulting into a triangular cross section. Of course, there are secondary diagonal connections. Two groups of three guys are connected to the same ground support points at a distance of 70 m from the centroidal line of the mast and are attached to the mast at a height of 60 m and 120 m from the bottom. In each group of guys, they are at an angle of 120 degrees.

We seek to formulate the equations of motion of the overall structural system describing small-amplitude vibrations of the mast whereas the amplitude of oscillation of the pendula may not be necessarily small due to the high flexibility and light damping inherent in the pendula. Moreover, we employ an energy approach to obtain the computational model, as discussed in the next section.

Clearly, the mast can be reasonably modeled as an Euler-Bernoulli beam. Due to the geometry of the truss, any pair of orthogonal planes is a pair of principal inertial and bending planes. It is also desirable to model the cable guys as equivalent springs. It is convenient to refer to Fig. 4 where we observe a 3D view of one group of stays with the Cartesian reference frame $\{O, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$. We choose the axes as in Fig. 4(a) and evaluate the equivalent stiffnesses K_1^i and K_2^i relating to the planes $(\mathbf{a}_1, \mathbf{a}_3)$ and $(\mathbf{a}_2, \mathbf{a}_3)$, respectively, as

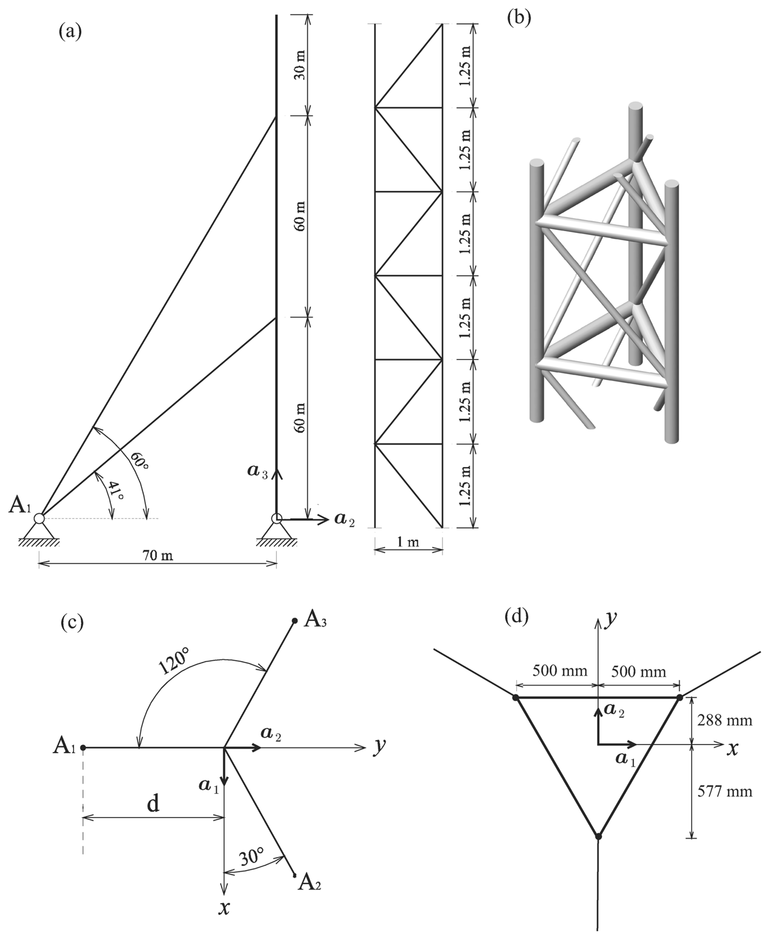

Fig. 3 Structural scheme of the mast: (a) lateral view of the mast and guys, (b) scheme of the truss, (c) top view of the guyed mast, (d) cross section of the mast

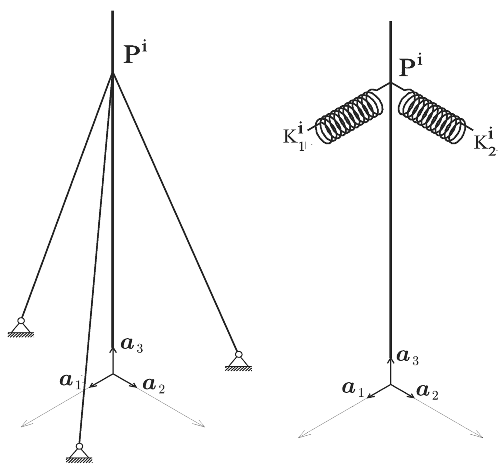

Fig. 4 Generic group of three guys and their equivalent springs

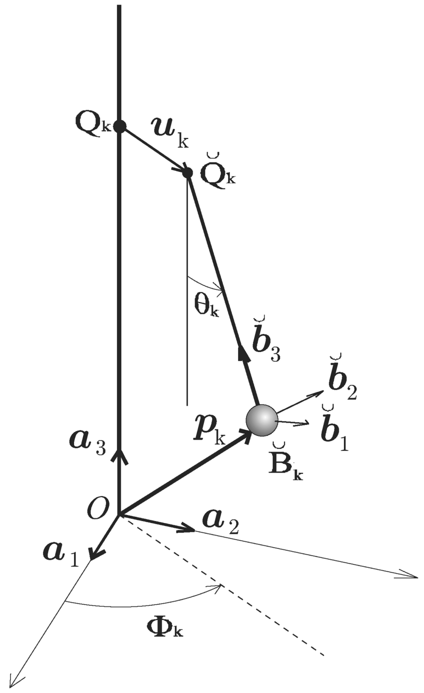

Fig. 5 Generic *k*th pendulum damper with the local reference frame

$$K_1^i = K_1^e + K_1^g = \frac{3}{2}\frac{E_s A_s}{L_i}\frac{d^2}{L_i^2} + \frac{N^0}{L_i} \tag{3}$$

$$K_2^i = K_2^e + K_2^g = \frac{3}{2}\frac{E_s A_s}{L_i}\frac{d^2}{L_i^2} + 3\frac{N^0}{L_i} \tag{4}$$

where K_j^e and K_j^g denote the elastic and geometric stiffnesses, respectively, L_i is the length of the generic guy of the ith group, $E_s A_s$ is the axial stiffness of the generic guy, d is the horizontal distance between the ground stay support and the centroidal mast line, N^0 is the pre-tension in the guys.

Next, we describe the kinematics and equations of motion of the kth spherical pendulum of length l_k whose position vector in the current configuration (see Fig. 5) is

$$\mathbf{p}_k = \breve{Q}_k - l_k \breve{\mathbf{b}}_3 = u_{1k}\mathbf{a}_1 + u_{2k}\mathbf{a}_2 + z_k\mathbf{a}_3 - l_k\breve{\mathbf{b}}_3 \tag{5}$$

where $\breve{Q}_k$ is the current position of the pendulum suspension point whose coordinate in the reference configuration is z_k, $(\breve{\mathbf{b}}_1, \breve{\mathbf{b}}_2, \breve{\mathbf{b}}_3)$ is the moving local frame attached to the pendulum, $\mathbf{u}_k = u_{1k}\mathbf{a}_1 + u_{2k}\mathbf{a}_2$ is the displacement vector of the support point Q_k. Hence, the ensuing kinetic and potential energy of the pendulum are

$$\begin{aligned}\mathcal{T}_k = \frac{1}{2}m_k(\dot{\mathbf{p}}_k \cdot \dot{\mathbf{p}}_k) = \frac{1}{2}m_k\{\dot{u}_{1k}^2 + \dot{u}_{2k}^2 + l_k^2(\dot{\theta}_k^2 + \dot{\phi}_k^2\sin^2\theta_k) + 2l_k[\dot{u}_{1k}(\cos\theta_k\dot{\theta}_k\cos\phi_k - \sin\theta_k\sin\phi_k\dot{\phi}_k) \\ + \dot{u}_{2k}(\cos\theta_k\dot{\theta}_k\sin\phi_k + \sin\theta_k\cos\phi_k\dot{\phi}_k)]\}\end{aligned} \tag{6}$$

$$\mathcal{V}_k = m_k g \mathbf{p}_k \cdot \mathbf{a}_3 = m_k g(z_k - l_k \cos\theta_k) \tag{7}$$

where m_k is the mass of the pendulum and θ_k and ϕ_k are the angles shown in Fig. 5. Therefore, the Euler-Lagrange's equations deliver the equations of motion of the pendulum as

$$\begin{aligned} m_k l_k^2(\ddot{\theta}_k - \dot{\phi}_k^2 \sin\theta_k \cos\theta_k) + m_k l_k(\ddot{u}_{1k}\cos\theta_k\cos\phi_k + \ddot{u}_{2k}\cos\theta_k\sin\phi_k + g\sin\theta_k) &= 0 \\ m_k l_k^2(\ddot{\phi}_k \sin^2\theta_k + 2\dot{\phi}_k\cos\theta_k\dot{\theta}_k) + m_k l_k(-\ddot{u}_{1k}\sin\theta_k\sin\phi_k + \ddot{u}_{2k}\sin\theta_k\cos\phi_k) &= 0 \end{aligned} \tag{8}$$

The tension N_k in the pendulum arm is expressed as

$$N_k = m_k[\ddot{u}_{1k}\sin\theta_k\cos\phi_k + \ddot{u}_{2k}\sin\theta_k\sin\phi_k - l_k(\dot{\theta}_k^2 + \dot{\phi}_k^2\sin^2\theta_k) - g\cos\theta_k] \tag{9}$$

Projecting the tension of the pendulum arm along the directions $(\mathbf{a}_1, \mathbf{a}_2)$ yields the horizontal forces that the pendulum transfers to the mast

$$\begin{aligned} F_{1k} = (N_k\breve{\mathbf{b}}_3)\cdot\mathbf{a}_1 = -m_k[&\ddot{u}_{1k}\sin^2\theta_k\cos^2\phi_k + \ddot{u}_{2k}\sin^2\theta_k\sin\phi_k\cos^2\phi_k \\ &- l_k(\dot{\theta}_k^2\sin\theta_k\cos\phi_k + \dot{\phi}_k^2\sin^3\theta_k\cos\phi_k) - g\sin\theta_k\cos\theta_k\cos\phi_k] \end{aligned} \tag{10}$$

$$\begin{aligned} F_{2k} = (N_k\breve{\mathbf{b}}_3)\cdot\mathbf{a}_2 = -m_k[&\ddot{u}_{1k}\sin^2\theta_k\sin\phi_k\cos\phi_k + \ddot{u}_{2k}\sin^2\theta_k\sin^2\phi_k \\ &+ l_k(\dot{\theta}_k^2\sin\theta_k\sin\phi_k + \dot{\phi}_k^2\sin^3\theta_k\sin\phi_k) - g\sin\theta_k\cos\theta_k\sin\phi_k] \end{aligned} \tag{11}$$

These point forces are the actual control forces that, under suitable conditions, are designed to damp the mast vibrations.

4. Computational model

A planar model of the guyed mast is obtained considering forces whose resultants lie in one of the principal planes, say $(\mathbf{a}_2, \mathbf{a}_3)$. Fig. 6 shows the first-order planar varied configuration of the mast with the pendula along with the geometric and mechanical data, and the kinematic descriptors. Namely, $v(z,t)$ is the displacement of the mast along $\mathbf{a}_2$, $\theta_k(t)$ is the angle of the kth pendulum supported from the pivot Q_k whose coordinate is z_k, the equivalent ith spring is attached to the mast at point P_i whose distance from the bottom is h_i, the overall height of the mast is H.

The energy approach is employed to construct the mass, damping and stiffness matrix of the guyed mast with and without the pendula. An exact solution of the linearized model may be achieved with significant efforts under the assumption of uniform elastic and inertial properties of the mast. However, here a more general computational approach is preferred in view of more general geometries and loading conditions.

The kinetic energy of the system, comprising the kinetic energy of the mast and that of the pendula, can be expressed as

$$\mathcal{T} = \mathcal{T}_m + \mathcal{T}_p = \frac{1}{2}\int_0^H \rho A(z)\dot{v}^2(z)dz + \frac{1}{2}\sum_{k=1}^{N_p} m_k(\dot{v}_k^2 + 2l_k\dot{v}_k\dot{\theta}_k + l_k^2\dot{\theta}_k^2) \tag{12}$$

where $\rho A(z)$ is the mass per unit length of the mast, N_p is the number of pendula, and the first-

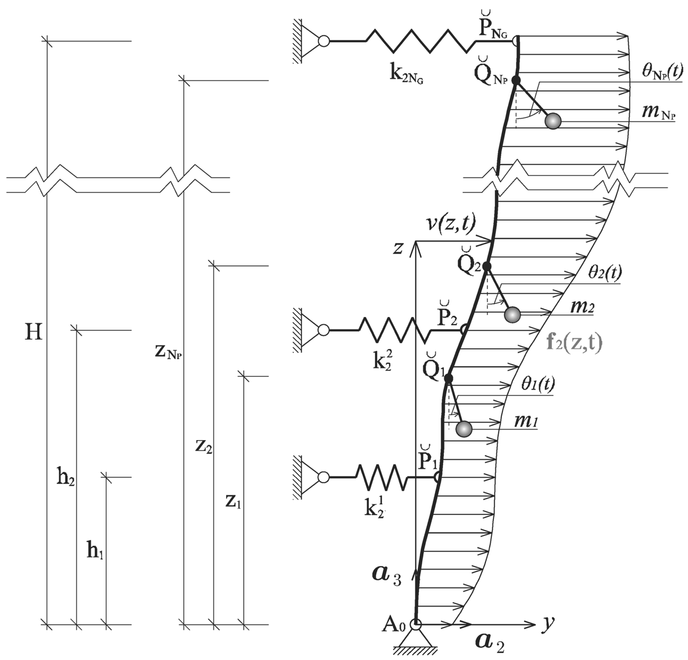

Fig. 6 Varied conguration of the equivalent planar mast under a distributed transverse force

order velocities of the pendula were considered. On the other hand, the elastic and geometric potential energy of the system, to within second-order terms, is expressed as

$$V = V_m + V_s + V_p = \frac{1}{2}\int_0^H EJ_1 v_{zz}^2 dz + \frac{1}{2}\sum_{i=1}^{N_G} K_2^{(i)} v_i^2 + g\sum_{k=1}^{N_p} m_k l_k \frac{\theta_k^2}{2} \tag{13}$$

where EJ_1 is the bending stiffness of the mast around $\mathbf{a}_1$, $K^{(i)}$ is the overall stiffness of the ith group of guys in the pertinent direction, and the linear curvature-displacement relationship, $\mu = z_{zz}$, has been used. The viscous dissipation of the guyed mast and pendula is introduced via a Rayleigh dissipation function representing the dissipation in the mast and the dissipation in the pendulum dampers as

$$\mathcal{D} = \mathcal{D}_m + \mathcal{D}_p = \frac{1}{2}\int_0^H c(z)\dot{v}^2(z)dz + \frac{1}{2}\sum_{k=1}^{N_P} c_k \dot{\theta}_k^2 \tag{14}$$

The Ritz-Galerkin method is employed letting

$$v(z,t) = \sum_{i=1}^{N} \xi_i(t)\phi_i(z) = \boldsymbol{\xi}^T\boldsymbol{\Phi} \tag{15}$$

where $\boldsymbol{\xi}$ indicates the vector of the generalized coordinates associated with the displacement of the mast and $\boldsymbol{\Phi}$ is the vector collecting the trial functions. Here, the trial functions have been determined so as to satisfy all boundary conditions, namely, vanishing of the displacement and moment at the spherical bottom hinge and vanishing of the moment and shear force at the free top section

$$\phi_j(\bar{z}) = \sin(j\pi\bar{z}) + \frac{(j\pi)^3\cos(j\pi)}{12}(2-\bar{z})\bar{z}^3 \tag{16}$$

where $\bar{z} = z/H$.

The vector of the pendulum angles is denoted $\boldsymbol{\theta}^T = [\theta_1, \ldots, \theta_{N_p}]$. Furthermore, by introducing a global generalized vector $\mathbf{q}^T = [\xi_1, \ldots, \xi_N | \theta_1, \ldots, \theta_{N_p}] = [\boldsymbol{\xi}^T | \boldsymbol{\theta}^T]$, we let

$$\boldsymbol{\xi} = \mathbf{A}\mathbf{q}, \quad \boldsymbol{\theta} = \mathbf{B}\mathbf{q} \tag{17}$$

where **A** and **B** indicate Boolean matrices. Therefore,

$$v(z,t) = \mathbf{q}^T(t)\mathbf{A}^T\boldsymbol{\Phi}(z), \quad \theta_k = \mathbf{B}_k\mathbf{q} \quad \text{with} \quad k = 1, 2, \ldots, N_p \tag{18}$$

Substituting the velocity vector into the kinetic energy and the dissipation function, and further substituting the displacement vector into the potential energy, yields the mass, damping, and stiffness matrices, respectively, as

$$\begin{aligned}\mathbf{M} &= \mathbf{M}_m + \mathbf{M}_p \\ &= \mathbf{A}^T\Big[\int_0^H \mu(z)\boldsymbol{\Phi}\boldsymbol{\Phi}^T dz\Big]\mathbf{A} \\ &\quad + \mathbf{A}^T\left[\sum_{k=1}^{N_P} m_k\boldsymbol{\Phi}(z_k)\boldsymbol{\Phi}^T(z_k)\right]\mathbf{A} + \sum_{k=1}^{N_P} m_k l_k^2\mathbf{B}_k^T\mathbf{B}_k \\ &\quad + \mathbf{A}^T\sum_{k=1}^{N_P} m_k\boldsymbol{\Phi}(z_k)\mathbf{B}_k + \sum_{k=1}^{N_P} m_k\mathbf{B}_k^T\boldsymbol{\Phi}(z_k)^T\mathbf{A}\end{aligned} \tag{19}$$

$$\mathbf{C} = \mathbf{C}_m + \mathbf{C}_p = \mathbf{A}^T\Big[\int_0^H c(z)\boldsymbol{\Phi}\boldsymbol{\Phi}^T dz\Big]\mathbf{A} + \sum_{k=1}^{N_P} c_k l_k^2\mathbf{B}_k^T\mathbf{B}_k \tag{20}$$

$$\begin{aligned}\mathbf{K} &= \mathbf{K}_m + \mathbf{K}_s + \mathbf{K}_p \\ &= \mathbf{A}^T\Big[\int_0^H EJ_2(z)\boldsymbol{\Phi}_{zz}\boldsymbol{\Phi}_{zz}^T dz\Big]\mathbf{A} + \mathbf{A}^T\left[\sum_{i=1}^{N_G} K_2^{(i)}\boldsymbol{\Phi}(z_i)\boldsymbol{\Phi}^T(z_i)\right]\mathbf{A} + g\sum_{k=1}^{N_P} m_k l_k\mathbf{B}_k^T\mathbf{B}_k\end{aligned} \tag{21}$$

Then, the resulting equations of motion are expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{P}(t) \tag{22}$$

where the vector of the generalized forces is obtained as

$$\mathbf{P}(t) = \int_0^H q(z,t)\mathbf{A}^T\boldsymbol{\Phi}(z)dz \tag{23}$$

An algorithm in MATHEMATICA was implemented to calculate all the system matrices. The natural frequencies and modes of the guyed mast were calculated solving the eigenvalue problem $\mathbf{K}\mathbf{U} = \omega^2\mathbf{M}$ assuming synchronous solutions $\mathbf{q}(t) = \mathbf{U}\exp(i\omega t)$.

5. Tuning of the pendulum dampers and control performance

The only parameters affecting the pendula are, besides the reference position coordinate z_k, the suspended mass m_k and the length of the pendulum arm l_k. The frequency of the kth pendulum is $\omega_k^p = \sqrt{g/l_k}$. When the kth pendulum is tuned to the frequency of the nth mode, denoted ω_n, the pendulum length is determined as $l_k = g/\omega_n^2$. On the other hand, the collocation of the kth pendulum is typically where the nth mode, which it is tuned with, exhibits the maximum modal displacement. However, an optimization study has to be conducted under the prevailing loading conditions. As to the mass of the pendulum, clearly the higher the pendulum mass the more effective the damper is although the secondary mass cannot exceed certain values. We let $r_k = m_k/(\rho A H)$ denote the mass ratio between the mass of the pendulum and the total mass of the mast.

To assess the effectiveness of the tuned pendulum dampers, we calculated the frequency-response functions of the guyed mast without and with the dampers. To this end, we considered a pulsating distributed force quadratically varying with z, representing the wind pressure force, as

$$f(z,t) = f_m(2-\bar{z})\bar{z}e^{i\Omega t} \tag{24}$$

Hence, $\mathbf{P}(t) = \mathbf{F}\exp i\Omega t$, Ω is the frequency of the external forcing.

The top section of the mast is the point whose motion and acceleration are sought to be minimized. Therefore, by considering the geometrical and mechanical data of the guyed mast in Table 1, we calculated the frequency-response functions of the displacement and acceleration. The pendulum damper was tuned to the lowest mode and the length of the pendulum was determined to be 1.29 m. We determined the optimal position of the pendulum considering various positions along the span of the mast; it turned out to be equal to 89 m.

Fig. 7 shows the frequency-response curve of the nondimensional displacement of the top point assuming vanishing damping in the pendulum. The addition of the pendulum damper slightly reduces the frequency of the lowest mode and reduces the peak amplitude at resonance although a second resonance peak is exhibited at the frequency of the second mode of the modified structure. However, if the excitation is narrow-banded with the band localized around the frequency of the lowest mode, the control objective is satisfactorily achieved. As expected, the resonance peak reduction is enhanced increasing the mass ratio, for example, it varies from about 6% for r = 6% to about 21% when r = 15%. Higher reductions are observed for the acceleration as shown in Fig. 8.

Table 1 Geometrical and mechanical properties of the guyed mast

σ^0 (MPa)	250
A_{s1} (mm^2)	269
A_{s2} (mm^2)	165
E_s (MPa)	$1.20 \cdot 10^5$
EJ (N·mm^2)	$2.93 \cdot 10^{14}$
ρA (kg/m)	97.6

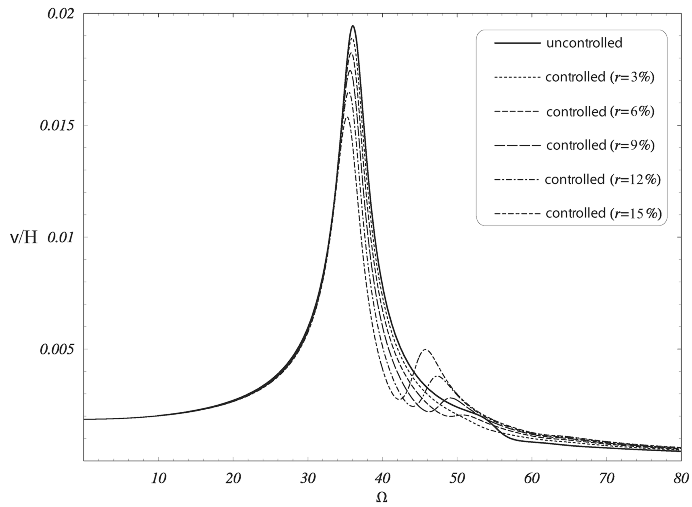

Fig. 7 Frequency-response functions of the nondimensional displacement of the top point of the uncontrolled and controlled mast for various mass ratios r and $c_p = 0$

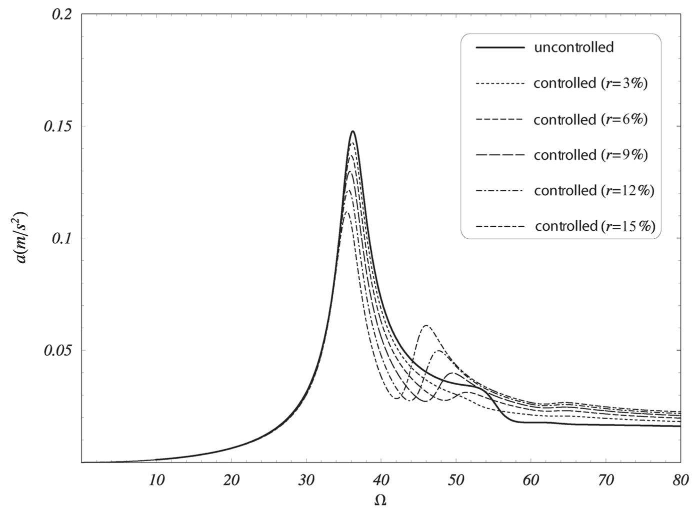

Fig. 8 Frequency-response functions of the acceleration of the top point of the uncontrolled and controlled mast for various mass ratios r and $c_p = 0$

Fig. 9 shows the lowest three mode shapes of the guyed mast without the pendulum and with the pendulum. The frequencies (periods) of the three modes of the uncontrolled structure are 0.44 Hz (2.28 s), 0.67 Hz (1.5 s), 0.77 Hz (1.29 s). On the other hand, the frequencies (periods) of the three modes of the controlled structure with r = 15% are 0.43 Hz (2.30 s), 0.55 Hz (1.82 s), 0.77 Hz (1.29 s). Clearly, the reaction force exerted at the pivot of the pendulum acts as a restoring stabilizing force that reduces the displacement and acceleration of the top point of the mast.

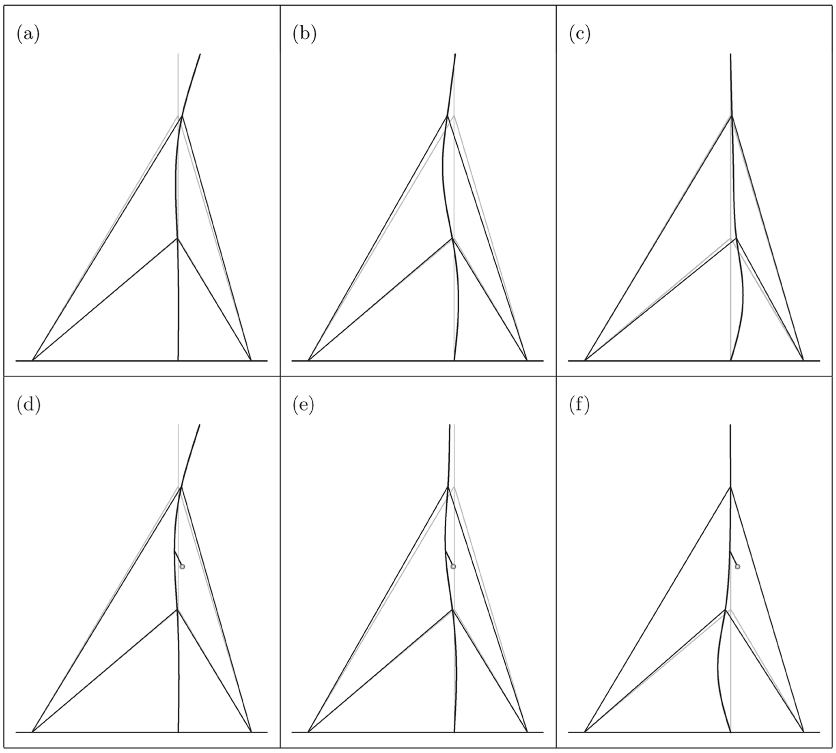

Fig. 9 Lowest three mode shapes of the guyed mast: (a)-(b)-(c) uncontrolled and (d)-(e)-(f) controlled with r = 15%

6. Conclusions

The equations of motion of a guyed mast have been obtained and a computational model has been constructed employing the Ritz-Galerkin method. The modal properties and the frequency-response function, by considering a harmonic transverse distributed force, have been calculated considering the transverse bending modes only. Then, the effectiveness of a passive control architecture based on inertial pendulum absorbers has been demonstrated. The equations of motion have been obtained in a rather general 3D framework and considering finite motions of the pendula still assuming a small-amplitude vibration regime for the mast in view of future investigations into the nonlinear vibration regime.

The control objective has been set to minimize the displacement and acceleration of the top point of the mast where typically the weather measurement sensors or electronic communication devices are placed. An optimization study has indicated to place one pendulum where the modal displacement associated with the second mode attains its maximum. A good attenuation of the displacement and, more sensibly, of the acceleration, has been demonstrated using a relatively simple passive control architecture. A full 3D control of the guyed mast can be attained using pairs of suitably tuned pendula acting in the principal planes of the mast.

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