

A comparative analysis of structural damage detection techniques by wavelet, kurtosis and pseudofractal methods

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Abstract. The aim of this paper is to compare wavelet, kurtosis and pseudofractal based techniques for structural health monitoring in the presence of measurement noise. A detailed comparison and assessment of these techniques have been carried out in this paper through numerical experiments for the calibration of damage extent of a simply supported beam with an open crack serving as an illustrative example. The numerical experiments are deemed critical due to limited amount of experimental data available in the field of singularity based detection of damage. A continuous detectability map has been proposed for comparing various techniques qualitatively. Efficiency surfaces have been constructed for wavelet, kurtosis and pseudofractal based calibration of damage extent as a function of damage location and measurement noise level. Levels of noise have been identified for each technique where a sudden drop of calibration efficiency is observed marking the onset of damage masking regime by measurement noise.

Keywords: wavelet; kurtosis; fractal; structural damage detection; signal to noise ratio.

1. Introduction

The detection of the presence, location and extent of singularity in measured noisy signals are often important for identifying and characterizing various engineering, social and medical phenomena (Dimarogonas 1996, Ide and Sornette 2002, Addison 2005). Various methods are

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available for the detection and characterization of these singularities (Robertson *et al.* 2003). Of these, wavelet based (Reda Taha *et al.* 2006), kurtosis based (Hadjileontiadis *et al.* 2005, Hadjileontiadis and Douka 2007) and pseudofractal based (Hadjileontiadis *et al.* 2005, Hadjileontiadis and Douka 2007, Wang and Qiao 2007) methods have gained considerable importance in the recent times.

The wavelet based method depends on the fact that the wavelet transform of a signal containing a singularity in itself or in any of its derivatives forms a local extremum at the location of the singularity. The number of vanishing moments of the analyzing mother wavelet must be higher than the derivative number where the singularity is present within the signal. The absolute value at the extremum at the point of the location of the singularity can be related to the degree of the singularity present in the signal (Mallat 2001).

The kurtosis and the pseudofractal based methods, on the other hand, seek local deviation of a signal from Gaussianity and depend on the computation of kurtosis or pseudofractal values of the measured response signal respectively for a number of overlapping windows with preselected width and percentage overlaps. Here too, the local extrema formed at the location of singularity is observed and the absolute value of the extrema can be related to the degree of singularity.

The presence of crack in a structure introduces a singularity at the crack tip and brings about a sharp change in the displacement and the stress-strain fields in the neighbourhood of the location of the crack (Carneiro and Inman 2002). As a result, the first derivative of the typical spatial responses of the structures, like modeshapes or static and dynamic displaced shapes contain associated singularity within them. As a result, the wavelet, kurtosis and pseudofractal based methods of singularity detection in noisy signals have a definite potential for structural health monitoring and assessment. The identification of the existence, location and extent of a beam with an open crack has been a popular example in this regard for the illustration of the effectiveness of these methods for a considerable period of time (Hadjileontiadis *et al.* 2005, Hadjileontiadis *et al.* 2005, Gentile and Messina 2003) where the identification of the existence and the presence of an open crack in a beam have been well dealt with from both theoretical and experimental aspects along with some reports on calibration of damage extents. The wavelet based damage identification method has gained more popularity than the other methods of singularity detection for its ease and flexibility. Although the importance of measurement noise in the damage data has been acknowledged in most of the studies (Gentile and Messina 2003), few of them have shown the effects of measurement noise on damage calibration (Pakrashi *et al.* 2007, Zhong and Oyadiji 2007).

With the advent of novel and sophisticated devices and techniques a significant number of publications have considered the experimental detection of an open crack in a beam from spatial data (Patsias and Staszewski 2002, Poudel *et al.* 2005, Chukwujekwu Okafor and Dutta 2000, Rucka and Wilde 2006, Poudel *et al.* 2007, Pakrashi *et al.* 2007). Under these circumstances, the effectiveness of these various emerging and fast growing methods need to be characterized from the viewpoint of detection probability due to the presence of noise. Unlike the usual receiver operating characteristics (ROC) based assessment of singularity detection techniques (Chabert *et al.* 2000), the problem of damage calibration essentially focuses on the average error in calibration due to the presence of noise and is dependent on the location and the extent of the damage, the nature and the extent of the corruptive noise and the resolution of the measurement device. The idea of false alarm is more relevant for the damage location identification problem. As a result, a detailed discussion on the effects of noise on the various assessment techniques and a relative comparison among them is deemed important. These findings provide a basis for the choice of on-site measurements for

various detection techniques and help ranking, interpreting and gathering a confidence about the obtained results. This approach of characterizing various damage detection methodologies is inspired by Hou *et al.* (2000), where they constructed a discrete detectability map on a very basic qualitative level for wavelet based detection of stiffness reduction of a structural system consisting of a mass connected by multiple breakable springs and acted on by recorded earthquake response data. The studies available in literature indicate that the characterization of the various methods of damage detection process is heavily dependent on computer based numerical simulations due the limited availability of data either from experiments or from the real structure.

This paper thus compares wavelet based, kurtosis based and pseudofractal based damage extent calibration processes through numerical experiments as the comparison requires several realizations of response data corrupted by measurement noise for various damage location and extent. The comparison is demonstrated on an example problem of a simply supported beam with an open crack. Various damage locations and extents are considered in this regard for a range of associated measurement noise. The effects of noise in the calibration of damage extent have been discussed in detail and the levels of noise where a severe and sudden loss of damage calibration efficiency occurs have been identified. The comparison method and the general qualitative findings pointed out in this paper are not limited to beams, open cracks, linear structures or space domain damage identification. It is expected, that a wide range of structural health monitoring applications involving the identification of singularities in the measured data (including bridge-vehicle interaction (Zhu and Law 2006) and measured harmonic response of structures (Khatam *et al.* 2007) in the time domain, where the singularities in space represented as Dirac delta functions are transferred to time due to the sampling property of Dirac delta function) will benefit from this study.

2. Damage model

The first modeshape of a simply supported beam of length 'L' and depth 'h' with an open crack of depth 'c' at a distance 'a' from the left hand support has been considered in this paper for the purpose of illustration. The first modeshape is also comparatively simpler to obtain from a real structure. The choice of a damage model serves the purpose of obtaining phenomenological consistency with various laboratory based studies (Patsias and Staszewski 2002, Poudel *et al.* 2005, Chukwujekwu Okafor and Dutta 2000, Rucka and Wilde 2006, Pakrashi *et al.* 2007) and the findings from continuous (Carneiro and Inman 2002) and smeared (Bovsunovsky and Matveev 2000) crack models of other researchers that the presence of a singularity in the modeshape or its derivatives due to the damage is the key to wavelet, kurtosis or pseudofractal based damage detection.

The cracked beam is modelled as an assembly of two sub-beams joined by a rotational spring at the location of the damage assuming the effects of damage to be localized in its immediate neighbourhood whereby the change of global modal properties are not significant. The free vibration equation for the beams on either side of the crack is given as

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

where E , I , A and ρ are the Young's modulus, the moment of inertia, the cross sectional area and the density of the material of the beam on either side of the crack. The displacement of the beam

from its static equilibrium position is $y(x, t)$, at a distance of x from the left hand support along the length of the beam at an instant of time t . Continuities in displacement, moment and shear are present at the location of the crack while a discontinuity for slope is present at that location and is given in terms of the non dimensional crack section flexibility θ (Narkis 1998) dependent on crack depth ratio ($\delta = c/h$) as

$$\Phi_R'(a) - \Phi_L'(a) = \theta L \Phi_R''(a) \quad (2)$$

where Φ represents the mode shape and the subscripts R and L represent the right and the left hand side of the crack respectively. The term θ is expressed as a polynomial of δ as

$$\theta = 6\pi\delta^2(h/L)(0.5033 - 0.9022\delta + 3.412\delta^2 - 3.181\delta^3 + 5.793\delta^4) \quad (3)$$

The modeshape derived from the damage model contains singularity in its derivative at the damage location.

3. Wavelet, kurtosis and pseudofractal based singularity detection

3.1 Wavelet based detection

For a wavelet with no more than m number of vanishing moments, it can be shown (Mallat 2001) that for very small values of scales in the domain of interest, the continuous wavelet transform of a function $f(x)$ in the square integrable function space can be related to the m^{th} derivative of the signal. For any wavelet basis function $\psi(x)$, this relationship can be expressed as

$$\lim_{s \rightarrow 0} \frac{Wf(b, s)}{s^{(m+1/2)}} \propto \frac{d^m f(x)}{dx^m} \quad (4)$$

where $W(\cdot)$ is the continuous wavelet transform of $f(x)$ and b and s are the translation and the scale parameters respectively. Hence it is possible for a wavelet to detect singularities in a signal or its derivatives through the incorporation of a proper choice of basis function.

The measure of the local regularity in the neighbourhood of a point in a function can be related to the local Lipschitz exponent around that point (Mallat 2001). A function $f(x)$ in the square integrable space is pointwise Lipschitz $\kappa \geq 0$ at a point x if there exists a $K > 0$ and a polynomial p_ν of degree \check{m} such that

$$\forall x \in \mathbb{R}, |f(x) - p_\nu(x)| \leq K|x - \nu|^\kappa \quad (5)$$

The term κ provides the degree of singularity in the neighbourhood of the point x . If the function $f(x)$ is uniformly Lipschitz $k < \check{n}$ over an interval $[\check{a}, \check{b}]$, then there exists an $\check{A} > 0$ such that

$$\forall (b, s) \in [\check{a}, \check{b}] \times \mathbb{R}^+, |Wf(b, s)| \leq \check{A}s^{\kappa + \frac{1}{2}} \left(1 + \left|\frac{b - \nu}{s}\right|^\kappa\right) \quad (6)$$

Thus, the magnitude of the wavelet coefficients around a point can be related to the local Lipschitz exponent, and hence to the degree of singularity present at that point.

3.2 Kurtosis and pseudofractal based detection

Both kurtosis and pseudofractal based detections of singularity consider a cumulant based scheme incorporating a moving window. The kurtoses of an empirically chosen width of window within the signal with nearly 99 percent overlap of the windows are computed (Hadjileontiadis *et al.* 2005). The absolute deviation of the number of kurtosis values for each position of window from the mean indicates the damage at its location by forming an extremum at the location of damage. The formula for kurtosis has been taken in this paper to be

$$\beta = \frac{\int_0^L (x - \bar{\mu})^4 \Phi(x) dx}{\left(\int_0^L (x - \bar{\mu})^2 \Phi(x) dx \right)^2} \quad (7)$$

and the Kurtosis Crack Detector (KCD) is computed as

$$KCD = |\beta_i - \bar{\beta}| \quad (8)$$

where β_i are the values of kurtosis at each position of the sliding window and $\bar{\beta}$ is the average of the kurtosis values. The mean value of the mode shape $\Phi(x)$ over the length of the beam is represented by $\bar{\mu}$.

The pseudofractal dimension based crack detection scheme (Hadjileontiadis *et al.* 2005) is similar to the Kurtosis Crack Detector (KCD) and is referred to as Fractal Crack Detector (FCD) in this paper. The pseudofractal based crack detection scheme is considered in this paper as the computed measure

$$FCD = abs \left(\frac{\log\left(\frac{L_1}{L_2}\right)}{\log\left(\frac{S_1}{S_2}\right)} \right) \quad (9)$$

where $L_{()}$ are the respective lengths of each section of the curve within the overlapping windows as computed by employing a step size of $S_{()}$. The subscripts of L and S in Eq. (9) represent the cases corresponding to two different step sizes used. Both KCD and FCD are essentially a measure of the local deviation of a measured signal from Gaussianity.

4. Concepts of damage calibration efficiency and detectability map

Damaged modeshape data is simulated for a beam of length 1 m. The cross sectional area (A), depth (h) and the moment of inertia (I) of the square beam are taken as 0.0001 m², 0.01 m and 8.33×10^{-10} m⁴ respectively. The Young's modulus (E) and the density of the beam (ρ) are assumed to be 190×10^9 N/m² and 7900 kg/m³. The modeshape is analysed by Coif4 wavelet basis function, which has eight vanishing moments, and is hence is suitable for damage detecting the singularity

present in the derivative of the modeshape. The modeshape data is multiplied by a Hanning window of length equal to that of the modeshape data to reduce edge effects and enhance the performance of wavelet based identification of damage (Gentile and Messina 2003). Data for kurtosis and pseudofractal based detection is not windowed.

Small and edge cracks are usually more difficult to detect and non-detection is quite common in the presence of significant noise due to the phenomenon of masking (Gentile and Messina 2003, Pakrashi *et al.* 2007). In fact, the performance of various detectors in the presence of noise is a central and critical issue in structural health monitoring, assessment and damage detection. This paper considers synthetic additive Gaussian noise, a major source of problem in various measurements, for the purposes of simulation and discussion. The presence of noise alters the magnitudes of local maxima of damage calibration coefficients (employing different techniques) near the damage location. A criterion for efficiency for the calibration of damage is defined here by considering a noise free ideal damage calibration of the crack depth ratio to be a benchmark. Deviations from the ideal calibration values are considered as error in calibration. This forms the idea of a continuous detectability map. A perfect match is colour coded to be white (digitized to a value of 1) and a non-detection (corresponding to a value of 0) is coded as black. The efficiency of any realization of the damage calibration due to the presence of noise, if detected, lies between 0 and 1 according to its percentage deviation from the ideal calibration values. Thus, if the magnitude of local maxima of the wavelet coefficients near the damage location for the damaged modeshapes with and without noise are $W(\Phi_N)_{(s,b)}$ and $W(\Phi_{WN})_{(s,b)}$ respectively then the efficiency of damage calibration η can be defined as

$$\eta = \left(1 - \left(\frac{W(\Phi_{WN})_{(s,b)} - W(\Phi_N)_{(s,b)}}{W(\Phi_{WN})_{(s,b)}} \right) \right) \quad (10)$$

Considering a range of values of crack depth ratio, position and SNR, for all practical purposes the value of η lies between 0 and 1. Thus, for a successfully detected crack, the calibration does not deviate beyond 100 percent. Very high values of SNR or low value of crack depth ratio result in a non-detection of the location of damage and thus the corresponding point in the continuous detectability map assumes a digitised value of zero represented by the colour black. Thus, the colour coded continuous detectability map can take up the digitised values 1 (white) and 0 (black) respectively in a consistent fashion. Fig. 1 illustrates some examples of the proposed continuous detectability map. The map depicts the evolution of relative efficiency of wavelet, kurtosis and pseudofractal based calibration of damage and is indicative of the probability of detection (POD) of the method at the damage calibration level. As expected, the small and edge cracks and the finer scales are affected easily by noise and remain in the non-detected zone. Windowing the measured data is observed to lower the efficiency of kurtosis and pseudofractal based detection methods. An improved damage calibration scheme would require penetrating within the dark zones indicated in Fig. 1 and open up the bright zone more broadly. This map thus provides a simple way to qualitatively compare various detection systems. In fact, the detectability map for a single realization of a measurement allows assessing the quality of the obtained data. The calibrated efficiency surfaces can be approximated with polynomial and exponential models. A particular example for wavelet based damage calibration is investigated for an SNR 95 dB in Fig. 2. The calibration efficiency η is conjectured in this paper to approximately follow the expression

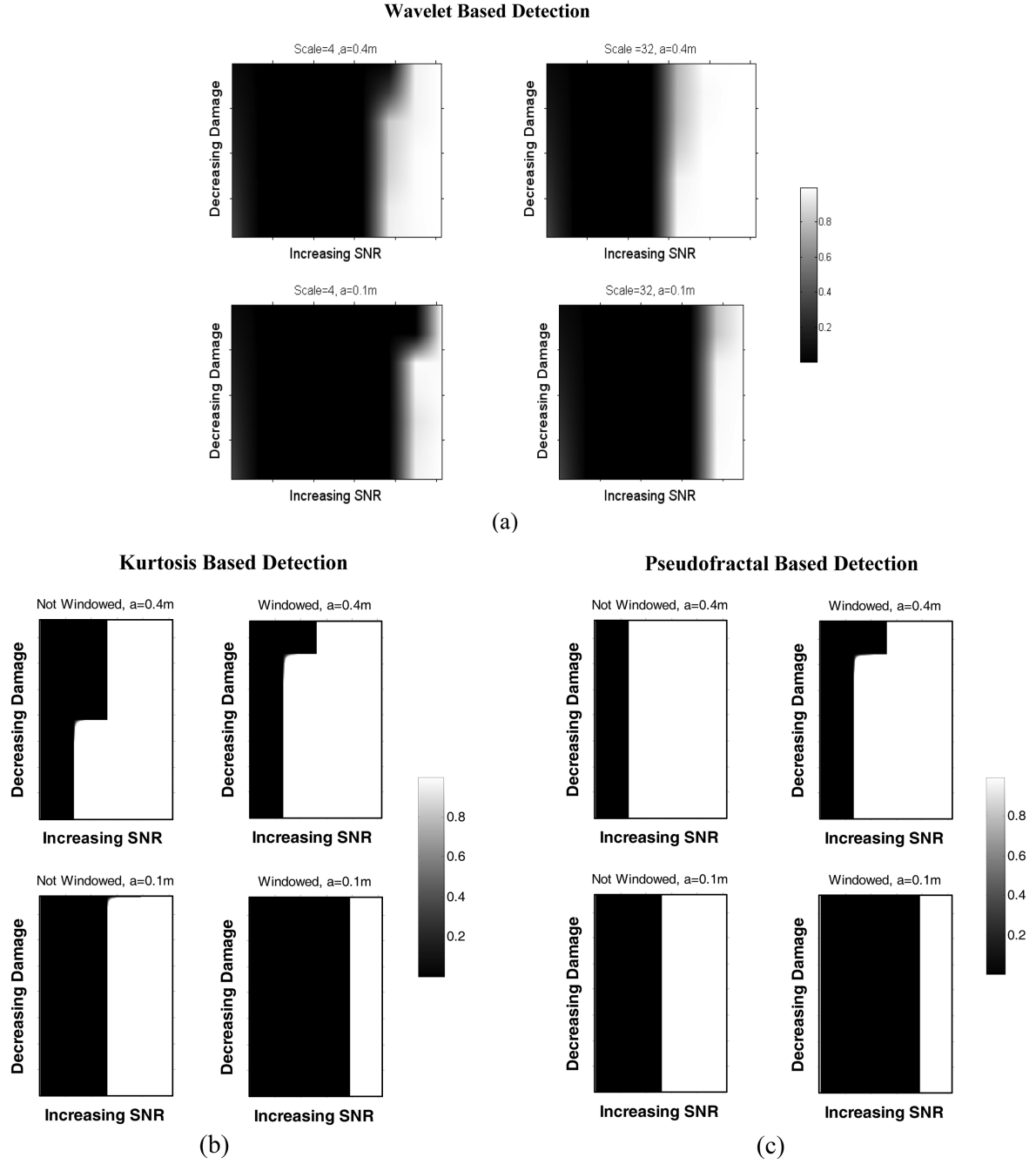


Fig. 1 Detectability map for (a) wavelet, (b) kurtosis and (c) pseudofractal based damage calibration

$$\eta = \begin{cases} e^{P(\delta)} & \text{if } c \geq c_{th} \\ 0 & \text{if } c < c_{th} \end{cases} \quad (11)$$

where $P(\delta)$ is a polynomial of the crack depth ratio

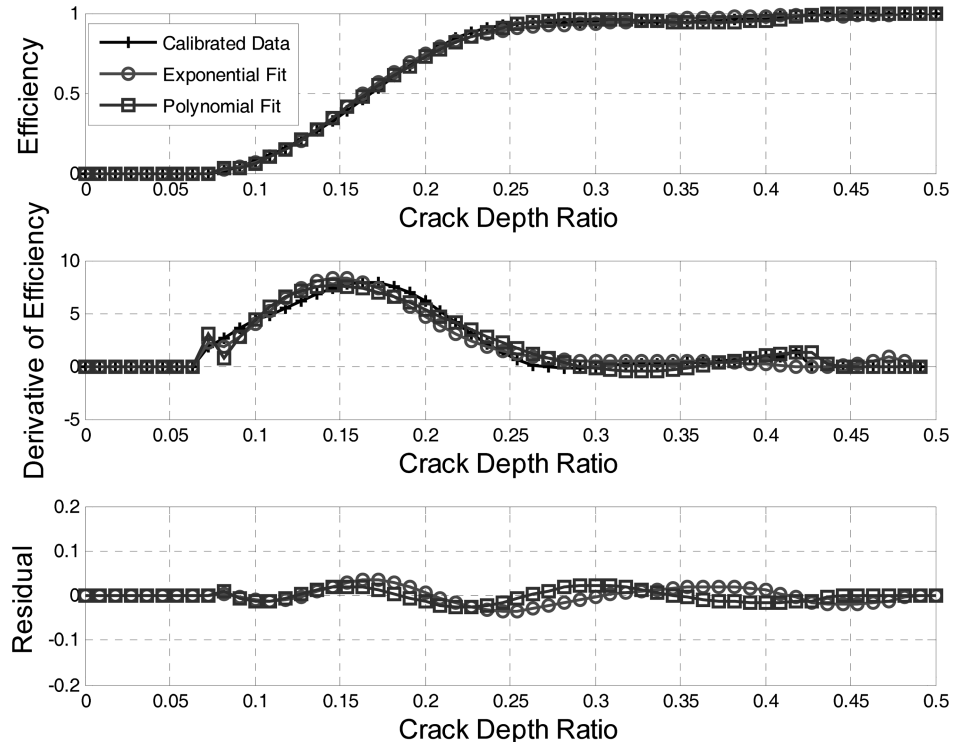


Fig. 2 Model fit of the nature of efficiency curves

$$P(\delta) = \alpha_0 + \sum_{i=1}^n \alpha_i \delta^i \quad (12)$$

and $\alpha_{(i)}$ being the fitted coefficients of the polynomial. The polynomial fitted in the exponential model in Fig. 2 is of the order 5. The derivative of the efficiency in Fig. 2 gives an idea about the sensitivity of the wavelet based damage calibration. The calibration is found relatively less sensitive for very low or very high crack depth ratios and comparatively high for medium damage conditions. As a result, the transition from a low to medium damage level or from a medium to a high damage level is very distinct. The residuals indicate the effectiveness of the fitted plots (Fig. 2). The damage detection and subsequent calibration using wavelets is more sensitive to noise at finer scales, while at coarser scales it is difficult to locate the position of small and edge cracks. As a result, the best scale for analysis depends on the SNR, the crack depth ratio and the extent of damage.

5. Comparison of damage calibration efficiency surfaces

The efficiency of damage calibration using wavelet, kurtosis and pseudofractal based methods are presented for a wide range of damage locations, extent and noise level. Each efficiency value for a certain location, extent of damage and a background noise level has been simulated 1000 realizations and averaged. Figs. 3(a) and 3(b) show the efficiency surfaces of wavelet based damage

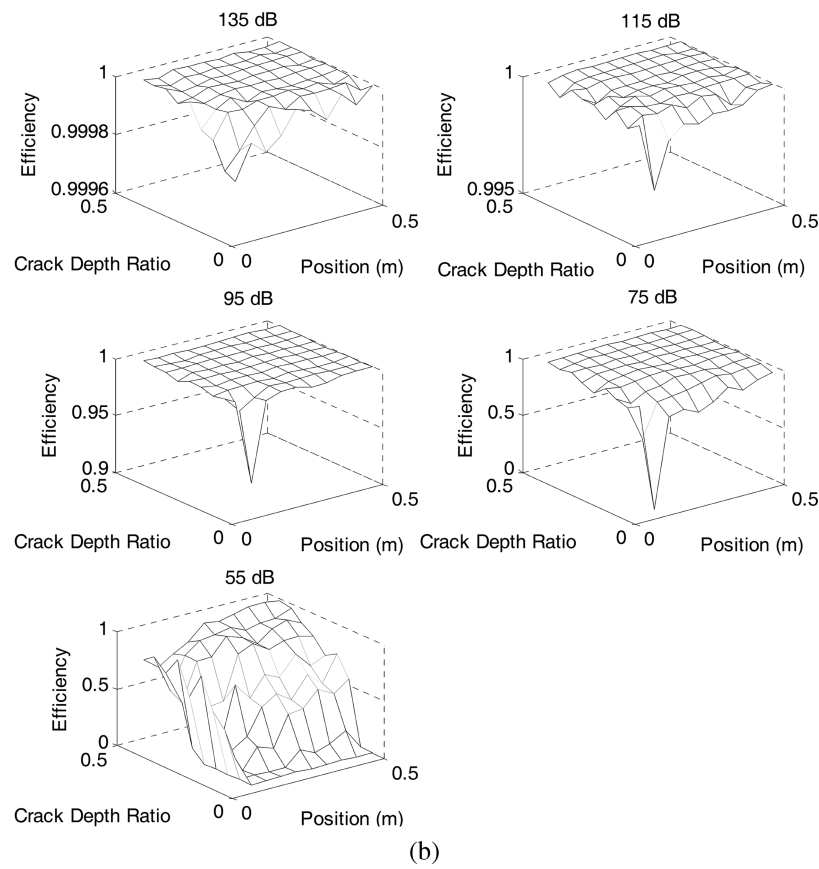
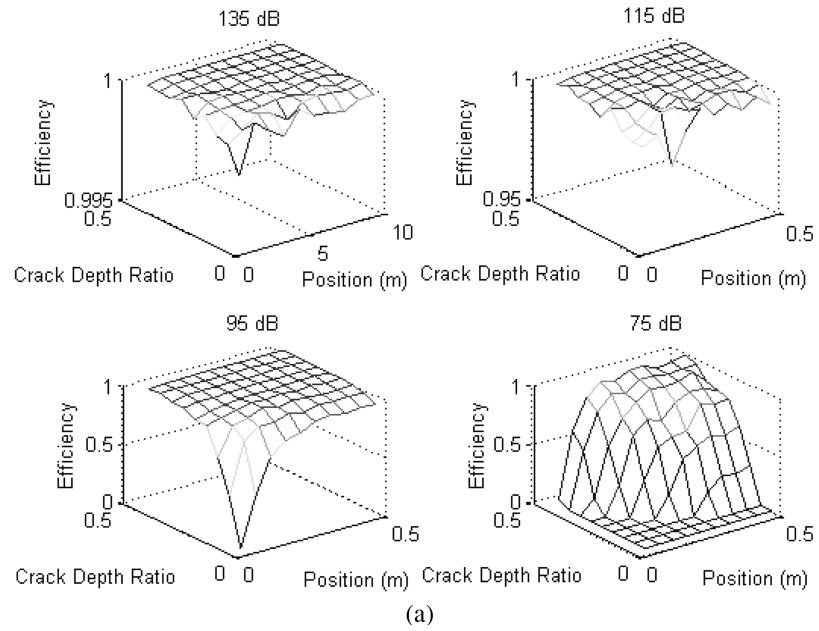


Fig. 3 Efficiency surfaces for wavelet based damage detection at (a) Scale 8 and (b) Scale 32

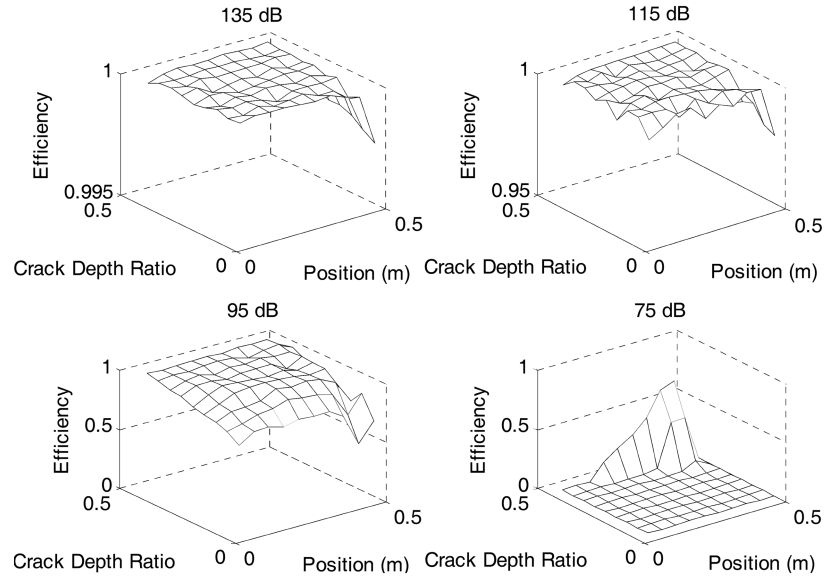


Fig. 4 Efficiency surfaces for kurtosis based damage detection

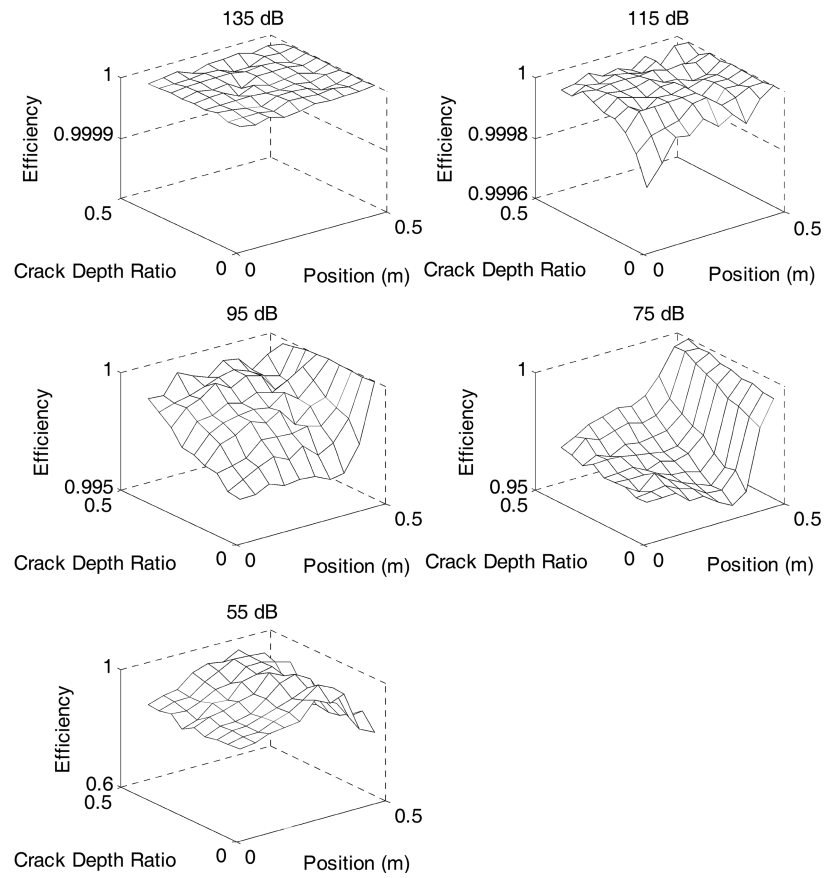


Fig. 5 Efficiency surfaces for pseudofractal based damage detection

calibration for scales 8 and 32 respectively. A sharp transition from an efficient calibration to a comparatively inefficient calibration is observed. It is also important, that simply by employing a coarser scale (scale 32 instead of scale 8) the stability domain against noise level can be extended significantly. However, it should also be kept in mind that a very coarse scale would have a low capability locating the position of damage in a precise fashion. This flexibility cannot be obtained from kurtosis or pseudofractal calibration methods. Efficiency surfaces are shown in Figs. 4 and 5 respectively for kurtosis and pseudofractal based damage detection. The sharp transition to inefficiency with changing noise is observed. The pseudofractal based calibration is found to be more stable against noise for kurtosis based and even the wavelet based calibration at scales 8 and 32.

6. Conclusions

This paper compares the various techniques of damage detection in structures which can find a sharp local change or a singularity in any measured noisy response of a damaged structure or in any of its derivatives. A detailed numerical investigation has been carried out to compare the effectiveness of the damage detection methods under measurement noise for calibration of the extent of damage. The efficiency surfaces for damage calibration have been constructed for a wide range of damage location, extent and level of associated noise. Wavelet based damage detection is very flexible due to the capability of analyses at various scales, robust against noise and consistent in terms of calibration. The performance of a kurtosis based method is poor in terms of noise robustness. Pseudofractal based damage detection is not as flexible as a wavelet based detection but is observed to be extremely robust against noise. Both kurtosis and pseudofractal methods suffer from the problem of being subjective in terms of choosing the number of discrete points in the moving window employed for computation. The detection methods compared in this paper considered possess a narrow domain of SNR where the masking due to measurement noise suddenly becomes significant.

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