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Remaining life prediction of concrete structural components accounting for tension softening and size effects under fatigue loading

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Abstract. This paper presents analytical methodologies for remaining life prediction of plain concrete structural components considering tension softening and size effects. Non-linear fracture mechanics principles (NLFM) have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. Size effect has been accounted for by modifying the Paris law, leading to a size adjusted Paris law, which gives crack length increment per cycle as a power function of the amplitude of a size adjusted stress intensity factor (SIF). Details of tension softening effects and size effect in the computation of SIF and remaining life prediction have been presented. Numerical studies have been conducted on three point bending concrete beams under constant amplitude loading. The predicted remaining life values with the combination of tension softening & size effects are in close agreement with the corresponding experimental values available in the literature for all the tension softening models.

Keywords: concrete fracture; fatigue loading; tension softening; stress intensity factor; size effect; crack growth; remaining life.

1. Introduction

Concrete is a widely used material that is required to withstand a large number of cycles of repeated loading in structures such as highways, dams, airports, bridges and offshore structures. The present state-of-the-art of designing such structures against the fatigue mode of distress is largely empirical, gained by many years of experience. Fracture mechanics is a rapidly developing field that has great potential for application to concrete structural design. As long as the designer is dealing with structures made of similar to those for which the relationships were derived, the performance can be reasonably well predicted. However, as conditions change, a need exists for a rational approach. Concrete contains numerous flaws, such as holes or air pockets, precracked aggregates, lack of bond between aggregate and matrix, etc., from which cracks may originate. Cracks generally propagate in a direction, which is perpendicular to the maximum tensile stress. The fracture behavior of concrete is greatly influenced by the fracture process zone (FPZ). The variation of FPZ

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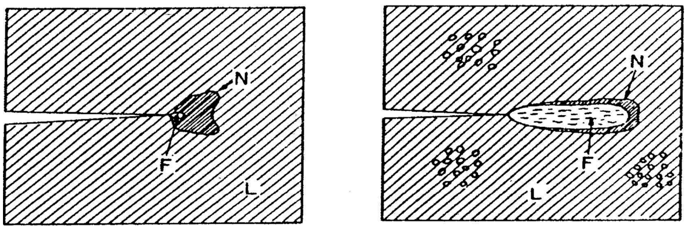


(a) Ductile-brittle (Metals) (b) Quasi-brittle (concrete)

Fig. 1 FPZ in ductile and brittle materials (Bazant 2002)

along the structure thickness or width is usually neglected. The inelastic fracture response due to the presence of FPZ may then be taken into account by cohesive pressure acting on the crack faces. Fig. 1 shows FPZ in ductile-brittle materials and quasi-brittle materials (Bazant 2002).

Few experimental investigations on fatigue crack propagation in concrete have been reported (Ingraffea 1977). The rate of fatigue crack growth in concrete exhibits an acceleration stage that follows an initial deceleration stage. In the deceleration stage the rate of crack growth decreases with increasing crack length, whereas in the acceleration stage there is a steady increase in crack growth rate up to failure. Fracture mechanics principles were applied to describe the crack growth during the acceleration stage of fatigue crack growth in concrete. It has been observed that the Paris law coefficients are dependent on the material composition potentially explaining the large differences in the values of the Paris law coefficients. Prasad and Krishnamoorthy (2002) developed a 2D computational model for investigation of crack formation and crack growth in plain and RC plane stress members. Thomas *et al.* (2005) described methodologies for modeling 3D crack propagation in unreinforced concrete. It was mentioned that tensile failure involves progressive micro-cracking, debonding and other complex irreversible processes of internal damage. Wu *et al.* (2006) proposed an analytical model to predict the effective fracture toughness of concrete based on the fictitious crack model. The equilibrium equations of forces in the section were derived in combination with the plane section assumption. Slowik *et al.* (2006) presented a method for determining tension softening curves of cementitious materials based on an evolutionary algorithm. Extensive research work was carried out towards numerical modelling of fracture and size effect in plain concrete using lattice model by Raghu Prasad *et al.* (2006). From literature, it has also been observed that the research work towards crack growth analysis and remaining life prediction of concrete structural components considering tension softening is limited (Ingraffea 1977, Stuart 1982, Baluch *et al*. 1987, Bazant and Xu 1991, Ramsamooj 1994, Toumi and Turatsinze 1998, Matsumoto and Li 1999, Subramaniam *et al*. 2000, Prasad and Krishnamoorthy 2002, Gasser and Holzapfel 2005, Wu *et al*. 2006, Slowik *et al*. 2006, Raghu Prasad and Vidya Sagar 2006). There is a scope to conduct crack growth analysis and remaining life prediction of concrete structural components considering tension softening effect in to account under fatigue loading. Further, it has been observed from the literature that there is numerous tension softening models to account for softening effect. There is scanty information in choosing the appropriate softening model for reliable remaining life prediction.

The change of a structural property when the size of a structural specimen changes is known as size effect related to this property. Most structures experience an increase in brittleness and a decrease is strength as their size increases. The size effect on fatigue in bending is strongly dependent on the structural size. Under the same flexural stress levels, the smaller the beam height, the longer is the fatigue life. The deformation characteristics such as fatigue crack growth history as well as final critical fatigue crack length at fatigue failure are also dependent on the beam size (Bazant and Schell 1993, Bazant 1984, Bazant and Kazemi 1990). Bazant (2000) presented most of the results related to size effect on the nominal strength of structures. Hanson and Ingraffea (2003) conducted numerical studies to compare the fracture toughness values for concrete using the size effect, two parameter and fictitious crack models considering tension softening effect. From their analyses, it was observed that the fracture toughness values for the size effect and two parameter models tend to be less than those of fictitious crack model. Karihaloo *et al*. (2006) examined the deterministic size effect in the strength of cracked concrete structures and observed that the deterministic strength size effect becomes stronger as the size of the cracked structure increases but weakens as the size of the crack reduces relative to the size of the structure. Yi *et al*. (2007) carried out experiments to evaluate the size effect on the flexural compressive strength of RC flexural members. They observed that size effect is apparent where the flexural compressive strength at failure and the corresponding strain value and the ultimate strain decrease as the specimen size increases. From the literature, it is observed that the research on crack growth analysis and remaining life prediction of concrete structural components accounting for size effect under fatigue loading is limited (Bazant and Schell 1993, Bazant 1984, Bazant and Kazemi 1990, Bazant 2000, Hanson and Ingraffea 2003, Karihaloo *et al*. 2006, Yi *et al*. 2007).

This paper presents methodologies for remaining life prediction of plain concrete structural components considering tension softening and size effects. Non-linear fracture mechanics principles (NLFM) have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. Size effect has been accounted for by modifying the Paris law, leading to a size adjusted Paris law, which gives crack length increment per cycle as a power function of the amplitude of a size adjusted stress intensity factor (SIF). Details of tension softening effects and size effect in the computation of SIF and remaining life prediction have been presented. Numerical studies have been conducted on three point bending concrete beams under constant amplitude loading for four cases namely, (i) considering linear elastic fracture mechanics (LEFM) (ii) combination of LEFM and size effect (iii) NLFM and (iv) combination of NLFM and size effect. It is observed that the predicted remaining life values using LEFM principles are significantly lesser compared to the corresponding experimental values. The predicted remaining life values are lesser in the case of combined LEFM and size effect when compared with those of the values predicted in the case of LEFM alone. The predicted remaining life values with the combination of NLFM & size effects are in close agreement with the corresponding experimental values for all the tension softening models. From the studies, it can be concluded that the difference in the predicted remaining life with and without considering size effect is not very significant.

2. Concrete fracture models

Based on different energy dissipation mechanisms, NLFM models for quasi-brittle materials can be classified as a fictitious crack approach (cohesive crack model) and an equivalent-elastic crack approach. Fracture mechanics models using only the Dugdale - Barenblatt energy dissipation mechanism are usually referred to as the fictitious crack approach, whereas fracture mechanics models using only the Griffth-Irwin energy dissipation mechanism are usually referred to as the effective-elastic crack approach or equivalent-elastic crack approach.

The strain energy release rate for a mode I crack, G_q, can be expressed as (Shah and Swartz 1995)

$$G_q = G_{Ic} + G_\sigma \tag{1}$$

The value of G_{Ic}, can be evaluated based on LEFM and is called the critical energy release rate. G_σ is equal to the work done by the cohesive pressure over a unit length of the crack for a structure with a unit thickness.

Brief description of fictitious crack model is presented below (Shah and Swartz 1995).

2.1 Fictitious crack approach (Cohesive crack model)

The fictitious crack approach assumes that energy to create the new surface is small compared to that required to separate them, and the energy rate term G_{IC} vanishes in Eq. (1). Fig. 2 shows the simulation of a newly formed crack structures and the corresponding fracture process zone (Shah and Swartz 1995). As a result, the energy dissipation for crack propagation can be completely characterized by the cohesive stress-separation relationship $\sigma(w)$. Since all energy produced by the applied load is completely balanced by the cohesive pressure, Eq. (1) is reduced to (with $G_{Ic} = 0$)

$$G_q = \int_0^{w_t} \sigma(w)\,dw \tag{2}$$

w_t is the crack opening displacement at the initial crack tip.

Eq. (2) is valid for structures with a constant thickness. The fictitious crack is assumed to propagate when the principal tensile stress reaches the tensile strength of material, f_t.

Cohesive crack model requires a unique σ - w curve to quantify the value of energy dissipation. The choice of the σ - w function influences the prediction of the structural response significantly, and the local fracture behaviour, for example the crack opening displacement, is particularly sensitive to the shape of σ - w. Many different shapes σ - w curves, including linear, bilinear, trilinear, exponential, and power functions, have been used in the literature. Some of the widely used σ - w curves with appropriate expressions are listed in Table 1.

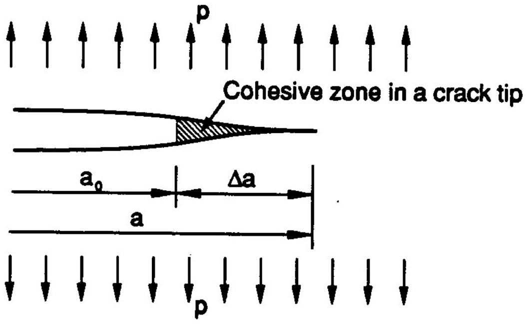


Fig. 2 Mode I crack for fictitious crack approach (Shah and Swartz 1995)

Table 1 Different types of closing pressure for FPZ

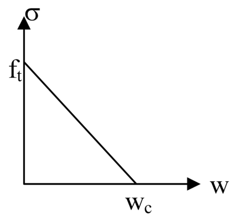
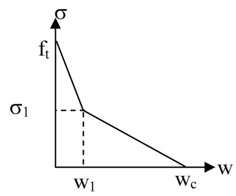
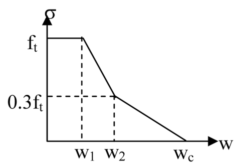
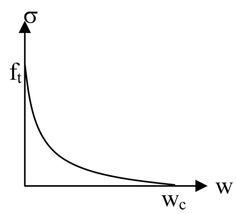
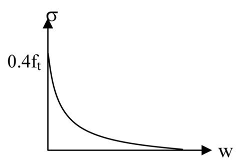
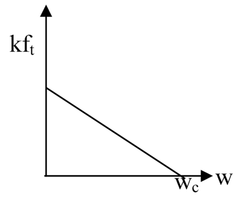
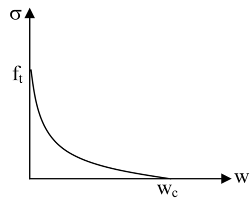
Type	Expression	Shape
Linear curve - Hillerborg *et al.*, 1976	$\sigma = f_t(1 - w/w_c)$	
Bilinear curve - Roelfstra and Wittmann, 1986	$\sigma = \begin{cases} f_t - (f_t - \sigma_1)w/w_1 & \text{for} \quad w \le w_1 \\ \sigma_1 - \sigma_1(w - w_1)/(w_c - w_1) & \text{for} \quad w_1 > w \end{cases}$	
Trilinear curve - Liaw *et al.*, 1990	$\sigma = \begin{cases} f_t & \text{for} \quad w \le w_1 \\ f_t - 0.7f_t(w - w_1)(w_2 - w_1) & \text{for} \quad w_1 < w \le w_2 \\ 0.3f_t(w_c - w)/(w_c - w_2) & \text{for} \quad w_2 < w \le w_c \end{cases}$	
Exponential curve - Footer *et al.*, 1986	$\sigma = f_t\left(1 - \frac{w}{w_c}\right)^n$ where n is a fitting parameter	
Reinhardt, 1985	$\sigma = f_t\left\{1 - \left(\frac{w}{w_c}\right)^n\right\}$ where $0 < n < 1$ is a fitting parameter	
Gopalaratnam and Shah, 1985 similar relationship was also suggested by Cedolin *et al.*, 1987	$\sigma = f_t \exp(kw^{\lambda})$ where k and λ are material parameters $k = -0.06163$ and $\lambda = 1.01$ for concrete with f_c' values of 33-47 MPa.	
Power curve - Du *et al.*, 1990	$\sigma = 0.4f_t(1 - w/w_c)^{1.5}$	

Table 1 Continued

Type	Expression	Shape
Bilinear curve with $w_1 = 0$ - Figueiras and Owen, 1984	$\sigma = kf_t(1 - w/w_c)$ Where, k = constant	
Power curve - Hordijk, 1991	$\sigma = f_c\left\{\left[1 + \left(a_1\frac{w}{w_3}\right)^3\right]_{\exp}\left(-a_2\frac{w}{w_c}\right) - \frac{w}{w_c}(1 + a_1^3)\exp(-a_2)\right\}$ where a_1 and a_2 are fitting parameters	

3. SIF accounting for tension softening and remaining life prediction

The mechanism under fatigue loading in plain concrete may be attributed to progressive bond deterioration between aggregates and matrix or by development of cracks existing in the concrete matrix. These two mechanisms may act together or separately, leading to complexity of the fatigue mechanism. It is well known fact that concrete typically exhibits nonlinear fracture processes because of the large FPZ, leading to LEFM based approach objectionable. Hence an analytical model for assessing the fatigue life of concrete accounting the tension softening effect is required. The following are the basic assumptions of tension softening.

<u>Modelling assumptions</u>

i. Plane sections of the beam remain plane after deformation
ii. Fictitious crack surface remains plane after deformation
iii. Normal closing tractions acting on the fictitious crack follow the linear stress crack opening displacement
iv. Bending stress in the concrete along the bottom of the beam is equal to the traction normal to the crack mouth at the bottom of the beam.

To incorporate the tension softening behavior, based on the principle of superposition the stress intensity factor (SIF) has to be modified as (Fig. 3)

$$K_I = K_I^P - K_I^q \tag{3}$$

where K_I^P is SIF for the concentrated load P in a three point bending beam geometry, and K_I^q is SIF due to the closing force applied on the effective crack face inside the process zone, which can be obtained through Green's function approach by knowing the appropriate softening relation. Superposition principle is used by accounting for the nonlinearity in incremental form. SIF due to applied load and due to closing force will act in opposite directions. K_I will not become zero as the magnitude of K_I^q is around 10 to 20% of K_I^P.

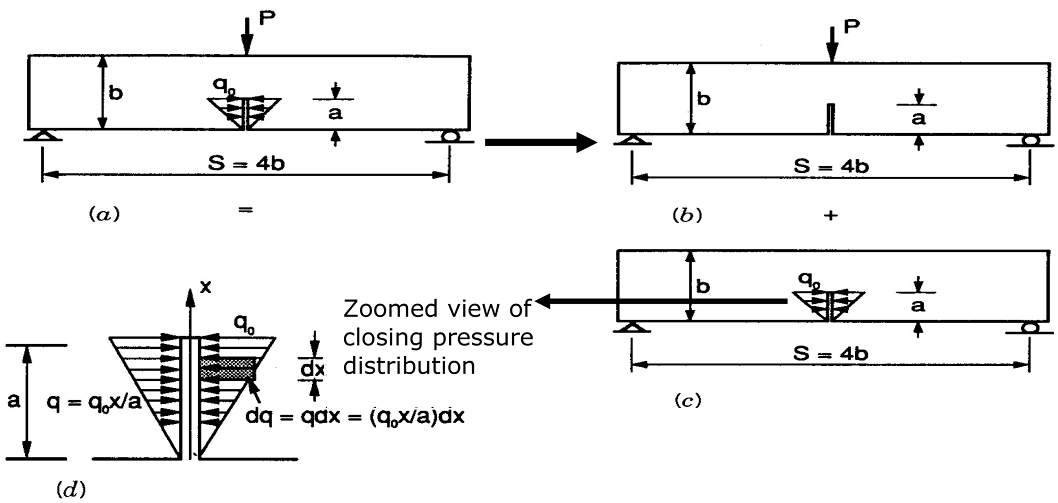


Fig. 3 Illustration of superposition principle (Shah and Swartz 1995)

3.1 Computation of K_I^P

SIF due to the concentrated load P can be calculated by using LEFM principles. A three-point bending beam is shown in Fig. 3(b). The SIF for the beam can be expressed as

$$K_I^P = \sigma\sqrt{\pi a}\, g_1\left(\frac{a}{b}\right) \tag{4a}$$

$$\text{where, } \sigma = \frac{3PS}{2b^2 t} \tag{4b}$$

where P = applied load, a = crack length, b = depth of the beam, t = thickness and $g_1(a/b)$ = geometry factor, depends on the ratio of span to depth of the beam and is given below for S/b = 2.5 (Tada *et al.* 1985)

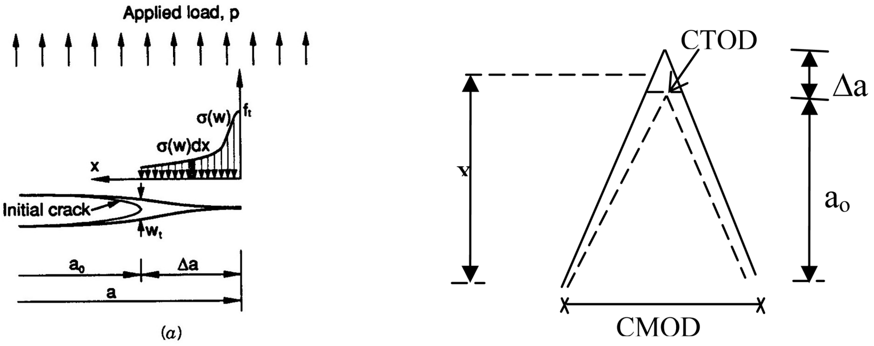
$$g_1\left(\frac{a}{b}\right) = \frac{1.0 - 2.5a/b + 4.49(a/b)^2 - 3.98(a/b)^3 + 1.33(a/b)^4}{(1-a/b)^{3/2}} \tag{5}$$

3.2 Computation of K_I^q

The incremental SIF due to the closing force dq can be written as (Shah and Swartz 1995)

$$dK_I^q = \frac{2}{\sqrt{\pi \Delta a}} dq\; g\left(\frac{a}{D}, \frac{x}{a}\right) \tag{6}$$

where dq can be expressed as function of softening stress distribution over the crack length Δa; the function 'g' represents the geometry factor.



(a) Modelling of quasi-brittle crack with crack surfaces in contact (Shah and Swartz, 1995)

(b) Schematic diagram of crack opening displacement

Fig. 4 Cohesive crack modelling

Calculation of 'dq'

By using the above concept (Fig. 3(d)), cohesive crack can be modelled in the following manner Fig. 4).

The crack opening displacement w at any point x is assumed to follow linear relationship (Fig. 4) and can be expressed as

$$w = \delta\left(\frac{a_0 - x}{\Delta a} + 1\right) \qquad a_0 \le x \le a_{eff} \tag{7}$$

where δ is the crack tip opening displacement, a_0 is initial crack length and

$$a_{eff} = a_0 + \Delta_a$$

As an example, let us consider linear softening law (refer Table 1)

$$\sigma = f_t(1 - w/w_c) \tag{8}$$

Where f_t = tensile strength of concrete and w_c= critical crack opening displacement

Substituting for w from Eq. (7) in the linear softening law given by Eq. (8) one can obtain

$$dq = \sigma = f_t\left\{1 - \frac{\delta}{w_c}\left(\frac{a_0 - x}{\Delta a} + 1\right)\right\} \tag{9}$$

The crack opening displacement at any point $\delta(x)$ can be calculated using the following equation

$$\delta(x) = \mathrm{CMOD}\, g_3\left(\frac{a}{b}, \frac{x}{a}\right) \text{ where}$$

$$g_3\left(\frac{a}{b}, \frac{x}{a}\right) = \left\{\left(1 - \frac{x}{a}\right)^2 + \left(1.081 - 1.149\frac{a}{b}\right)\left[\frac{x}{a} - \left(\frac{x}{a}\right)^2\right]\right\}^{1/2} \tag{10}$$

where CMOD is crack mouth opening displacement and is calculated using the following formula.

$$\text{CMOD} = \frac{4\sigma a}{E} g_2\left(\frac{a}{b}\right) \tag{11}$$

where $g_2(a/b)$ is geometric factor, depends on the ratio of span to depth of the beam and is given below for $S = 2.5b$

$$g_2\left(\frac{a}{b}\right) = \frac{1.73 - 8.56a/b + 31.2(a/b)^2 - 46.3(a/b)^3 + 25.1(a/b)^4}{(1-a/b)^{3/2}} \tag{12}$$

Hence, replacing dq in Eq. (6) and integrating over length Δa, K_I^q can be obtained as

$$K_I^q = \int_{a_0}^{a_{eff}} \frac{2f_t}{\sqrt{\pi\Delta a}}\left\{1 - \frac{\delta}{w_c}\left(\frac{a_0 - x}{\Delta a} + 1\right)\right\} g\left(\frac{a}{b}, \frac{x}{a}\right) dx \tag{13}$$

where

$$g\left(\frac{a}{b}, \frac{x}{a}\right) = \frac{3.52(1-x/a)}{(1-a/b)^{3/2}} - \frac{4.35 - 5.28x/a}{(1-a/b)^{1/2}} + \left[\frac{1.30 - 0.30(x/a)^{3/2}}{\sqrt{1-(x/a)^2}} + 0.83 - 1.76\frac{x}{a}\right]\left[1 - \left(1 - \frac{x}{a}\right)\frac{a}{b}\right] \tag{14}$$

Similar expressions are obtained for other models such as bilinear, trilinear, exponential, power curve model etc.,

Remaining life can be predicted by using any one of the standard crack growth equations (such as Paris, Erdogan - Ratwani, etc.)

$$\frac{da}{dN} = f(\Delta K) \tag{15}$$

Here ΔK can be computed by using following expression

$$\Delta K = K_{\max} - K_{\min}, \quad \text{where} \quad K_{\max} = K^p - K^q \tag{16}$$

4. Accounting for size effect

In general, Paris law is of the form

$$\frac{da}{dN} = C(\Delta k)^m \tag{17}$$

where C, m are crack growth constants. This law can also be expressed as

$$\frac{da}{dN} = K\left(\frac{\Delta k}{K_{Ic}}\right)^m \tag{18}$$

where $K = C \cdot K_{Ic}^m$ and K_{Ic} = fracture toughness under monotonic loading.

Since it is known that Paris law is valid only for one specimen size (crack growth parameters being adjusted for that size) or asymptotically for very large specimens. Paris law is combined with the size effect law, leading to a size adjusted Paris law, which gives the crack length increment per cycle as a power function of the amplitude of a size adjusted SIF. The size adjustment is based on the relative specimen size, also called as brittleness number of the structure. Hence, K_{Ic} will become,

$$K_{Ic} = K_{If}\sqrt{\frac{\beta}{1+\beta}} \tag{19}$$

in which $\beta = D/D_0$ = relative specimen size, where, D is beam depth, K_{If} is a constant which represents the asymptotic value of fracture toughness for infinitely large specimen coinciding with the asymptotic value of the R-curve and D_0 is an empirical constant that may be interpreted as the size in the middle of the transition between the strength theory and linear elastic fracture mechanics.

4.1 Calculation of D_o

The length of fracture process zone for an infinitely large specimen, c_f can be written as (Bazant and Kazemi 1990)

$$c_f = \frac{g(a_0/b)}{g'(a_0/b)}D_0 \quad \text{assume, } \frac{a_0}{b} = x \tag{20}$$

The geometric factor $g(x)$ for a three point bending specimen as shown in Fig. 5 can be calculated using the following expression (RILEM 1990)

$$g(x) = \left(\frac{s}{b}\right)^2 \pi x[1.5g_1(x)]^2 \tag{21}$$

where, $g_1(x)$ is the geometry factor corresponding to S/b ratio.

For S/b = 2.5, the geometric factor ($g_1(x)$), can be calculated using Eq. (5). The first derivative of $g(x)$ w. r. t x is

$$g'(x) = \left(\frac{s}{b}\right)^2 \pi\frac{d}{dx}x[1.5g_1(x)]^2 \tag{22}$$

where

$$\frac{d}{dx}x[1.5g_1(x)]^2 = \frac{\frac{9}{4}(1-2.5x+4.49x^2-3.98x^3+1.33x^4)^2}{(1-x)^4} + \frac{\frac{9}{4}x(1-2.5x+4.49x^2-3.98x^3+1.33x^4)}{(1-x)^3}$$
$$(1-2.5+8.89x-11.94x^2+5.32x^3) + \frac{\frac{27}{4}x(1-2.5x+4.49x^2-3.98x^3+1.33x^4)^2}{(1-x)^4} \tag{23}$$

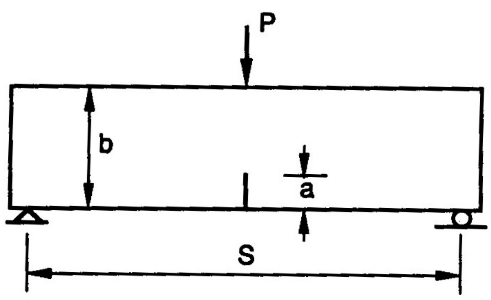


Fig. 5 Three point bending specimen

Similarly for s/b = 4.0, the geometric factor can be calculated using the following expression (Tada *et al*. 1985)

$$g_1(x) = \frac{1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2)}{\sqrt{\pi}(1+2x)(1-x)^{3/2}} \tag{24}$$

The first derivative of $g(x)$ w. r. t x is same as given in Eq. (22) and

$$\frac{d}{dx}x[1.5g_1(x)]^2 = \frac{\frac{9}{4}(1.99 - x(1-x)(2.15 - 3.93 + 2.7x^2)^2}{(1.7724 + 3.5448x)^2(1-x)^3} + \frac{\frac{9}{2}x(1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2)}{(1-x)^3(1.7724 + 3.5448x)^2}$$

$$(-(1-x)(2.15 - 3.93x + 2.7x^2) + x(2.15 - 3.93x + 2.7x^2) - x(1-x) - (-3.93 + 5.4x) -$$

$$\frac{15.952x(1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2))^2}{(1.7724 + 3.5448x)^3(1-x)^3} + \frac{6.75x(1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2))^2}{(1.7724 + 3.5448x)^2(1-x)^4} \tag{25}$$

5. Numerical studies

Crack growth studies and remaining life prediction has been carried out for three point bending concrete beams using LEFM, NLFM and size effect principles under constant amplitude loading. Three example problems are presented below.

5.1 Example 1: Toumi and Turatsinze (1998)

This problem was experimentally studied by Toumi and Turatsinze (1998) and the details of the problem are given below.

Max.stress = 1.125 MPa	Length (s) = 320 mm
Min.stress = 0.298 MPa	depth (b) = 80 mm
Initial crack length = 4 mm	thickness (t) = 50 mm
Final crack length = 0.5b = 40 mm	tensile strength (f_t) = 4.2 MPa
Fract. toughness = 0.63 MPa$\sqrt{m}$	

w_c = 0.2 mm, w_1 = 0.03 mm, w_2 = 0.05 mm, σ_1 = 1.2 MPa
Modulus of elasticity = 37749 MPa and $n_1 = 0.61$
The bending tensile stress (σ_b) can be calculated by using Eq. 4(b).

Using the above data, crack growth analysis and remaining life prediction has been carried out using LEFM, NLFM and size effect principles. Tables 2(a) and 2(b) shows the predicted remaining life values for various loading cases along with the experimental values presented by Toumi and Turatsinze (1998). The following observations can be made from Tables 2(a) and 2(b).

- The predicted remaining life values with LEFM are within about 12% of the corresponding experimental values.

Table 2(a) Predicted remaining life with LEFM and NLFM

Max. stress MPa	LEFM	Linear	Bi-Linear	Trilinear	Expo-Footer	Expo-Reinhardt	Power model	Exptl. Toumi Bascos and Turatsinze (1998)
1.125	28689 10.96%	33304 -3.36%	32252 -0.09%	32941 -2.23%	33310 -3.38%	33102 -2.73%	30887 4.14%	32222
1.05	57251 9.99%	66747 -4.93%	63892 -0.44%	65032 -2.23%	66781 -4.98%	65348 -2.73 %	61011 4.09%	63611
0.975	62603 9.85%	74775 -7.68%	69692 -0.36%	70998 -2.24%	74791 -7.70%	71346 -2.74%	66592 4.11%	69444
0.9	16188 11.70%	19102 -4.19%	18479 -0.80%	18801 -2.55%	19116 -4.27%	18892 -3.05%	17612 3.93%	18333

Table 2(b) Predicted remaining life with the combination of LEFM/NLFM & Size effect

Max. stress MPa	LEFM	Linear	Bi-Linear	Trilinear	Expo-Footer	Expo-Reinhardt	Power model	Exptl. Toumi Bascos and Turatsinze (1998)
1.125	27937 13.29%	32434 -0.66%	31409 2.52%	32080 0.44%	32439 -0.67%	32236 -0.04%	30077 6.66%	32222
1.05	54081 14.98%	64893 -2.02%	62351 1.98%	63331 0.44%	64821 -1.90%	63741 -0.20%	59012 7.23%	63611
0.975	59861 13.80%	71092 -2.37%	67892 2.23%	69140 0.44%	72126 -3.86%	69692 -0.36%	64242 7.49%	69444
0.9	15802 13.81%	18602 -1.47%	17891 2.41%	18320 0.07%	18714 -2.08%	18495 -0.88%	17011 7.21%	18333

- The predicted remaining life values with the combination of LEFM and size effect are within about 15% of the corresponding experimental values.
- The predicted remaining life values accounting for tension softening effect (NLFM) are within about +4% to −8% of the experimental values.
- In the case of NLFM, the predicted remaining life values with linear, bi-linear, tri-linear, exponential-Footer and exponential-Reinhardt models are larger compared to the experimental values whereas power curve model predicts lesser values compared to the experimental values.
- The predicted remaining life values accounting for combination of NLFM and size effect are within about +8% to −4% of the experimental values.
- In the case of combination of NLFM and size effect, the predicted remaining life values with linear, exponential – Footer and exponential-Reinhardt models are larger compared to the experimental values and bi-linear, tri-linear and power curve models predicts lesser values compared to the experimental values.
- In general, the predicted remaining life values with the combination of NLFM & size effects are in close agreement with the corresponding experimental values for all the tension softening models.

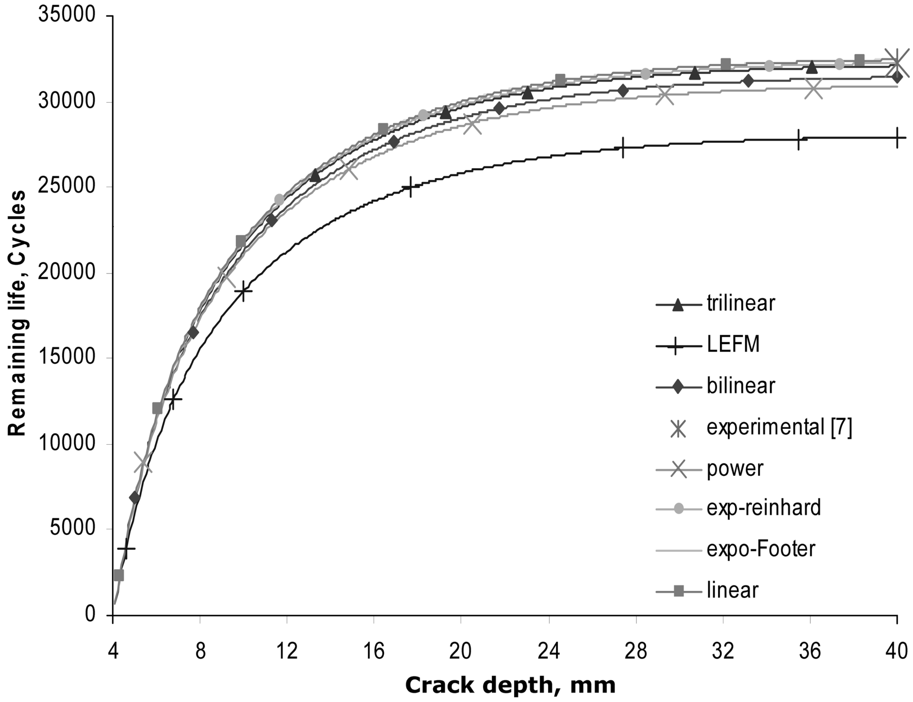


Fig. 6 Remaining life with combination of LEFM/NLFM & Size effect

- From the studies, it can be concluded that the difference in the predicted remaining life with and without considering size effect is not very significant.

Fig. 6 shows a plot of crack length vs. remaining life for a loading case with stress =1.125 N/mm^2 using LEFM/NLFM and size effect principles.

5.2 Example 2: Bazant and Schell (1993)

This problem was experimentally studied by Bazant and Schell (1993). The details of the problem are presented below. Different beam depths and spans were considered for experimental studies.

Beam depth (b) = 38.1, 107.8, 304.8 mm
Span (S) = 2.5 * beam depth
Thickness (t) = 38.1 mm
Initial crack length = b/6 mm
Modulus of elasticity = 38,300 MPa
Tensile strength = 8.9 MPa

Tables 3(a) and 3(b) shows the predicted remaining life for the different loading cases using LEFM, NLFM and combination of these with size effect principles. Tables 3(a) and (b) also shows the experimentally found remaining life values for the various loading cases reported by Bazant and Schell (1993). The following observations can be made from Tables 3(a) and (b).

- The predicted remaining life values with LEFM are within about 14% of the corresponding experimental values.
- The predicted remaining life values with the combination of LEFM and size effect are within

Table 3(a) Predicted remaining life with LEFM and NLFM

Details of beam, mm	Max. and Min stress, MPa	Linear	Bi-Linear	Trilinear	Expo. model by Footer	Expo. model by Reinhardt	Power model	LEFM	Exptl. Bazant and William (1993)
b=38.1 S=95.25 t=38.1	0.291 0.0279	34862 -4.35%	33496 -0.26%	34129 -2.16%	34982 -4.70%	34672 -3.78%	31982 4.27%	29010 13.17%	33409
b=107.8 S=268.75 t=38.1	0.07422 0.00675	7789 -4.55%	7498 -0.64%	7662 -2.84%	7801 -4.71%	7754 -4.08%	7162 3.87%	6779 9.00%	7450
b=304.8 S=762 t=38.1	0.01915 0.00174	42812 -4.76	41146 -0.68%	41970 -2.70%	42798 -4.73%	42486 -3.96%	39102 4.32%	36642 10.34%	40867

Table 3(b) Predicted remaining life with the combination of LEFM/NLFM & Size effect

Details of beam, mm	Max. and Min stress, MPa	Linear	Bi-Linear	Trilinear	Expo. model by Footer	Expo. model by Reinhardt	Power model	LEFM	Exptl. Bazant and William (1993)
b=38.1 S=95.25 t=38.1	0.291 0.0279	33864 -1.36%	32614 2.38%	33256 0.46%	34125 -2.14%	33812 -1.21%	31102 6.90%	28264 15.4%	33409
b=107.8 S=268.75 t=38.1	0.07422 0.00675	7592 -1.90%	7289 2.16%	7432 0.24%	7599 -2.00%	7561 -1.49%	6902 7.36%	6583 11.64%	7450
b=304.8 S=762 t=38.1	0.01915 0.00174	41632 -1.87%	40158 1.73%	40632 0.58%	41802 -2.29%	41381 -1.26%	38001 7.01%	35201 13.86%	40867

about 16% of the corresponding experimental values.

- The predicted remaining life values accounting for tension softening effect (NLFM) are within about +5% to -7% of the experimental values.
- In the case of NLFM alone, the predicted remaining life values with linear, bi-linear, tri-linear, exponential-Footer and exponential-Reinhardt models are larger compared to the experimental values whereas power curve model predicts lesser values compared to the experimental values.
- The predicted remaining life values accounting for combination of NLFM and size effect are within about +7.0% to 3% of the experimental values.
- In the case of combination of NLFM and size effect, the predicted remaining life with linear, exponential – Footer and exponential - Reinhardt models is larger compared to the experimental values and bi-linear, tri-linear and power models predicts lesser values compared to the experimental values.
- In general, the predicted remaining life values with the combination of NLFM & size effects are in close agreement with the corresponding experimental values for all the tension softening models.

Table 4 Remaining life using combination of LEFM/NLFM and (Bilinear and trilinear) Size effect

Max. stress MPa	Stress ratio	initial Crack Length	Crack Growth Constants C m	Remaining life predicted by using		
				LEFM	Bi-Linear	Tri-Linear
0.5194	0.1	75mm	7.71e-25, 3.12	38078*	41768	42641
	0.2		5.78e-24, 3.12	33176	36415	37123
	0.3		1.72e-24, 3.15	25436	27968	28501
0.692	0.1	75mm	7.71e-25, 3.12	24536	26912	27492
	0.2		5.78e-24, 3.12	21987	24115	24618
	0.3		1.72e-24, 3.15	14789	16208	16584
0.4328	0.1	85 mm	7.71e-25, 3.12	25123	30984	28112
	0.2		5.78e-24, 3.12	22453	24601	25214
	0.3		1.72e-24, 3.15	17936	19692	20102

*-Experimental value 44000 (Baluch *et al.* 1987) (stress ratio = 0.1 and maximum stress = 0.5194 MPa)

- From the studies, it can be concluded that the difference in the predicted remaining life with and without considering size effect is not very significant.

5.3 Example 3 Baluch et al. (1987)

Another example problem has been chosen for crack growth analysis and remaining life prediction. This problem was experimentally studied by Baluch *et al.* (1987)

Length of supported span (S) = 1360 mm, Thickness (t) = 51 mm

Depth (b) = 152 mm, Fracture toughness = 1.16×10^6 N/m$^{3/2}$

Table 4 shows the predicted remaining life for different loading cases using LEFM and NLFM principles considering size effect. From Table 4, it can be observed that there is about 11% difference between the predicted value and the corresponding experimental value in the case of combination of LEFM and size effect. For other loading cases, the experimental values are not available in the literature for comparison. The methodologies for crack growth analysis and remaining life prediction accounting for tension softening effect have been tested and verified for the previous examples. For this example, the remaining life has been predicted employing bi-linear and tri-linear models with the combination of NLFM and size effect.

6. Conclusions

Methodologies for remaining life prediction of plain concrete structural components considering tension softening and size effects have been presented. NLFM principles have been used for crack growth analysis and remaining life prediction. Various tension softening models such as linear, bi-linear, bi-linear, tri-linear, exponential and power curve have been presented with appropriate expressions. Size effect has been accounted for by modifying the Paris law, leading to a size adjusted Paris law, which gives crack length increment cycle as a power function of the amplitude of a size adjusted SIF. The size adjustment is based on the brittleness number of the structure, representing the ratio of the structure size D to the transitional size D_o. Details of tension softening effects and size effect in the computation of SIF and remaining life prediction of concrete structural components has been presented.

Numerical studies have been conducted on three point bending concrete beams under constant amplitude loading for four cases namely, (i) LEFM (ii) combination of LEFM and size effect (iii) NLFM and (iv) combination of NLFM and size effect. Remaining life has been predicted for the above cases and the predicted values have been compared with the corresponding experimental observations. The predicted remaining life with LEFM is significantly lesser than those of the corresponding experimental values. In the case of NLFM, the predicted remaining life values with linear, bi-linear, tri-linear, exponential-footer and exponential-reinhardt models are larger compared to the experimental values whereas power curve model predicts lesser values compared to the experimental value. In the case of combination of NLFM and size effect, the predicted remaining life values with linear, exponential – Footer and exponential - Reinhardt models are larger compared to the experimental values and bi-linear, tri-linear and power curve models predicts lesser values compared to the experimental values. The predicted remaining life values with the combination of NLFM & size effects are in close agreement with the corresponding experimental values for all the tension softening models. From the studies, it can be concluded that the difference in the predicted remaining life with and without considering size effect is not very significant.

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