Structural Engineering and Mechanics, Vol. 32, No. 1 (2009) 37-53
DOI: http://dx.doi.org/10.12989/sem.2009.32.1.037

Reliability analysis of wind-excited structures using domain decomposition method and line sampling

L.S. Katafygiotis† and Jia Wang‡

Department of Civil Engineering, Hong Kong University of Science and Technology, Hong Kong, P.R. China

(Received July 4, 2008, Accepted October 28, 2009)

Abstract. In this paper the problem of calculating the probability that the responses of a wind-excited structure exceed specified thresholds within a given time interval is considered. The failure domain of the problem can be expressed as a union of elementary failure domains whose boundaries are of quadratic form. The Domain Decomposition Method (DDM) is employed, after being appropriately extended, to solve this problem. The probability estimate of the overall failure domain is given by the sum of the probabilities of the elementary failure domains multiplied by a reduction factor accounting for the overlapping degree of the different elementary failure domains. The DDM is extended with the help of Line Sampling (LS), from its original presentation where the boundary of the elementary failure domains are of linear form, to the current case involving quadratic elementary failure domains. An example involving an along-wind excited steel building shows the accuracy and efficiency of the proposed methodology as compared with that obtained using standard Monte Carlo simulations (MCS).

Keywords: reliability analysis; wind excitation; domain decomposition method; line sampling.

1. Introduction

In reliability problems of wind-excited structures where a deterministic linear structural model and a stochastic excitation model are assumed, the random variables used to model the random wind excitation can be combined in an N_Z-dimensional random vector $\mathbf{Z} = [Z_1, \ldots, Z_{N_Z}]$ with joint PDF $f(\mathbf{Z})$. For a given realization of the excitation, corresponding to a specific sample $\mathbf{Z}$, the structural response is computed through dynamic analysis and is checked as to whether it satisfies the adopted failure criteria. The failure probability can be written as

$$P_F = \int_{\mathbb{R}^{N_Z}} I_F(\mathbf{Z}) f(\mathbf{Z}) d\mathbf{Z} \tag{1}$$

where $I_F(\mathbf{Z})$ indicates the state of the structure for a given $\mathbf{Z}$: $I_F(\mathbf{Z}) = 1$ if the structure fails, in the sense that the structural responses exceed some pre-specified thresholds, and $I_F(\mathbf{Z}) = 0$ otherwise. Due to the large number N_Z of random variables and the complexity of the failure domain $\{\mathbf{Z} | I_F(\mathbf{Z}) = 1\}$, one needs to resort to statistical methodologies to evaluate the above reliability integral.

---

† Associate Professor, E-mail: lambros@ust.hk
‡ Graduate Student, E-mail: jayson@ust.hk

Monte Carlo simulations (MCS) (Rubinstein 1981, Proppe *et al*. 2003) offer a robust methodology well suited for solving such high-dimensional reliability problems. In MCS, the failure probability is estimated by the arithmetic average of the indicator function $I_F(\mathbf{Z})$ over the samples $\{\mathbf{Z}^{(r)}, r = 1, \ldots, N_s\}$ generated according to $f(\mathbf{Z})$. The accuracy and efficiency of MCS does not depend on the geometry of the failure domain or the dimension of the problem. Instead, it depends only on the failure probability $P(F)$ and the number of generated samples N_s. The coefficient of variation (COV) δ of the MCS estimator for $P(F)$ is

$$\delta = \sqrt{\frac{1-P(F)}{P(F)N_s}} \approx \sqrt{\frac{1}{P(F)N_s}} \qquad (P(F) \ll 1) \tag{2}$$

In engineering applications, the failure probability is expected to be small. In this case, the above equation implies that the computational effort is prohibitively high in order to obtain an estimate with acceptable accuracy. For example, if $P(F) = 10^{-4}$, one needs $N_s = 10^6$ samples (structural dynamic analyses) to achieve the accuracy level of 10% COV.

The excessive computational requirement of MCS is its main shortcoming and has been the reason for pursuing the development of alternative more efficient simulation algorithms. However, it is noted that several commonly used simulation algorithms, such as importance sampling, encounter serious difficulties when considering high-dimensional reliability problems such as the one at hand. Some of the pioneering works highlighting the problems encountered when dealing with high-dimensional problems are (Au and Beck 2003, Schuëller *et al.* 2004, Schuëller and Pradlwarter 2007, Katafygiotis and Zuev 2008).

In this paper a very efficient algorithm, called Domain Decomposition Method (DDM) (Katafygiotis and Cheung 2006), is used for solving the given reliability problem, i.e., calculating the probability of the wind-excited structural responses exceeding some specified thresholds within a given time duration. Expressing the failure domain as a union of elementary failure domains, the DDM estimates the probability of the overall failure domain as the sum of the probabilities of these elementary failure domains multiplied by a reduction factor which reflects the overlapping degree of the different elementary failure domains. The key difference of the investigated problem from the original formulation of the DDM is that the elementary failure domains are quadratic rather than linear. Herein we extend the DDM with the help of Line Sampling (LS) (Schuëller *et al.* 2003, Koutsourelakis *et al.* 2004, Schuëller *et al.* 2004, Pradlwarter *et al.* 2007) so as to handle the specific type of reliability problem considered. An illustrative example involving an along-wind excited steel building shows the accuracy and efficiency of the proposed methodology.

2. Formulation

In this part the reliability problem corresponding to wind-excited structural response(s) exceeding some predetermined threshold(s) during a given time interval is formulated. Using the spectral representation method (Grigoriu 2002), the wind velocity fluctuation is simulated as a sum of harmonic waves with amplitudes modeled by standard normal random variables. Utilizing the unit impulse response functions, the responses of the structure are expressed as quadratic functions of a standard normal random vector used to simulate the wind velocities. In the corresponding standard normal space the failure domain can be expressed as a union of elementary failure domains whose boundaries are of quadratic form.

2.1 Stochastic modeling of multivariate wind velocity field

The generation of along-wind velocity field is considered. At a given point located at height h from the ground, the velocity is

$$V(h;t) = \overline{V}(h) + v(h;t) \tag{3}$$

where $\overline{V}(h)$ is the mean value and $v(h;\, t)$ is the fluctuating component of the wind velocity. In the discretization of the wind velocity field we aim to generate N_u fluctuating components $v_j(t)$, $j = 1, \ldots, N_u$, at points with corresponding heights h_j, $j = 1, \ldots, N_u$, where the discretized pressures and then forces are to be evaluated. The simulation of the fluctuating components amounts to simulating a one-dimensional N_u-variate zero-mean stationary stochastic vector process $\mathbf{v}(t) = [v_1(t), \ldots, v_{N_u}(t)]^T$ according to its cross-power spectral density (one-sided) matrix

$$\mathbf{S}^0(\omega) = \begin{bmatrix} S^0_{11}(\omega) & S^0_{12}(\omega) & \ldots & S^0_{1N_u}(\omega) \\ S^0_{21}(\omega) & S^0_{22}(\omega) & \ldots & S^0_{2N_u}(\omega) \\ \vdots & \vdots & \ldots & \vdots \\ S^0_{N_u1}(\omega) & S^0_{N_u2}(\omega) & \ldots & S^0_{N_uN_u}(\omega) \end{bmatrix} \tag{4}$$

which is assumed to be real due to the negligibility of the quadrature spectrum (Simiu and Scanlan 1986), symmetric and positive definite (Di Paola and Gullo 2001) at each frequency ω.

Among various algorithms for simulating the above stochastic vector process, the spectral representation method (Shinozuka and Jan 1972, Shinozuka and Deodatis 1991, Deodatis 1996, Grigoriu 2002) is one of the most widely used. According to this method, the stochastic vector process is simulated as a superposition of harmonic waves with random phases or random amplitudes. Herein the algorithm using random amplitudes is adopted and is briefly described below.

According to Cholesky's method, the matrix $\mathbf{S}^0(\omega)$ can be decomposed as follows

$$\mathbf{S}^0(\omega) = \mathbf{H}(\omega)\mathbf{H}(\omega)^T \tag{5}$$

where $\mathbf{H}(\omega)$ is a lower triangular real matrix due to the principal axis theorem (Jacob 1990). Let $\mathbf{H}_d(\omega)$, $d = 1, \ldots, N_u$, denote the d-th column vector of $\mathbf{H}(\omega)$, that is

$$\mathbf{H}(\omega) = [\mathbf{H}_1(\omega)\ \ \mathbf{H}_2(\omega)\ \ \ldots\ \ \mathbf{H}_{N_u}(\omega)] \tag{6}$$

Let ω_u denote the cutoff frequency above which all components $S^0_{jk}(\omega)$, j, $k = 1, \ldots, N_u$, of $\mathbf{S}^0(\omega)$ are insignificant for practical purposes. That is

$$\int_0^{\omega_u} S^0_{jk}(\omega)d\omega = (1 - r_{u,jk})\int_0^{\infty} S^0_{jk}(\omega)d\omega \tag{7}$$

where $r_{u,jk}$ is the ratio of the neglected power spectrum content over the total content and ω_u is such that all r_u's are smaller than a predefined threshold. Divide the interval $[0,\, \omega_u]$ into N_ω equal segments, each having length $\Delta\omega = \omega_u/N_\omega$, and consider a sequence of frequencies based on the centroid rule as follows: $\omega_l = (2l - 1)\,\Delta\omega/2$, $l = 1, \ldots, N_\omega$. Then, the stochastic vector process $\mathbf{v}(t)$ is simulated using the following formula (Grigoriu 2002)

$$\mathbf{v}(t) = \sqrt{\Delta\omega}\sum_{l=1}^{N_\omega}\sum_{d=1}^{N_u}[\mathbf{H}_d(\omega_l)Z_{1ld}\cos(\omega_l t) + \mathbf{H}_d(\omega_l)Z_{2ld}\sin(\omega_l t)] \tag{8}$$

where Z_{1ld} and Z_{2ld}, $l = 1, \ldots, N_\omega$, $d = 1, \ldots, N_u$, are independent standard normal random variables. Thus, each component $v_j(t)$, $j = 1, \ldots, N_u$ is simulated according to

$$v_j(t) = \sqrt{\Delta\omega}\sum_{l=1}^{N_\omega}\sum_{d=1}^{N_u}[H_{jd}(\omega_l)Z_{1ld}\cos(\omega_l t) + H_{jd}(\omega_l)Z_{2ld}\sin(\omega_l t)] \tag{9}$$

where $H_{jd}(\omega_l)$ is the j-th component of $\mathbf{H}_d(\omega_l)$. It can be easily shown that the probabilistic properties of the simulated stochastic vector process given by Eq. (8) are identical to the required targets. In order to reduce the cost of digitally generating the stochastic vector process, the Fast Fourier Transform (FFT) (Brigham 1988) technique can be utilized.

2.2 Geometric description of the reliability problem in high dimension

Consider a linear wind-excited structure with N_u wind excitation forces $U_j(t)$, $j = 1, \ldots, N_u$, and N_y dynamic responses of interest $Y_i(t)$, $i = 1, \ldots, N_y$. The relationship between the responses and the excitation forces is given by

$$\begin{aligned} Y_i(t) &= \sum_{j=1}^{N_u}\int_0^t q_{ij}(t, \tau)U_j(\tau)d\tau \\ &= \sum_{j=1}^{N_u}\int_0^t q_{ij}(t-\tau)U_j(\tau)d\tau \end{aligned} \tag{10}$$

where $q_{ij}(t, \tau)$ is the response function for Y_i at time t due to a unit impulse excitation for U_j at time τ. The system is assumed to start with zero initial conditions, is time-invariant so that $q_{ij}(t, \tau) = q_{ij}(t - \tau)$, and is causal, i.e., $q_{ij}(t, \tau) \equiv 0$ for $t < \tau$, the latter explaining why the integration in Eq. (10) is taken from 0 to t instead of from 0 to ∞. The wind excitations $U_j(t)$, $j = 1, \ldots, N_u$, are random, and thus the responses $Y_i(t)$, $i = 1, \ldots, N_y$, are random as well. The problem considered herein is to estimate the probability that any one of the N_y output responses $Y_i(t)$, $i = 1, \ldots, N_y$, exceeds in magnitude some specified threshold b_i within a given time duration T

$$F = \bigcup_{i=1}^{N_y}\{\exists t \in [0, T] : |Y_i(t)| > b_i\} \tag{11}$$

In this paper only along-wind excitations are considered. Based on the stochastic modeling of the wind velocity field presented in the last section, the wind excitations $U_j(t)$, $j = 1, \ldots, N_u$, can be expressed as

$$U_j(t) = \frac{1}{2}\rho L_j V_j(t)^2 = \frac{1}{2}\rho L_j(\overline{V}_j + v_j(t))^2 \tag{12}$$

where ρ is the air density, taken to be 1.2 kg/m^3 in the illustrative example of the paper, and L_j is the area upon which the discretized force $U_j(t)$ is assumed to act. Expanding the above equation, we obtain

$$U_j(t) = \frac{1}{2}\rho L_j \overline{V}_j^2 + \rho L_j \overline{V}_j v_j(t) + \frac{1}{2}\rho L_j v_j(t)^2 \tag{13}$$

Thus, the responses $Y_i(t)$, $i = 1, \ldots, N_y$, can be written as

$$Y_i(t) = Y_i^0 + Y_i^{(1)}(t) + Y_i^{(2)}(t) \tag{14}$$

where $Y_i^{(0)}$ is the deterministic part of the i-th response corresponding to the mean wind speed excitations, which can be obtained by a single structural static analysis; $Y_i^{(1)}(t)$ and $Y_i^{(2)}(t)$, respectively, correspond to the excitations due to the linear and the quadratic terms of the wind velocity fluctuation

$$Y_i^{(1)}(t) = \sum_{j=1}^{N_u} \int_0^t \rho L_j \overline{V}_j q_{ij}(t-\tau) v_j(\tau) d\tau \tag{15}$$

$$Y_i^{(2)}(t) = \sum_{j=1}^{N_u} \int_0^t \frac{1}{2}\rho L_j q_{ij}(t-\tau) v_j(\tau)^2 d\tau \tag{16}$$

Substituting Eq. (9) into Eq. (15), $Y_i^{(1)}(t)$, $i = 1, \ldots, N_y$, can be expressed as a linear function of the standard normal random vector $\mathbf{Z}$ comprising all the Z_{1ld} and Z_{2ld} random variables

$$Y_i^{(1)}(t) = \mathbf{a}^{(i)}(t)^T \mathbf{Z} = \sum_{l=1}^{N_\omega} \sum_{d=1}^{N_u} (a_{1ld}^{(i)}(t) Z_{1ld} + a_{2ld}^{(i)}(t) Z_{2ld}) \tag{17}$$

with the elements of the coefficient vector $\mathbf{a}^{(i)}(t)$ given by

$$a_{1ld}^{(i)}(t) = \sum_{j=1}^{N_u} \sqrt{\Delta\omega}\rho L_j \overline{V}_j H_{jd}(\omega_l) |Q_{ij}(\omega_l)| \cos(\omega_l t + \theta\{Q_{ij}(\omega_l)\}) \tag{18}$$

$$a_{2ld}^{(i)}(t) = \sum_{j=1}^{N_u} \sqrt{\Delta\omega}\rho L_j \overline{V}_j H_{jd}(\omega_l) |Q_{ij}(\omega_l)| \sin(\omega_l t + \theta\{Q_{ij}(\omega_l)\}) \tag{19}$$

where $|Q_{ij}(\omega_l)|$ and $\theta\{Q_{ij}(\omega_l)\}$ are the magnitude and the phase angle of the Fourier transform of $q_{ij}(t)$ at frequency ω_l. Herein we assume stationary response and consider the steady state dynamic response since the duration of interest is usually long (of the order of an hour). That is, we disregard the transient part and consider only the steady state part in Eqs. (18) and (19).

Similarly, by substituting Eq. (9) into Eq. (16), $Y_i^{(2)}(t)$, $i = 1, \ldots, N_y$, can expressed as a quadratic function of $\mathbf{Z}$

$$\begin{aligned} Y_i^{(2)}(t) = \mathbf{Z}^T \mathbf{B}^{(i)}(t) \mathbf{Z} = \sum_{l_1=1}^{N_\omega} \sum_{l_2=1}^{N_\omega} \sum_{d_1=1}^{N_u} \sum_{d_2=1}^{N_u} [&Z_{1l_1d_1} Z_{1l_2d_2} B_{1l_1d_1,1l_2d_2}^{(i)}(t) + Z_{1l_1d_1} Z_{2l_2d_2} B_{1l_1d_1,2l_2d_2}^{(i)}(t) \\ &+ Z_{2l_1d_1} Z_{1l_2d_2} B_{2l_1d_1,2l_2d_2}^{(i)}(t) + Z_{2l_1d_1} Z_{2l_2d_2} B_{2l_1d_1,2l_2d_2}^{(i)}(t)] \end{aligned} \tag{20}$$

with the elements of the matrix $\mathbf{B}^{(i)}(t)$ given by

$$B^{(i)}_{1l_1d_1,1l_2d_2}(t) = \sum_{j=1}^{N_u}\frac{1}{4}\rho L_j\Delta\omega H_{jd_1}(\omega_{l_1})H_{jd_2}(\omega_{l_2})[|Q_{ij}(\omega_{l_2+l_1})|\cos(\omega_{l_2+l_1}t+\theta\{Q_{ij}(\omega_{l_2+l_1})\}) + |Q_{ij}(\omega_{l_2-l_1})|\cos(\omega_{l_2-l_1}t+\theta\{Q_{ij}(\omega_{l_2-l_1})\})] \quad (21)$$

$$B^{(i)}_{1l_1d_1,2l_2d_2}(t) = \sum_{j=1}^{N_u}\frac{1}{4}\rho L_j\Delta\omega H_{jd_1}(\omega_{l_1})H_{jd_2}(\omega_{l_2})[|Q_{ij}(\omega_{l_2+l_1})|\sin(\omega_{l_2+l_1}t+\theta\{Q_{ij}(\omega_{l_2+l_1})\}) + |Q_{ij}(\omega_{l_2-l_1})|\sin(\omega_{l_2-l_1}t+\theta\{Q_{ij}(\omega_{l_2-l_1})\})] \quad (22)$$

$$B^{(i)}_{2l_1d_1,1l_2d_2}(t) = \sum_{j=1}^{N_u}\frac{1}{4}\rho L_j\Delta\omega H_{jd_1}(\omega_{l_1})H_{jd_2}(\omega_{l_2})[|Q_{ij}(\omega_{l_2+l_1})|\sin(\omega_{l_2+l_1}t+\theta\{Q_{ij}(\omega_{l_2+l_1})\}) - |Q_{ij}(\omega_{l_2-l_1})|\sin(\omega_{l_2-l_1}t+\theta\{Q_{ij}(\omega_{l_2-l_1})\})] \quad (23)$$

$$B^{(i)}_{2l_1d_1,2l_2d_2}(t) = \sum_{j=1}^{N_u}\frac{1}{4}\rho L_j\Delta\omega H_{jd_1}(\omega_{l_1})H_{jd_2}(\omega_{l_2})[|Q_{ij}(\omega_{l_2-l_1})|\cos(\omega_{l_2-l_1}t+\theta\{Q_{ij}(\omega_{l_2-l_1})\}) - |Q_{ij}(\omega_{l_2+l_1})|\cos(\omega_{l_2+l_1}t+\theta\{Q_{ij}(\omega_{l_2+l_1})\})] \quad (24)$$

where $|Q_{ij}(\omega_{l_2+l_1})|$ and $\theta\{Q_{ij}(\omega_{l_2+l_1})\}$ are the magnitude and the phase angle of the Fourier transform of $q_{ij}(t)$ at frequency $\omega_{l_2}+\omega_{l_1}$; $|Q_{ij}(\omega_{l_2-l_1})|$ and $\theta\{Q_{ij}(\omega_{l_2-l_1})\}$ are the magnitude and the phase angle of the Fourier transform of $q_{ij}(t)$ at frequency $\omega_{l_2}-\omega_{l_1}$.

Thus, the structural response $Y_i(t)$, $i = 1, \ldots, N_y$, can be expressed as a quadratic function of **Z**

$$Y_i(t) = Y_i^0 + \mathbf{a}^{(i)}(t)^T\mathbf{Z} + \mathbf{Z}^T\mathbf{B}^{(i)}(t)\mathbf{Z} \quad (25)$$

In the discrete time formulation, the responses $Y_i(t)$, $i = 1, \ldots, N_y$, are calculated at discrete time instants $t_k = k\Delta t$, $k = 1, \ldots, N_t$

$$Y_i(k) = Y_i^0 + \mathbf{a}^{(i)}(k)^T\mathbf{Z} + \mathbf{Z}^T\mathbf{B}^{(i)}(k)\mathbf{Z} \quad (26)$$

where the index k is used to represent the time t_k. The failure event defined earlier in Eq. (11) can be expressed as a union of $N = 2N_yN_t$ elementary failure events

$$F = \bigcup_{i=1}^{N_y}\bigcup_{k=1}^{N_t}\{|Y_i(k)| > b_i\} = \bigcup_{i=1}^{N_y}\bigcup_{k=1}^{N_t}\bigcup_{s=1}^{2}F_{iks} \quad (27)$$

where each elementary failure event is $F_{iks} = \{(-1)^s\, Y_i(k) > b_i\}$. In the space of standard normal random variables **Z**, the failure domain corresponding to the failure event F_{iks} is defined as follows

$$F_{iks} = \{\mathbf{Z}|g_{iks}(\mathbf{Z}) < 0\} \quad (28)$$

where g_{iks} (**Z**) denotes the quadratic limit state function (LSF)

$$g_{iks}(\mathbf{Z}) = \mathbf{Z}^T\mathbf{B}^{(i)}(k)\mathbf{Z} + \mathbf{a}^{(i)}(k)^T\mathbf{Z} + b_i + Y_i^0; \quad s = 1 \quad (29)$$

$$= -\mathbf{Z}^T\mathbf{B}^{(i)}(k)\mathbf{Z} - \mathbf{a}^{(i)}(k)^T\mathbf{Z} + b_i - Y_i^0; \quad s = 2 \quad (30)$$

3. Proposed methodology using Domain Decomposition Method and line sampling

3.1 Domain Decomposition Method

The Domain Decomposition Method (DDM) (Katafygiotis and Cheung 2006) is briefly reviewed in this section. DDM is similar to Importance Sampling using Elementary Events (ISEE) (Au and Beck 2001) and a method presented in Yuen and Katafygiotis (2004), although the approaches leading to each of these methods are different. ISEE stems from IS with a suitably selected IS density, while the DDM is based on estimating the overlapping degree of different elementary failure domains comprising the failure domain.

Consider a failure domain expressed as a union of elementary failure domains, $F = \cup_{r=1}^{N} F_r$. If the elementary failure domains F_r, $r = 1, \ldots, N$, are mutually exclusive, the probability $P(F)$ is simply the sum of all the $P(F_r)$. The difficulty in calculating the probability of the overall failure domain arises from that the elementary failure domains overlap each other and the overlapping degree is unknown. The DDM focuses on the estimation of the overlapping degree and estimates the failure probability $P(F)$ as

$$\hat{P}(F) = \hat{\eta}\sum_{r=1}^{N}\hat{P}(F_r) \tag{31}$$

where $\hat{P}(F_r)$ is the estimate of $P(F_r)$, and $\hat{\eta}$ is the reduction factor which estimates the overlapping degree of different elementary failure domains. The overlapping reduction factor estimate $\hat{\eta}$ is obtained as follows: First select K indices $r_j, j = 1, \ldots, K$, from the set $\{1, \ldots, N\}$ according to a probability mass function (PMF) $g(i)$ given by

$$g(i) = \frac{\hat{P}(F_i)}{\sum_{j=1}^{N}\hat{P}(F_j)} \tag{32}$$

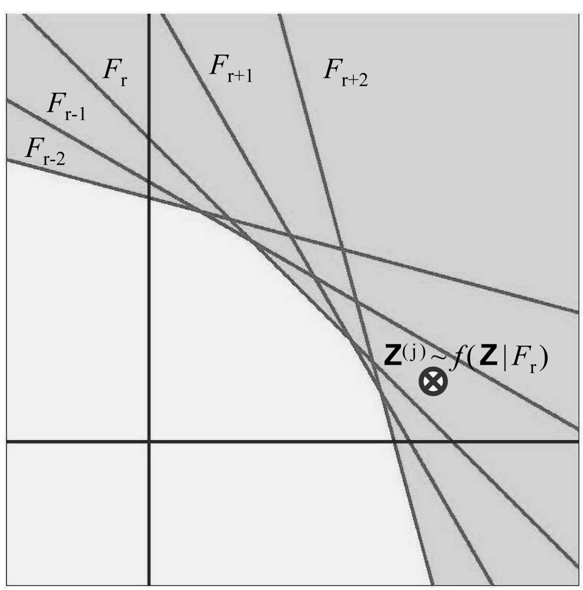

Fig. 1 Schematic illustration of how to count $M(\mathbf{Z}^{(j)})$ in DDM

Based on the selected indices, generate K sample points $\mathbf{Z}^{(j)}$, $j = 1, \ldots, K$, such that $\mathbf{Z}^{(j)} \in F_{r_j}$ is distributed according to the conditional distribution $f(\mathbf{Z}|F_{r_j})$. Then, for each $\mathbf{Z}^{(j)}$ the number $M(\mathbf{Z}^{(j)})$ of failure domains to which $\mathbf{Z}^{(j)}$ belongs is counted, e.g., in Fig. 1 the point $\mathbf{Z}^{(j)}$ following the conditional distribution $f(\mathbf{Z}|F_r)$ belongs to the failure domains F_r, F_{r+1} and F_{r+2} and, thus, $M(\mathbf{Z}^{(j)}) = 3$. To determine $M(\mathbf{Z}^{(j)})$, a dynamic analysis is performed for the given $\mathbf{Z}^{(j)}$ and the number of discrete time instants at which the response(s) of interest exceed the given threshold(s) is counted. Finally, the overlapping reduction factor estimate $\hat{\eta}$ is given by

$$\hat{\eta} = \frac{1}{K}\sum_{j=1}^{K}\frac{1}{M(\mathbf{Z}^{(j)})} \tag{33}$$

It can be seen that the application of the DDM requires successfully performing two key tasks: the estimation of $P(F_r)$ and the generation of sample points distributed according to the conditional PDF $f(\mathbf{Z}|F_r)$.

For the reliability problem at hand, if the part of response due to the the quadratic terms of wind velocity fluctuation, $Y_i^{(2)}(t)$, is neglected in the expression of $Y_i(t)$, $i = 1, \ldots, N_y$, the boundary of the elementary failure domains will be of linear form, which is the case considered in the original formulation of DDM, and the algorithms for the above two tasks are straightforward (Katafygiotis and Cheung 2006). Specifically, for an elementary failure domain $F = \{\mathbf{Z}|g(\mathbf{Z}) < 0\}$ with linear LSF

$$g(\mathbf{Z}) = \mathbf{a}^T\mathbf{Z} + c \tag{34}$$

the corresponding failure probability is readily obtained as

$$P(F) = 1 - \Phi(\beta) \tag{35}$$

where $\Phi(.)$ denotes the standard normal cumulative distribution function and β is given by

$$\beta = \frac{c}{\|\mathbf{a}\|} \tag{36}$$

where $\|\mathbf{a}\|$ is the Euclidean norm of the vector $\mathbf{a}$. To generate a point $\mathbf{Z}$ in F according to the conditional PDF $f(\mathbf{Z}|F)$, simulate a standard normal vector $\mathbf{Z}'$ first. Next, simulate a scalar x according to the conditional standard normal PDF given that $x < -\beta$, which can be done by first simulating a random number λ that is uniformly distributed in the interval [0, 1] and then calculating $x = \Phi^{-1}[\Phi(-\beta) - \lambda\Phi(-\beta)]$. Finally the required point is given by $\mathbf{Z} = \mathbf{Z}' + (\lambda - \mathbf{u}^T\mathbf{Z}')\mathbf{u}$ where $\mathbf{u} = \mathbf{a}/\|\mathbf{a}\|$ is the unit direction vector along $\mathbf{a}$.

3.2 Line sampling method

For the reliability problem considered herein, where the boundary of the elementary failure domains are of quadratic form, the tasks of estimating $P(F_r)$ and generating sample points distributed according to the conditional PDF $f(\mathbf{Z}|F_r)$ are nontrivial. This section presents the procedures for performing these two key tasks by applying Line Sampling (LS) (Koutsourelakis *et al.* 2004, Schuëller *et al.* 2004, Pradlwarter *et al.* 2007).

Consider the failure domain F with quadratic LSF given by

$$g(\mathbf{Z}) = \mathbf{Z}^T\mathbf{B}\mathbf{Z} + \mathbf{a}^T\mathbf{Z} + c \tag{37}$$

LS starts with choosing an important direction vector $\mathbf{u}$ characterizing the failure domain. The selection of the direction vector $\mathbf{u}$ depends on the problem under study, and plays a key role in the performance of the method (Koutsourelakis *et al*. 2004). Recall that in our problem, where the LSF is given by Eq. (37), the quadratic term represents the structural response corresponding to the quadratic terms of the wind velocity fluctuation. This is small compared to the linear term corresponding to the structural response due to the linear terms of the wind velocity fluctuation. Thus, this quadratic LSF is weakly nonlinear and, therefore, it is reasonable to choose $\mathbf{u}$ to be equal to $\mathbf{a}/\|\mathbf{a}\|$, i.e., the unit direction vector along $\mathbf{a}$. Based on the chosen $\mathbf{u}$, the N_Z-dimensional space $\mathbb{R}^{N_Z}$ of the standard normal random variables $\mathbf{Z}$ is divided into two subspaces: the one dimensional subspace $\mathbb{R}^1$ which is parallel to $\mathbf{u}$, and the $(N_Z - 1)$-dimensional subspace $\mathbb{R}_\perp$ which is orthogonal to $\mathbf{u}$. That is, each vector $\mathbf{Z}$ can be expressed as $\mathbf{Z} = \mathbf{Z}_\perp + x\mathbf{u}$ where $\mathbf{Z}_\perp \in \mathbb{R}_\perp$. Note that the scalar $x = \mathbf{Z}^T\mathbf{u}$ follows a standard normal distribution because the norm of $\mathbf{u}$ is one and $\mathbf{Z}$ is a standard normal random vector.

The LS algorithm for estimating the probability of the failure domain $F = \{\mathbf{Z}|g(\mathbf{Z}) < 0\}$ with $g(\mathbf{Z})$ given by Eq. (37) is as follows: Firstly, generate N_l independent sample points $\mathbf{Z}_\perp^{(r)}$ $r = 1, ..., N_l$, in $\mathbb{R}_\perp$ according to $f(\mathbf{Z}|\mathbb{R}_\perp)$. This can be done by generating points $\mathbf{Z}^{(r)}$, $r = 1, \ldots, N_l$, according to $f(\mathbf{Z})$ and then taking their projections in the subspace $\mathbb{R}_\perp$

$$\mathbf{Z}_\perp^{(r)} = \mathbf{Z}^{(r)} - (\mathbf{u}^T\mathbf{Z}^{(r)})\mathbf{u} \tag{38}$$

Secondly, calculate the conditional failure probability given $\mathbf{Z}_\perp^{(r)}$, that is, $P_f^{(r)} = P\{\mathbf{Z}|g(\mathbf{Z}) < 0;$ $\mathbf{Z}_\perp = \mathbf{Z}_\perp^{(r)}\}$. The conditional failure event

$$g(x\mathbf{u} + \mathbf{Z}_\perp^{(r)}) = (\mathbf{u}^T\mathbf{B}\mathbf{u})x^2 + (2\mathbf{u}^T\mathbf{B}\mathbf{Z}_\perp^{(r)} + \mathbf{u}^T\mathbf{a})x + [(\mathbf{Z}_\perp^{(r)})^T\mathbf{B}\mathbf{Z}_\perp^{(r)} + c] < 0 \tag{39}$$

is a quadratic inequality with respect to the scalar x, and can be simplified further to be explicit, in order that its probability can be easily computed according to the standard normal distribution of x. Fig. 2 illustrates Line Sampling for the case we have been encountering in our numerical example, where the equation

$$g(x\mathbf{u} + \mathbf{Z}_\perp^{(r)}) = (\mathbf{u}^T\mathbf{B}\mathbf{u})x^2 + (2\mathbf{u}^T\mathbf{B}\mathbf{Z}_\perp^{(r)} + \mathbf{u}^T\mathbf{a})x + [(\mathbf{Z}_\perp^{(r)})^T\mathbf{B}\mathbf{Z}_\perp^{(r)} + c] = 0 \tag{40}$$

has a negative quadratic coefficient, i.e., $\mathbf{u}^T\mathbf{B}\mathbf{u} < 0$, and two real roots denoted as $x_l^{(r)}$ and $x_u^{(r)}$ where $x_l^{(r)} < x_u^{(r)}$. In this case, the conditional failure event $\{\mathbf{Z}|g(\mathbf{Z}_\perp^{(r)} + x\mathbf{u}) < 0\}$ can be simplified as $\{x < x_l^{(r)}\} \cup \{x > x_u^{(r)}\}$ and thus $P_f^{(r)}$ is given by

$$P_f^{(r)} = \Phi(-x_u^{(r)}) + \Phi(x_l^{(r)}) \tag{41}$$

Finally, the probability of the failure event $F = \{\mathbf{Z}|g(\mathbf{Z}) < 0\}$ is estimated by the arithmetic average of the N_l conditional failure probabilities

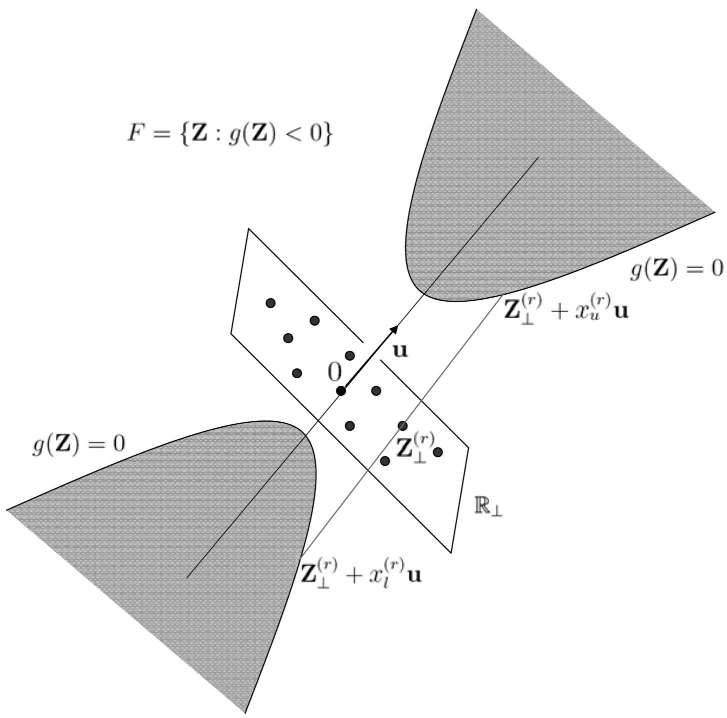

Fig. 2 Schematic illustration of line sampling

$$\hat{P}(F) = \frac{1}{N_l}\sum_{r=1}^{N_l} P_f^{(r)} \tag{42}$$

The procedure for performing the second task, i.e., generating a point according to $f(\mathbf{Z}|F)$, is as follows: Firstly, generate N_l points $\mathbf{Z}_\perp^{(r)}$, $r = 1, \ldots, N_l$, in the subspace $\mathbb{R}_\perp$ and compute the corresponding conditional failure probabilities $P_f^{(r)}, r = 1, \ldots, N_l$, as above. Secondly, choose a point $\mathbf{Z}_\perp$ from the set $\mathbf{Z}_\perp^{(r)}$, $r = 1, \ldots, N_l$, according to the PMF

$$g(i) = \frac{P_f^{(i)}}{\sum_{r=1}^{N_l} P_f^{(r)}} \tag{43}$$

Finally, based on the chosen point $\mathbf{Z}_\perp$ simulate a random number x following the conditional standard normal distribution given that it corresponds to failure, i.e., $\{x|g(\mathbf{Z}_\perp + x\mathbf{u}) < 0\}$. The final simulated point is then given by

$$\mathbf{Z} = \mathbf{Z}_\perp + x\mathbf{u} \tag{44}$$

The simulation of x is explained further below. If the conditional failure event $\{x|g(\mathbf{Z}_\perp + x\mathbf{u}) < 0\}$ is simplified as $\{x < x_l\} \cup \{x > x_u\}$, then x should be generated from the conditional standard normal PDF given $x \in (-\infty, x_l) \cup (x_u, \infty)$. To simulate such x, one of the regions $(-\infty, x_l)$ and (x_u, ∞) is selected first according to their probabilities, that is, the region $(-\infty, x_l)$ is selected with probability $\Phi(x_l)/(\Phi(x_l) + \Phi(-x_u))$, and the region (x_u, ∞) is selected with the remaining probability. Then x is obtained by first simulating a random number λ that is uniformly distributed in the interval [0, 1] and then calculating $x = \Phi^{-1}[\Phi(x_l) - \lambda\Phi(x_l)]$ if the selected region is $(-\infty, x_l)$ or $x = \Phi^{-1}[\Phi(x_u) + \lambda\Phi(-x_u)]$ if the selected region is (x_u, ∞).

3.3 Summary of the proposed methodology

In the following we present a summary of the procedures using the DDM and LS to estimate $P(F)$, where $F = \cup_{i=1}^{N_y} \cup_{k=1}^{N_t} \{|Y_i(k)| > b_i\}$. Note that herein the responses $Y_i(t)$, $i = 1, \ldots, N_y$, are assumed to be stationary because they are assumed to be the steady state responses of a linear structure subjected to stationary excitations. This implies that the probability of failure is the same at each time instant, and the overlapping degree among the various elementary failure domains is time invariant. With this observation, the procedures are summarized as follows:

1. Arbitrarily choose an index p from the set of time instants $\{1, \ldots, N_t\}$, and establish the quadratic LSF for the $2N_y$ elementary failure domains F_{ips}, $i = 1, \ldots, N_y$, $s = 1, 2$:

$$g_{ips}(\mathbf{Z}) = \mathbf{Z}^T\mathbf{B}^{(i)}(p)\mathbf{Z} + \mathbf{a}^{(i)}(p)^T\mathbf{Z} + b_i + Y_i^0; \quad s = 1 \tag{45}$$

$$= -\mathbf{Z}^T\mathbf{B}^{(i)}(p)\mathbf{Z} - \mathbf{a}^{(i)}(p)^T\mathbf{Z} + b_i - Y_i^0; \quad s = 2 \tag{46}$$

   where the coefficient vector $\mathbf{a}^{(i)}(p)$ and the coefficient matrix $\mathbf{B}^{(i)}(p)$ are determined respectively by Eqs. (18)-(19) and Eqs. (21)-(24).

2. Estimate $\hat{P}(F_{ips})$, the probability of the failure domain F_{ips}, $i = 1, \ldots, N_y$, $s = 1, 2$, using the LS method.

3. Select K index pairs $\{(i_j, s_j), j = 1, \ldots, K\}$ from the set $\{(i, s), i = 1, \ldots, N_y, s = 1, 2\}$ according to the PMF $g(i, s)$ given by

$$g(i, s) = \frac{\hat{P}(F_{ips})}{\sum_{j=1}^{N_y}\sum_{k=1}^{2}\hat{P}(F_{jpk})} \tag{47}$$

4. Based on the above-selected index pairs $\{(i_j, s_j), j = 1, \ldots, K\}$, simulate K sample points $\mathbf{Z}^{(j)}$, $j = 1, \ldots, K$, using the described LS-based procedure such that $\mathbf{Z}^{(j)} \in F_{i_j p s_j}$ follows the conditional distribution $f(\mathbf{Z}|F_{i_j p s_j})$. For each point $\mathbf{Z}^{(j)}$, count the number $M(\mathbf{Z}^{(j)})$ of the elementary failure domains F_{iks}, $i = 1, \ldots, N_y$, $k = 1, \ldots, N_t$, $s = 1, 2$, to which the point $\mathbf{Z}^{(j)}$ belongs.

5. The failure probability $P(F)$ is estimated by

$$\hat{P}(F) = \hat{\eta}N_t\sum_{i=1}^{N_y}\sum_{s=1}^{2}\hat{P}(F_{ips}), \quad \hat{\eta} = \frac{1}{K}\sum_{j=1}^{K}\frac{1}{M(\mathbf{Z}^{(j)})} \tag{48}$$

4. Illustrative example

We consider an along-wind excited steel building (Fig. 3), which has the same geometric shape as the Commonwealth Advisory Aeronautical Research Council (CAARC) standard tall building model (Melbourne 1980). A 45-story, 10-bay by 15-bay rectangular tubular framework is used to model

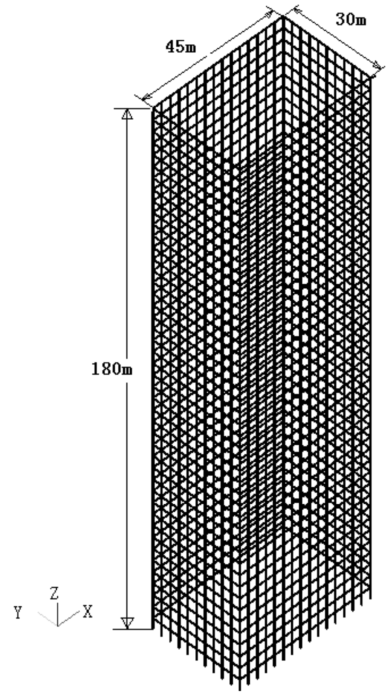

Fig. 3 Along-wind excited steel building in the example

Table 1 Design of colum members and beam members

Floor zone	Column members	Beam members
1~9F	W14X550	W30X357
10~18F	W14X500	W30X326
19~27F	W14X370	W30X292
28~36F	W14X257	W30X261
37~45F	W14X159	W30X221

this building. With story height of 4 m and bay width of 3 m, the building has a total height of 180 m and a rectangular floor with dimension 30 m by 45 m. Each floor is assumed to be rigid and has lumped swaying mass of 6.75×10^5 kg and rotational mass moment of inertia of 1.645×10^8 kg.m^2 at the geometric center of the floor. The members of beams and columns have standard AISC steel sections, and the details of the design are presented in Table 1. With the above configurations, the established building model has the following first three modal frequencies: 0.197 Hz, 0.251 Hz and 0.422 Hz.

The along-wind excitation in the Y-direction of the building is considered. The excitation field is discretized using two schemes: In Scheme 1(a) fine spatial discretization involving N_u = 45 excitation forces is considered; in Scheme 2(a) coarser discretization involving N_u = 6 excitation forces is assumed. The acting heights and acting areas for Scheme 1 and 2 are shown in Table 2 and Table 3, respectively.

According to the Hong Kong wind code, the mean wind speed $\overline{V}_j$ (m/s) at the height h_j(m), j = 1,

Table 2 Acting height sand acting areas of 45 excitation forces in discretization Scheme 1

Excitation	Acting height (m)	Acting area (m^2)
$U_1(t)$	4	6×45
$U_2(t)$~$U_{44}(t)$	8~176(interval of 4)	4×45
$U_{45}(t)$	180	2×45

Table 3 Acting heights and acting areas of 6 excitation forces in discretization Scheme 2

Excitation	Acting height (m)	Acting area (m^2)
$U_1(t)$	24	45×45
$U_2(t)$	68	45×45
$U_3(t)$	112	33.75×45
$U_4(t)$	136	22.5×45
$U_5(t)$	156	22.5×45
$U_6(t)$	176	11.25×45

…, N_u, is given by the power law (Simiu and Scanlan 1986)

$$\overline{V}_j = 41\left(\frac{h_j}{180}\right)^{0.25} \tag{49}$$

The formulas proposed by Davenport (1961, 1968) are used to model the one-sided cross spectral density matrix $\mathbf{S}^0(\omega)$ of the fluctuating wind velocity components $\mathbf{v}(t) = [v_1(t), \ldots, v_{N_u}(t)]^T$. The power spectral density function $\mathbf{S}^0_{jj}(\omega)$ of $v_j(t)$, $j = 1, \ldots, N_u$, is given by

$$\mathbf{S}^0_{jj}(\omega) = \frac{\overline{V}_j^2 k^2}{\left(\ln\frac{h_j}{h_0}\right)^2} \frac{8\pi x^2}{\omega(1+x^2)^{4/3}} \tag{50}$$

$$x = \frac{600\omega}{\pi\overline{V}(10)} \tag{51}$$

where ω (rad/s) is the frequency, $k = 0.4$ is Von Karman's constant, $h_0 = 0.05$ m is the roughness length, and $\overline{V}_{10} = 19.9$ m/s is the mean wind velocity at the height of 10 m. The cross-power spectral density function $\mathbf{S}^0_{jk}(\omega)$ of $v_j(t)$ and $v_k(t)$ is given by

$$\mathbf{S}^0_{jk}(\omega) = \sqrt{\mathbf{S}^0_{jj}(\omega)\mathbf{S}^0_{kk}(\omega)}\gamma_{jk}(\omega), \quad j, k = 1, \ldots, N_u, \quad j \neq k \tag{52}$$

$$\gamma_{jk}(\omega) = \exp\left[-\frac{\omega}{2\pi}\frac{C_h|h_j - h_k|}{\frac{1}{2}(\overline{V}_j + \overline{V}_k)}\right] \tag{53}$$

where $\gamma_{jk}(\omega)$ is the coherence function between $v_j(t)$ and $v_k(t)$, and C_h is a constant that can be set equal to 10 for structural design purposes (Simiu and Scanlan 1986).

To perform the generation of the wind velocity fluctuations according to Eq. (9), the cutoff frequency is taken as $\omega_u = 0.8\pi$ rad/s so that the ratio r_u of the neglected power spectrum content is less than 10% for each component $\mathbf{S}_{jk}^{0}(\omega)$, $j,k = 1,\dots,N_u$. The frequency step $\Delta\omega = \pi/900$ is selected and the period of the fluctuating wind velocity components $\mathbf{v}(t) = [v_1(t),\dots,v_{N_u}(t)]^T$ is $4\pi/\Delta\omega = 3600$ s. Based on the above chosen parameters, the number of standard normal random variables involved in the simulation, namely, the dimension of $\mathbf{Z}$, is $2 \times N_u \times \omega_u/\Delta\omega = 64{,}800$ and 8,640 for discretization Scheme 1 and Scheme 2, respectively.

In this example we assume that the displacement response $Y(t)$ at the top floor of the building is of interest. The relationship between the excitation forces and the responses are provided by the impulse response functions $q_j(t)$, $j = 1, \dots, N_u$, where $q_j(t)$ denotes the response $Y(t)$ due to unit impulse excitation for $U_j(t)$. These N_u required impulse response functions are obtained through N_u dynamic analyses of the established finite element model of the building using the software SAP 2000. The failure event is defined as the response $Y(t)$ exceeding in magnitude a specified threshold b within one hour, i.e., the assumed time duration is $T = 3{,}600$ s. This time duration is conventionally used in wind engineering, for consistence with the duration of actual strong winds. In the discrete time formulation where the sampling time interval is chosen to be $\Delta t = 0.01$ s and the number of time instants is $N_t = T/\Delta t = 360{,}000$, the failure event is expressed by

$$F = \cup_{k=1}^{N_t}[|Y(k)| > b] = \{\cup_{k=1}^{N_t}[|Y(k)| > b]\} \cup \{\cup_{k=1}^{N_t}[|Y(k)| > -b]\} \tag{54}$$

In the space of standard normal random variables, the corresponding failure domain is a union of $2N_t = 720{,}000$ elementary failure domains.

The failure events with thresholds $b = 1.244$ m and $b = 1.368$ m are considered. These two thresholds have been chosen such that MCS with 10^4 samples for the discretization Scheme 1 gives the estimates $\hat{P}(F) = 0.01$ (with COV $\delta = 9.9\%$) and $\hat{P}(F) = 0.001$ (with COV $\delta = 31.6\%$). The statistical properties of the estimator for the failure probability by the proposed methodology are assessed by 40 simulation runs, in each of which 200 samples are used in the LS to estimate the failure probability at a randomly selected time instant and 100 samples are used in the DDM to estimate the overlapping reduction factor. Note that each sample used in the LS corresponds to a process of solving Eq. (40) and does not require a structural dynamic analysis. Thus, the computational effort for one run is controlled by the number of samples used in the DDM, i.e., 100 in our case. For each such sample a dynamic analysis is required in order to determine the number of time instants at which the response exceeds the specified threshold that defines the failure event. Thus, one run by our proposed methodology requires an effort equivalent to MCS with 145 and 106 samples (including the effort for calculating the impulse response functions) for Scheme 1 and 2, respectively.

The sample mean and the sample COV of the failure probability estimates for Scheme 1 using the proposed method are shown in the fourth and fifth rows of Table 4. It can be seen that the estimate by the proposed methodology is very accurate. The sixth row of Table 4 shows the number of MCS samples required in order to get the same COV as that obtained by the proposed method, i.e., $\delta = 24.2\%$ for the estimation of $P(F) = 0.01$ and $\delta = 33.8\%$ for the estimation of $P(F) = 0.001$. According to Eq. (2), this number is given by

$$N_{MCS} = \frac{1 - P(F)}{\delta^2 P(F)} \tag{55}$$

Thus the proposed methodology requires 145 samples versus 1,535 samples in the MCS to get the same accuracy for estimating $P(F) = 0.01$, i.e., a factor of around ten times in efficiency

Table 4 Comparsion of the failure probability estimates

				b=1.244 m	b=1.368 m
Discretization Scheme 1	Quadratic LSF	MCS	$\hat{P}(F)$	1.0×10^{-2}	1.0×10^{-3}
			COV	9.9%	31.6%
		Proposed Method	Sample mean	1.10×10^{-2}	1.32×10^{-3}
			Sample COV	24.2%	33.8%
			N_{MCS}	1,535	6,622
	Linear LSF	DDM	Sample mean	1.92×10^{-5}	2.59×10^{-7}
			Sample COV	6.5%	6.5%
Discretization Scheme 2	Quadratic LSF	MCS	$\hat{P}(F)$	3.60×10^{-2}	4.40×10^{-3}
			COV	5.2%	15.0%
		Proposed Method	Sample mean	3.42×10^{-2}	4.79×10^{-3}
			Sample COV	18.8%	28.2%
			N_{MCS}	799	2,613
	Linear LSF	DDM	Sample mean	2.27×10^{-4}	5.68×10^{-6}
			Sample COV	7.7%	6.8%

improvement. For the smaller failure probability $P(F)$ = 0.001, the advantage in computational efficiency becomes further pronounced, with the efficiency improved by a factor of around fourty five times.

If one neglects in the expression of $Y(t)$ the part of the response due to the quadratic terms of the wind velocity fluctuation, $Y^{(2)}(t)$, and approximates the quadratic LSF by the underlying linear LSF, DDM in its original version can be directly used for estimating $P(F)$. The sample mean and sample COV of the failure probability estimates by DDM using 40 simulation runs with 100 samples for each run are shown in the rows marked "Linear LSF" in Table 4. It can be seen that neglecting $Y^{(2)}(t)$ in the expression of $Y(t)$ can cause severe underestimation of the failure probability. Therefore, the design based on such linear approximation of the response is not conservative and must be avoided.

The failure probability estimates using the discretization Scheme 2 (six excitation forces) are shown in the last seven rows of Table 4. Similarly, it can be observed that the proposed method provides good estimates and its efficiency improvement (compared with MCS) increases with the decrease of the failure probability. Also, as in the case of Scheme 1, if the excitation in the quadratic term of wind velocity fluctuation is neglected, the failure probability is severely underestimated.

By comparing the results for discretization Scheme 2 with those for discretization Scheme 1, it can be seen that the failure probability estimates (with quadratic LSF) for the coarser discretization Scheme 2 are larger by a factor of three to four times. This is because the coarser discretization neglects the spatial correlation within the rather large height segments considered, and, therefore, assumes that the velocity fluctuations are fully correlated over each such segment. As a result, the total force, calculated using such coarse spatial discretization, has a larger variability, i.e., larger variance than the corresponding total force over the same segment when a finer spatial discretization is considered. The resulting larger variance of the total force leads to higher estimates of the failure probability, which however are conservative.

5. Conclusions

The problem of calculating the probability that the wind-excited structural responses exceed specified thresholds within a given time duration is considered. The Domain Decomposition Method (DDM) is used to solve this reliability problem, estimating the failure probability as the sum of the probabilities of the elementary failure domains multiplied by a reduction factor accounting for the overlapping degree of different elementary failure domains. The DDM has been extended to tackle the problem at hand involving quadratic elementary failure domains with the help of Line Sampling (LS). In particular, LS is used for the estimation of the probability of each quadratic elementary failure domain and also for the simulation of sample points within such a domain according to the conditional probability distribution given that elementary failure domain. As shown in the illustrative example involving an along-wind excited steel building the proposed methodology is accurate and highly efficient compared with standard MCS.

Acknowledgements

This research has been supported by the Hong Kong Research Grants Council under grants 614305 and CA04/05.EG01. This support is gratefully acknowledged.

References

Au, S.K. and Beck, J.L. (2001), "First excursion probabilities for linear systems by very efficient importance sampling", *Probabilist. Eng. Mech.*, **16**(3), 193-207.

Au, S.K. and Beck, J.L. (2003), "Importance sampling in high dimensions", *Struct. Safety*, **25**(2), 139-163.

Brigham, E.O. (1988), *The Fast Fourier Transform and its Applications*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Davenport, A.G. (1961), "The spectrum of horizontal gustiness near the ground in high winds", *J. R. Meteorol. Soc.*, **87**, 194-211.

Davenport, A.G. (1968), "The dependence of wind load upon meteorological parameters", *Proceedings of the International Research Seminar on Wind Effects on Buildings and Structures*, Toronto.

Deodatis, G. (1996), "Simulation of ergodic multivariate stochastic processes", *J. Eng. Mech.*, **122**(8), 778-787.

Di Paola, M. and Gullo, I. (2001) "Digital generation of multivariate wind field processes", *Probabilist. Eng. Mech.*, **16**, 1-10.

Grigoriu, M. (2002), *Stochastic Calculus: Applications in Science and Engineering*, Birkhauser Boston, New York.

Jacob, B. (1990), *Linear Algebra*, W.H. Freeman and Company, New York.

Katafygiotis, L.S. and Cheung, S.H. (2006), "Domain decomposition method for calculating the failure probability of linear dynamic systems subjected to Gaussian stochastic loads", *J. Eng. Mech.*, **132**(5), 475-486.

Katafygiotis, L.S. and Zuev, K.M. (2008), "Geometric insight to the challenges of solving high-dimensional reliability problems", *Probabilist. Eng. Mech.*, **23**(2-3), 208-218.

Koutsourelakis, P.S., Pradlwarter, H.J. and Schuëller, G.I. (2004), "Reliability of structures in high dimensions, part I: algorithms and applications", *Probabilist. Eng. Mech.*, **19**(4), 409-417.

Melbourne, W.H. (1980), "Comparison of measurements on the CAARC standard tall building model in simulated model wind flows", *J. Wind. Eng. Ind. Aerod.*, **6**, 73-88.

Pradlwarter, H.J., Schuëller, G.I., Koutsourelakis, P.S. and Charmpis, D.C. (2007), "Application of line sampling simulation method to reliability benchmark problems", *Struct. Safety*, **29**, 208-221.

Proppe, C., Pradlwarter, H.J. and Schuëller, G.I. (2003) "Equivalent linearization and Monte Carlo simulations in stochastic dynamics", *Probabilist. Eng. Mech.*, **18**(1), 1-15.

Rubinstein, R.Y. (1981), *Simulation and the Monte-Carlo Method*, New York: Wiley.

Schuëller, G.I., Pradlwarter, H.J. and Koutsourelakis, P.S. (2003), "A comparative study of reliability estimation procedures for high dimensions", *16th ASCE Engineering Mechanics Conference*, University of Washington, Seattle.

Schuëller, G.I., Pradlwarter, H.J. and Koutsourelakis, P.S. (2004), "A critical appraisal of reliability estimation procedures for high dimensions", *Probabilist. Eng. Mech.*, **19**(4), 463-474.

Schuëller, G.I. and Pradlwarter, H.J. (2007), "Benchmark study on reliability estimation in higher dimensions of structural systems – An overview ", *Struct. Safety*, **29**(3), 167-182.

Shinozuka, M. and Jan, C.M. (1972), "Digital simulation of random processes and its applications", *J. Sound Vib.*, **25**(1), 111-128.

Shinozuka, M. and Deodatis, G. (1991), "Simulation of stochastic processes by spectral representation", *Appl. Mech. Rev.*, **44**(4), 191-204.

Simiu, E. and Scanlan, R.H. (1986), *Wind Effects on Structures*, John Wiley & Sons, Inc., New York.

Yuen, K.V. and Katafygiotis, L.S. (2004), "An efficient simulation method for reliability analysis using simple additive rules of probability", *Probabilist. Eng. Mech.*, **20**(1), 109-114.