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Stochastic response spectra for an actively-controlled structure

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Abstract. A stochastic response spectrum method is proposed for simple evaluation of the structural response of an actively controlled aseismic structure. The response spectrum is constructed assuming a linear structure with an active mass damper (AMD) system, and an earthquake wave model given by the product of a non-stationary envelope function and a stationary Gaussian random process with Kanai-Tajimi power spectral density. The control design is executed using a linear quadratic Gaussian control strategy for an enlarged state space system, and the response amplification factor is given by the combination of the obtained statistical response values and extreme value theory. The response spectrum thus produced can be used for simple dynamical analyses. The response factors obtained by this method for a multi-degree-of-freedom structure are shown to be comparable with those determined by numerical simulations, demonstrating the validity and utility of the proposed technique as a simple design tool. This method is expected to be useful for engineers in the initial design stage for structures with active aseismic control.

Keywords: active control; stochastic response spectra; multi-DOF; peak factor.

1. Introduction

Active control technology is one of the approaches employed for improving the seismic resistance of structures (Chang and Soong 1980, Yang 1982, Pantelides 1990, Singh *et al.* 1997). Dynamic control analysis for the design of such active control is executed from the initial design stage, requiring both examination of structural integrity and also estimation of control characteristics for the structure. Dynamic seismic design of structures is often carried out using response spectra given as functions of the damping ratio and natural period of the structure. A comprehensive response spectrum analysis is rarely executed because of the large number of parameters required for control design, whereas the ordinary response spectrum analysis involves only two relatively simple design parameters such as damping ratio and natural period. In the present study, a new response spectrum method is proposed for evaluating the structural response of a multiple degree-of-freedom (DOF) structure in the design of an active control system for seismic resistance.

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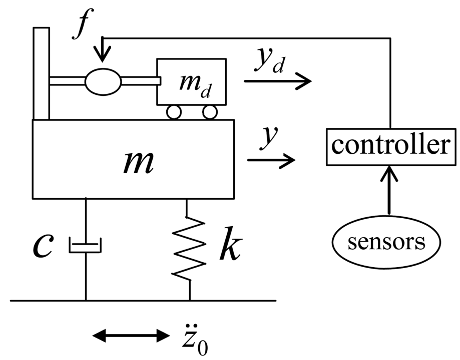

Fig. 1 Mathematical model for response spectrum generation

2. Mathematical model for response spectrum analysis

2.1 Equations of motion governing dynamics of actively-controlled structure

The fundamental mathematical model adopted to construct the response spectrum is shown in Fig. 1. This model estimates the dynamics of the actively controlled structure in response to ground motion. Although several active control systems have been proposed for aseismic building design, the active mass damper (AMD) method is adopted here as one of the simplest systems in order to demonstrate the potential of the proposed response spectrum method.

In Fig. 1, y, y_d and z_0 describe the absolute displacements of the main structure, the AMD, and the ground, and m, c, and k denote the mass, damping coefficient, and stiffness of the structure, respectively. The parameters m_d and f denote the mass of the AMD and the control force. The system equations for the mathematical model shown in Fig. 1 are then given as follows:

$$m\ddot{y} + c(\dot{y} - \dot{z}_0) + k(y - z_0) = -f \tag{1}$$

$$m_d \ddot{y}_d = f \tag{2}$$

$$f = gu \tag{3}$$

where u is the control signal and g is the transformed gain from u to f.

2.2 Earthquake model

The following non-stationary stochastic model describing earthquake motion is assumed in the present study

$$\ddot{z}_0(t) = a(t) \cdot \ddot{w}(t) \tag{4}$$

where $a(t)$ is a deterministic time-dependent function, with $a(t) = A_m(e^{-c_1 t} - e^{-c_2 t})$. The envelope function $a(t)$ is defined in order to realize the non-stationary state for amplitude. The term $\ddot{w}(t)$ is a narrow-band stationary Gaussian process that is frequently modeled as a absolute acceleration response in a single-DOF(SDOF) system subject to stationary Gaussian white noise given by $\ddot{z}_g(t)$. This model is a well-known artificial earthquake model having a Kanai-Tajimi spectrum (Tajimi

1960), and is adopted in the present study. The governing equation is expressed by

$$M\ddot{w} + C(\dot{w} - \dot{z}_g) + K(w - z_g) = 0 \tag{5}$$

Substituting $\nu = w - z_g$ into Eq. (5) yields

$$\ddot{\nu} + 2h\Omega\dot{\nu} + \Omega^2\nu = -\ddot{z}_g; \quad 2h\Omega = \left(\frac{C}{M}\right), \quad \Omega^2 = \left(\frac{K}{M}\right) \tag{6}$$

3. Control rule and covariance of structural response

3.1 State space expression of active control system

By defining the relative displacements as

$$x = y - z_0, \quad x_a = y_d - y \tag{7}$$

and coupling these variables with Eqs. (1)-(4) and (6), the following extended state equation for an actively-controlled structure can be obtained

$$\dot{\mathbf{X}}(t) = \mathbf{A}_0(t)\mathbf{X}(t) + \mathbf{B}u(t) + \mathbf{D}\ddot{z}_g(t) \tag{8}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x_a \\ v \\ \dot{x} \\ \dot{x}_a \\ \dot{v} \end{bmatrix}, \quad \mathbf{A}_0(t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\omega_0^2 & 0 & a(t)\Omega^2 & -2\xi\omega_0 & 0 & 2a(t)h\Omega \\ \omega_0^2 & 0 & 0 & 2\xi\omega_0 & 0 & 0 \\ 0 & 0 & -\Omega^2 & 0 & 0 & -2h\Omega \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{g}{m} \\ \dfrac{g}{m}\left\{1 + \dfrac{1}{\left(\dfrac{m_d}{m}\right)}\right\} \\ 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad \begin{aligned} 2\xi\omega_0 &= \left(\frac{c}{m}\right) \\ \omega_0^2 &= \left(\frac{k}{m}\right) \end{aligned}$$

3.2 Stochastic control rule

To achieve active vibration control, the linear quadratic Gaussian (LQG) algorithm (Wu and Yang 2000) is employed. Applying the LQG algorithm to the state space equation, which consists of time-dependent coefficients associated with Gaussian white noise (Eq. (8)), the optimal control signal $u_0(t)$ is derived as follows

$$u_0(t) = -\frac{1}{r}\mathbf{B}^T\mathbf{P}(t)\mathbf{X}(t) \equiv -\mathbf{F}(t)\mathbf{X}(t) = -[k_1(t) \;\; k_2(t) \;\; k_3(t) \;\; k_4(t) \;\; k_5(t) \;\; k_6(t)]\mathbf{X}(t) \tag{9}$$

Here, $\boldsymbol{P}(t)$ is the solution satisfying the following Riccati equation (Preumont 1997)

$$-\frac{d}{dt}\mathbf{P}(t) = \mathbf{P}(t)\mathbf{A}_0(t) + \mathbf{A}_0^T(t)\mathbf{P}(t) - r^{-1}\mathbf{P}(t)\mathbf{B}\mathbf{B}^T\mathbf{P}(t) + \mathbf{Q} \tag{10}$$

and $\mathbf{Q}$ and r are defined as control parameters within the following performance index

$$J = E\left[\int_0^{T_0}\{\mathbf{X}^T(t)\mathbf{Q}\mathbf{X}(t) + ru^2(t)\}dt\right] \tag{11}$$

where $E[\bullet]$ denotes the expected value.

The time-dependent feedback gain vector $\mathbf{F}(t)$ can be obtained by solving $\mathbf{P}(t)$ in Eq. (10). However, the application of such a time-dependent gain to on-site active control is not always useful, as the calculation of the time-dependent gain is computationally intensive. The present treatment therefore adopts a time-invariant feedback gain that is estimated at some appropriate time at which the envelope function $a(t)$ is maximum, that is, at $t = \log(c_2/c_1)/(c_2-c_1)$. This estimation is defined from the viewpoint of obtaining the best effect on vibration suppression in the larger response region. Despite the use of such a time-invariant gain, good vibration control can be achieved as shown later, owing to the improved robustness of the LQG strategy.

Using the time-invariant feedback gain vector, i.e., $\boldsymbol{F}(t) \equiv \boldsymbol{F}$, Eq. (8) can be arranged as follows

$$\dot{\mathbf{X}}(t) = (\mathbf{A}_0(t) - \mathbf{B}\mathbf{F})\mathbf{X}(t) + \mathbf{D}\ddot{z}_g(t) \equiv \mathbf{A}(t)\mathbf{X}(t) + \mathbf{D}\ddot{z}_g(t) \tag{12}$$

Supposing the initial values of all state variables to be zero, the equation to be satisfied by the response covariance matrix $\mathbf{V}(t)$ with $\mathbf{V}(t=0) = \mathbf{0}$ (Preumont 1990), is given by

$$\dot{\mathbf{V}}(t) = \mathbf{A}(t)\mathbf{V}(t) + \mathbf{V}(t)\mathbf{A}^T(t) + 2\pi\zeta\mathbf{D}\mathbf{D}^T \tag{13}$$

The covariance of the structural response can thus be obtained by solving Eq. (13).

4. Derivation of response spectrum

4.1 Basic strategy for evaluation of the response spectrum

The expression of the response spectrum describing the absolute acceleration response of a structure generally has one of the following forms

$$1)\quad R_a(T,\xi) = |\ddot{y}(t)|_{\max} \tag{14}$$

$$2)\quad R_r(T,\xi) = \frac{|\ddot{y}(t)|_{\max}}{|\ddot{z}_0(t)|_{\max}} \quad (= \text{response factor}) \tag{15}$$

where $|\ddot{y}(t)|_{\max}$ and $|\ddot{z}_0(t)|_{\max}$ are the maximum values of absolute acceleration relating to structural response (see Fig. 1), T is the natural period of the structure, and ξ is the damping ratio of the structure. Eq. (15) is adopted in the present treatment, because the maximum level of earthquake acceleration is not always decided in the initial design stage.

To obtain the response spectrum stochastically, Eq. (15) is approximated by

$$R_r(T,\xi) = \frac{|\sigma_{\ddot{y}}(t)|_{\max} p_{\ddot{y}}}{|\sigma_{\ddot{z}_0}(t)|_{\max} p_{\ddot{z}_0}} \tag{16}$$

where $\sigma(t)$ is the non-stationary standard deviation, and $p_{\ddot{y}}$ and $p_{\ddot{z}_0}$ are the peak factors relating to $\ddot{y}$ and $\ddot{z}_0$, which are approximated as time-invariant parameters under the assumption that these parameters are not affected by the non-stationary state of the random process or by the active control force. The specific expression for the peak factor adopted in the present treatment is that of Kiureghian (1980), and is given by

$$p = \sqrt{2\log \nu_e T_0} + \frac{0.5772}{\sqrt{2\log \nu_e T_0}}; \quad \nu_e = \begin{cases} (1.63\tilde{\delta}^{0.45} - 0.38)\tilde{\nu} & : \tilde{\delta} < 0.69 \\ \tilde{\nu} & : \tilde{\delta} \geq 0.69 \end{cases}$$
$$\tilde{\nu} = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \qquad \tilde{\delta} = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \tag{17}$$

Here, T_0 is the duration, and λ_m is the mth-order spectral moment, which is given by

$$\lambda_m = \int_0^\infty \omega^m G_F(\omega)|H(\omega)|^2 d\omega \tag{18}$$

where $G_F(\omega)$ is the one-sided power spectral density of the input acceleration, and $H(\omega)$ is the frequency transfer function from input acceleration to response acceleration.

4.2 Calculation of peak factors

As it is assumed that the peak factor is not affected by the non-stationary state, the derivation of the peak factor for $\ddot{z}_0(t)$ is identical to that for $\ddot{w}(t)$ through Eq. (4). The peak factors $p_{\ddot{w}}$ and $p_{\ddot{z}_0}$ can thus be easily obtained.

To obtain $p_{\ddot{y}}$, some additional manipulation is required. As the non-stationary state is neglected in estimation of the peak factor, stationary analysis of random vibration may be executed using $\ddot{w}(t)$ instead of $\ddot{z}_0(t)$. The applicable equations of motion are then revised as follows

$$m\ddot{y} + c(\dot{y} - \dot{w}) + k(y - w) = -f \tag{19}$$

$$m_d \ddot{y}_d = f \tag{2}$$

$$f = gu \tag{3}$$

Introducing Laplace transforms for Eqs. (2), (3), (6), (9), and (19), the transfer function $G_2(s)$ from $\ddot{z}_g(t)$ to $\ddot{y}(t)$ is obtained utilizing the transfer function $G_1(s)$ from $\ddot{z}_g(t)$ to $\ddot{w}(t)$, as follows.

$$G_2(s) = \frac{\left\{\left[\left(\frac{m_d}{g}s^2+k_5s+k_2\right)(2\xi\omega_0 s+\omega_0^2)+\frac{m_d}{m}s^2(-k_1+k_3-k_4s+k_6s)\right]G_1(s)+\frac{m_d}{m}s^2(-k_3-k_6s)\right\}}{\left[\left(\frac{m_d}{g}s^2+k_5s+k_2\right)(s^2+2\xi\omega_0 s+\omega_0^2)+\frac{m_d}{m}s^2(-k_1+k_2-k_4s+k_5s)\right]} \tag{20}$$

The mth-order spectral moment required for peak factor $p_{\ddot{y}}$ can then be obtained easily.

4.3 Estimation of maximum value concerning non-stationary standard deviation

The maximum value related to the non-stationary standard deviation of $\ddot{z}_0(t)$ is estimated as follows. The square of the non-stationary standard deviation of $\ddot{z}_0(t)$ is represented by Eq. (4) as follows

$$\sigma_{\ddot{z}_0}^2(t) = E[a^2(t)\ddot{w}^2(t)] = a^2(t)E[\ddot{w}^2(t)] \tag{21}$$

The maximum value thus occurs at the time at which the envelope function $a(t)$ reaches a maximum, since $\ddot{w}(t)$ is a stationary random process and $E[\ddot{w}^2(t)](\equiv\sigma_{\ddot{w}}^2)$ becomes constant. This peak value can thus be easily acquired.

The maximum value related to the non-stationary standard deviation of $\ddot{y}(t)$ is then estimated from the square of the non-stationary standard deviation of $\ddot{y}(t)$, given by $\sigma_{\ddot{y}}^2(t)$, which can be readily obtained through simple statistical calculations using Eqs. (1) and (3).

The response spectrum $R_r(T, \xi)$ required for calculating the structural dynamics of a multi-DOF structure using an AMD system can then be approximately obtained through Eq. (16).

5. Extension of response spectra to multi-dof structure

The dynamic characteristics of a multi-DOF structure with an active control system can be obtained analytically by utilizing the extended state space equation and covariance matrix method as shown above. It is to be demonstrated in the present study, however, that approximate values of each maximum response can be estimated using an appropriate stochastic response spectra, allowing the peak responses to be calculated easily using known modal parameters without computers.

The mathematical model shown in Fig. 2 is assumed for a multi-DOF structure. The system equations for this model are given by

$$\mathbf{M}\ddot{\tilde{\mathbf{x}}}+\mathbf{C}\dot{\tilde{\mathbf{x}}}+\mathbf{K}\tilde{\mathbf{x}} = -\mathbf{MI}\ddot{z}_0-\mathbf{E}f \tag{22}$$

$$m_d\ddot{y}_d = f \tag{2}$$

$$f = gu \tag{3}$$

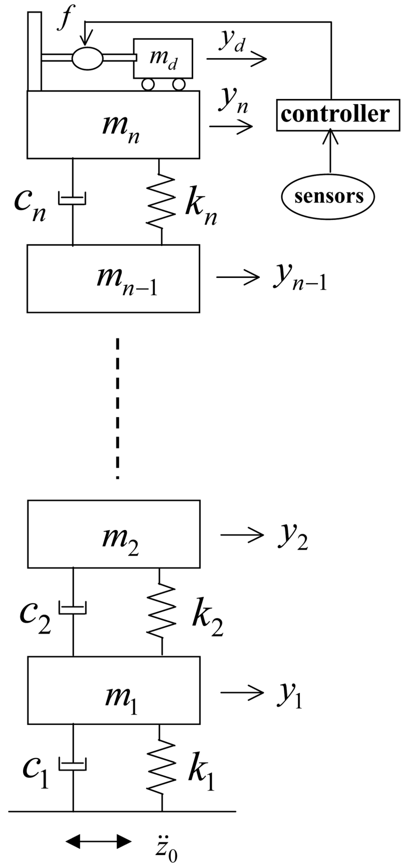

Fig. 2 Mathematical model for multi-DOF structure

where

$$\tilde{\mathbf{x}}^T = [x_1\ x_2\ \dots\ \dots\ x_n]\ ; \quad x_j = y_j - z_0 (j = 1 \sim n), \quad x_a = y_d - y_n$$

$$\mathbf{E}^T = [0\ 0\ \dots\ 0\ 1] \quad \mathbf{I}^T = [1\ 1\ \dots\ 1\ 1]$$

and **M**, **C**, and **K** are the mass, damping, and stiffness matrices for the main structure.

By applying modal analysis (e.g., $\tilde{\mathbf{x}} = \mathbf{\Phi q}$) to Eq. (22) under the assumption of orthogonality for the damping matrix, a set of equations to be analyzed in ith-order modal space can be obtained approximately as follows

$$\ddot{q}_i + 2\tilde{\xi}_i \tilde{\omega}_{0_i} \dot{q}_i + \tilde{\omega}_{0_i}^2 q_i = -\beta_i \ddot{z}_0 - \phi_{ni} \frac{g}{\tilde{m}_i} u_i \tag{23}$$

$$\ddot{y}_d = \frac{g}{m_d} u_i \tag{24}$$

$$\ddot{v} = -2h\Omega\dot{v} - \Omega^2 v - \ddot{z}_g \tag{6}$$

where u_i gives the modal control signal derived under the assumption of independent modal space control (Meirovitch and Silverberg 1983) being valid. In Eq.(23), q_i, $\tilde{\xi}_i$, $\tilde{\omega}_{0_i}$, β_i and $\tilde{m}_i$ are the modal coordinate, modal damping, modal frequency, participation factor, and modal mass for the ith-order mode, and ϕ_{ni} (normalized to one in this treatment) denotes the ith-order modal shape at the top floor (n).

Eq. (24) can also be rewritten as

$$\ddot{x}_a = -\ddot{x}_n - \ddot{z}_0 + \frac{g}{m_d} u_i \tag{25}$$

where $\ddot{x}_n$ is exactly equal to $\sum_{i=1}^{n} \phi_{ni} \ddot{q}_i$. The approximated expression for $\ddot{x}_n$, however, is introduced in order to realize the same calculation as the ordinary response spectrum method against an actively-controlled multi-DOF structure, by neglecting modal coupling effects and considering the limited case where Eq. (25) is used at the ith-order modal coordinate, i.e.

$$\ddot{x}_n \cong \phi_{ni} \ddot{q}_i = \ddot{q}_i \tag{26}$$

Thus, Eq. (25) becomes

$$\ddot{x}_a = -\ddot{q}_i - \ddot{z}_0 + \frac{g}{m_d} u_i \tag{27}$$

which is formally identical to Eq. (2).

Using Eqs. (2) and (3), Eq. (1) can be transformed into another expression as follows

$$\left(1 + \frac{m_d}{m}\right)\ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x + \left(\frac{m_d}{m}\right)\ddot{x}_a = -\left(1 + \frac{m_d}{m}\right)\ddot{z}_0 \tag{28}$$

where the right side of Eq. (28) may be approximately represented as $-\ddot{z}_0$, because of $1 >> (m_d/m)$ for many actively-controlled structures. Through utilizing Eqs. (25) and (26), similarly, Eq. (23) is also transformed into

$$\left(1 + \frac{m_d}{\tilde{m}_i}\right)\ddot{q}_i + 2\tilde{\xi}_i\tilde{\omega}_{0_i}\dot{q}_i + \tilde{\omega}_{0_i}^2 q_i + \left(\frac{m_d}{\tilde{m}_i}\right)\ddot{x}_a = -\left(\beta_i + \frac{m_d}{\tilde{m}_i}\right)\ddot{z}_0 \tag{29}$$

where the right side of Eq. (29) may be designated as $-\beta_i\ddot{z}_0$ at lower modes, because there is higher possibility of $|\beta_i| >> (m_d/\tilde{m}_i)$ at those modes. In such a case, the maximum values of ith-order mode can be easily obtained by scaling of β_i with the response spectrum already calculated from the SDOF structure with the active control system, by the same methodology as the well-known response spectrum method. β_i at higher modes, on the other hand, frequently becomes small. There are, however, considerable cases such as $|\beta_i| > (m_d/\tilde{m}_i)$, because $\tilde{m}_i$ becomes larger at higher modes owing to the normalization of $\phi_{ni} = 1$. Without regard to $|\beta_i| > (m_d/\tilde{m}_i)$ being correct or not, total value of $(\beta_i + m_d/\tilde{m}_i)$ becomes smaller at higher modes, and it means that modal responses at higher modes do not affect the dynamics of structural response too much.

This method, therefore, allows the response factor for absolute acceleration of structure to be determined by conventional approximation methods, which are frequently used in aseismic design for non-actively controlled structures.

6. Evaluation of proposed method

The response spectrum, produced by SDOF actively-controlled structure, as the fundamental data in order to estimate approximately the dynamics of multi-DOF structure with an active control system is derived from Eq. (16). The validity of the proposed technique is examined here through a comparison between analytical results and numerical simulations. The parameters for the reference case are as follows:

1) parameters for mass and control

$$\left(\frac{m_d}{m}\right) = 0.02, \quad \left(\frac{g}{m}\right) = 0.98$$

$$r = 1, \quad \mathbf{Q} = \begin{bmatrix} 100 & & & & & \\ & 1 & & & \mathbf{0} & \\ & & 1 & & & \\ & & & 100 & & \\ & \mathbf{0} & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

2) parameters for structural dynamics

$$\xi = 0.01$$

3) parameters for input

$$\Omega = 5\pi, h = 0.6, \zeta = 1, T_0 = 10, A_m = 5.379, c_1 = 0.3, c_2 = 0.5$$

where h = 0.6 means the tendency of the input wave to be a broadband process, and this value is especially selected, from the viewpoint of giving higher response amplifications against all the natural frequencies shown in Table 1, in order to examine the effects of multi-DOF system.

Fig. 3 shows a comparison between the analytical results obtained using Eq. (16), with numerical results obtained by Monte Carlo simulation of 500 artificial earthquakes. The analytical results are in good agreement with the numerical results, confirming that the proposed method based on Eq. (16) is useful as a simple approach for obtaining the response spectrum for an actively controlled structure under seismic loading.

The effect of active control can be evaluated by comparing the response spectra obtained with and without control, as shown in Fig. 4 for the same set of parameters. It can be seen that the absolute acceleration response of the actively controlled structure is reduced with respect to that without control in the shorter natural period region. In the longer period region, active control results in slightly larger response, since the control rule adopted in the present paper aims to reduce the response level of $x, \dot{x}$ by considering the corresponding weighting parameters at Eq. (11) being larger.

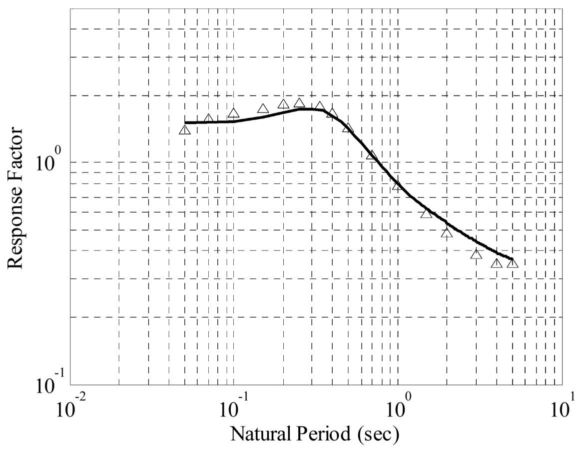

Fig. 3 Comparison of analytical (line) and numerical (symbols) response spectra

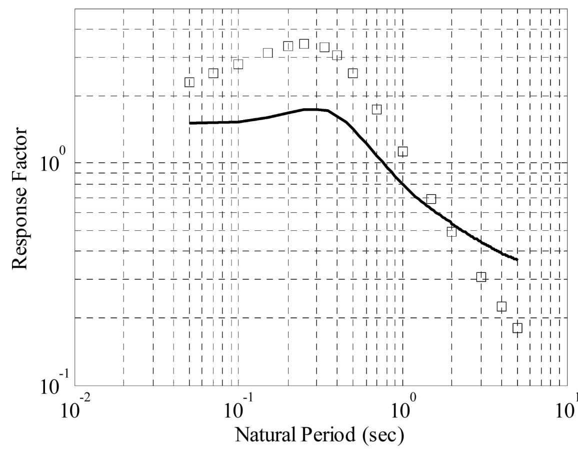

Fig. 4 Comparison of response spectra with (line) and without (symbols) active control

7. Parametric study

7.1 Influence of structural damping on response acceleration

The response spectra $R_r(T, \xi)$ for structural damping (ξ) of 0.005, 0.010, and 0.050 are shown in Fig. 5. Note that the $\xi = 0.050$ case corresponds to the reference case. It is clear that the response of the structure without active control is markedly affected by structural damping in the resonance region. With active control, however, there is no remarkable influence on the building response, since the equivalent damping force introduced by the active control system has a more pronounced effect on the structural dynamics compared to the natural structural damping.

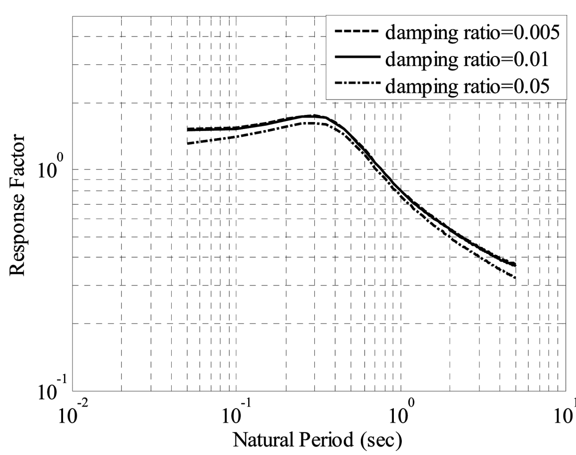

Fig. 5 Influence of structural damping on response spectrum

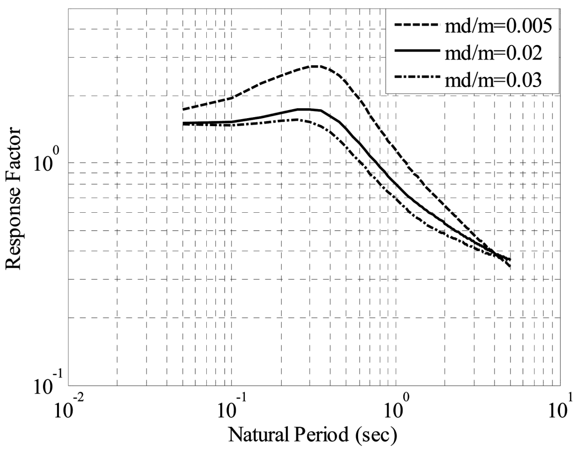

Fig. 6 Influence of (m_d/m) on response spectrum with active control

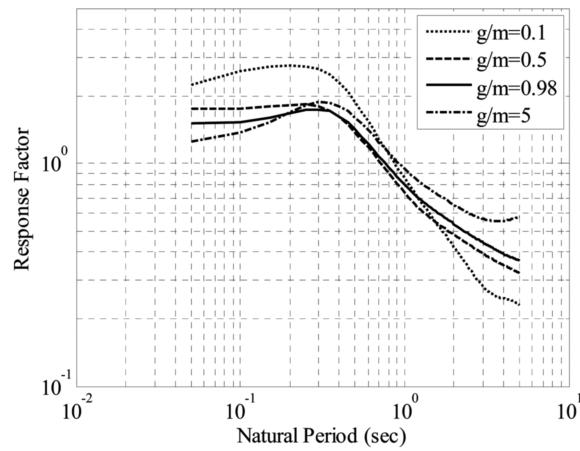

Fig. 7 Influence of (g/m) on response spectrum with active control

7.2 Control parameters affecting structure response

Response spectra for active control using various values of m_d/m and g/m are shown in Figs. 6 and 7. The damping ratio in both cases is 0.01, and all other parameters are the same as for the reference case. The parameters m_d/m and g/m can be seen to have a pronounced influence on the performance of active control. An increase in either of these parameters generally implies an improvement in performance for suppressing the absolute response in the resonance region or the relative response at higher natural periods. Conversely, the response spectrum with active control moves toward that without active control as each of these parameters tends to zero. This analysis demonstrates how the response spectra can be used to evaluate the dynamic response of a structure and assess the structural vibration suppression achieved by various implementations of active control.

8. Example of application to a multi-dof structure

As a typical application of the proposed response spectrum analysis, the response of a 3-DOF main structure ($n = 3$ in Fig. 2) is evaluated. The following parameters are employed:

1) *Modal parameters*

Table 1 Modal parameters used in typical example

	$\tilde{\xi}_i$	$\tilde{\omega}_{0_i}$	$\tilde{m}_i$	$\frac{g}{\tilde{m}_i}$	β_i	ϕ_{3i}	ϕ_{2i}	ϕ_{1i}
first mode	0.01	15.58	368.2	0.532	1.22	1	0.802	0.445
second mode	0.01	43.66	572.6	0.342	-0.28	1	-0.555	-1.247
third mode	0.01	63.10	1859	0.105	0.06	1	-2.247	1.802

2) *Control parameters*
Weighting function ; the same as the reference case
State space vector concerned ; $\tilde{\mathbf{X}}^T = [q_i\ x_a\ v\ \dot{q}_i\ \dot{x}_a\ \dot{v}]$

3) *Other parameters*
Same as the reference case

The response factors for absolute acceleration at the top floor obtained by the present spectral analysis using the square root of sum of squares (SRSS) and absolute sum (ABS) methods are compared in Table 2 with the results of Monte Carlo simulation. The results obtained by the spectral analysis are in reasonable agreement with the simulation result, demonstrating that it is possible to apply the proposed response spectrum analysis to the design of an active control system for a multi-DOF structure in a similar manner to existing approaches.

Table 2 Calculated response factors for absolute acceleration at the top floor under active control

Method	Response Factor	Algorithm*
SRSS	2.1	$SRSS = \sqrt{\sum_{i=1}^{n} \{\phi_{ji}\beta_i\ {}_iS_a\}^2}$
ABS	2.7	$ABS = \sum_{i=1}^{n} \lvert\phi_{ji}\beta_i\ {}_iS_a\rvert$
Monte Carlo simulation	2.62	

* ${}_iS_a = {}_iS_a\left(\tilde{\xi}_i, \tilde{\omega}_{0_i}, \frac{g}{\tilde{m}_i}\right)$ denotes the mean value of the response factor for absolute acceleration related to the *i*th mode, and is obtained from stochastic response spectra such as Figs. 5-7.

9. Conclusions

Active control technology is often applied to increase the seismic resistance of structures. In the design of such an actively controlled structure, detailed dynamical analysis is executed from the initial design stage, requiring not only examination of structural integrity but also estimation of control characteristics. The dynamical aseismic design is often carried out in a simplified manner using response spectra given as functions of the damping ratio and natural period of the structure. In the present study, a new response spectrum method that accounts for active control was proposed. The response spectrum was constructed considering a linear SDOF structure with an active mass damper system as a fundamental mathematical model, from which the dynamics for a multi-DOF structure with active control are estimated. The derivation includes an earthquake wave given by a product of a non-stationary envelope function and a stationary Gaussian random process with the Kanai-Tajimi spectrum, and a control design executed using a linear quadratic Gaussian control strategy. Comparison of the results of the response spectrum analysis with numerical simulations for a 3-DOF structure with AMD demonstrates that the proposed approach is useful as a simplified design method for actively controlled structures. The proposed technique is expected to be readily applicable for simple estimations in the initial seismic design stage for an actively controlled structure.

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