Structural Engineering and Mechanics, Vol. 31, No. 6 (2009) 737-749
DOI: http://dx.doi.org/10.12989/sem.2009.31.6.737

Vibration analysis of a uniform beam traversed by a moving vehicle with random mass and random velocity

T-P. Chang†

Department of Construction Engineering, National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan, ROC

M-F. Liu‡

Department of Applied Mathematics, I-Shou University, Kaohsiung, Taiwan, ROC

H-W. O‡†

Department of Construction Engineering, National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan, ROC

(Received February 27, 2008, Accepted March 20, 2009)

Abstract. The problem of estimating the dynamic response of a distributed parameter system excited by a moving vehicle with random initial velocity and random vehicle body mass is investigated. By adopting the Galerkin's method and modal analysis, a set of approximate governing equations of motion possessing time-dependent uncertain coefficients and forcing function is obtained, and then the dynamic response of the coupled system can be calculated in deterministic sense. The statistical characteristics of the responses of the system are computed by using improved perturbation approach with respect to mean value. This method is simple and useful to gather the stochastic structural response due to the vehicle-passenger-bridge interaction. Furthermore, some of the statistical numerical results calculated from the perturbation technique are checked by Monte Carlo simulation.

Keywords: Galerkin's method; statistical characteristics; Vehicle-Passenger-Bridge Interaction; Improved Perturbation Approach; Monte Carlo Simulation.

1. Introduction

The problem of estimating the response of a distributed parameter system due to a moving loads or moving masses is very important in current engineering applications such as, to the analysis and design of bridge, highway, and railway bridges. The response of structural systems subjected to

† Professor, Corresponding author, E-mail: tpchang@ccms.nkfust.edu.tw
‡ Associate Professor, E-mail: meifeng@isu.edu.tw
‡† Graduate Student, E-mail: u9012820@ccms.nkfust.edu.tw

moving loads depends on numerous parameters, such as the mass, stiffness, and damping in structures and vehicles, the velocity of moving vehicles, the track irregularities.

The evaluation of dynamic response of distributed parameter systems subjected to a moving load has been extensively studied in the past. Early Timoshenko (1922) presented the classical solution of a vibrating load passing over a beam and for the analysis of trains crossing a bridge. Sadiku and Leipholz (1987) compared the solutions for both the moving-mass and moving-force problems by utilizing series solutions involving the Green function. Based on the Euler-Bernoulli, Rayleigh and Timoshenko beam theories, Katz *et al*. (1988) investigated the dynamic response of a constant-velocity moving load acting on a rotating shaft. Esmailzadeh and Ghorashi (1995) have coped with the problem of transverse vibration of simply supported beams subjected to uniform partially distributed moving mass. Lee (1994) estimated the dynamic response of a beam with intermediate point constraints subjected to a moving load by using Hamilton's principle. Fryba (1999) investigated the vibration of simply supported beams with a single, lumped load moving at constant speed along its span. Recently Esmailzadeha and Jalilib (2003) deeply studied the dynamics of vehicle-passenger-structure interaction of bridges traversed by moving vehicles.

The stochastic analysis of distributed parameter system under moving loads has been assumed as stochastic the intensity of moving force (Bolotin 1965) or been assumed as stochastic the force amplitudes and the time arrivals on the system (Ricciardi 1994). In both cases, the velocity of moving forces has been considered constant. Zibdeh (1995) dealt with the random vibration of an elastic beam subjected to random loads moving with time-varying velocity. Sobczyk *et al*. (1996) studied the dynamics of structural systems with randomly varying parameters; the analysis and formulations were based on the theory of random integral equations. Chang and Liu (1996) investigated the stochastic response of a nonlinear beam subjected to a moving load by using finite element analysis. Recently Sniady *et al*. (1999) studied the vibration of the beam by considering the velocity of the moving force as stochastic. ElBeheiry (2000) adopted the perturbation criteria to investigate the effects of small travel speed variations on active suspensions of vehicles. More recently, Wang *et al*. (2002) proposed a new technique to obtain the approximate probability density for the resonance response of finite-damping nonlinear vibration system under random disturbances. Zibdeh and Abu-Hilal (2003) performed the random vibration analysis of laminated composite coated beam traversed by a random moving load. Feng and He (2003) described the random vibro-impact systems by mean of impact Poincare' map. Chang *et al*. (2006) adopted the Galerkin's technique to investigate the dynamic response of a beam with internal hinge subjected to a random moving oscillator.

In the present investigation, both the deterministic and stochastic analysis of the vehicle-passenger-bridge coupled system will be performed. First the deterministic analysis of the coupled system will be performed before we proceed to deal with the stochastic analysis. In stochastic analysis, the improved perturbation technique with respect to mean value recently proposed by Muscolino *et al*. (2000, 2002) is utilized. This method adopts the first-order and second-order probabilistic information of stochastic parameters to compute the statistical response of the system. The stochastic analysis can be readily performed through a numerical procedure once the transition matrix of the system has been obtained; the complete procedure was described in details by Muscolino (1996). The probabilistic characteristics of the stochastic parameters are obtained by the improved perturbation method and the results are checked by Monte Carlo simulation, also, the reliability analysis of structure is performed based on certain failure criteria of the structure described by Chang and Chang (1994).

2. Governing equation of motion

The goal of this study is to perform the deterministic and stochastic dynamic analysis of the mathematical model for the passenger-vehicle-bridge interaction. For simplicity, only linear models are assumed to simulate the dynamics of both bridge and vehicle suspension systems. We adopt a uniform Euler-Bernoulli beam to characterize the bridge that is assumed as horizontal at the equilibrium position under its own weight.

Moreover, the steady state displacements of the vehicle are also measured from their static equilibrium positions generated just before the vehicle enters the bridge. Therefore, the gravitational effect of the vehicle weight is considered as an extra portion of variable moving loads acting on the bridge. We assume that the vehicle travels along the bridge with the velocity $\dot{u}(t)$, where $u(t)$ is the position of the center of gravity (c.g.) of the vehicle body measured from the left-end support of the bridge, as shown in Fig. 1. The vehicle is characterized as a half-car planar model with six degrees-of-freedom, which is composed of a body mass, a driver and a passenger, and two axles. The body mass is considered to have the vertical deflection and rotational deflection, while the driver and passenger are restrained to have only their own vertical motion, meanwhile, each axle has its own

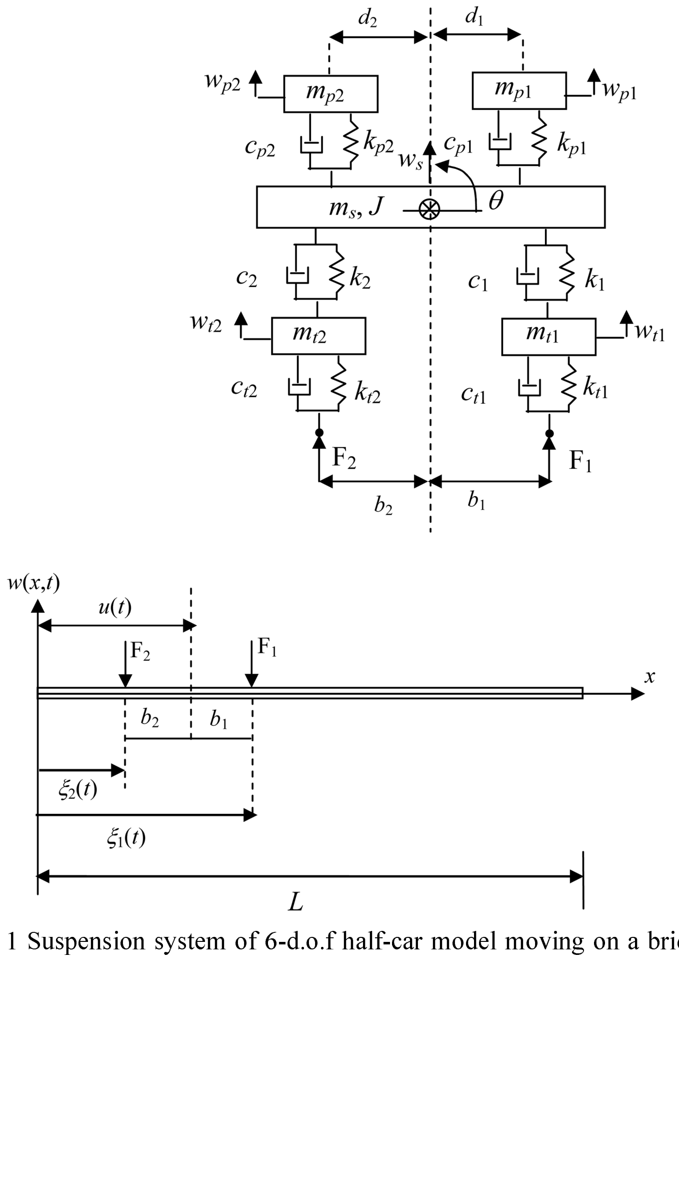

Fig. 1 Suspension system of 6-d.o.f half-car model moving on a bridge

vertical oscillation. The compositions of the suspension system, the tires, and the passenger seats are characterized by the combination of linear springs and viscous dampers connected in parallel arrangements as shown in Fig. 1. To derive the governing equation of motion for the coupled system, the Lagrange's equation is adopted. First of all, in order to obtain the approximate solution of equations of motion of the coupled system, the deflection of the beam $w(x, t)$ can be expressed as a series expansion as follows

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t) \tag{1}$$

where $q_i(t)$ are the generalized time-dependent coordinate, and $\phi_i(t)$ are the transverse Eigenfunctions (mode shapes) of a beam satisfying the boundary conditions.

By considering only the first n eigenfunctions of Eq. (1), the orthogonality conditions among these mode shapes can be derived as follows

$$\int_0^L \rho \phi_i(x) \phi_j(x) dx = N_i \delta_{ij}, \quad \int_0^L EI \phi_i''(x) \phi_j''(x) dx = S_i \delta_{ij}, \quad \int_0^L c \phi_i(x) \phi_j(x) dx = O_i \delta_{ij} \tag{2}$$

where c is the equivalent linear viscous damping ratio of the bridge. δ_{ij} is the Kronecker delta function for $i, j = 1, 2, \ldots, n$, and N_i, S_i, and O_i are, respectively, the generalized mass, generalized stiffness, and generalized damping coefficients in the ith mode.

In Lagrange's formulation, we need the time-dependent contact force between the tires and the beam, which can be expressed in terms of the Heaviside function in the following

$$\begin{aligned} f_g(x,t) &= -\left(m_{t1} + m_s \frac{b_2}{b_1 + b_2} + m_{p1} \frac{b_2 + d_1}{b_1 + b_2} + m_{p2} \frac{b_2 - d_2}{b_1 + b_2}\right) g H(x - \xi_1(t)) \\ &\quad -\left(m_{t2} + m_s \frac{b_1}{b_1 + b_2} + m_{p1} \frac{b_2 - d_1}{b_1 + b_2} + m_{p2} \frac{b_2 + d_2}{b_1 + b_2}\right) g H(x - \xi_2(t)) \\ &= -(f_{g1} H(x - \xi_1(t)) + f_{g2} H(x - \xi_2(t))) \end{aligned} \tag{3}$$

In Eq. (3), $\xi_1(t)$ and $\xi_2(t)$ denote the locations of the contact points of the front and rear tires along the bridge at any instant t, which can be expressed as

$$\xi_1(t) = u(t) + b_1, \quad \xi_2(t) = u(t) - b_2 \tag{4}$$

where $u(t)$ is denoted as the position of the center of gravity of the moving vehicle measured frm the left-end support of the beam, as presented in Fig. 1. Based on the orthogonality conditions given by Eq. (2) and the Galerkin approximation of Eq. (1), the governing equations of motion of the coupled system can then be derived readily. The model is governed by six linear second order differential equations of motion, which are formulated in the following.

The equation of the vertical motion for the vehicle body is

$$\begin{aligned} \ddot{w}_s(t) &= \Omega_1 w_s(t) + \Omega_2 \theta(t) + \frac{k_{p1}}{m_s} w_{p1}(t) + \frac{k_{p2}}{m_s} w_{p2}(t) + \frac{k_1}{m_s} w_{t1}(t) + \frac{k_2}{m_s} w_{t2}(t) \\ &\quad \Omega_3 \dot{w}_s(t) + \Omega_4 \dot{\theta}(t) + \frac{c_{p1}}{m_s} \dot{w}_{p1}(t) + \frac{c_{p2}}{m_s} \dot{w}_{p2}(t) + \frac{c_1}{m_s} \dot{w}_{t1}(t) + \frac{c_2}{m_s} \dot{w}_{t2}(t) \end{aligned} \tag{5}$$

and the equation of the rotational motion of the vehicle body is governed by

$$\ddot{\theta}(t) = \Omega_5 w_s(t) + \Omega_6 \theta(t) + \frac{k_{p1}d_1}{J} w_{p1}(t) - \frac{k_{p2}d_2}{J} w_{p2}(t) + \frac{k_1 b_1}{J} w_{t1}(t) - \frac{k_2 b_2}{J} w_{t2}(t)$$

$$+ \Omega_7 \dot{w}_s(t) + \Omega_8 \theta(t) + \frac{c_{p1}d_1}{J} \dot{w}_{p1}(t) - \frac{c_{p2}d_2}{J} \dot{w}_{p2}(t) + \frac{c_1 b_1}{J} \dot{w}_{t1}(t) - \frac{c_2 b_2}{J} \dot{w}_{t2}(t) \tag{6}$$

The equation of the vertical motion of the driver is

$$\ddot{w}_{p1}(t) = \frac{k_{p1}}{m_{p1}} w_s(t) + \frac{k_{p1}d_1}{m_{p1}} \theta(t) - \frac{k_{p1}}{m_{p1}} w_{p1}(t) + \frac{c_{p1}}{m_{p1}} \dot{w}_s(t) + \frac{c_{p1}d_1}{m_{p1}} \dot{\theta}(t) - \frac{c_{p1}}{m_{p1}} \dot{w}_{p1}(t) \tag{7}$$

while the vertical motion of the passenger is expressed as

$$\ddot{w}_{p2}(t) = \frac{k_{p2}}{m_{p2}} w_s(t) - \frac{k_{p2}d_2}{m_{p2}} \theta(t) - \frac{k_{p2}}{m_{p2}} w_{p2}(t) + \frac{c_{p2}}{m_{p2}} \dot{w}_s(t) - \frac{c_{p2}d_2}{m_{p2}} \dot{\theta}(t) - \frac{c_{p2}}{m_{p2}} \dot{w}_{p2}(t) \tag{8}$$

The equation of the vertical motion for the front axle is

$$\ddot{w}_{t1}(t) = \frac{k_1}{m_{t1}} w_s(t) + \frac{k_1 b_1}{m_{t1}} \theta(t) - \frac{k_1 + k_{t1}}{m_{t1}} w_{t1}(t) + \frac{k_{t1}}{m_{t1}} w(\xi_1(t), t) D_1$$

$$+ \frac{c_1}{m_{t1}} \dot{w}_s(t) + \frac{c_1 b_1}{m_{t1}} \dot{\theta}(t) - \frac{c_1 + c_{t1}}{m_{t1}} \dot{w}_{t1}(t) + \frac{c_{t1}}{m_{t1}} \dot{w}(\xi_1(t), t) D_1 \tag{9}$$

and the vertical motion of the rear axle is expressed as

$$\ddot{w}_{t2}(t) = \frac{k_2}{m_{t2}} w_s(t) - \frac{k_2 b_2}{m_{t2}} \theta(t) - \frac{k_2 + k_{t2}}{m_{t2}} w_{t2}(t) + \frac{k_{t2}}{m_{t2}} w(\xi_2(t), t) D_2$$

$$+ \frac{c_2}{m_{t2}} \dot{w}_s(t) - \frac{c_2 b_2}{m_{t2}} \dot{\theta}(t) - \frac{c_2 + c_{t2}}{m_{t2}} \dot{w}_{t2}(t) + \frac{c_{t2}}{m_{t2}} \dot{w}(\xi_2(t), t) D_2 \tag{10}$$

The governing of equation of motion of the bridge is formulated by n second order differential equations as follows

$$N_i \ddot{q}_i(t) - (N_i \Omega_{10i} - O_i)\dot{q}_i t - (N_i \Omega_{9i} - S_i) q_i(t)$$

$$= k_{t1}\phi_i(\xi_1(t)) D_1 w_{t1}(t) + k_{t2}\phi_i(\xi_2(t)) D_2 w_{t2}(t) + c_{t1}\phi_i(\xi_1(t)) D_1 \dot{w}_{t1}(t), \quad i = 1, 2, \ldots, n$$

$$+ c_{t2}\phi_i(\xi_2(t)) D_2 \dot{w}_{t2}(t) - \phi_i(\xi_1(t)) D_1 f_{g1} - \phi_i(\xi_2(t)) D_2 f_{g2} \tag{11}$$

where the coefficients Ω_1 to Ω_8 in Eqs. (5) and (6) are

$$\Omega_1 = \frac{-1}{m_s}(k_{p1} + k_{p2} + k_1 + k_2), \quad \Omega_2 = \frac{1}{m_s}(-k_{p1}d_1 + k_{p2}d_2 - k_1 b_1 + k_2 b_2)$$

$$\Omega_3 = \frac{-1}{m_s}(c_{p1} + c_{p2} + c_1 + c_2), \quad \Omega_4 = \frac{1}{m_s}(-c_{p1}d_1 + c_{p2}d_2 - c_1 b_1 + c_2 b_2)$$

$$\Omega_5 = \frac{1}{J}(-k_{p1}d_1 + k_{p2}d_2 - k_1 b_1 + k_2 b_2), \quad \Omega_6 = \frac{-1}{J}(k_{p1}d_1^2 + k_{p2}d_2^2 + k_1 b_1^2 + k_2 b_2^2)$$

$$\Omega_7 = \frac{1}{J}(-c_{p1}d_1 + c_{p2}d_2 - c_1 b_1 + c_2 b_2), \quad \Omega_8 = \frac{-1}{J}(-c_{p1}d_1^2 + c_{p2}d_2^2 - c_1 b_1^2 + c_2 b_2^2) \tag{12}$$

and the ith element of Ω_9 and Ω_{10} in Eq. (11) can be written in the following forms

$$\Omega_{9,i}(t) = \frac{-1}{N_i}[k_{t1}\phi_i^2(\xi_1(t))D_1^2 + k_{t2}\phi_i^2(\xi_2(t))D_2^2] \tag{13}$$

$$\Omega_{10,i}(t) = \frac{-1}{N_i}[c_{t1}\phi_i^2(\xi_1(t))D_1^2 + c_{t2}\phi_i^2(\xi_2(t))D_2^2] \tag{14}$$

The coefficients D_1 and D_2 are utilized to determine whether the front or rear tires stay on the bridge, for instance, $D_1 = 1$ denotes the front tire is on the bridge while $D_1 = 0$ means the front tire is off the bridge.

We can rearrange Eqs. (5)-(11) to form a system of $(n+6)$ second order coupled differential equations with time-varying coefficients. Obviously, the two coefficients D_1 and D_2 and the Eigen-functions $\phi_i(\xi_1(t))$ and $\phi_i(\xi_2(t))$ represent these time-varying coefficients in the governing equations of motion. Eqs. (5)-(11) can be represented in the state-space form very readily as follows

$$\dot{\mathbf{z}}(t) = \mathbf{B}(t)\mathbf{z}(t) + \mathbf{V}\mathbf{G}(t) \tag{15}$$

where

$$\mathbf{z}(t) = [\mathbf{p}(t)\ \ \mathbf{q}(t)\ \ \dot{\mathbf{p}}(t)\ \ \dot{\mathbf{q}}(t)]^T_{2(n+6)*1}, \quad \mathbf{q}(t) = [q_1(t)\ \dots\ q_n(t)]^T_n \tag{16}$$

and the state variables vector for the coupled system is

$$\mathbf{p}(t) = \{w_s(t)\ \ \theta(t)\ \ w_{p1}(t)\ \ w_{p2}(t)\ \ w_{t1}(t)\ \ w_{t2}(t)\}^T \tag{17}$$

Once Eq. (15) has been solved, it is quite straightforward to compute the dynamic response of the coupled system, for example, we can obtain the vertical deflection of the bridge, the vertical deflection and rotational displacement of the vehicle body mass, the vertical deflections of the driver and passenger, etc…

3. Stochastic analysis

In the stochastic analysis, we assume the initial velocity (v) and body mass (m_s) of the moving oscillator to be uncertain to make their characterization in probabilistic sense. The equations governing the problem possess both random force and coefficients, and therefore the response becomes a random process. Denoting by E[v] and E[m_s] the mean value of initial velocity and moving body mass for the vehicle, and by β_i the fluctuation of ith ($i = 1, 2$) uncertain parameter with respect to its mean value and under the assumption that $\|\beta_i\| \ll 1$, the random variables are described by the following laws of variation:

$$v = \mathrm{E}[v](1+\beta_1), \quad m_s = \mathrm{E}[m_s](1+\beta_2) \tag{18}$$

Here β_i is assumed as uniformly distributed random variables with zero mean. For simplicity, E[$\beta_1\beta_2$] is assumed as zero in the present study although the assumption is not necessary.

In this study, the improved perturbation technique is adopted to solve the stochastic equations.

This method was originally proposed by Elishakoff *et al.* (1995) to solve static problems as an improved approach to the traditional stochastic finite element method (Kleiber and Hein 1992), and then extended to perform the dynamic analysis of linear systems with random coefficients by Muscolino *et al.* (2000), and to investigate bridge-vehicle interaction under random moving mass by Muscolino *et al.* (1999). The basic concept of the improved perturbation technique uses the Taylor series expansion of the uncertain parameters and ignores third and higher order terms. In addition, the state-space vector is used to write the second order differential equations into a set of first order differential equations. Consequently, it transforms the second order stochastic differential equation into a set of first order deterministic differential equations. According to the philosophy of this method, keeping track of the relationship (18) and introducing the vector $\boldsymbol{\beta}$ of uncertain parameters, the coupled system of Eq. (15) is rewritten in terms of state variables as follows

$$\dot{\mathbf{z}}(\boldsymbol{\beta}, t) = \mathbf{B}(\boldsymbol{\beta}, t)\mathbf{z}(\boldsymbol{\beta}, t) + \mathbf{V}\mathbf{G}(\boldsymbol{\beta}, t) \tag{19}$$

Eq. (19) consists of $(2n+12)$ equations since the vectors $\mathbf{q}$ and $\mathbf{p}$ are, respectively, $n \times 1$ and 6×1 order, with random and time-dependent coefficients. According to the mean-value perturbation approach the matrix $\mathbf{B}(\boldsymbol{\beta}, t)$, the vector $\mathbf{G}(\boldsymbol{\beta}, t)$ and $\mathbf{z}(\boldsymbol{\beta}, t)$ can be rewritten in approximate forms as follows (Muscolino *et al.* 2002)

$$\mathbf{B}(\boldsymbol{\beta}, t) = \overline{\mathbf{B}}(t) + \sum_{i=1}^{2} \mathbf{B}_i(t)\beta_i \tag{20}$$

$$\mathbf{G}(\boldsymbol{\beta}, t) = \overline{\mathbf{G}}(t) + \sum_{i=1}^{2} \mathbf{G}_i(t)\beta_i \tag{21}$$

$$\mathbf{z}(\boldsymbol{\beta}, t) = \overline{\mathbf{z}}(t) + \sum_{i=1}^{2} \mathbf{z}_i(t)\beta_i \tag{22}$$

where the over bar denotes mean value, while the symbol with the index indicates the deviation from the mean value. Substituting Eqs. (20)-(22) into Eq. (19) and taking the average of Eq. (19), finally we can wind up with the following equations

$$\dot{\overline{\mathbf{z}}}(t) = \overline{\mathbf{B}}(t)\overline{\mathbf{z}}(t) + \sum_{i}^{2} \mathbf{B}_i\mathbf{z}_i(t)\mathrm{E}[\beta_i^2] + \mathbf{V}\overline{\mathbf{G}}(t) \tag{23}$$

$$\dot{\mathbf{z}}_i(t) = \overline{\mathbf{B}}(t)\mathbf{z}_i(t) + \mathbf{B}_i(t)\overline{\mathbf{z}}(t) + \mathbf{V}\mathbf{G}_i(t), \quad i = 1, 2 \tag{24}$$

The solution of these coupled Eqs. (23)-(24) can be computed by using the numerical procedure proposed in Muscolino (1996). Once these equations are solved, the stochastic response of the coupled system can be readily calculated as described by Muscolino *et al.* (2002). Furthermore, it is possible to draw the statistical moments of higher order of the beam deflection by means of the following relationship

$$M_r(\boldsymbol{\beta}) = \mathrm{E}[w^{\mathrm{r}}] = \mathrm{E}[\{\boldsymbol{\phi}^T\mathbf{z}(\boldsymbol{\beta}, t)\}^{\mathrm{r}}] \tag{25}$$

In the present study, the results by the mean-value perturbation method will be checked by Monte

Carlo simulation. The parameters used to test the applicability of this proposed approach are the coefficient of variation and the coefficient of excess defined by the following relationships, respectively (Lewis 1987).

$$\text{c.o.v.} = \left.\frac{\sigma(x,t)}{\overline{\Re}(x,t)}\right|_{(x=L/2;\ t=t_{L/2})}, \quad \sigma^2 = \mathrm{E}[(\Re-\overline{\Re})^2] \tag{26}$$

$$\text{c.e} = \left.\frac{k_4(x,t)}{\sigma^4(x,t)}\right|_{(x=L/2;\ t=t_{L/2})}, \quad k_4 = \mathrm{E}[(\Re-\overline{\Re})^4] \tag{27}$$

The first parameter defined as the ratio between the standard deviation $\sigma(x, t)$ and the mean value $\overline{\Re}(x,t)$ of the displacement or stress at $x = L/2$ at the time when the vehicle is located at the middle point position of the beam. The second parameter is defined as the ratio between the fourth cumulate $k_4(x, t)$ and the square power of displacement or stress variance evaluated at $x = L/2$ at the time when the vehicle is located at the mid-point of the beam.

4. Numerical examples and discussions

The proposed procedure has been used to calculate the response of a simply supported beam subjected to a moving vehicle as shown in Fig. 1. The bridge is characterized as a uniform beam and is considered free of any load or deflection initially, and therefore is horizontal at the equilibrium position under its own weight. The Eigenfunctions $\phi_i(x)$, based on the Euler-Bernoulli beam theory, are adopted in the numerical computations, specifically the normalized Eigenfunctions of a simply supported beam are given by

$$\phi_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi x}{L}\right), \quad i = 1, 2, \ldots, n \tag{28}$$

4.1 Deterministic analysis

In deterministic analysis, a vehicle traveling at a constant speed on a bridge is presented here. The 6-d.o.f. passenger-vehicle planar model shown in Fig. 1 is considered here. The numerical values of the parameters as proposed by Esmailzadeh, *et al.* (2003) are depicted as follows:

Bridge: $L = 100$ m, $E = 207$ Gpa, $I = 0.174$ m^4, $\rho = 20000$ kg m^{-1}, $c = 1750$ N s m^{-1}.
Vehicle: $m_s = 1794.4$ kg, $m_{t1} = 87.15$ kg, $m_{t2} = 140.4$ kg, $m_{p1} = 75$ kg, $m_{p2} = 75$ kg,
$J = 3443.05$ kg m^2, $b_1 = 1.271$ m, $b_2 = 1.716$ m, $d_1 = 0.481$ m, $d_2 = 1.313$ m,
$k_1 = 66.824$ KN m^{-1}, $k_2 = 18.615$ KN m^{-1}, $k_{t1} = k_{t2} = 101.115$ KN m^{-1},
$k_{p1} = k_{p2} = 14.0$ KN m^{-1}, $c_1 = 1190$ N s m^{-1}, $c_2 = 1000$ N s m^{-1},
$c_{t1} = c_{t2} = 14.6$ N s m^{-1}, $c_{p1} = 50.2$ N s m^{-1}, $c_{p2} = 62.1$ N s m^{-1} (29)

The dynamic deflections of the mid-point of the bridge are shown in Fig. 2 for three different values of the vehicle speed. Meanwhile, the transient response for the deflection motion of the vehicle body is illustrated in Fig. 3. Moreover, the variations of the deflection motion of the driver, the passenger, the front and rear tires are presented in Figs. 4-7 respectively. It should be noted that the numerical results shown in Figs. 2-7 match with those presented in Esmailzadeh, *et al.* (2003).

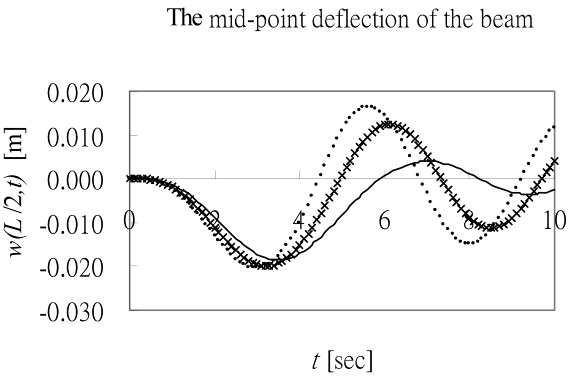

Fig. 2 Time history of mid-point deflection $w(L/2, t)$ of the beam for $v = 15.55$ m/sec (—), $v = 20$ m/sec (-+-), and $v = 24.44$ m/sec (···)

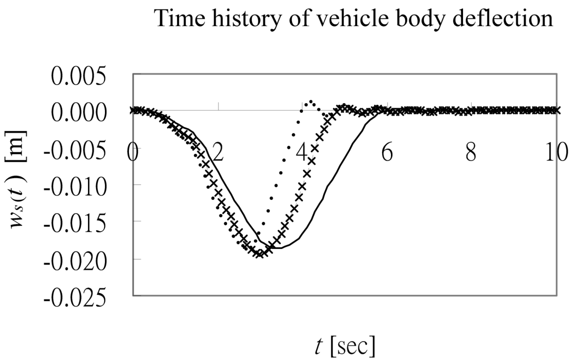

Fig. 3 Time history of vehicle body deflection $w_s(t)$ for $v = 15.55$ m/sec (—), $v = 20$ m/sec (-+-), and $v = 24.44$ m/sec (···)

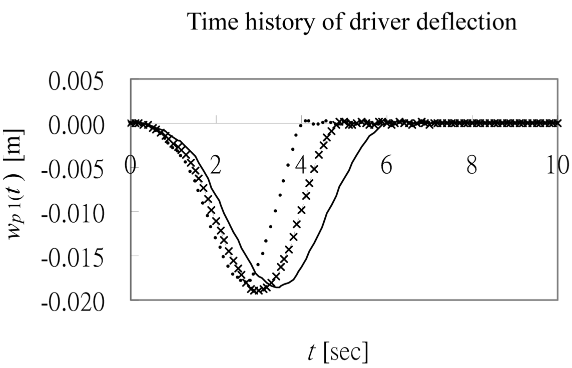

Fig. 4 Time history of driver deflection $w_{p1}(t)$ for $v = 15.55$ m/sec (—), v=20 m/sec (-+-), and $v = 24.44$ m/sec (···)

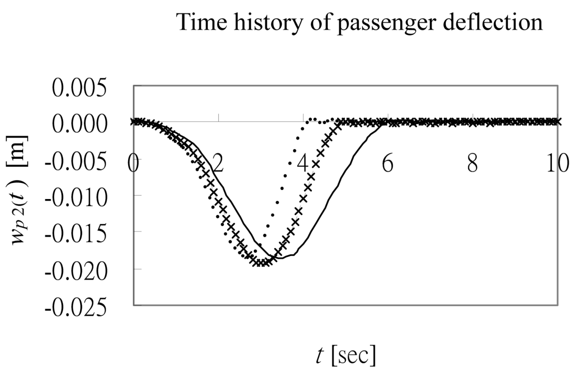

Fig. 5 Time history of passenger deflection $w_{p2}(t)$ for $v = 15.55$ m/sec (—), $v = 20$ m/sec (-+-), and $v = 24.44$ m/sec (···)

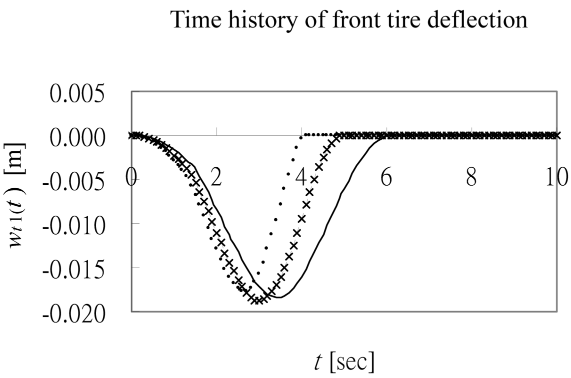

Fig. 6 Time history of front tire deflection $w_{t1}(t)$ for $v = 15.55$ m/sec (—), $v = 20$ m/sec (-+-), and $v = 24.44$ m/sec (···)

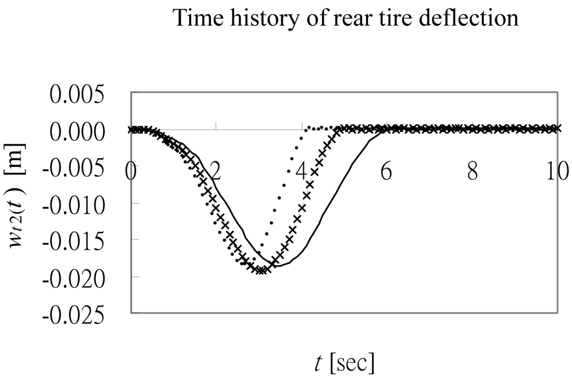

Fig. 7 Time history of rear tire deflection $w_{t2}(t)$ for $v = 15.55$ m/sec (—), $v = 20$ m/sec (-+-), and $v = 24.44$ m/sec (···)

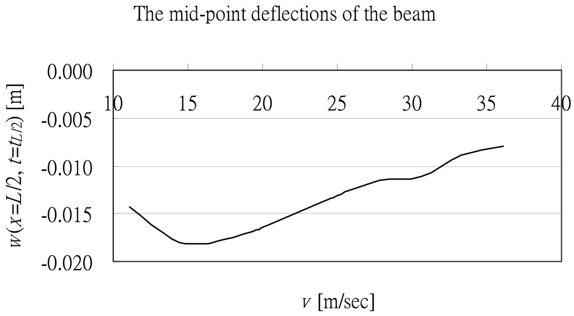

Fig. 8 The mid-point dynamic deflection

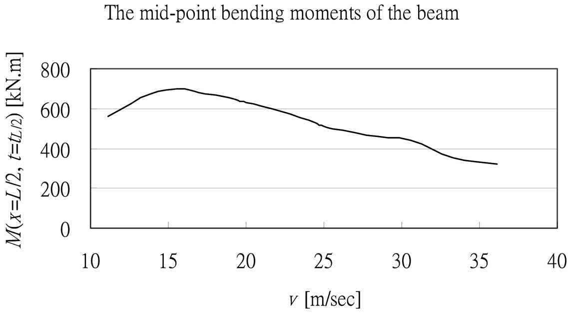

Fig. 9 The mid-point bending moment

The variations of the mid-point value of the transversal dynamic deflection and the bridge bending moment, when the vehicle is located at the mid-point of the beam, with respect to the vehicle speed, are shown in Figs. 8 and 9 respectively. It is observed that when the vehicle travels at 15 m/sec, the deflection of the mid-point of the beam where the vehicle is located reaches the maximum value.

4.2 Stochastic analysis

In order to simulate another bridge-vehicle interaction case, the following numerical values for the parameters are adopted here.

Bridge: $L = 100$ m, $E = 200$ Gpa, $I = 0.271$ m^4, $\rho = 15000$ kg m^{-1}, $c = 3000$ N s m^{-1}.
Vehicle: $m_s = 2000$ kg, $m_{t1} = 90$ kg, $m_{t2} = 140$ kg, $m_{p1} = 80$ kg, $m_{p2} = 80$ kg,
$J = 3387.5$ kg m^2, $b_1 = 1$ m, $b_2 = 2$ m, $d_1 = 0.5$ m, $d_2 = 1.5$ m,
$k_1 = 65.0$ KN m^{-1}, $k_2 = 20.0$ KN m^{-1}, $k_{t1} = k_{t2} = 100.0$ KN m^{-1},
$k_{p1} = k_{p2} = 140$ KN m^{-1},$c_1 = 1190$ N s m^{-1}, $c_2 = 1000$ N s m^{-1},
$c_{t1} = {}_{t2} = 15$ N s m^{-1}, $c_{p1} = 50$ N s m^{-1}, $c_{p2} = 60$ N s m^{-1} (30)

The deterministic analysis will be performed before we proceed to confront with the stochastic analysis. In order to achieve the possible maximal deflection and bending moment of the mid-point of the bridge, we employ $v = 22.22$ m/s to carry out the following numerical computations. The variations of the mid-point value of the transversal dynamic deflection and the bridge bending moment, when the vehicle is located at the mid-point of the beam, with respect to the vehicle speed,

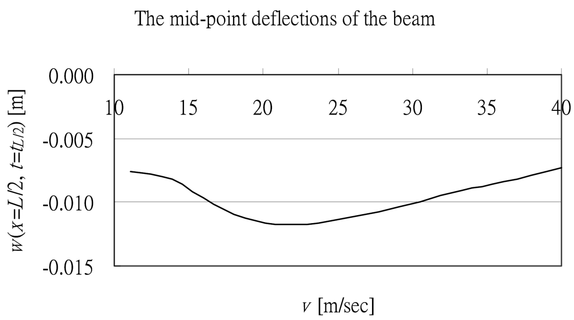

Fig. 10 The mid-point dynamic deflection

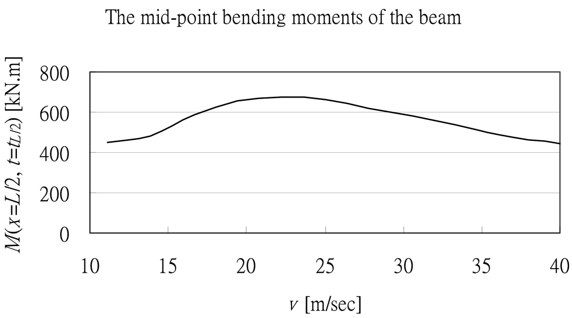

Fig. 11 The mid-point bending moment

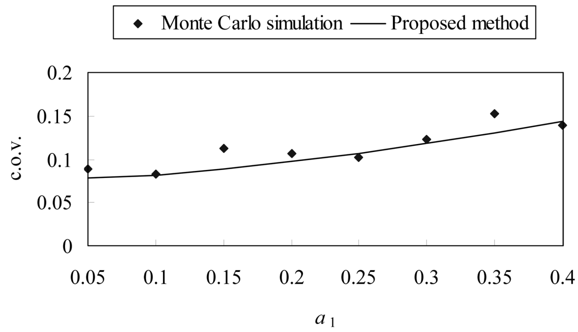

Fig. 12 Absolute value of coefficient of variation c.o.v. of mid-point deflection versus different fluctuations a_1 of stochastic initial velocity

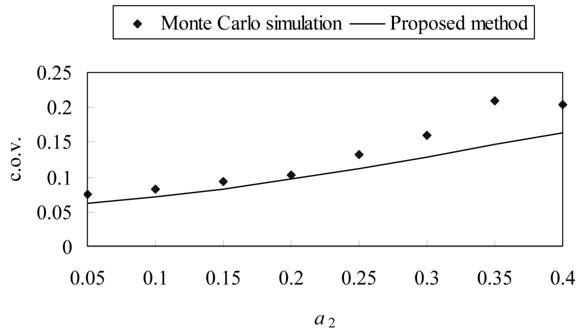

Fig. 13 Absolute value of coefficient of variation c.o.v. of mid-point deflection versus different fluctuations a_2 of stochastic vehicle mass

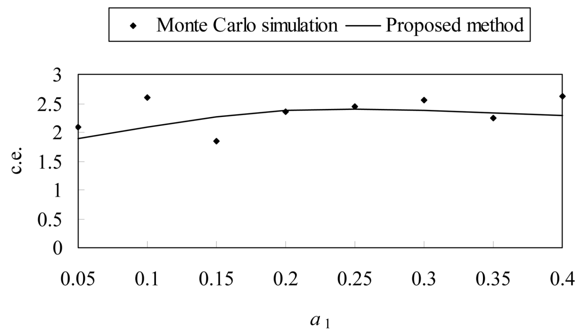

Fig. 14 Absolute value of excess coefficient c.e. of mid-point deflection versus different fluctuations a_1 of stochastic initial velocity

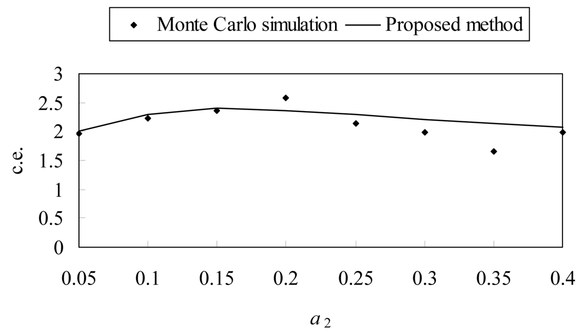

Fig. 15 Absolute value of excess coefficient c.e. of mid-point deflection versus different fluctuations a_2 of stochastic vehicle mass

are presented in Figs. 10 and 11 respectively. It is observed that when the vehicle travels at 22.22 m/sec, the deflection of the mid-point of the beam where the vehicle is located reaches maximum value. In stochastic analysis, the stochastic body mass is assumed to have mean value $E[m_s] = 2000$ kg, and the mean value of the initial velocity is assumed to be $E[v] = 22.22$ m/s. It is noted that, as stated previously, the initial velocity and vehicle body mass has been assumed as stochastic variables. The uncertain parameters β_i are assumed to be uniformly distributed with zero mean in the intervals $[-a_i, a_i]$ $(i = 1,2)$ with a_i varying between 0 and 0.4. The field of applicability of the proposed method is investigated and checked by Monte Carlo simulation. The parameters used to test the applicability of this proposed approach are the coefficient of variation and the coefficient of excess defined by Eqs. (26) and (27) respectively. With the purpose to observe the contribution of the single uncertain parameter, the cases of random velocity and vehicle body mass have been considered separately. As it can be detected from Figs. 12 to 15, the coefficient of variation (c.o.v.) and the coefficient of excess (c.e.) calculated from the perturbation approach are in a good agreement with those computed from Monte Carlo simulation. In Figs. 12-15, the values of a_i were fixed to be equal to 0.2 for all the uncertainties unless it is a varying parameter in that particular figure.

5. Conclusions

The problem of calculating the dynamic response of a distributed parameter system excited by a moving vehicle with random initial velocity and vehicle body mass is investigated. The vehicle, including the driver and passenger, is modeled as a half-car planar model, which is moving on a wide span uniform bridge modeled in the form of a simply supported Euler-Bernoulli beam. The system response is a stochastic process although its characteristics are assumed to be deterministic. By adopting the modal analysis and Galerkin's method, a set of approximate governing equations of motion possessing time-dependent uncertain coefficients and forcing function is obtained. The statistical characteristics of the response of the beam are computed by using improved perturbation approach. This method is simple and useful to gather the stochastic structural response due to the vehicle-passenger-bridge interaction. Furthermore, some of the statistical numerical results calculated from the perturbation technique are checked by Monte Carlo simulation.

Acknowledgements

This work was partially supported by the National Science Council of the Republic of China under grant NSC 94-2211-E327-016. The authors are grateful for this support.

References

Bolotin, V.V. (1965), *Statistical Methods in Engineering Mechanics*, Stroiizdat, Moscow.
Chang, T.P. and Chang, H.C. (1994), "Stochastic dynamic finite element analysis of a nonuniform beam", *Int. J. Solids Struct.,* **31**, 587-597.
Chang, T.P. and Liu, Y.N. (1996), "Dynamic finite element analysis of a nonlinear beam subjected to a moving load", *Int. J. Solids Struct.,* **33**, 1673-1688.
Chang, T.P., Lin, G.L. and Chang, E. (2006), "Vibration analysis of a beam with an internal hinge subjected to a random moving oscillator", *Int. J. Solids Struct.,* **43**, 6398-6412.
ElBeheiry, E.M. (2000), "Effects of small travel speed variations on active vibration control in modern vehicles", *J. Sound Vib.*, **232**, 857-875.
Elishakoff, I., Ren, Y.J. and Shinozuka, M. (1995), "Improved finite element method for stochastic problems", *Chaos Soliton. Fract.,* **5**, 833-846.
Esmailzadeh, E. and Ghorashi, M. (1995), "Vibration analysis of beams traversed by uniform partially distributed moving masses", *J. Sound Vib.*, **184**, 9-17.
Esmailzadeh, E. and Jalilib, N. (2003), "Vehicle-passenger-structure interaction of uniform bridges traversed by moving vehicles", *J. Sound Vib.*, **260**, 611-635.
Feng, Q. and He, H. (2003), "Modeling of the mean Poincare' map on a class of random impact oscillators", *Eur. J. Mech. A-Solid,* **22**, 267-281.
Fryba, L. (1999), *Vibration of Solids and Structures Under Moving Loads*, Telford, London.
Katz, R., Lee, C.W., Ulsoy, A.G. and Scott, R.A. (1988), "The dynamic response of rotating shaft subject to a moving load", *J. Sound Vib.*, **122**, 131-148.
Kleiber, M. and Hein, T.D. (1992), *The Stochastic Finite Element Method,* Wiley, Chichester.
Lee, H.P. (1994), "Dynamic response of a beam with intermediate point constraints subject to a moving load", *J. Sound Vib.*, **171**, 361-368.
Lewis, E.E. (1987), *Introduction to Reliability Engineering*, John Wiley & Sons, New York.
Muscolino, G. (1996), "Dynamically modified linear structures: deterministic and stochastic response", *J. Struct.*

Eng., ASCE, **122**, 1044-1051.

Muscolino, G. and Sidoti A. (1999), "Dynamics of railway bridges subjected to moving mass with stochastic velocity", *Structural Dynamics Eurodyn'99*, 711-716.

Muscolino, G., Ricciardi, G. and Impollonia, N. (2000), "Improved dynamic analysis of structures with mechanical uncertainties under deterministic input", *Prob. Eng. Mech.,* **15**, 199-212.

Muscolino, G., Benfratello, S. and Sidoti, A. (2002), "Dynamics analysis of distributed parameter system subjected to a moving oscillator with random mass, velocity and acceleration", *Prob. Eng. Mech.,* **17**, 63-72.

Ricciardi, G. (1994), "Random vibration of beam under moving loads", *J. Struct. Eng., ASCE,* **120**, 2361-2381.

Sadiku, S. and Leipholz, H.H.E. (1987), "On the dynamics of elastic systems with moving concentrated masses", *Ing. Arch.*, **57**, 223-242.

Sniady, P., Biemat, S., Sieniawska, R. and Zukowski, S. (1999), "Vibrations of the beam due to a load moving with stochastic velocity", Spanos, P.D. *et al.*, editors. *Proceedings of Computational Stochastic Mechanics (CSM'98),* Amsterdam, Balkema.

Sobczyk, K., Wedrychowicz, S. and Spencer B.F. (1996), "Dynamics of structural systems with spatial randomness", *Int. J. Solids Struct.,* **33**, 1651-1669.

Timoshenko, S. (1922), "On the forced vibration of bridge", *Philos. Mag. Series,* **43**, 1018.

Wang, R., Duan, Y.B. and Zang, Z. (2002), "Resonance analysis of finite-damping nonlinear vibration system under random disturbances", *Eur. J. Mech. A-Solid,* **21**, 1083-1088.

Zibdeh, H.S. (1995), "Stochastic vibration of an elastic beam due to random moving loads and deterministic axial forces", *Eng. Struct.,* **17**, 530-535.

Zibdeh, H.S. and Abu-Hilal, M. (2003), "Stochastic vibration of laminated composite coated beam traversed by a random moving load", *Eng. Struct.,* **25**, 397-404.