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# Optimal design of composite laminates for minimizing delamination stresses by particle swarm optimization combined with FEM

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**Abstract.** The present paper addresses the optimal design of composite laminates with the aim of minimizing free-edge delamination stresses. A technique involving the application of particle swarm optimization (PSO) integrated with FEM was developed for the optimization. Optimization was also conducted with the zero-order method (ZOM) included in ANSYS. The semi-analytical method, which provides an approximation of the interlaminar normal stress of laminates under in-plane load, was used to partially validate the optimization results. It was found that optimal results based on ZOM are sensitive to the starting design points, and an unsuitable initial design set will lead to a result far from global solution. By contrast, the proposed method can find the global optimal solution regardless of initial designs, and the solutions were better than those obtained by ZOM in all the cases investigated.

**Keywords:** laminates; interlaminar stress; particle swarm optimization; delamination; FEM.

## 1. Introduction

Laminated structures have found many fields of application in recent decades owing to their advantages of combined high strength and low weight as well as good flexibility. In order to fully exploit the potential of composite laminates, it is essential to become acquainted with the stress distribution features in a laminate under external action. Traditional classical lamination theory (CLT) provides a good prediction for in-plane stresses and strains for some simple cases. In general, however, owing to the mismatch of the mechanical properties of the adjacent layers, interlaminar singular stress fields will develop in the vicinity of the free edges of the laminate. This so-called "free-edge effect", i.e., localized high interlaminar stresses, is directly attributed to delamination in a laminate and causes laminate fracture to take place earlier (Peng and Chen 2006, Sleight 1999).

Peng and Chen (2006) pointed out that for the laminates $[(\pm\theta)_2, 90^o]_S$ and $[90^o, (\pm\theta)_2]_S$, when the location of the $90^o$ layer changes from inside to outside, the interlaminar normal stress at all the interfaces of the free-edge changes from a relatively high tensile magnitude to a compressive stress and, as a result, the delamination failure is suppressed and the ultimate strength is enhanced

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accordingly. Sancho and Miravete (2006) reported that the interlaminar normal stress $\sigma_z$ is responsible for most delamination failures, based on experimentation and stress analyses of laminates. Ferreira *et al*. (1995) formulated the interlaminar normal and shear stresses as functions of the transverse normal stress $\sigma_y$ based on the bilinear approximation method developed by Pagano and Pipes (1970). Thereafter, a multi-objective optimization was executed to minimize the interlaminar stresses. Since the calculation was based upon CLT, the model and the load conditions were restricted to simple parameters. Lindemann *et al*. (2002) assessed the delamination tendency by using the square value of the interlaminar normal stress $\sigma_z^2$ and minimized it by two optimization algorithms, i.e., the zero order method and the generalized reduced gradient algorithm. These gradient-based algorithms are computationally efficient, but they do not guarantee global optimum results. Therefore, it is necessary to start the optimization from different initial designs and compare the results to ensure that the optimization is global.

In the structural optimization field, much attention has recently been paid to evolution-based algorithms. These algorithms do not require continuity and derivative existence of the objective function or constraint function. They are robust and can provide a more reliable approach to obtain the global optimum in non-smooth problems as compared to the gradient-based methods. Genetic algorithms (GAs) have been applied successfully to composite structure problems (Aligeigloo *et al*. 2007, Paluch *et al*. 2008, Park *et al*. 2001, Muc and Gurba 2001, Murugan *et al*. 2007, Naik *et al*. 2008, Walker and Smith 2003, Rahul *et al*. 2005). Very recently, particle swarm optimization (PSO) (Kennedy and Eberhart 1995), which is also an evolutionary global algorithm, has gained popularity. The PSO algorithm is based on a simplified social model and mimics the behavior of a bird flock in search for food. PSO has also been successfully applied to composite structure problems (Chen *et al*. 2008, Kathiravan *et al*. 2007, Omkar *et al*. 2008). PSO shares several similarities with GA. For example, both PSO and GA start with a randomly generated population, evaluate the population for fitness values, update the population, and use random methods to search for the optimal solution. The main advantage of PSO over GA is that it does not need complicated encoding, decoding, or special genetic operators such as mutation and crossover, and thus it has fewer parameters to be adjusted. It is therefore more effective in terms of CPU time (Dong *et al*. 2005, Ge *et al*. 2007, Suresh *et al*. 2007) and offers quite straightforward implementation.

Although there has been much research concerning the optimization of composite structures (Ganguli and Chopra 1995, 1996, 1997, Marannano and Mariotti 2008, Murugan and Ganguli 2005, Rao and Arvind 2007, Topal and Uzman 2006), only a few works among them aimed to suppress delamination in a laminate. In the present paper, a commercial finite element code, ANSYS, is used to calculate interlaminar stresses at the free-edge of laminates. Optimization for minimizing interlaminar normal stresses is then conducted using the zero-order method (ZOM) incorporated in ANSYS. Results are presented for two different loading conditions: in-plane tensile load and uniform bending load. Numerical results demonstrate that the optimal solutions based on ZOM are sensitive to the initial designs. In order to overcome this disadvantage, a technique of applying the PSO algorithm integrated with FEM is developed. Examples of minimizing, respectively, the maximum, the mean, and the variation of the absolute peak interlaminar normal stresses in a laminate are presented. The ability to derive the global optimal and the potential of dealing with complicated problems in terms of practical time constraints are discussed and demonstrated.

## 2. Interlaminar stresses analysis

In recent decades numerous methods have been developed to explore interlaminar stresses, including the finite-difference method, finite element method, and boundary layer method. Overviews of different approaches have been given by Mittelstedt and Becker (2007) and Kant and Swaminathan (2000). FEM is an effective and convenient approach to assess interlaminar stresses for various boundary and/or loading conditions. In this paper, the general FE code ANSYS is utilized for analyzing stresses in a laminate, as shown in Fig. 1. Eight-node anisotropy brick elements are used in order to obtain the three-dimensional stress fields. As the free-edge effect is restricted to a distance of approximately the total laminate thickness from the free-edge to the interior, it is necessary to refine the mesh at these regions, as shown in Fig. 2.

To verify the accuracy of the interlaminar stresses calculation, a FE analysis was first performed on a sample plate [0/45º/−45º/90º]s made of T300/5208 (Sancho and Miravete 2006). The plate analyzed is 120 mm in length and 30 mm in width with a total thickness of 1 mm. Ply properties are listed in Table 1. The load introduced is an axial deformation of 0.66% along the length. Since a

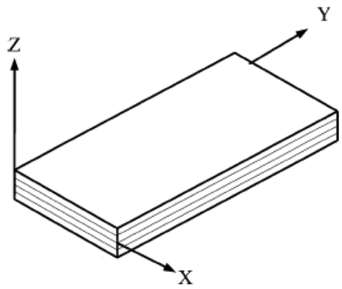


Fig. 1 A laminate model

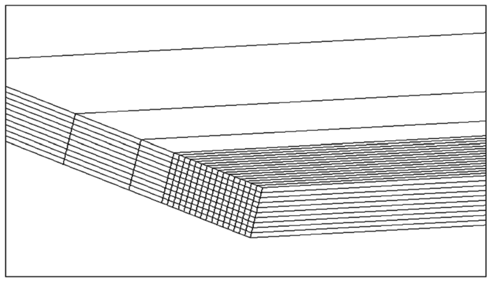

Fig. 2 FE mesh at the free-edge

Table 1 Material properties (modules expressed in MPa)

| $E_x$ | $E_y = E_z$ | $G_{xy} = G_{xz}$ | $G_{yz}$ | $u_{xy} = u_{xz}$ | $u_{yz}$ |
|---|---|---|---|---|---|
| 181000 | 10300 | 7170 | 4130 | 0.28 | 0.51 |

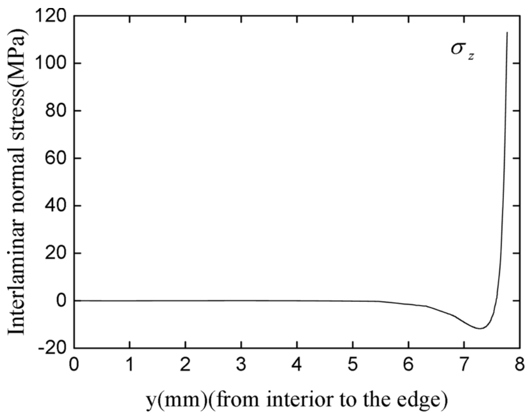


Fig. 3 $\sigma_z$ distribution at the interface (90º/90º)

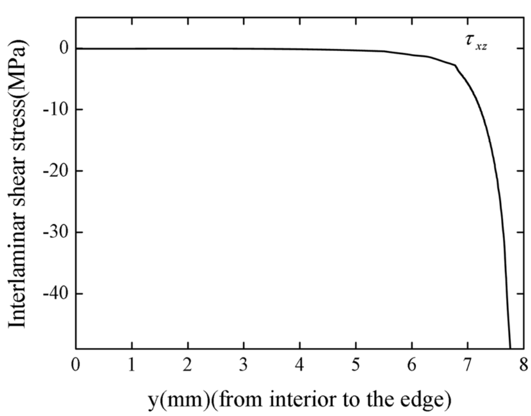


Fig. 4 $\tau_{xz}$ distribution at the interface (45º/−45º) symmetric stacking sequence is employed, only half of the plate is modeled. The calculated interlaminar normal stress $\sigma_z$ in the symmetry interface (90°/90°) and interlaminar shear stresses $\tau_{xz}$ in the interface (45°/−45°) are plotted in Figs. 3-4. The stress distribution in Figs. 3-4 is very similar to that reported by Pipes and Pagano (1970). The peak values of $\sigma_z$ and $\tau_{xz}$ are 116 MPa and −48 MPa, respectively, which are very close to the values (113 MPa and −45 MPa) reported by Sancho and Miravete (2006).

## 3. Optimization problem

It is known that compressive normal stress is beneficial with respect to restraining free-edge delamination. However, high compressive normal stresses at the free edges go along with high tensile normal stresses at some distance within the interior of the laminates, which in regard to delamination is undesirable (Lindemann *et al.* 2002). Given this, the present paper aims to minimize the absolute value of the interlaminar normal stresses at the free-edges. For a laminate with $N$ layers, there are $n = N - 1$ interfaces and $n$ interlaminar peak normal stresses that have to be minimized. Let the absolute value of the peak interlaminar normal stress at the $i$th interface be expressed as $\sigma_{zi}(\theta)$; three kinds of functions, i.e., the maximum, the mean, and the variation of $\sigma_{zi}(\theta)$, are respectively taken as objectives. The optimization problems are formulated as follows

$$1)\ \underset{\theta}{\text{Min}}: f(\theta) = \underset{(i=1,\ldots,n)}{\text{Max}}\,[\sigma_{zi}(\theta)] \tag{1}$$

$$2)\ \underset{\theta}{\text{Min}}: \mu(\theta) = \frac{1}{n}\sum_{i=1}^{n}\sigma_{zi}(\theta) \tag{2}$$

$$3)\ \underset{\theta}{\text{Min}}: S(\theta) = \frac{1}{\sqrt{n}}\sqrt{\sum_{i=1}^{n}(\sigma_{zi}(\theta) - \mu(\theta))^2} \tag{3}$$

where $f$, $\mu$, and $S$ are respectively, the maximum, the mean, and the variation of $\sigma_{zi}(\theta)$ at the $n$ interfaces. $\theta$ is the ply orientation angle. In the following, $f$, $\mu$, and $S$ in Eqs. (1), (2), and (3) will be taken as the objective functions separately and each of them is a single-objective optimization problem.

## 4. Optimization procedure

### 4.1 Zero-order method

There are two optimization methods provided by ANSYS, the zero-order method (also called the subproblem approximation method) and the first-order method. The first-order method uses gradients of the dependant variables. In each iteration, a search direction is selected by using a steepest descent or conjugate direction method, and the unconstrained problem is minimized along the specified direction. As expected, each iteration consists of a large number of subiterations calculating search directions as well as gradients. Hence, the first-order method performs several analysis loops and requires substantially much longer CPU time in comparison to that required by ZOM.

The zero-order method (ZOM), due to its independency from using derivatives of the problem variables, is the first candidate for the optimization subroutine. It can be efficiently applied to most engineering problems (Bayandor *et al.* 2002, Hasan *et al.* 2003, Hossain *et al.* 2007, Kayabasi and Ekici 2007). For this method, the dependent variables are replaced with the response surface (RS) approximations by means of least squares fitting and minimization is performed every iteration on the penalized function. Each iteration is equivalent to one complete analysis loop. ANSYS DO module generates and utilizes the polynomial RS approximation for the objective or constraint function as follows (ANSYS Manual 2003)

$$\tilde{f} = a_0 + \sum_{n=1}^{N} a_n x_n + \sum_{n=1}^{N} b_n x_n^2 + \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} c_{mn} x_m x_n \quad (4)$$

where $a$, $b$, and $c$ are coefficients determined by a weighted least squares technique. $\tilde{f}$ is the approximation of the objective or constraint function, $N$ is the number of design variables, and $x_i$ ($i = 1, 2, \ldots N$) is the design variable.

For the ZOM, convergence is assumed if any one of the following conditions is satisfied.

$$\left|f^{(j)} - f^{(j-1)}\right| \le \tau \quad (5a)$$

$$\left|f^{(j)} - f^{(b)}\right| \le \tau \quad (5b)$$

$$\left|x_i^{(j)} - x_i^{(j-1)}\right| = \rho \quad (i = 1, 2, 3, \ldots, N) \quad (5c)$$

$$\left|x_i^{(j)} - x_i^{(b)}\right| = \rho \quad (i = 1, 2, 3, \ldots, N) \quad (5d)$$

where $f^{(j)}$ is the objective function at iteration $j$ and $f^{(b)}$ is the best objective function at the current iteration, and $x^{(j)}$ and $x^{(b)}$ are the design variable vectors corresponding to $f^{(j)}$ and $f^{(b)}$. $\tau$ and $\rho$ are the objective function and design variable tolerances, respectively. In the following optimization examples, both the objective function tolerance and the design variable tolerance are set to be 10e-8.

### 4.2 Particle swarm optimization

PSO, proposed by Kennedy and Eberhart (1995), was basically developed through simulation of bird flocking in two-dimensional space. It is a population-based search algorithm where each individual is referred to as a particle and represents a candidate solution. To discover the optimal solution, each particle changes its searching direction according to two factors, its own best previous experience (pBest) and the best experience of all other members (gBest). Every swarm continuously updates itself through the aforementioned best solutions. Thus, a new generation of community comes into being, which has moved closer towards the best solution, ultimately converging onto the optimal solution. In practical operation, one assesses the quality of a particle through its fitness function value, which is determined by the optimization objective. Let the position of the ith particle in a D-dimensional space be expressed as $X_i = (x_{i1}, x_{i2} \ldots x_{iD})^T$ and the velocity $V_i = (v_{i1}, v_{i2} \ldots v_{iD})^T$. The particles are renewed according to Eqs. (6) and (7) given below.

$$v_{id}^{k+1} = \chi \times [w v_{id}^k + c_1 randd_1^k()(pBest_{id}^k - x_{id}^k) + c_2 rand_2^k()(gBest_{id}^k - x_{id}^k)] \quad (6)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \tag{7}$$

where the superscript $i$ denotes the particle and the subscript $k$ denotes the iteration number. $rand_1^k()$ and $rand_2^k()$ are random numbers uniformly distributed in the range [0, 1]. $c_1$ and $c_2$ are the acceleration constants, both taking values around 2 in general cases, and $w$ is the inertia weight parameter, being adjusted dynamically during the optimization (Shi and Eberhart 1998)

$$w = w_{\max} - t(w_{\max} - w_{\min})/t_{\max} \tag{8}$$

where $w_{\max}$ is the initial weight factor and $w_{\min}$ is the final weight factor. An initial greater value of $w$ may result in greater population diversity at the beginning of the optimization in order to promote global exploration of the search space. At a later stage, $w$ takes a smaller value, since a focused exploration can be realized and more refined solutions can be obtained. In this work, we set $w_{\max} = 0.9$ and $w_{\min} = 0.4$. $t$ is the current iteration number and $t_{\max}$ is the maximum number of iterations. In Eq. (6), $\chi$ is the constriction factor (Clerc 1999), introduced for ensuring convergence: $\chi = 2/\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|$, $\varphi = c_1 + c_2$, $\varphi > 4$.

For a constrained optimization problem, the dynamic penalty function method is used to handle the constraints as follows.

$$\underset{x}{\mathrm{Min}}: F(x) = \mathrm{Min}\left\{f(x) + \sigma\left[\sum_{i=1}^{p}(h_i(x))^2 + \sum_{j=1}^{q}[\max(0, g_j(x))]^2\right]\right\} \tag{9}$$

S.t: $h_i(x) = 0, \quad i = 1, 2, \ldots, p$
$g_j(x) \le 0, \quad j = 1, 2, \ldots, q$

Here, $\sigma$ is the penalty factor. To improve the performance of the algorithm, the penalty factor $\sigma$ is taken to be correlated with the iteration number

$$\sigma(t) = e^{m \cdot t^n / t_{\max} + \omega_0}$$

where $m$ and $n$ are positive coefficients used to adjust the changing rate of $\sigma$; $\omega_0$ is the initial penalty factor. In the present paper, $m$ and $n$ are assigned to be 5 and 1.2, respectively, and $\omega_0$ is set to 10. The dynamic penalty factor is small at the beginning of iteration, which may help to search for the optimum solution in a larger design space. As the iteration number increases, the penalty factor gradually becomes larger, enforcing the constraint to be satisfied.

### *4.3 Optimization procedure combining PSO and FEM*

The optimization procedure is essentially written in the software MATLAB. In the optimization process, the procedure first initializes the swarm. ANSYS is then called on the back stage to evaluate the objective and constraint functions of each particle through APDL commands written beforehand. ANSYS outputs the values of the functions to an external file. MATLAB reads them and computes the penalty function in Eq. (9) as the fitness of each particle. Every particle updates itself through Eqs. (6) and (7). The updated swarm is then returned to ANSYS for the next iteration. This process is repeated until the number of iterations reaches the pre-determined maximum iteration number. In the whole optimization process, data from ANSYS and MATLAB are exchanged back and forth with each other, as shown in Fig. 5. The number of times ANSYS is

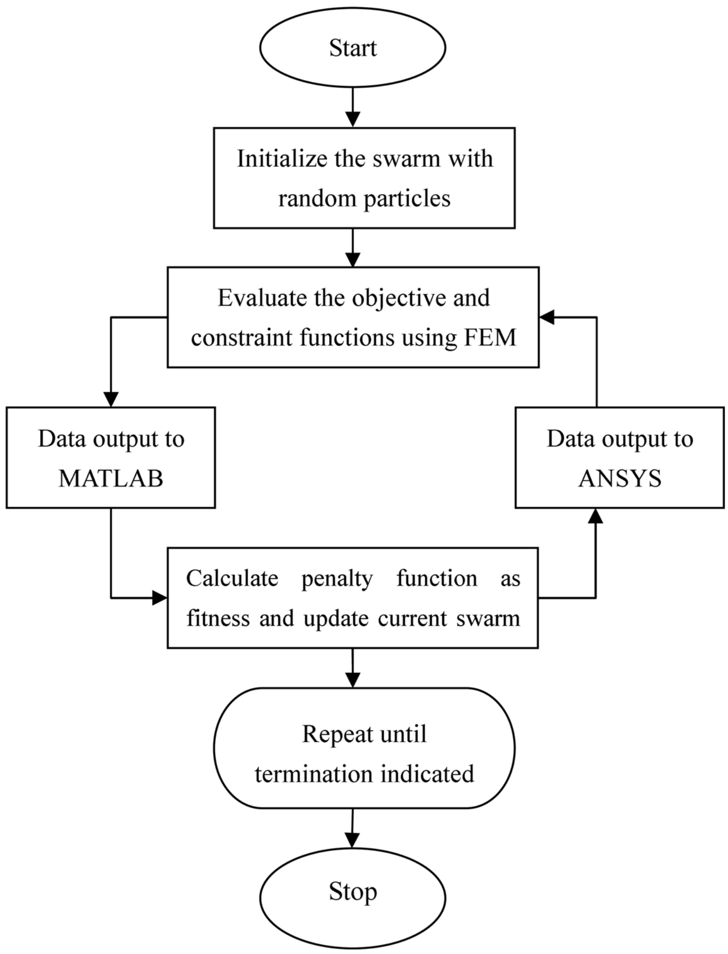


Fig. 5 Flow chart of optimization process by PSO and FE code

invoked for analysis equals the maximum iteration number multiplied by the number of particles. For PSO, the typical range for the number of particles is 20~40. For most problems, 10 particles are sufficient to obtain good results (Kathiravan *et al*. 2007).

It is well known that the calculated stress value by FE code depends upon the denseness of the finite element mesh. With a denser mesh, a quantitatively more precise analysis of the objective function value can be obtained. For optimization problems, however, the difference of alternative candidates is the main concern. Consequently, a moderate mesh is used for the stress analysis in order to save computational effort.

## 5. Stacking sequence optimization examples

Two kinds of laminate optimization problems are solved. The laminate with one design variable has a stacking sequence of $[0/\theta/-\theta/0]_s$, where $\theta$ is the design variable. The laminate with two design variables has a stacking sequence of $[0/\pm\theta/\pm\beta]_s$, where $\theta$ and $\beta$ are the design variables. Two

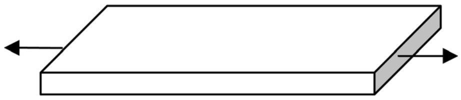

Fig. 6 Load Case 1: In-plane load

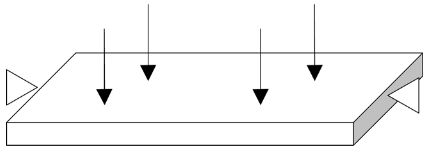

Fig. 7 Load Case 2: Out of plane load

different loading cases are considered: uniaxial tensile load and transverse uniform bending load, as shown in Fig. 6 and Fig. 7. For the case of in-plane load, $n$ in Eqs. (1)-(3) is taken as $N/2$ due to symmetry ($N$ is the total number of plies). For the out of plane load, two opposite edges of the length are fixed and the other two edges are free, and $n$ in Eqs. (1)-(3) is taken as $N-1$.

### 5.1 One design variable problem

The optimization model with one design variable has a symmetric $[0/\theta/-\theta/0]$s lay-up. The laminate is made from 700S/2500 and the ply properties (Takezono *et al.* 2001) are listed in Table 2. The laminate has a length of 90 mm, width of 20 mm, and thickness of 1 mm. The in-plane load is uniform axial tensile stress of 100 MPa and the out of plane load is a transverse uniform bending load of 0.1 MPa.

Since the basic reason for the occurrence of the free-edge effect is the layerwise different elastic properties, it is expected that the optimization will try to minimize the difference in the ply angles of two adjacent plies. In order to avoid an optimization result leading to a lay-up where all plies are $0^o$, which gives rather poor properties in the transverse direction, the ratio of the extensional stiffness $A_{22}$ perpendicular to the loading direction and $A_{11}$ in the loading direction is constrained as (Lindemann *et al.* 2002)

$$0.4 \leq A_{22}/A_{11} \leq 1.0 \tag{10}$$

Table 2 Ply properties of CF/EP (T700S/2500) (modules expressed in MPa)

| $E_X$ | $E_Y = E_Z$ | $G_{XY} = G_{XZ}$ | $G_{YZ}$ | $u_{xy} = u_{xz} = u_{yz}$ |
|---|---|---|---|---|
| 135000 | 8000 | 4500 | 3700 | 0.34 |

Table 3 Optimization results (one design variable case)

| Load case | Method | Optimization results | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\theta_{opt}$ | $f_{opt}$ | $\theta_{opt}$ | $\mu_{opt}$ | $\theta_{opt}$ | $S_{opt}$ |
| In-plane | ZOM(10) | 65.7 | 1.03 | 65.8 | 0.74 | 65.7 | 0.25 |
| | ZOM(45) | 59.1 | 0.28 | 58.9 | 0.18 | 58.8 | 0.07 |
| | ZOM(85) | 59.7 | 0.29 | 58.9 | 0.18 | 59.7 | 0.04 |
| | PSO | 58.8 | 0.28 | 58.5 | 0.18 | 59.7 | 0.04 |
| Out of plane load | ZOM(10) | 65.4 | 1.90 | 65.5 | 0.98 | 65.6 | 0.63 |
| | ZOM(45) | 73.5 | 1.98 | 60.8 | 0.99 | 66.5 | 0.63 |
| | ZOM(85) | 59.8 | 1.93 | 67.9 | 1.01 | 65.5 | 0.63 |
| | PSO | 61.7 | 1.86 | 61.7 | 0.94 | 61.6 | 0.62 |

For the one design variable case, when optimizing with PSO, a swarm size of 10 particles is chosen and the maximum number of iterations is assigned to be 10. For ZOM, the maximum iteration number is set to 100 so that ZOM and PSO would entail a comparable number of FE analyses. Three different initial designs are used for ZOM: $\theta$ = 10º, 45º, and 85º. The optimization results are listed in Table 3. $\theta_{opt}(^0)$ is the optimum design solution, and $f_{opt}$, $\mu_{opt}$, and $S_{opt}$ are the three optimum objectives corresponding to Eqs. (1)-(3) (unit in MPa).

Table 3 shows that under an in-plane load, when the initial design is 10º, the optimization results based on ZOM are unfeasible and are far from global solutions. For other cases, the best results obtained with ZOM are equivalent to those yielded by PSO. Interestingly, it is shown that for both load cases the three different objectives obtained by PSO reach their minimal values at nearly the same point. In order to validate the optimization results, the semi-analytical method reported in Ferreira *et al*. (1995) was adopted to calculate the interlaminar normal stresses under an in-plane load. In this method, the peak value of interlaminar normal stress at an interface was formulated as a weighted function of the in-plane transverse normal stress $\sigma_y$ in all the laminae outboard of the interface. The formulation is described in detail in Ferreira *et al*. (1995). The in-plane transverse normal stresses were calculated with CLT and the interlaminar normal stresses can be obtained through the formulation. The calculated peak values of interlaminar normal stresses at interfaces under $N_x$ = 100 N/m are plotted in Fig. 8. The interfaces 1, 2…$n$ are counted from the uppermost interface (0/$\theta$) to the mid-plane (0/0). Fig. 9 shows the calculated results for $f(\theta)$, $\mu(\theta)$, and $S(\theta)$, as defined in Eqs. (1)-(3). Fig. 8 shows that the absolute values of the peak interlaminar normal stress at all the interfaces reach their minimum when $\theta$ is located at about 59º (0º not included). This is the reason that the three different objectives reach their minimal values at nearly the same point, as shown in Fig. 9, and the calculated results are in good agreement with those given in Table 3.

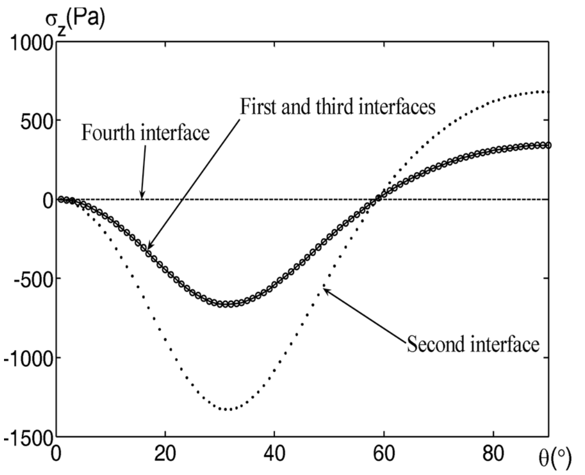


Fig. 8 Calculated peak $\sigma_z$ at interfaces based the on analytical approach (Ferreira *et al.* 1995)

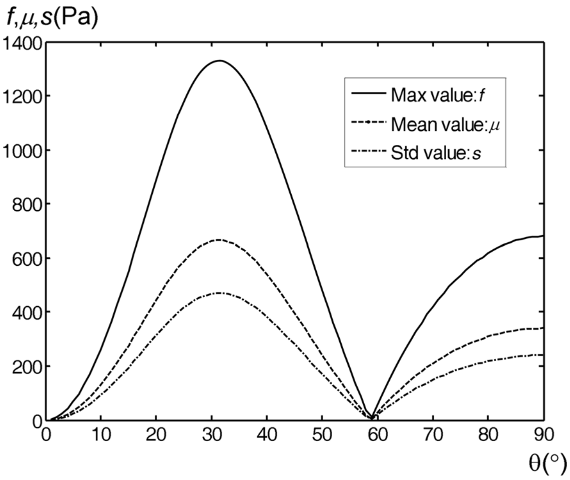


Fig. 9 Calculated $f(\theta)$, $\mu(\theta)$ and $S(\theta)$

Table 4 Results of maximum $f(\theta)$, $\mu(\theta)$ and $S(\theta)$ for in-plane load by PSO

| $\theta$ | $f_{max}$ | $\theta$ | $\mu_{max}$ | $\theta$ | $S_{max}$ |
|---|---|---|---|---|---|
| 29.7 | 6.06 | 30.9 | 3.19 | 29.2 | 1.76 |

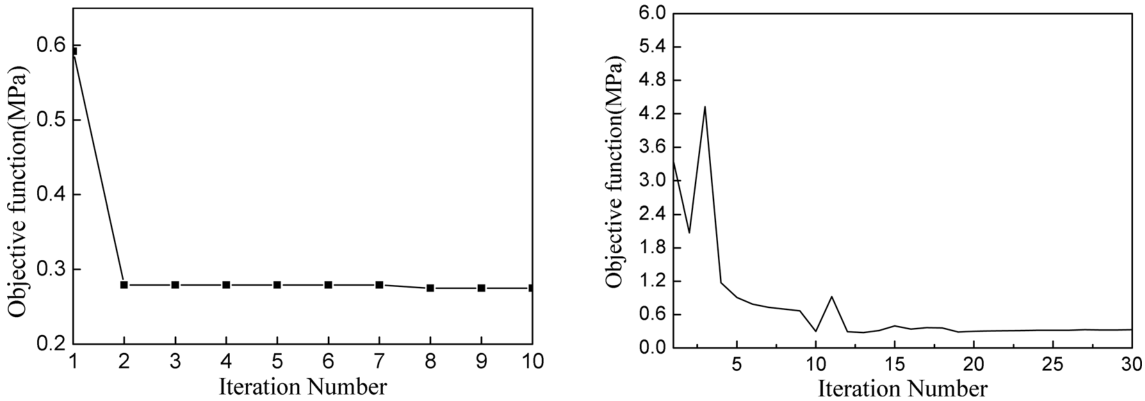


Fig. 10 Iteration history for optimization of $f$ under in-plane load using PSO (left) and ZOM (right, initial design: 45°)

The solutions that resulted in maximum $f$, $\mu$, and $S$ were additionally obtained through PSO, as shown in Table 4. We found that all the three objectives reach the maxima when $\theta$ takes a value near 30°. These results also agree with those illustrated in Fig. 9.

In order to compare the convergence performances of PSO and ZOM, we take the optimization of $f$ under an in-plane load as an example. The variations of the objective function versus the iteration number by PSO and ZOM, respectively, are presented in Fig. 10. It can be seen that for PSO an optimal result equivalent to that given by ZOM is obtained at 2 iterations, and the corresponding number of ANSYS evoked for the FE analysis is $2 \times 10 = 20$. For other cases, the optimum solution is reached at 1~3 iterations. For ZOM, all the optimum solutions are achieved at the former 30 iterations. Thus, for the case of one design variable, PSO and ZOM (a suitable initial design set provided) yield equivalent solutions with a comparable number of FE analyses executed.

From the one design variable case with a relatively simple design space, the results given by the proposed method show excellent agreement with the analytical results and they are superior or equivalent to the best results among the three different initial sets by ZOM with a comparable number of FE analyses regardless of the initial designs.

### *5.2 Two design variables case*

Lindemann *et al.* (2002) used two different approaches to minimize the maximum absolute value of interlaminar normal stresses. For comparison, the same laminate model is chosen. The thickness and the width were 2 mm and 50 mm, respectively. The laminate lay-up is $[0/\pm\theta/\pm\beta]$s. The in-plane load in the literature is uniform axial strain of $\varepsilon_x = 0.001$ and the out of plane load is taken as 0.1 MPa in this work. The ply properties are listed in Table 5.

The length of the laminate was not given in the literature. In this paper, it is assigned as 150 mm. Likewise, in order to refrain from an optimum result of a lay-up in which all plies are 0°, Eq. (10)

Table 5 Ply properties of CF/EP (T700S/2500) (modules expressed in MPa)

| $E_X$ | $E_Y = E_Z$ | $G_{XY} = G_{XZ}$ | $G_{YZ}$ | $u_{xy} = u_{xz} = u_{yz}$ |
|---|---|---|---|---|
| 135000 | 10000 | 5000 | 3972 | 0.27 |

Table 6 Optimization results (Two design variables case)

| Load case | Method | Optimization results | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\theta_{opt}$ | $\beta_{opt}$ | $f_{opt}$ | $\theta_{opt}$ | $\beta_{opt}$ | $\mu_{opt}$ | $\theta_{opt}$ | $\beta_{opt}$ | $S_{opt}$ |
| In-plane | ZOM(45,45) | 57.6 | 57.1 | 0.52 | 56.9 | 57.8 | 0.31 | 48.2 | 47.4 | 0.29 |
| | ZOM(36,72) | 45.3 | 46.5 | 2.51 | 55.4 | 56.1 | 0.44 | 55.5 | 55.3 | 0.09 |
| | ZOM(60,60) | 48.5 | 48.0 | 1.69 | 61.5 | 0.3 | 0.044 | 48.2 | 48.3 | 0.33 |
| | PSO | 61.4 | 0.3 | 0.03 | 0.5 | 61.7 | 0.022 | 61.6 | 0.4 | 0.003 |
| Out of plane load | ZOM(45,45) | 60.3 | 54.1 | 1.94 | 67.4 | 0.3 | 0.70 | 70.2 | 5.7 | 0.37 |
| | ZOM(36,72) | 68.0 | 0.3 | 1.35 | 54.7 | 56.7 | 1.78 | 70.6 | 3.5 | 0.40 |
| | ZOM(60,60) | 63.1 | 3.0 | 1.36 | 63.2 | 0.3 | 0.72 | 63.1 | 2.8 | 0.41 |
| | PSO | 64.5 | 0.07 | 1.31 | 61.5 | 0.4 | 0.68 | 0.04 | 77.8 | 0.24 |

Table 7 Optimization results for in-plane with initial design of (15°, 85°)

| $\theta_{opt}$ | $\beta_{opt}$ | $f_{opt}$ | $\theta_{opt}$ | $\beta_{opt}$ | $\mu_{opt}$ | $\theta_{opt}$ | $\beta_{opt}$ | $S_{opt}$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 62.1 | 0.05 | 0.3 | 61.80 | 0.04 | 0.3 | 62.0 | 0.017 |

is adopted as a constraint function. For the two design variables case, a swarm size of 15 particles is taken and the maximum number of iterations is assigned to be 10. For ZOM, the maximum iteration number is set at 150. The same initial designs, [0/±45°/±45°]s, [0/±36°/±72°]s, and [0/±60°/±60°]s, as in the literature are used for ZOM in the present example. The optimization results with two design variables are listed in Table 6. $\theta_{opt}(°)$ and $\beta_{opt}(°)$ are the optimum design solutions, and $f_{opt}$, $\mu_{opt}$ and $S_{opt}$ are the three optimum objectives (unit in MPa).

Table 6 shows that for an out of plane load, the optimum objective function results obtained by ZOM are dependent on the initial designs, and the best among the three solutions are close to the results based on PSO. For an in-plane load, however, ZOM provides considerably worse results relative to those based on PSO, and the results of ZOM are more sensitive to the initial designs. If an initial design [0/±15°/±85°]s is utilized, a better solution from ZOM can be reached, as given in Table 7.

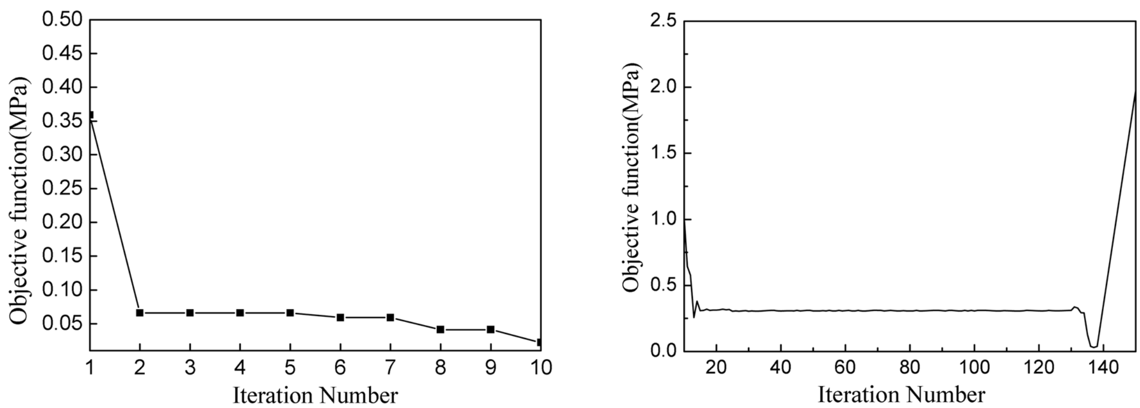


Fig. 11 Iteration history for optimization of $\mu$ under in-plane load using PSO (left) and ZOM (right, initial design: 15°, 85°)

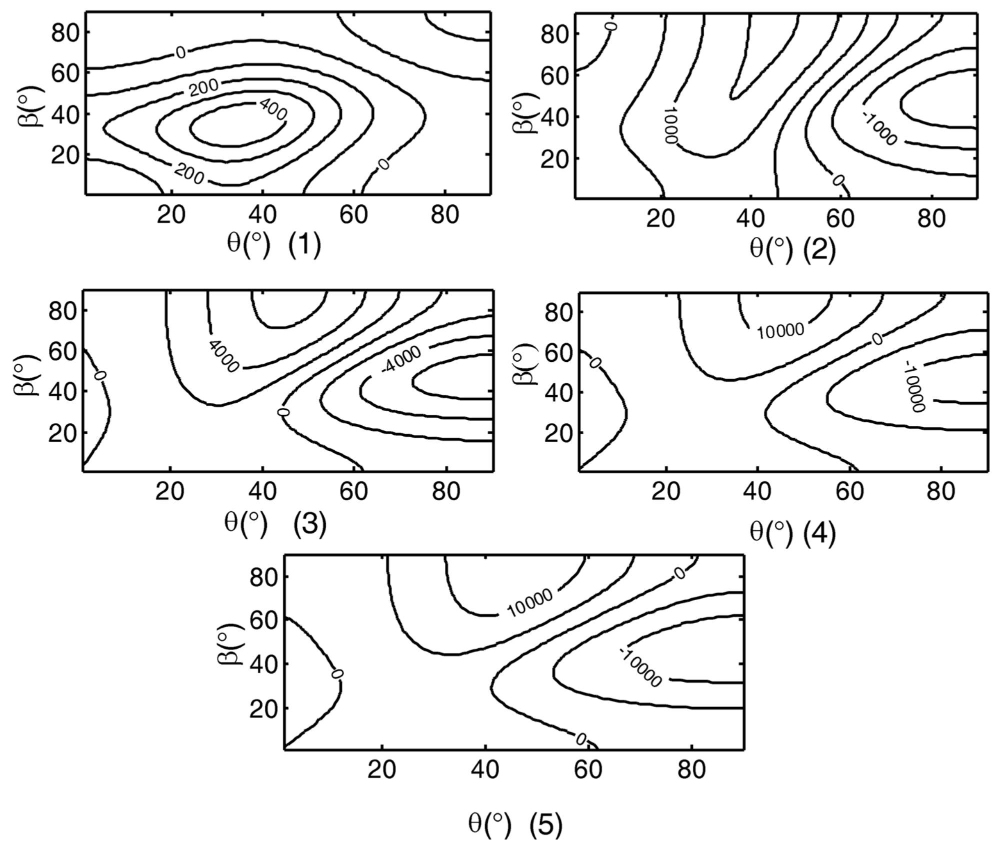


Fig. 12 Interlaminar normal stresses at each interface obtained using the analytical approach (Ferreira *et al.* 1995)

Fig. 11 displays the convergence performances of PSO and ZOM. The objective functions $\mu$ under an in-plane load versus the iteration number are demonstrated in the figure. We observed that for PSO a satisfactory optimum solution is obtained at 2 iterations, and the corresponding number of ANSYS evoked for the FE analysis is $2 \times 15 = 30$. For other cases, the optimum solutions are reached at 2~7 iterations. For ZOM, as shown at the right side of Fig. 11, the optimal solution is achieved at 139 iterations, corresponding to 139 FE analyses, and for other cases, the optimum solution is also reached at around 130 iterations. It is concluded that for two design variables, PSO yielded better optimal solutions with fewer FE analyses than ZOM. In the literature (Lindemann *et al.* 2002), an optimum lay-up of [0/±52°/±52°] by gradient-based algorithms was obtained, and the corresponding value of $f$ under an in-plane load was 1.52 MPa, which is much worse than the present result based on PSO (Table 6).

Adopting the analytical method in the literature (Ferreira *et al.* 1995), the contour lines of $\sigma_z$ at each interface of $[0/\pm\theta/\pm\beta]$s under an in-plane tensile load of $N_x = 100$ N/m are plotted in Fig. 12, in which the unit of $\sigma_z$ is Pa and the unit of $\theta$ and $\beta$ are degree (°). The label (1)-(5) corresponds to the interface number 1, 2…5 counting from the uppermost interface $(0/\theta)$ to the symmetric interface $(-\beta/-\beta)$. From Fig. 12, it is observed that for each interface the absolute value of $\sigma_z$ reaches 0 at the points around (62°, 0) and (0, 62°). This is the reason why the optimal design variables $\theta_{opt}$ and $\beta_{opt}$ vary between (62°, 0) and (0, 62°), as shown in Tables 6 and 7. The plots in Fig. 12 also show that the problem is multimodal with different local minimum points.

For some practical engineering optimization problems, the design variables behave as a discrete form for manufacturing reasons. For example, the thickness of a ply is defined as a set of discrete values [0.1 mm, 0.2 mm, 0.3 mm…] and the fiber orientations of the plies are defined as [0, 15°, 30°, 45°…]. This greatly reduces the design space. In some cases, we can convert the optimum values of continuous results to the nearest discrete manufacturable values. Meanwhile, for some other problems, the discrete optimization problems can be strictly solved with PSO by using a special coding technology, as done in our previous work (Chen *et al.* 2008). Hence, it is expected that the proposed methodology is applicable to engineering problems with continuous, discrete or mixed variables.

## 6. Conclusions

Optimal stacking sequence design of composite laminates is carried out for in-plane load and out-of plane load with the aim of minimizing the maximum, the mean, and the variation of the absolute peak value of the interlaminar normal stresses in laminates. A technique involving the application of the exterior optimization algorithm PSO integrated with the FEM was developed for the optimization. The semi-analytical method, which provides an approximation of the interlaminar normal stress of laminates under an in-plane load, was used to partially validate the optimization results. ZOM incorporated in ANSYS was also used for comparison. It was found that PSO gives better results than ZOM. For a one design variable case, the results yielded by PSO are superior or equivalent to the best results among the three different initial sets by ZOM with a comparable number of FE analyses. For the case of two design variables with a lager design space, PSO can yield results better than ZOM with fewer FE analyses and the result is also better than that available in the literature.

As the optimization results based on ZOM are sensitive to the initial design sets, when conducting optimization with ZOM, one should choose different initial design sets and compare the optimization solutions. The central problem is that it is not known which initial design can lead to a satisfactory optimum and whether another initial design exists from which a better result can be obtained. By contrast, with PSO it is not necessary to specify initial designs manually as it is a global stochastic method. Thus, it can be implemented in a straightforward manner and improvement of the objective function can be achieved simply by choosing a larger number of particles or iterations.

It is believed that by merging the general FE package and evolution-based optimization algorithm with a parallel computing scheme, the proposed approach provides designers a feasible and efficient methodology with great potential for developing tailoring applications in composite structural design and other complex engineering designs.

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