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Optimization of a telescope movable support structure by means of Volumetric Displacements

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Abstract. The Purpose of this paper is to show the applicability of a methodology, developed by the authors, with which to perform the mechanical optimization of space truss structures strongly restricted. This methodology use a parameter call "Volumetric Displacement", as the Objective Function of the optimization process. This parameter considers altogether the structure weight and deformation whose effects are opposed. The Finite Element Method is employed to calculate the stress/strain state and the natural frequency of the structure through a structural linear static and natural frequency analysis. In order to show the potentially of this simple methodology, its application on a large diameter telescope structure (10 m) considering the strongly restriction that became of its use, is presented. This methodology, applied in previous works on continuous structures, such as shell roof and fluid storage vessels, is applied in this case to a space truss structure, with the purpose of generalize its applicability to different structural topology. This technique could be useful in the morphology design of deployable and retractable roof structures, whose use has extensively spread in the last years.

Keywords: structural analysis; optimization; telescope.

1. Introduction

Different optimization processes try to minimize certain parameters, such as those associated with the mechanical behavior and/or weight of the structure. To execute this process boundary conditions should be considered such as the restrictions that are directly associated with the functional details of the system.

The herein proposed optimization methodology is similar to the one formerly used for continuous structures, such as those of laminar covers (Ortega and Arias 1998, Robles and Ortega 2000, Robles and Ortega 2001) and fluid storage vessels (Ortega and Robles 2005).

A model of such technique applied to a movable metallic space truss structure, in this case the structure of a large diameter (10 m) reflecting telescope, where the constraints resulting from the optical system components, had to be considered. It is worth mentioning that this level of complexity is not frequent with other types of structures.

The primary function of a telescope structure is to support the optical system with all its

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instrumentation. However, it is to be taken into account that extremely accurate alignment and aim conditions are to be maintained while tracking the path of any object moving through the space. This leads to a rather critical situation in which many constraints are to be evaluated regarding the displacement of the support structure in order to prevent optical aberration, to which the complexity associated with a moving space structure is to be added. In addition, the minimization of the structure weight, closely related with the reduction of thermal inertia, the cost of the structure and the driving mechanisms should be also considered when trying to design a low-weight structure while in order to reduce displacement, heavy structures should be adopted. In order to solve this dilemma between these two opposed approaches a parameter call "Volumetric Displacement", as the Objective Function of the optimization process, is proposed, because its considers altogether the two parameters - structure weight and deformation.

2. Description of the telescope

The studied telescope is quite similar to the one currently being erected at Las Palmas, Canarias Islands, Spain (Linares *et al*. 2000, Matín Alvarez *et al*. 1997, Pan *et al*. 2000). The adopted typology is mostly employed for large-diameter telescopes (Mendoza *et al.* 2004) and there are similar units at the Texas McDonald observatory, the erection of which was completed in 1997 (Abel 2000, Medwadowski 1986) and the one currently being completed at Southland, Northern Cape, South Africa (Meiring *et al.* 2003, Willem *et al.* 2003). It is worth mentioning that at present a new telescopes generation with diameters between 20 to 30 m are at its design stage, for example the Giant Magellan Telescope (GMT) (Gunnels *et al.* 2004) and the California Extremely Large Telescope (CELT) (Abel 2000) project, by a team leaded by Dr. S.J. Medwadowski, designer of the McDonald Observatory telescope.

Regarding the Canarias telescope, Fig. 1, the site of emplacement was selected in view of its

Fig. 1 Structure of the Canarias Telescope

excellent visibility conditions. It is located inside the crater of an extinct volcano at a height of 2000 m over sea level, where the atmosphere is normally dry, stable and transparent. Due to the height at which it is located, the site is subjected to temperatures ranging between −15 and 35°C. However, during 98% of the year, the prevailing weather conditions under which temperatures range between −2 and 19°C, allow to operate the unit. Light pollution is virtually non existent, as there isn't any closely located urban area and public lighting is legally controlled under strict restrictions, due to the existence in the island of a significant number of telescopes owned by different countries of the European Community.

2.1 The optical system

The herein is a reflecting telescope, with two mirrors arranged in Cassegrain configuration. It has also a third mirror, located at the center line of the space truss structure, which reflects the light beams to the two Nasmyth foci and the four Folded Cassegrain foci located on the central ring (Fig. 2).

The primary mirror has an area equivalent to that of a circular 10 m diameter disk and is composed of 36 hexagonal segments, each 0.936 m in side length. The focal length of the mirror is 16.50 m. As the instrumentation metallic support structure will be subjected to unavoidable distortions, and to prevent the resulting image optical aberration that may and shall occur at the different attitudes at which the telescope may be aimed to perform astronomical observations, the position of the individual primary mirror segments is controlled by means of a special system to maintain a nominally spherical reflecting surface. Its total weight is 16560 kg and the support system weight is 5100 kg.

The secondary mirror is a single hexagonal piece 1.177 m in side length and a 3.90 m curvature radius. This is also actively controlled in order to keep its optical alignment. It can be moved with 5

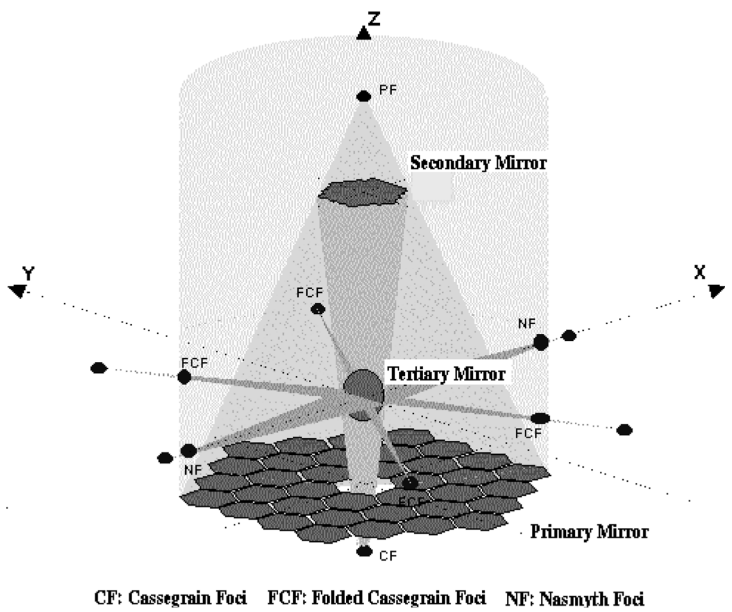

Fig. 2 Optical system (Pan *et al.* 2000)

degrees of freedom in order to keep its alignment and other corrections can be made as well to control image movement, independently from thermal and gravitational deformations. The distance between the primary and secondary mirrors is 14.739 m. The weight of the secondary mirrors is 60 kg.

Finally, the tertiary mirror is a flat unit, from 1.062 m to 1.511 m in diameter, 60 kg in weight and its primary function is to direct images to the different foci.

The optical system of the telescope includes a number of instruments the purpose of which is to correct the different optical aberrations before images are captured by the astronomical system.

2.2 Primary mirror support and positioning system

The segments that make up the primary mirror are not sufficiently rigid as to be able to support their own weight without causing deformations that may distort the reflected images (Roberts *et al.* 2005). This is why a support system able to afford proper rigidity is to be used. This support is connected to the structure by means of three positioning devices that allow a maximum displacement of 1.2 mm perpendicular to the mirror surface.

This support system should allow the easy removal of the primary mirrors to carry out the necessary maintenance surface treatment procedures.

2.3 Metallic support structure model

To facilitate the analysis, the telescope structure can be split into two main components: 1) The mount, that along with other elements provides two rotation axes, supports the telescope truss structure and transfers the load to the support base. This part of the structure is not included in this analysis. 2) The space truss structure that supports the mirrors and their associated instruments and maintains optical alignment within an allowable range.

A tubular space truss structure was selected due to its great structural efficiency and also because it has lower thermal inertia than other types of structures of the same strength, allows good air circulation, offers low resistance to wind loads and this, in turn, reduce turbulence and minimizes structure vibration.

It is to be noted that for the space tubular truss structure, tubes of different thickness were used, with those of greater thickness located in the more loaded zones.

At the central area of the space truss structure, a box section ring is located, that was modeled with 12.5 mm wall thickness and vertical stiffeners located in a radial direction spaced at 0.85 m. This ring exhibits a great rigidity and good conveyance of the flexural and torsional loads. Above and below this ring there are two space truss structures that are subjected to the prevailing axial stresses.

The upper truss structure, Fig. 3, comprises the following subsystems: upper truss structure; Secondary mirror structure; Spider structure; Upper hexagonal ring; Medium hexagonal ring. The lower truss structure, Fig. 4, comprises the following: Primary mirror cell; Tubular ring; Tertiary mirror structure.

These two parts have almost mutually independent individual functions, and thus they can be designed and analyzed separately. Obviously, this bring about important benefits from the computer analysis viewpoint. This is why here they have been separately analyzed just as it was made when the Canarias (Martín Alvarez *et al.* 1997, Linares *et al*. 2000) structure was analyzed.

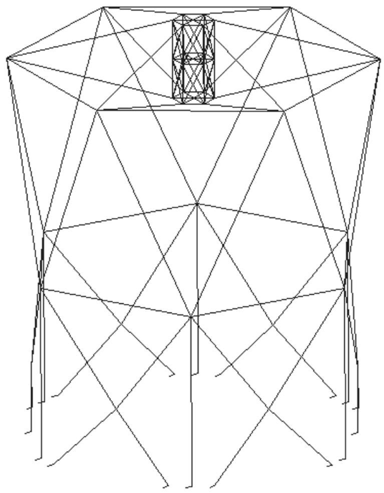

Fig. 3 Upper Structure model

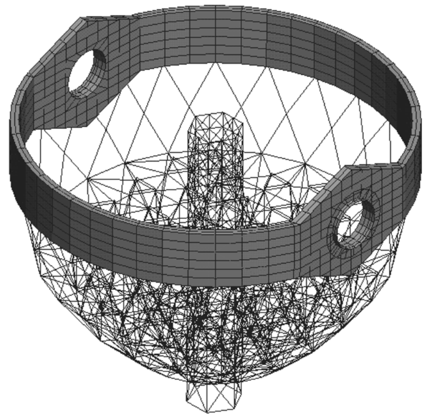

Fig. 4 Lower Structure model

The primary mirror cell is made up of a three layer truss structure. The upper layer is in charge of supporting the servomechanisms that control the position of the hexagonal mirrors. This structure is connected to the metallic ring by means of truss.

The upper structure design is very similar to that of the already built with the only difference, that here a hexagonal ring was incorporated to improve its stiffness. An additional hexagonal ring is located at the end of the upper structure, surrounding the primary mirror. This ring is bigger and tilted to a 30^{o} angle. The ring supports the spider, which comprises six pairs of truss that support the secondary mirror structure. These truss have been placed such that the obstruction of the light beams reflected to the primary mirror is minimized. This design takes into account that it is desirable that this structure has a very thin section, which was made by rotating the position of the upper hexagonal ring. The shadow cast by the spider is limited to only 0.5% of the primary mirror surface area.

2.4 Support structure specific design requirements

Added to the regular structural requirements, the herein described are those that this particular structure should also comply with, considering its intended use.

- The stiffness of the space truss structure should be maximized to avoid optical misalignments due to the bending effects caused by the gravitational loads.
- The total weight of the structure should be reduced in order to minimize cost, reduce thermal inertia and avoid the induction of thermal turbulence that would degrade image quality (Blaurock *et al.* 2005).
- The structure should have a low and stable infrared emission. This is attained by using certain materials or reducing the telescope mass (Jessen *et al.* 2007).
- The area of the structure exposed to air drafts should be small, to prevent the generation of turbulence in the flow of air in order to avoid image oscillation (Kan and Eggers 2006).
- The structure of the telescope should allow a slight air current around and through the structure elements, specially those located above the primary mirror. This allows a reduction in the

temperature gradients of the different structural elements thus minimizing optical misalignment caused by this effect.

- The telescope mechanism should allow three degrees of freedom with high accurate and rigidity, considering that, on the basis of the distance between the earth and the celestial objects it may be necessary to make prolonged exposures of the same image for several hours. To facilitate this, it is convenient to reduce the weight of the instrument.
- The operation of the telescope in case of seismic occurrences has not been anticipated, though it has to be prepared to withstand such loads along its life span.

Synthesizing, the first six requirements it can be concluded that a low weight and highly rigid structure should be adopted such that it maintains a level of rigidity compatible with its design intention. Regarding the last requirement and the vibration that may be caused by the action of wind loads and telescope rotations, these shall not be herein taken into account. Something similar occurs with thermal actions (Smith *et al.* 2004).

To the effect of simplifying the optimization process, we shall work with gravitational static loads which are the prevailing ones. Then the pre-designed structure should be checked for the dynamic actions resulting from telescope rotation and those originated from wind loads. On the basis of this verification structural reinforcements shall then be incorporated.

3. Methodology

3.1 Theoretical basis of the optimization process

The herein optimization process is applied to the geometry of the different elements that make up the metallic space truss structure of the telescope being studied. It is to be mentioned that the thin wall box-type ring that connects the Upper and Lower structures, supports the primary mirror and is in charge of conveying stresses to the mount, was included in the model but was not considered for the optimization.

The geometry and weight of the ring are very similar to those adopted for the Canarias telescope.

In order to determined the fitness of two different structures is not sufficient to compare the maximum displacements, due to the limited scope of this analysis that is reduced to the behavior at a single point. Conversely, a technique should be adopted that makes a more general consideration of such displacements.

With the aim of minimizing structure displacements a "Volumetric Displacement" parameter was used that is obtained as the summation of the product of the displacement occurred at each point *i* (δi) times its volume of influence ($L_i \times A_i$), according to the following formula (Robles and Ortega 2001)

$$DV = \sum_{i=1}^{n} \delta_i \times L_i \times A_i \qquad (1)$$

The volume of influence is defined as the volume of the tubs that concur to the node in consideration, and is directly related to the weight of the structure.

To determine displacement values, the own weight of the structure was adopted as its load status together with the weight of the optical system and the instrumentation. The analysis was made by using a software based on the Finite Element Method, commercially available under the name of

Algor15 (Algor15 Professional Mech/VE 2002). A linear stress analysis was made with beam type elements for the metallic space truss structure and the ring was modeled with plate elements. As can be seen the implementation of this technique is relatively simple.

The mathematical formulation of the optimization, for the particular case of minimizing the displacements of these metallic space truss structure, can be formulated as follows:

$$\text{Minimize}\, DV(x) \tag{2}$$

where:

DV: Volumetric Displacement

x: design variables vector

This is subjected to the following restrictions:

- The maximum relative displacement between primary mirror segment supports;
- The angular deviations between the normal of the mirrors; and
- The relative shift between mirrors

It is to be remarked that these relative displacements are limited for the maximum stroke of the system that controls primary mirror segments, which has been reduced in order to have a certain safety margin.

4. Numerical results and discussion

This case is resolved by splitting the space truss structure into two parts, namely an upper truss structure (Fig. 3) and a lower truss structure (Fig. 4) into which the box section metallic ring is located.

4.1 Upper structure

Due to the fact that the structure is movable, at a first stage the mechanical analysis was made for the telescope in three positions: horizontal, tilted at a 45° angle and vertical, in order to find the most unfavorable load conditions that it is put in evidence with the Volumetric Displacements.

From the Figs. 5, 6 and 7 it can be seen that the higher values and variations in the Volumetric

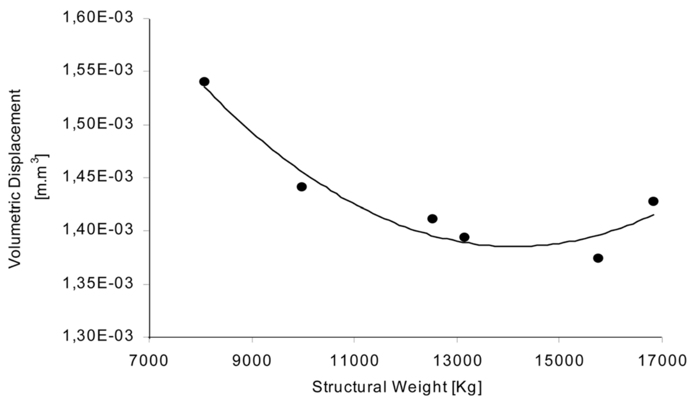

Fig. 5 Volumetric Displacement of the Upper Structure, in horizontal position, versus structural weight

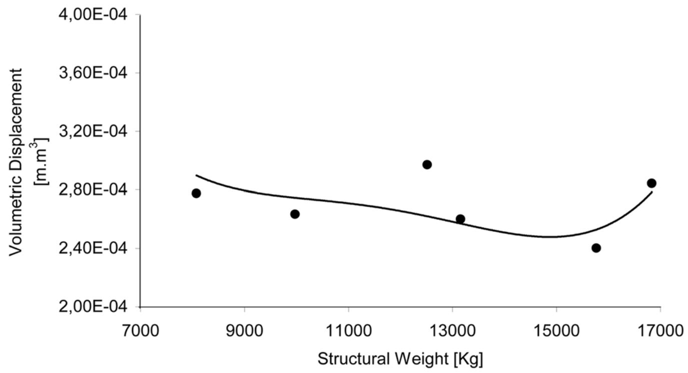

Fig. 6 Volumetric Displacement of the Upper Structure, tilted at 45° degree angle, versus structural weight

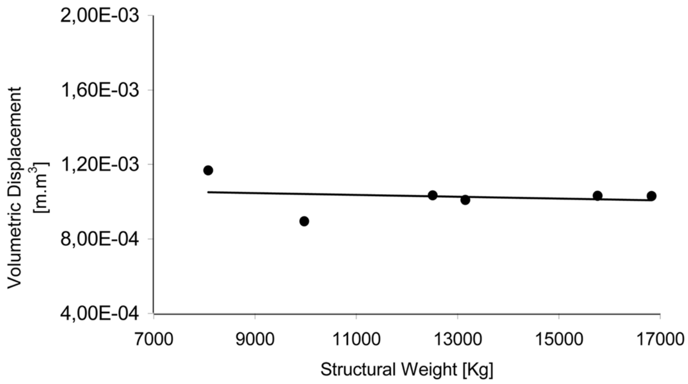

Fig. 7 Volumetric Displacement of the Upper Structure in vertical position, versus structural weight

Displacements are presented with the telescope in horizontal position. This is why the minimization of Volumetric Displacements shall be made with the structure in that position.

In Fig. 5 the optimum values of the Volumetric Displacements with the structure in horizontal position that are presented, belong to the interval of the structure weight that ranges from 13000 to 16000 kg. A similar behavior exhibit the Volumetric Displacements with the structure in upright position, while with the structure tilted at a 45° angle, the values are practically constant for different structure weights.

4.2 Lower structure

As mentioned before, due to the fact that this is a movable structure, the mechanical analysis is to be made for the three positions of the telescope, namely horizontal, tilted at 45° angle and vertical, in order to find the most unfavorable load conditions. Volumetric Displacements for each of the three positions are presented in Table I for one of the models that were analyzed, whose weight is 15250 kg (weight of the primary mirror cell: 4900 kg).

Here we shall only study the mechanical behavior of the lower structure in vertical position. However, the other two positions were analyzed and lower displacements were found.

As can be seen in Fig. 8, Volumetric Displacement variation is very small and increases as structure weight increases. From this viewpoint almost any model could be adopted but due to

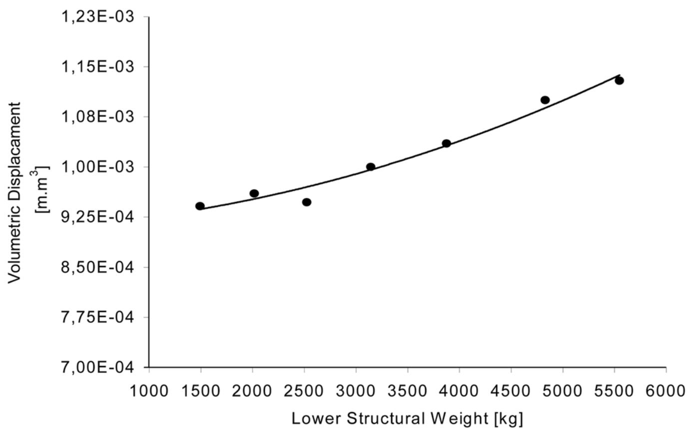

Fig. 8 Lower Structure Volumetric Displacement in vertical position, versus structure weight

Table 1 Lower Structure Volumetric Displacement values, determined for the 15250 kg model at the three selected positions

Model	Volumetric Displacement (m × m^3)
Horizontal	0.00056
45°	0.00088
Vertical	0.00112

economical reasons one of the lighter weight structures should be selected.

At this moment the restrictions resulting from the function of the structure should be considered. One of the constraints is the displacement that may be corrected by actions of the active control system to adjust the position of the primary mirror, which for this case has a maximum range of 1.2 mm. However, to the effect of ensuring a degree of safety, the models whose maximum differences in displacement at the mirror segment support points do not exceed 1 mm shall be considered valid. The models that have such conditions are: 18600 kg ($\Delta\delta$ = 0.8616 mm) y 15250 kg ($\Delta\delta$ = 0.8962 mm) (Table 1).

4.3 Optical restrictions affecting both structures

The angular deviations affecting alignment between the secondary mirror, located at the Upper Structure and the tertiary and secondary mirrors, located at the Lower Structure are presented in Tables 2 and 3 and the shift in mirror alignment are presented in Table 4.

Regarding angular deviations between mirrors, this can be observed by analyzing those belonging to the primary and the secondary mirror (Table 2) that are very small and equal for both Upper Structures. Regarding the Lower Structures, values are similar in all cases, the smaller value is however that of the 15250 kg structure.

Regarding angular deviations between secondary and tertiary mirrors (Table 3) these are very small and equal for both Upper Structures. In the case of the Lower Structures, the 18600 kg structure is the one that exhibits the smallest angular deviations.

In view of the so far obtained results, for the Upper Structure anyone could be selected, while for

Table 2 Angular deviations (radians) between primary and secondary mirrors

Lower Structure Weight (kg)	Upper Structure Weight (kg)	
	13200	15800
18600	5.90e-5	5.90e-5
15250	5.59e-5	5.59e-5

Table 3 Angular deviations (radians) between secondary and tertiary mirrors

Lower Structure Weight (kg)	Upper Structure Weight (kg)	
	13200	15800
18600	4.19e-5	4.19e-5
15250	5.88e-5	5.88e-5

Table 4 Shift in mirror alignment (mm) between secondary and tertiary mirrors

Lower Structure Weight (kg)	Upper Structure Weight (kg)	
	13200	15800
18600	7.10e-4	5.90e-4
15250	5.50e-4	4.30e-4

the lower one, as according to the considered restrictions different results shall be obtained, an analysis should then be made of some other border conditions which, in this case, could be the misalignment between the secondary and tertiary mirrors (Table 4). This table shows that the more convenient structure is the 18000 kg one, while for the Lower Structure, which is the one with lower values and significant differences, the one that should be selected is the 15250 kg one. This difference is due to the incorporation of a hexagonal ring at the middle part of the Upper Structure, in order to enhance its rigidity.

To compare the results with those presented in Martín Alvarez *et al*. (1997) and Pan *et al*. (2000), for the Canarias Telescope, it is possible to state that Lower Structures and rings are practically equal, while the Upper Structures show differences, being the herein discussed low weight structures. It is worth stating that the herein static analysis should be complemented by means of an analysis of the dynamic behavior of the structure that is influenced by telescope rotation and wind loads.

4.4 Dynamic response of the telescope structure

Here the dynamic response of the telescope structure is discussed. This analysis is to be made due to the fact that the structure is subjected to dynamic loads resulting from wind loads and its possible movements. Depending on the selected emplacement site for the telescope, it could also be subjected to seismic loads. In case of such an event, and due to the fact that during a seismic occurrence no astronomical observations shall be made, the resistance and stiffness of the structure should then be checked to ensure that the possible distortions shall be compatible with the instrumentation and shall not produce mirror damage.

Table 5 Natural Eigenfrecuency of the telescope (Tube in vertical position)

Mode Nº	Mode Shape Description	Eigenfrecuency [Hz]
1	Lateral mount bending	3.50
2	Torque of the spider and secondary mirror structure	5.66
3	Front bending of the mount	7.84
4	Torque of the mount	8.20

It is to be taken into account that the main requirement for this type of structure is to obtain high quality images and this in turn depends upon the optical errors of the instrument, some of which may be originated from mirror misalignment. In this regard image quality is greatly influenced by structure distortion. It is worth mentioning that natural frequencies (eigenvalues) are not only a direct measure of the dynamic behavior (Davison 1990, Lieber 2005) but also an indirect measure of the static performance, as these are proportional to the square root of the bendings due to gravity.

At this stage of the analysis no optimization process shall be made. The results of the calculations made to determine the natural frequencies of the structure, allow to determine the need of modifying the rigidity of any section of the telescope structure (optimized on the basis of its static behavior), which was not considered in the aforementioned optimization process. Due to the fact that the mount optimization process has not been considered, it is important to consider not only its analysis under the influence of static loads (eigenweight) but also the natural frequencies that complement the information regarding the rigidity of this part of the structure.

Table 5 shows the values of the first four natural frequencies, calculated by means of a software based on the Finite Element Method (Algor15 Professional Mech/VE 2002).

To analyze the natural frequency values included in Table 5, it is important to take into account that the maximum spectral wind energy for high mountain sites (Matín Alvarez *et al*. 1997) is of about 1 Hz and negligible for frequencies of about 5 Hz. This is why it is convenient that natural frequencies are over 5 Hz.

For the herein discussed structure, it is to be noted that the smallest eigenvalue that matches up with the lateral bending mode of the mount is within the frequency range, at which the wind load spectrum has enough energy as to excite the structure. From these facts we can draw the conclusion that the telescope mount structure should be optimized with the aim of increasing the said frequency as this would increase the structural rigidity.

It is worth mentioning that the torque mode of the spider and secondary mirror structure (5.66 Hz) is only excited by the wind, the vibration modes corresponding to front bending of the mount (7.84 Hz) and mount torque (8.20 Hz) are directly excited by the elevation and azimuth motors, respectively.

Regarding the different natural frequencies, it is important to remark that these have small influence on those that produce telescope movements that do not affect image quality, such as torsion along telescope center line.

Finally, though from the dynamic viewpoint wind load action is very important, considering the servomechanism in charge of enabling structure movements, the more rigid is the structure the higher acceleration rates it shall allow and this is important at the moment of aiming the telescope and tracking celestial objects.

5. Conclusions

The herein optimization methodology, based on the minimization of Volumetric Displacements, is a valuable tool for the metallic space truss structures designer as it enables to analyze simultaneously the displacements and volume of different structures.

This technique showed its potentiality when successfully applied to a movable space truss structure with strong displacement restrictions resulting from its use as the support structure for the optical system of a large diameter telescope. The technique can be easily applied on the basis of simple theoretical foundations.

It can be of help for the design and optimization of structures considering the prevailing loads.

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Notation

A : area over which the strain variation is extend
L : length over which the strain variation is extend
VD : volumetric Displacement. Objective function
X : vector of design variables
δ_i : structure displacement at each point i
$\Delta\delta$: displacement difference