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The design of reinforced concrete beams for shear in current practice: A new analytical model

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Abstract. The present paper reviews the shear design (of reinforced concrete beam) provisions of four different national codes and proposes a new but simplified shear strength empirical expression, incorporating variables such as compressive strength of concrete, percentage of longitudinal and vertical steel/s, depth of beam in terms of shear span-to-depth ratio, for reinforced concrete (RC) beams without shear reinforcement. The expression is based on the experimental investigation on RC beams without shear reinforcement. Further, the comparisons of shear design provisions of four National codes viz.: (i) IS 456-2000, (iii) BS 8110-1997, (iv) ACI 318-2002 (v) EuroCode-2-2002 and the proposed expression for the prediction of shear capacity of normal beam/s, have been made by solving a numerical example. The results of the numerical example worked out suggest that there is need for revision in the shear design procedure of different codes. Also, the proposed expression is less conservative among the IS, BS & Eurocode.

Keywords: shear strength of concrete; compressive strength of concrete; percentage of longitudinal and vertical steel/s; depth of beam in terms of shear span-to-depth ratio; code provisions.

1. Introduction

In reinforced concrete beam, shear does not produce failure directly on the vertical plane on which it acts, as one might anticipate. In fact, the major effect of the shear is to induce tensile stresses on the diagonal planes oriented at 45^o to the plane on which the shear acts. Since concrete has relatively low tensile strength as compared with its shear and compressive strength, overstress, therefore, will always initiated by tensile stresses. When the diagonal tension stresses in combination with bending stresses created by moment exceeds the tensile strength of concrete, diagonal cracking develops that can split the beam. Since these diagonal tensions occur without warning, hence must be prevented.

The design of reinforced concrete beams for shear is empirical and indirect and the uncertainties involved in predicting accurate values for shear stress and in establishing the diagonal tension strength of concrete make a rational design procedure for shear using stresses on diagonal planes impossible; therefore, the design procedure for shear is based on the assumption that a shear failure at a particular section occurs on a vertical plane when the shear force at that section due to factored

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service loads exceeds the concrete's fictitious vertical shear strength. The design of reinforced concrete beams for shear is empirical and indirect for three reasons: (1) Because of the heterogeneous nature of reinforced concrete and its inability to carry tension without cracking, no equation is available to compute accurately the maximum value of shear stress on a particular cross section of a beam. (2) Shear failures do not occur on vertical planes in the direction of the shear force; instead, tensile stresses associated with shear or with shear and moment cause failure on diagonal planes. (3) The tensile strength of concrete is highly variable, and the ability of a concrete beam to carry the diagonal tension associated with shear cannot be predicted accurately.

2. Shear strength in reinforced concrete beams without web reinforcement

2.1 Mechanism of Shear Transfer

There are several mechanisms by which shear are transmitted between two adjacent planes in a reinforced concrete beam. The prominent among these are identified as: shear stresses in uncracked concrete; interface shear transfer, often called "aggregate interlock" or "crack friction"; the dowel action of the longitudinal reinforcing bars; and arch action. The forces transferring shear across an inclined crack in a beam without stirrups are illustrated in Fig. 1.

The transverse (external) shear force is denoted as V (and has a maximum value near the support, equal to the support reaction). It is resisted by various mechanisms, the major ones (Fig. 1) being:

(1) Shear resistance V_{cz} of the uncracked portion of concrete;

(2) Vertical component Vay of the 'interface shear' (aggregate interlock) force V_a;

(3) Dowel force V_d in the tension reinforcement (due to dowel action); and

The interface shear Va is a tangential force transmitted along the inclined plane of the crack, resulting from the friction against relative slip between the interlocking surfaces of the crack. Its contribution can be significant, if the crack-width is limited. The dowel force V_d comes from 'dowel action' (Fig. 1).

The equilibrium of vertical forces in Fig. 1 results in the relation

$$V = Vcz + Vay + V_d \tag{1}$$

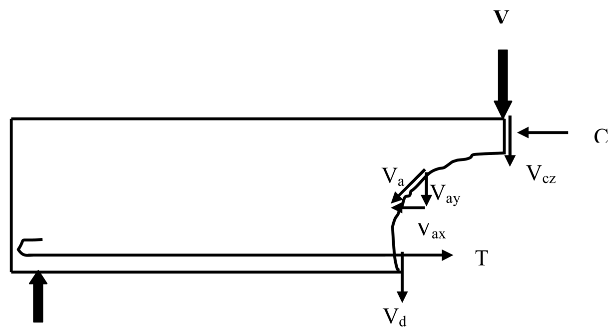

Fig. 1 Internal forces in a cracked beam without stirrups

3. Shear capacity of beams

Shear capacity is defined as the maximum shear force that a critical section can sustain. It is widely accepted that a main contributor to shear resistance in concrete is aggregate interlock. The use of web reinforcement to carry shear force is necessary when the concrete portion alone is unable to sustain the force. The presence of sufficient web reinforcement can help to prevent the brittle failure in a transfer beam. Moreover, design of transfer beam has to be handled very carefully, as failure of this one member would normally lead to the failure of the entire tower above it with little warning. Hence, a set of simple yet comprehensive shear design expression is required.

To illustrate the use of Code provisions of different country in respect of shear resistance of Transfer beams, a numerical example is worked out.

4. Design example

To illustrate the use of Code provisions in respect of shear resisted by concrete of different country and proposed expression, an example is worked out.

4.1 Example

Condition of Support: Simply supported
Uniformly Distributed Load (w): 20 kN/m
Concrete Grade (f_{ck}) M 30, Steel Grade (f_y) Fe 415
Effective Length of Beam (L): 5 m
Tension Reinforcement (A_{st}): 3 Nos. 12 mm ϕ
Depth of beam (D) = 300 mm
Effective depth of beam (d) = 275 mm
Width of beam (b) = 200 mm

Solution:

Since the support are simply supported, the critical section for shear will be at a distance, 'd' (200 mm) from the centre of support.

Maximum shear at support (R) = 50 kN
Maximum moment at center of span = 62.5 kN-m
Shear at the critical section = 46 kN
Moment at the critical section = 9.6 kN-m
Percentage of reinforcement (A_{st}) = 0.56%

The shear force diagram and bending moment diagram of a beam loaded with uniformly distributed load of intensity 20 kN/m, is shown in Fig. 2.

5. Shear capacity calculations by different shear design equations

Four design methods, namely, i) IS 456-2000, (iii) BS 8110-1997, (iv) ACI 318-2002 (v) EuroCode-2-2002 and the proposed expression are used to estimate the shear capacity of beams. The results are

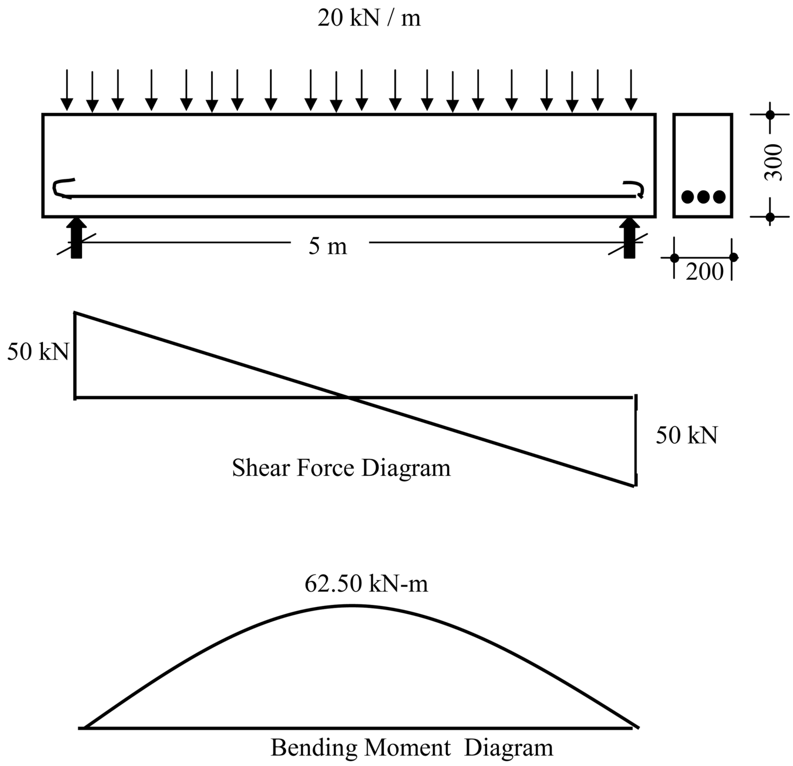

Fig. 2 Shear force and bending moment diagram

given in Table 1. The meanings of notations used are explained under each expression.

5.1 Shear resisted by concrete as per IS 456-2000

For Grade of concrete M30 the values of τ_c which are given in Table 19 of IS: 456-2000 are;

$\%A_{st}$	τ_c (MPa)
0.50	0.50
0.75	0.59

For $A_{st} = 0.56\%$, the value of τ_c which is found out from linear interpolation is $\tau_c = 0.52$ MPa. Hence the shear resisted by concrete is;

$$V_c = \tau_c bd \qquad (2)$$

$$V_c = 0.52 \times 200 \times 275/1000$$

$$V_c = 28.60 \text{ kN}$$

5.2 Shear resisted by concrete as per BS 8110-1985

Depending upon the Grade of concrete, percentage of tension steel A_{st}, and effective depth the values of τ_c for different percent of A_{st} is as follows.

Concrete Grade M30

% A_{st}	τ_c (MPa)
0.50	0.63
0.75	0.72

For A_{st} = 0.56%, the value of τ_c which is found out from linear interpolation is τ_c = 0.65 MPa. Hence the shear resisted by concrete is;

$$V_c = \tau_c \text{bd} \tag{3}$$
$$V_c = 0.65 \times 200 \times 275/1000$$
$$V_c = 35.75 \text{ kN}$$

5.3 Shear resisted by concrete as per ACI 318

The nominal ultimate shear stress to be resisted by concrete

$$v_c = 0.16\sqrt{f_c'} + 17.2\rho_w \frac{V_u d}{M_u} \leq 0.29\sqrt{f_c'} \tag{4}$$

where $\dfrac{V_u}{M_u} \leq 1$.

here $v_c = \tau_c$

$\rho_w = A_{st}/100 = 0.0056$

V_u and M_u are the shear and the moments at the critical section.

Hence,

$$v_c = 0.16 \times \sqrt{30 \times 0.8} + 17.2 \times 0.0056 \times \frac{46 \times 0.300}{9.6}$$
$$= 0.92 \text{ MPa}$$

Hence the shear resisted by concrete is;

$$V_c = v_c \text{ bd} \tag{5}$$
$$V_c = 0.88 \times 200 \times 275/1000$$
$$V_c = 52.8 \text{ kN}$$

5.4 Shear resisted by concrete as per Euro Code 2 : April 2002 final draft

The design value for the shear resistance in non-prestressed members not requiring design shear reinforcement is given by

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k(100\rho_l f_{ck})^{1/3} + 0.15\sigma_{cp}\right] b_w \text{d} \tag{6.1}$$

But, the minimum value is

$$V_{Rd,\min} = [0.035 k^{3/2} f_{ck}^{1/2}] b_w \text{d} \tag{6.2}$$

Hence the shear resisted by concrete is

$$V_{Rd,c} = V_c = v_c \text{ bd} \tag{6.3}$$
$$V_c = 0.568 \times 200 \times 275/1000$$
$$V_c = 31.24 \text{ kN}$$

where,

f_{ck} is in MPa and $f_{ck} \leq$ 100 MPa,

Depth factors: $k = 1 + \sqrt{\frac{200}{d}} \leq 2$, d is in mm

$\rho_l = \frac{A_{sl}}{b_w d} \leq 0.02$

A_{sl} The area of the anchored tensile reinforcement,

b_w is the smallest width of the cross- section in the tensile area (in mm),

σ_{cp} = Compressive strength for prestressed concrete beams

5.5 Author's proposed expression for estimating the shear strength of concrete

The proposed empirical expression for estimating the shear strength of normal beams (shear span-to-depth ratio > 1.8) is as follows

$$\tau_c = \left[\frac{0.22}{\gamma_{mc}}\sqrt{0.80 f_{ck}} + 0.16(p_t)\left(\frac{d}{a}\right)^{1/4}\right] \tag{7.1}$$

where

τ_c = shear strength of concrete in MPa

γ_{mc} = partial material safety factor for concrete of 1.5

f_{ck} = cube compressive strength of concrete (MPa)

p_t = percentage longitudinal tension steel ($100A_s$/b d)

d = effective depth (mm)

a = shear span = 400 mm

Eq. (7.1) does not take the concrete safety factor into account. If we factor it in, the resulting equation is:

$$\tau_c = \left[0.15\sqrt{0.80 f_{ck}} + 0.16(p_t)\left(\frac{d}{400}\right)^{1/4}\right] \tag{7.2}$$
$$= 0.735 + 8.159 \times 10^{-4}$$
$$= 0.739 \text{ MPa}$$

Hence the shear resisted by concrete is;

$$V_c = v_c \text{ bd} \tag{7.3}$$
$$V_c = 0.739 \times 200 \times 275/1000$$
$$V_c = 40.65 \text{ kN}$$

6. Comparison of results of shear strength of concrete

The comparisons of Code provisions of different country in respect of shear resistance of beams is worked out in detail and shown in Table 1.

Table 1 Comparisons results of shear strength of concrete

Sr. No.	Codes	Shear capacity of concrete (V_c) kN	% increase in shear capacity in comparison with IS456:2000
1	IS 456:2000	28.60	----
2	EuroCode2	31.24	10%
2	BS 8110-1985	35.75	25%
3	Proposed Eq.	40.64	42%
4	ACI 318-2002	48.40	69%

7. Experimental validation and comparison with analytical results

To verify the proposed analytical model, the experimental results are compared with the shear design provisions of four National codes viz.: (i) IS 456-2000, (iii) BS 8110-1997, (iv) ACI 318-2002 (v) EuroCode-2-2002 (Tables 3(a) to Tables 3(b), & Fig. 3.

7.1 Experimental program

In the present experimental investigation, the failure behavior of RC beams with varying distribution of longitudinal and vertical steel was studied.

The details of geometric properties and the materials for casting the test specimens are as follows:

OPC (Ordinary Portland cement of 43 grades locally available course aggregate made of Basalt and/or Granite was used. Table 2 shows the details of concrete mix design for 43 MPa compressive strength of concrete. To achieve the high-strength to the concrete mix and for workability

Table 2 Summary of concrete mix design

Sr. No.	Description	Value
1	Characteristic cube strength	43MPa
2	Cement type	Ordinary Portland Cement
3	Aggregate type	Crushed granite and natural river sand
4	Slump for concrete	125 to 180 mm
5	Free water content	128 kg/m^3
6	Cement content	440 kg/m^3
7	Coarse aggregate content	1290 kg/m^3
8	Fine aggregate content	717 kg/m^3
9	Water-cement ratio (w/c)	0.29
10	Aggregate-cement ratio (w/c)	4.50
11	Admixture (Highly water reducing)	0.80% by wt. of cement
12	Mix proportion	1 : 1.63 : 2.93:0.29

requirements, Plasticizers was used. For longitudinal and transverse reinforcement, Thermo-Mechanically treated (TMT) rebars of Fe 415 grade was used.

7.1 Test specimen dimensions

Each beam was of 1200 mm in length and simply supported on a span of 1000 mm. The width of beam (100 mm) was kept constant and the overall depth was changed from 150 mm to 400 mm with 50 mm increment. Therefore, the cross sectional sizes of test specimens were 100 × 150 mm, 100 × 200 mm, 100 × 250 mm. These have resulted into testing of about 340 beams yielding a large set of relevant and reliable data.

7.2 Test set-up

All the test beams were tested under four-point loading (2-active and 2-passive) system applied with the help of 500 kN capacity loading frame (Fig. 3).

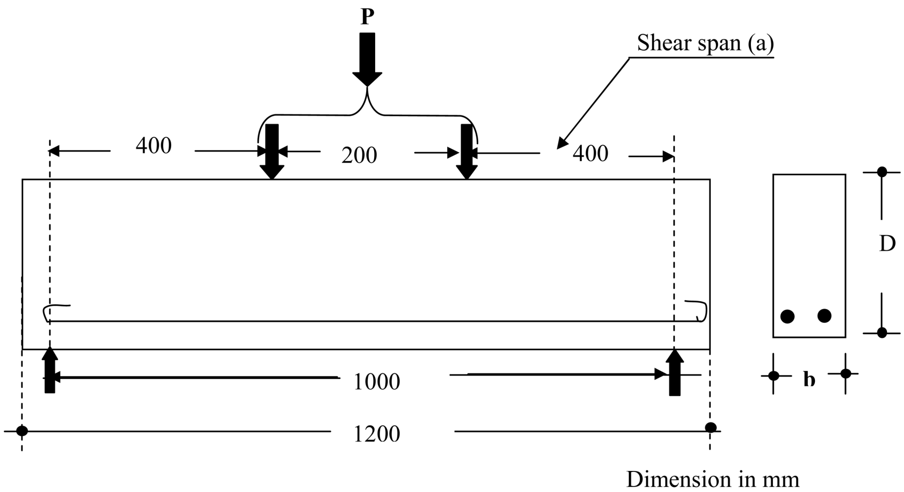

Fig. 3 Four point loading test set-up

Table 3(a) Shear strength of concrete without shear reinforcement for normal beams for (f_{ck} = 43 MPa): Effect of shear span-to-depth ratio

a/d ratio	Influence of shear span-to-depth ratio												
	Longitudinal tension steel (%): 1.50												
	Shear stress (MPa)							Shear strength ratio					
	V_{Test}	V_{IS}	V_{EC-2}	V_{BS}	V_{ACI} 11.5	V_{ACI} 11.3	$V_{EQ.}$ Eq. (7.1)	V_{Test} /V_{IS}	V_{Test}/ V_{EC2}	V_{Test}/ V_{BS}	V_{Test}/V_{ACI} 11.5	V_{Test}/V_{ACI} 11.3	V_{Test}/ V_{EQ}
1.78	1.85	0.79	0.89	0.95	1.06	0.94	1.09	2.34	2.10	1.95	1.75	1.97	1.70
2.29	1.72	0.79	0.94	1.01	1.02	0.94	1.08	2.18	1.83	1.70	1.69	1.83	1.59
3.20	1.65	0.79	0.94	1.08	0.99	0.94	1.06	2.10	1.76	1.53	1.67	1.76	1.57

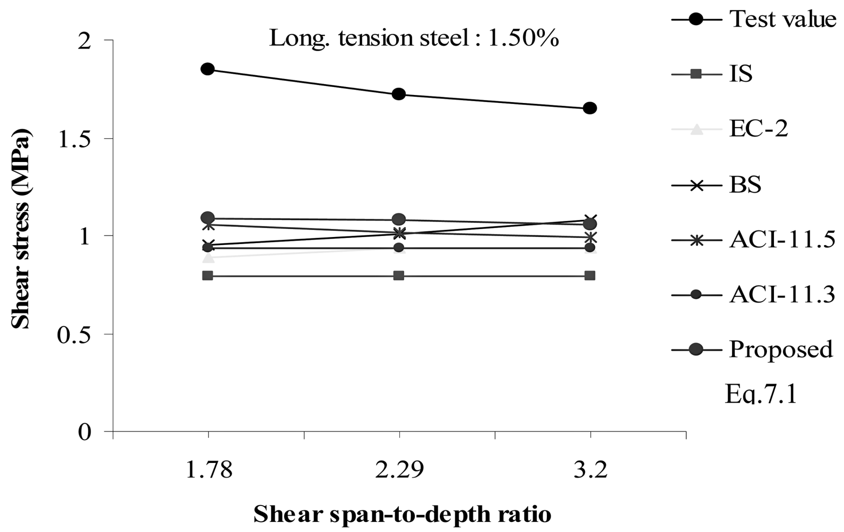

Fig. 4 Shear strength of normal beams without shear reinforcement - Influence of shear span-to-depth ratio

Table 3(b) Shear strength of concrete without shear reinforcement for normal beams for (f_{ck} = 43 MPa) : Effect of longitudinal tension steel (%)

Main tension steel (%)	Depth of beam (D) : 150 mm (a/d = 3.20)												
	Shear stress (MPa)							Shear strength ratio					
	V_{Test}	V_{IS}	V_{EC-2}	V_{BS}	V_{ACI} 11.5	V_{ACI} 11.3	$V_{EQ.}$ Eq. (7.1)	V_{Test}/V_{IS}	V_{Test}/V_{EC2}	V_{Test}/V_{BS}	V_{Test}/V_{ACI} 11.5	V_{Test}/V_{ACI} 11.3	V_{Test}/V_{EQ}
0.50	1.12	0.51	0.65	0.75	0.94	0.94	0.94	2.20	1.72	1.50	1.20	1.20	1.19
1.00	1.42	0.68	0.83	0.95	0.96	0.94	1.00	2.10	1.71	1.50	1.48	1.51	1.42
1.50	1.65	0.79	0.94	1.08	0.99	0.94	1.06	2.10	1.75	1.53	1.67	1.76	1.57
2.0.	1.85	0.88	1.03	1.19	1.01	0.94	1.12	2.10	1.80	1.55	1.83	1.97	1.65
2.50	2.10	0.95	1.03	1.25	1.04	0.94	1.18	2.20	2.00	1.70	2.00	2.23	1.80
3.00	2.36	1.01	1.03	1.36	1.07	0.94	1.24	2.35	2.30	1.74	2.21	2.51	1.90

8. Conclusions

From the brief review of several national codes, the codal recommendations for normal beams which are transferring the load through a flexural actions, are given in all the national codes namely Indian Standards (IS: 456), British Standards (BS: 8110), Eurocode-2 and ACI 318 Code. The BS Code and Eurocode considered the depth of section as an influencing parameter along with the percentage of tension reinforcement and grade of concrete in calculating concrete shear stress. While in IS Code it only depends on the grade of concrete and percentage of tension steel. In ACI Code, they have given empirical relationship to calculate the shear stress of concrete depending upon the grade of concrete, percentage of tension steel, ultimate shear force, and the ultimate moment at the section considered.

In the typical example adopted, the Euro code 2 value is about 10 percent higher the British Code

Table 3(c) Shear strength of concrete without shear reinforcement for normal beams : Effect of Compressive strength of concrete

Depth of Beam = 150 mm							
Sr.No	Long. tension steel (%)	Name of the code	Concrete Grade				
			M20	M25	M30	M35	M40
1	0.50	IS 456	0.48	0.49	0.50	0.50	0.51
2		Eurocode-2	0.53	0.55	0.59	0.62	0.65
3		BS8110	0.59	0.65	0.68	0.72	0.75
4		ACI 318 (Eq. 11.5)	0.66	0.75	0.80	0.88	0.94
5		ACI 318 (Eq. 11.3)	0.67	0.75	0.82	0.88	0.94
6		Proposed Eq. (7.1)	0.66	0.73	0.79	0.85	0.90
7	1.00	IS 456	0.62	0.64	0.66	0.67	0.68
8		Eurocode-2	0.68	0.70	0.75	0.79	0.83
9		BS8110	0.75	0.82	0.86	0.91	0.95
10		ACI 318 (Eq. 11.5)	0.69	0.77	0.83	0.90	0.96
11		ACI 318 (Eq. 11.3)	0.67	0.75	0.82	0.88	0.94
12		Proposed Eq. (7.1)	0.72	0.79	0.85	0.91	0.97
13	1.50	IS 456	0.72	0.74	0.76	0.78	0.79
14		Eurocode-2	0.76	0.80	0.85	0.89	0.94
15		BS8110	0.85	0.93	0.98	1.04	1.08
16		ACI 318 (Eq. 11.5)	0.72	0.80	0.85	0.92	0.99
17		ACI 318 (Eq. 11.3)	0.67	0.75	0.82	0.88	0.94
18		Proposed Eq. (7.1)	0.78	0.85	0.91	0.97	1.03
19	2.00	IS 456	0.79	0.82	0.84	0.86	0.88
20		Eurocode-2	0.85	0.88	0.94	0.99	1.03
21		BS8110	0.94	1.02	1.08	1.14	1.19
22		ACI 318 (Eq. 11.5)	0.74	0.82	0.88	0.95	1.01
23		ACI 318 (Eq. 11.3)	0.67	0.75	0.82	0.88	0.94
24		Proposed Eq. (7.1)	0.84	0.91	0.97	1.03	1.09

value is about 25 percent higher whereas the ACI value is about 69 percent higher than the value obtained as per IS Code. It amply suggests that IS Code is very conservative as compared to BS and ACI Codes. The results of the numerical example worked out suggest that there is need for upward revision in the shear design procedure of different codes. However, the proposed expression (7.1) is less conservative among the IS, BS & Eurocode.

Finally, the model was calibrated on the basis of experimental investigation work. The result of the model has demonstrated its ability to predict the shear strength of the beams with good approximation.

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