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Three dimensional analysis of reinforced concrete frames considering the cracking effect and geometric nonlinearity

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Abstract. In the design of tall reinforced concrete (R/C) buildings, the serviceability stiffness criteria in terms of maximum lateral displacement and inter-story drift must be satisfied to prevent large second-order P-delta effects. To accurately assess the lateral deflection and stiffness of tall R/C structures, cracked members in these structures need to be identified and their effective member flexural stiffness determined. In addition, the implementation of the geometric nonlinearity in the analysis can be significant for an accurate prediction of lateral deflection of the structure, particularly in the case of tall R/C building under lateral loading. It can therefore be important to consider the cracking effect together with the geometric nonlinearity in the analysis in order to obtain more accurate results. In the present study, a computer program based on the iterative procedure has been developed for the three dimensional analysis of reinforced concrete frames with cracked beam and column elements. Probability-based effective stiffness model is used for the effective flexural stiffness of a cracked member. In the analysis, the geometric nonlinearity due to the interaction of axial force and bending moment and the displacements of joints are also taken into account. The analytical procedure has been demonstrated through the application of R/C frame examples in which its accuracy and efficiency in comparison with experimental and other analytical results are verified. The effectiveness of the analytical procedure is also illustrated through a practical four story R/C frame example. The iterative procedure provides equally good and consistent prediction of lateral deflection and effective flexural member stiffness. The proposed analytical procedure is efficient from the viewpoints of computational effort and convergence rate.

Keywords: three dimensional analysis; reinforced concrete frames; effective moment of inertia; deflections.

1. Introduction

Reinforced concrete has become a universal structural material in the engineering construction in many respects, such as wide availability of reinforcing bars and consistent of concrete, its rigidity,

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and the economy of reinforced concrete compared to the other forms of construction. Therefore the use of R/C as a building material for complex structural system has increased in recent years, and the response of these members and structural system to a set of applied loads has been the subject of most investigation.

In reinforced concrete construction, a designer must satisfy not only the strength requirements but also the serviceability requirements. To assess the serviceability requirements of tall reinforced concrete buildings, it is necessary to accurately predict the deflection under lateral and gravity loads. In recent years, with the use of higher strength and new material in reinforced concrete, high rise and slender structures have been constructed around the world. In the design process of tall reinforced concrete structures, the control of the lateral drift is one of the most important design criteria, and therefore the serviceability stiffness criteria in terms of the maximum lateral displacement and inter-story drift must be satisfied to prevent large second-order P-delta effects. Due to the low tensile strength of concrete, cracking, which is primarily load dependant, is an inevitable phenomenon and can occur at service loads. The tensile cracking of concrete reduces the flexural stiffness of members and thus results in an increase in the deflection of reinforced concrete structures. For accurate determination of the deflection, cracked members in the reinforced concrete structures need to be identified and their effective flexural stiffness determined.

Geometric nonlinearity due to the interaction of axial force and bending moments, and the displacements of joints can be significant for an accurate prediction of lateral deflection of the structure under different loading conditions, especially in the case of tall reinforced concrete frames under lateral loading. The reduction in the flexural stiffness of members due to cracking increases the deflection of reinforced concrete structures. Also the action of the applied loads might be affected by the structure's deformation. Therefore geometric nonlinearity becomes significant factor when level of loading gets high enough to cause large displacements.

Several methods are available in the technical literature for computing the deflections in reinforced concrete structures, taking the nonlinear effects of concrete cracking into the consideration (Ngo and Scordelis 1967, Channakeshava and Sundara 1988). These methods consider the constitutive relationships of both steel and concrete together with the bond-slip relationship. Due to the complexities of the actual behavior of reinforced concrete frame, cumbersome computations are necessarily performed. Therefore, these methods can not be easily adopted in the design office.

In recent years, engineers and researches have also made significant advancement in the finite element procedures for the analysis of reinforced concrete frames. Attempts to model the stiffness reduction due to cracking using finite element procedures have been carried out by many investigators. Two approaches have been formed for reinforced concrete, namely microelement approach and macroelement approach. The microelement approach takes into account the structure to be divided in to many small finite elements including 2D-3D elements modeling concrete and bar elements modeling reinforcing steel. The discrete crack model and smeared crack model are also applied to simulate crack representation (Barzegar 1989, Ingraffea and Gerstle 1985). However, the macroelement approach incorporates such factor as the cracking effect and bonding behavior into the constitutive model of reinforced concrete. In this approach, each element represents both concrete and steel, and the local phenomena are incorporated into a constitutive model used to obtain stiffness matrix of macroelement (Chen 1982, Cauvin 1991, Vecchio and Emara 1992, Polak 1996, Chan *et al*. 2000). On the other hand most, if not all, of these analyses are expensive and time consuming for large scale multi-story reinforced concrete structures.

In the design of tall reinforced concrete structures, the moment of inertia of beams and columns are generally reduced at the specified ratios to compute the lateral drift by taking into consideration the cracking effect on the stiffness of structural frame. The gross moment of inertia of beams is generally reduced to 50% of their uncracked values while the gross moment of inertia of columns is reduced to 80%, without considering the type, history and magnitude of loading, and the reinforcement ratios in the members (Stafford and Coull 1991).

A simplified and computationally more efficient method for the three dimensional analysis of reinforced concrete frames with cracked beam and column elements was developed by Dundar and Kara (2007). ACI (1995), CEB (1985) and probability-based effective stiffness models were used to evaluate the effective moment of inertia of the cracked members. Geometric nonlinearity was not considered in the analysis. Whereas, cracking of structural concrete reduces the lateral stiffness and thus increases the lateral deflection of tall reinforced concrete frames. Hence, the implementation of the geometric nonlinearity in the analysis can be significant in order to accurately assess the lateral deflection of large multi-story reinforced concrete frames.

The analytical procedure was developed for quantitatively predicting the effects of cracking on the lateral deflection and stiffness characteristic of tall reinforced concrete building under service loading conditions (Chan *et al*. 2000). A general probability-based effective stiffness model was used to consider the effects of concrete cracking. The analytical procedure was based on the effective stiffness model and iterative algorithm with the linear finite element analysis. However geometric nonlinearity that becomes significant factor on the behavior of frames when level of loading is sufficient to cause large displacement was not considered in the analysis. Cracking of structural concrete generally leads to the reduction of the lateral stiffness and thus an increase in lateral deflection of tall building. It is therefore important to consider the geometric nonlinearity together with the cracking effect in the analysis in order to obtain more accurate results.

Shuraim developed a nonlinear analytical model for the analysis of reinforced concrete frames considering the material and geometric nonlinearities in the analysis (Shuraim 1997). The finite element formulations based on the layered approach and nonlinear constitutive laws can predict the nonlinear behavior of reinforced concrete frames with reasonable accuracy. However the analysis is expensive and time consuming for large scale tall reinforced concrete frames.

Current standards such as ACI Building Code (1995), CAN3-A23.3-M94 (1994) and TEC (1998) generally provide serviceability requirements together with empirical relationships that take into account cracking in simple members, especially those of beam type members, and the limitations of lateral drift of tall reinforced concrete frames. In contrast with member analysis, there is no methodology to be suggested in these codes of practice to consider the cracking effects for the analysis of tall reinforced concrete structures. In practice, the analysis of reinforced concrete frames are essentially carried out by using linear elastic models which consist of uncracked beam and column elements and ignores the geometric nonlinearity in the analysis. It is also quite possible that the design of tall reinforced concrete structures on the basis of linear elastic theory may not satisfy the serviceability requirements. Therefore it would be very useful to develop an analytical model for accurately predicting the lateral deflection of reinforced concrete frames, considering the cracking effect together with the geometric nonlinearity in the analysis.

In the present study, a computer program has been developed for the three dimensional analysis of reinforced concrete frames with cracked beam and column elements. Geometric nonlinearity is also taken into account in the analysis and direct stiffness method is applied to obtain the numerical solutions. The member stiffness matrix has been evaluated from the solution of the pertinent linear differential equation that governs the moment-curvature relation of a cracked member. The variation of the flexural rigidity of a cracked member is obtained by probability-based effective stiffness model. The results of the computer program have been verified with the experimental results available in the literature. Finally the effectiveness of the numerical analysis method is also illustrated through a practical four-storey reinforced concrete frame example.

2. Analysis of cracking effects

For tall reinforced concrete structures, lateral drift, which is significantly affected by an inevitable phenomenon of concrete cracking, becomes one of the most important design criteria, and hence cracking effects need to be considered in the analysis. The concrete members that contribute lateral stiffness have varying degrees of cracking ranging from the uncracked regions to the fully cracked regions. In general, the member has three cracked regions and two uncracked regions as seen in Fig. 1.

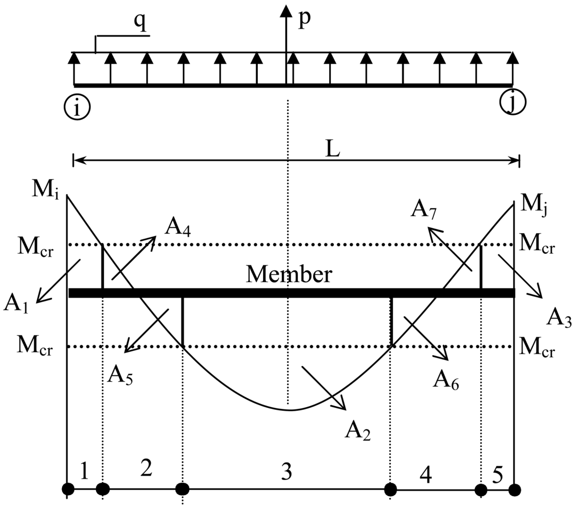
In the present study, probability-based effective stiffness model are used for the effective moment of inertia of the cracked members. In the probability-based effective stiffness model which accounts for the effects of concrete cracking with the stiffness reduction, the effective moment of inertia is obtained as the ratio of the area of moment diagram segment over which the working moment exceeds the cracking moment M_{cr} to the total area of moment diagram in the following form (Fig. 1).

$$A_{cr} = A_1 + A_2 + A_3 = \int_{M(x) \geq M_{cr}} M(x) \tag{1a}$$

$$A_{uncr} = A_4 + A_5 + A_6 + A_7 = \int_{M(x) \geq M_{cr}} M(x) \tag{1b}$$

$$A_t = A_{cr} + A_{uncr} \tag{1c}$$

$$P_{uncr}[M(x) < M_{cr}] = \frac{A_{uncr}}{A_t} \tag{1d}$$



1, 3, 5 cracked regions
2, 4 uncracked regions

Fig. 1 Cracked and uncracked regions of the member

$$P_{cr}[M(x) \geq M_{cr}] = \frac{A_{cr}}{A_t} \tag{1e}$$

$$I_{eff} = P_{uncr} I_1 + P_{cr} I_2 \tag{1f}$$

in which, I_1 and I_2 are the moments of inertia of the gross section and the cracked transformed section, respectively, M is the bending moment, M_{cr} is the moment corresponding to flexural cracking considered. The cracking moment, M_{cr} is calculated by the program using the following equation

$$M_{cr} = \frac{(f_r + \sigma_v) I_1}{y_t} \tag{2}$$

where σ_v is the axial compressive stress, f_r is the flexural tensile strength of concrete, and y_t is the distance from centroid of gross section to extreme fiber in tension.

In Eqs. (1), A_{cr} is the area of moment diagram segment over which the working moment exceeds the cracking moment M_{cr} and A_t is the total area of the moment diagram. In the same equation, P_{cr} and P_{uncr} are also the probability of occurrence of cracked and uncracked sections, respectively.

The flexural stiffness of a cracked member varies according to the amount of crack formation occurring in the members. Changes in the flexural stiffness of the cracked members cause certain transfer of the internal moments and forces of these members to the other uncracked member which will further result in the cracking of some of the otherwise uncracked members. The variation of the effective moment of inertia of the cracked members also necessitates the redistribution of the internal moments and forces in the structure. Hence iterative procedure should be applied to obtain the final deflections and internal forces of the structure.

3. Geometric nonlinearity

The implementation of geometric nonlinearity in the analysis can be significant for accurately assessing the lateral deflection of tall building when level of loading gets high enough to cause large displacement. In the present study, two different aspects of geometric nonlinearity are taken into consideration. These are the stability analysis considering the P-δ effect of the member, and overall structural stability considering P-Δ effect. The axial deformation of a member due to geometric deformations was found to be insignificant in reinforced concrete members. Therefore it is ignored in this study.

4. Member stiffness equation

In the three dimensional analysis of a beam or column element, the end force vector $\underline{P}$, the end displacement vector $\underline{d}$ and the fixed end force vector $\underline{P}_0$, each of 12 × 1 dimension, can be expressed, respectively, as follows

$$\underline{P}^T = [P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 P_{10} P_{11} P_{12}] \tag{3}$$

$$\underline{d}^T = [d_1 d_2 d_3 d_4 d_5 d_6 d_7 d_8 d_9 d_{10} d_{11} d_{12}] \tag{4}$$

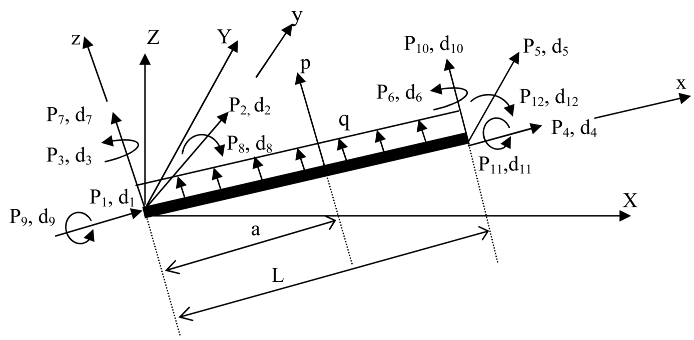


Fig. 2 A typical member subjected to a point and a uniformly distributed loads

$$\underline{P}_0^T = [P_{10} P_{20} P_{30} P_{40} P_{50} P_{60} P_{70} P_{80} P_{90} P_{100} P_{110} P_{120}] \tag{5}$$

A typical member subjected to a point and a uniformly distributed load, and positive end forces with corresponding displacements are also shown in Fig. 2.

To the knowledge of the authors, no study in the past has treated the geometric nonlinearity and cracking effect based on the probability-based effective stiffness model in the three dimensional analysis of R/C frames by employing the stiffness matrix method. In the present study, direct stiffness method is applied to obtain the numerical solutions, and the stiffness matrix of a three dimensional member has been obtained from the solution of pertinent linear differential equations that governs the moment curvature relation of a member, including the cracking effect end geometric nonlinearity. The member local stiffness matrix is also obtained, considering the P-δ effect of a member and cracking of structural concrete, as follows

$$\underline{k} = \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} \\ \underline{k}_{21} & \underline{k}_{22} \end{bmatrix} \tag{6a}$$

$$\underline{k}_{11} = \begin{bmatrix} \frac{AE_c}{L} & 0 & 0 & -\frac{AE_c}{L} & 0 & 0 \\ 0 & \frac{2(S_z+t_z)}{L^2} - \left(\gamma\frac{P}{L}\right) & \frac{S_z+t_z}{L} & 0 & -\frac{2(S_z+t_z)}{L^2} + \left(\gamma\frac{P}{L}\right) & \frac{S_z+t_z}{L} \\ 0 & \frac{S_z+t_z}{L} & S_z & 0 & -\frac{S_z+t_z}{L} & t_z \\ -\frac{AE_c}{L} & 0 & 0 & \frac{AE_c}{L} & 0 & 0 \\ 0 & -\frac{2(S_z+t_z)}{L^2} + \left(\gamma\frac{P}{L}\right) & -\frac{S_z+t_z}{L} & 0 & \frac{2(S_z+t_z)}{L^2} - \left(\gamma\frac{P}{L}\right) & -\frac{S_z+t_z}{L} \\ 0 & \frac{S_z+t_z}{L} & t_z & 0 & -\frac{S_z+t_z}{L} & S_z \end{bmatrix} \tag{6b}$$

$$\underline{k}_{22} = \begin{bmatrix} \frac{2(S_y+t_y)}{L^2}-\left(\gamma\frac{P}{L}\right) & -\frac{S_y+t_y}{L} & 0 & -\frac{2(S_y+t_y)}{L^2}+\left(\gamma\frac{P}{L}\right) & 0 & -\frac{S_y+t_y}{L} \\ -\frac{S_y+t_y}{L} & S_y & 0 & \frac{S_y+t_y}{L} & 0 & t_y \\ 0 & 0 & \frac{GJ}{L} & 0 & -\frac{GJ}{L} & 0 \\ -\frac{2(S_y+t_y)}{L^2}+\left(\gamma\frac{P}{L}\right) & \frac{S_y+t_y}{L} & 0 & \frac{2(S_y+t_y)}{L^2}-\left(\gamma\frac{P}{L}\right) & 0 & \frac{S_y+t_y}{L} \\ 0 & 0 & -\frac{GJ}{L} & 0 & \frac{GJ}{L} & 0 \\ -\frac{S_y+t_y}{L} & t_y & 0 & \frac{S_y+t_y}{L} & 0 & S_y \end{bmatrix} \tag{6c}$$

$$\underline{k}_{21} = \underline{k}_{12} = \underline{0} \tag{6d}$$

$$S_m = \frac{EI_{effm}(u_m(u_m c - s))}{H_m L} \qquad C_m = \frac{(s-u_m)}{u_m c - s} \qquad t_m = S_m C_m = \frac{EI_{effm}(u_m(s-u_m))}{H_m L} \tag{6e}$$

$$u_m = k_m L \qquad k_m = \sqrt{\frac{P}{E_c I_{effm}}} \qquad s = \sin u \qquad c = \cos u \qquad H_m = u_m s + (\gamma 2c) - \gamma 2 \qquad m = y, z \tag{6f}$$

In Eqs. (6), P, E_c and A are the axial force, modulus elasticity of concrete and cross sectional area respectively, G and J denote the shear modulus of concrete and torsional moment of inertia of the cross section, S_m and C_m are also the stability coefficients in the related directions, respectively. In the same equations, if the axial force is compressive $\gamma = 1$, and if the axial force is tensile $\gamma = -1$ and sinu and cosu equal to sinhu and coshu.

Member stiffness equation can finally be evaluated in the following form

$$\underline{k}\underline{d} + \underline{P}_0 = \underline{P} \tag{7}$$

Eq. (7) is given in the member coordinate system (x, y, z). Hence it should be transformed to the structure coordinate system (X, Y, Z).

In the present study, the full service load applied to the structure is divided into suitable number of load increments, and iterative procedure is applied in each loading step. Changes in geometry are considered in the analysis, and the joint global coordinates are modified after each iteration to consider the current joint displaced position. Since the member equilibrium equation and hence the member stiffness matrix are functions of the current nodal displacements, the member stiffness matrix has been developed with respect to the current displaced coordinates of joints. Therefore, in addition to the P-δ effect of a member, the formulation is based on the deformed structure, which means that further consideration of geometric nonlinearity will be achieved by updating the nodes' coordinates in each iteration.

In the three dimensional analysis of reinforced concrete frames developed in the present study, member equations are first obtained and then by considering the contributions which come from each element, the system stiffness matrix and system load vector are assembled. Finally, the system displacements and member end forces are obtained by solving the system equation. This procedure is repeated step by step in all iterations.

5. Computer program

A general purpose computer program developed for the three dimensional analysis of R/C frames

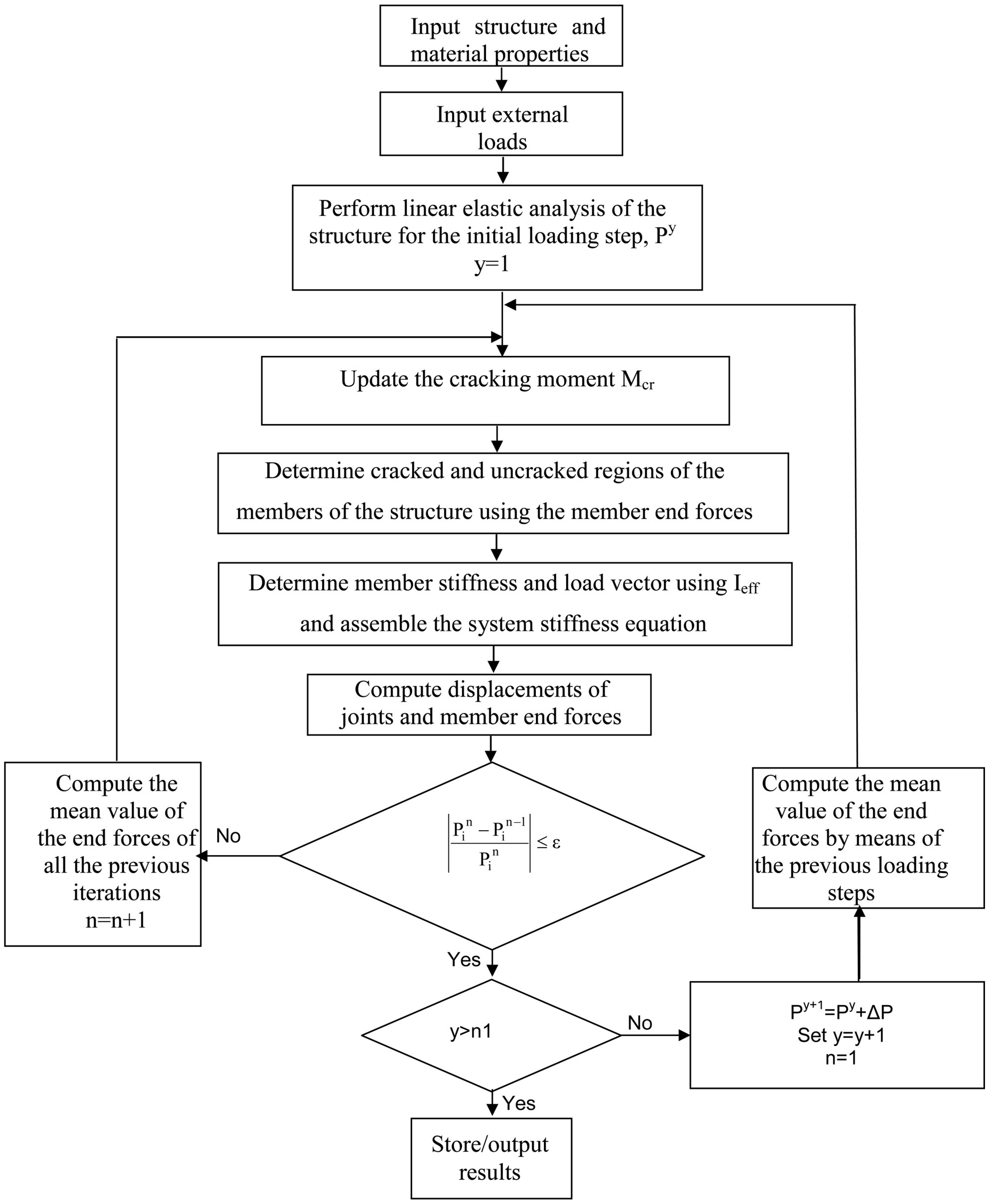


Fig. 3 Solution procedure of the program

with beams and columns in cracked state is coded in Fortran 77 language. The flow chart of the solution procedure of the program is given in Fig. 3. The proposed analytical procedure provides the history of nonlinear behavior of R/C frames due to cracking effect and geometric nonlinearity by applying the external load in an incremental manner. In the present study, the total load is divided into suitable number of loading steps (n1) and a proportional loading (ΔP) is applied step by step. Iterative procedure has also been adopted in each loading step.

In the solution procedure, using a procedure that evaluates the stiffness of cracked members by means of the flexural moment values by the previous iteration is not convergent in many cases. If the beams and columns are cracked in both positive and negative moment regions, it is indicated that both these regions will be more flexible in the next iteration; as a consequence, the moment will decrease either at midspan or at the end of the member, with a consequent decrease of the flexibility. On other hand over reductions of flexural stiffness in some beams or columns may cause smaller redistributions of internal forces for these members and thus excessive increase in the flexural stiffness of these cracked members in the subsequent iteration. Increase of flexural stiffness attracts the transfer of more internal forces to these members, thus leading to over reduction to occur again. The alternate increase and decrease in the flexural stiffness of members generates a generally non-convergent procedure. Therefore a procedure considering, in each iteration, the value of the end forces evaluated as the mean value of the forces of all previous iterations have been used in the present study (Cosenza 1990). This procedure decreases the fluctuations in the stiffness and accelerates the convergence of the algorithm. In the program,

$$\left|\frac{P_i^n - P_i^{n-1}}{P_i^n}\right| \le \varepsilon \tag{8}$$

is used as convergence criterion. Here, n is the iteration number, ε is the convergence factor and P_i^n ($i = 1, 12$) is the end forces of each member of the structure for n-th iteration.

Due to space limitation, the listing of the computer program is not given in the paper. A PC version and the manual of the program can be obtained free of charge from the authors upon request.

6. Verification of theoretical results and a four story frame example

In this section three examples are presented. In order to verify the applicability and determine the limitation of the proposed analytical procedure, the first two examples are taken from the literature. The third example also introduces a primary application of the proposed analytical method on a three dimensional four story R/C frame.

6.1 Example 1

In this example, the two story R/C frame tested by Vecchio and Emara (1992) for a comparison with their analytical solution has been analyzed by the present computer program. This reinforced concrete frame was designed with a center to center span of 3500 mm and a story height of 2000 mm (Fig. 4). All members of the frame were 300 mm wide by 400 mm deep and similarly reinforced with four No 20 deformed bars as both tensile and compressive steel. The concrete had a compressive strength of 30 MPa, and the reinforcing steel had a modulus of elasticity of 192500

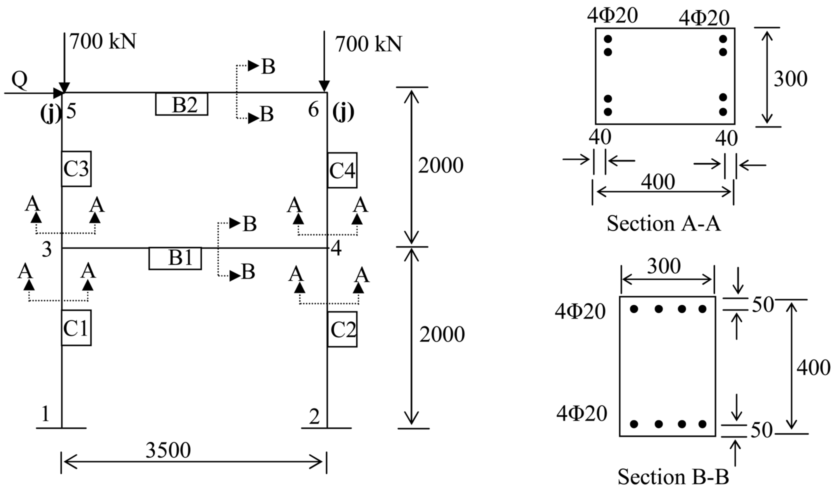


Fig. 4 Dimensions of reinforced concrete frame model, (dimensions in mm), (Vecchio and Emara 1992)

MPa (Vecchio and Emara 1992). The test procedure involved first applying a total axial load of 700 kN to each column and maintaining this load throughout the test. The lateral load (Q) was then monotonically applied until the ultimate capacity of the frame was achieved. The reinforced concrete frame is modeled by four columns and two beam elements as shown in Fig. 4. The cross-sections of the members, the axial loads and the span are also shown in the figure. In computing flexural tensile strength and modulus of elasticity of concrete, the following equations (ACI Code Eq. (1995)) are also used.

$$f_r = 0.62\sqrt{f_c'} \ \ (\text{N/mm}^2) \tag{9a}$$

$$E_c = 4730\sqrt{f_c'} \ \ (\text{N/mm}^2) \tag{9b}$$

in which, f_c' is the compressive strength of concrete.

To determine the applicability of the proposed analytical procedure, the comparison between the test and theoretical results for the lateral deflection of joint 5 obtained by the linear analysis, cracking analysis and the layered model developed by Vecchio and Emara (1992) is presented in Fig. 5 and Fig. 6. It is seen that the deflection calculated by the developed computer program agree well with the test results for applied loads within the 275 kN load level with maximum discrepancies of 12%. Linear elastic methods, not considering the cracking effect and geometric nonlinearity in the analysis, gives predictions of 42% of the test values of the lateral deflection of joint 5 at the 83% ultimate lateral load levels (i.e. 275 kN). However the proposed analytical procedure predicts 88% of the experimental deflection. At a load level of 50% of ultimate (i.e. 165 kN), the prediction of linear elastic method gives 56% of the experimental deflection. On the other hand the prediction of proposed analytical procedure which considers the cracking of structural concrete and geometric nonlinearity in the analysis gives 97% of the test values. The results show that even within the serviceability loading range the assumption of linear elastic behavior is not reliable and inaccurate since the first flexural stiffness reductions of members caused by initial cracking of structural members usually occurs at a very low load level. It can also be seen

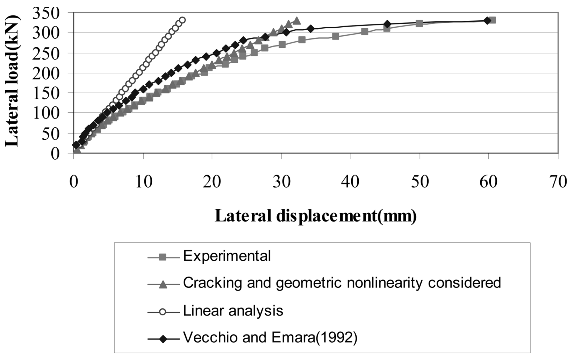


Fig. 5 Load deflection comparison between the experimental and analytical results of the lateral deflection of joint 5

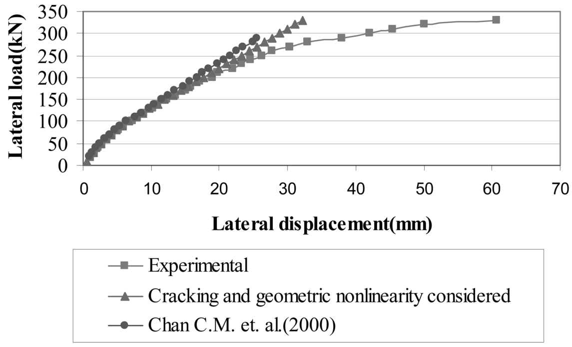


Fig. 6 Comparison of predicted deflections at joint 5 obtained by present and other analytical studies

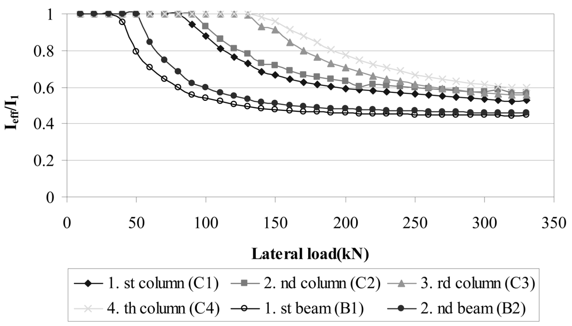


Fig. 7 Flexural stiffness reduction of each member versus lateral loads

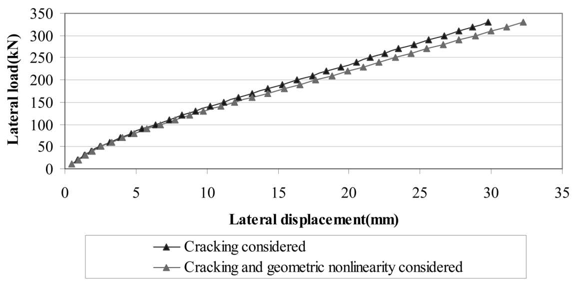


Fig. 8 Theoretical influence of geometric nonlinearity on the lateral deflection of frame

from the Fig. 6, the results of Vecchio and Emara's layer model (1992) are in good agreement with the test results especially after 83% of the ultimate load. On the other hand, the proposed analytical model shows an improved prediction over Vecchio and Emara's layer model (1992) for loads up to 275 kN. When the lateral load is beyond this load level the difference between the experimental and theoretical results becomes significant. Such differences are primarily due to the fact that steel and concrete materials become nonlinear beyond the serviceability loading level and also the plastic hinges are forming at the ends of the beams, and cracking occurs at the joints of the beam column near the ultimate limit state. However, the results of the proposed procedure which considers the geometric nonlinearity give better predictions than the other presented (Chan *et al*. 2000).

The theoretical results of the cracking sequence and flexural stiffness reductions of each member with respect to the lateral applied load are also shown in Fig. 7. As seen from the figure, the beams of the first and second stories crack first and then two columns at the first story, C1 and C2, start to crack followed by, in the final stage, the cracking of both columns at the second story, C3 and C4. Fig. 7 also shows that when the lateral load reaches 83% of the ultimate lateral load, the beams at the first and second stories have 45 and 47%, respectively, of the gross moment of inertia, and the two columns at the first story have 55 and 59% of their uncracked values. The other two columns at the second story have also 59 and 63% of their gross moment of inertia.

Fig. 8 shows the influence of geometric nonlinearity on the total lateral deflection (joint 5) of the

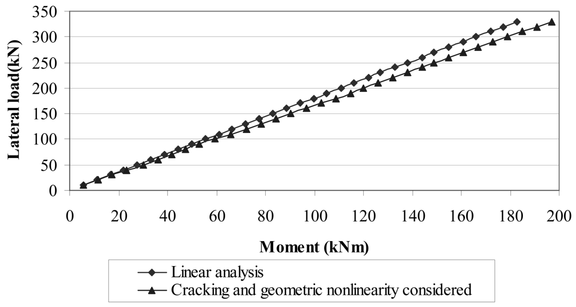


Fig. 9 The variation of the value of the moment of 3 rd column with increasing lateral load

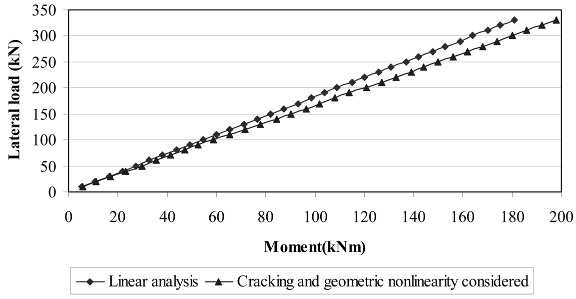


Fig. 10 The variation of the value of the moment of 4 th column with the lateral load

R/C frame. It can be seen that the contribution of the geometric nonlinearity to the total lateral deformation of the frame increases with increasing lateral loads, such as 9% for 83% of the ultimate load and 11% for the ultimate lateral load.

Fig. 9 and Fig. 10 present the variation of the value of the moment at j end of the 3 rd and 4 th columns (C3, C4) obtained by the linear and nonlinear analyses with respect to the lateral applied load. As seen from the figures, the influence of the geometric nonlinearity on the value of the flexural moment increases with the increase in the lateral load due to P-Δ effect.

6.2 Example 2

In the second example, the test results given by Chan *et al.* (2000) for a two story reinforced concrete frame have been compared with the results of the present computer program. This reinforced concrete frame was designed with a center to center span of 3000 mm, a first story height of 1170 mm and a second story height of 2000 mm (Fig. 11). The testing setup involved first applying a total axial load of 200 kN to each column which was maintained throughout the test. The lateral load (Q) was then monotonically applied until the ultimate capacity of the frame was

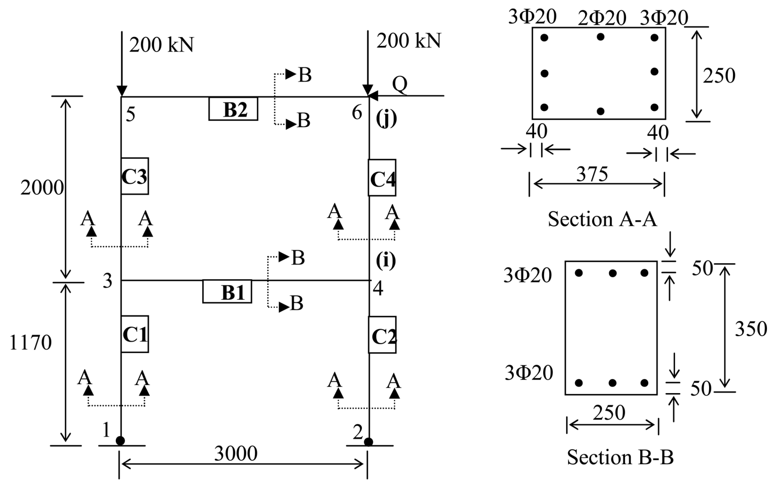


Fig. 11 Two story reinforced concrete frame model tested by Chan *et al.* (2000) (dimensions in mm)

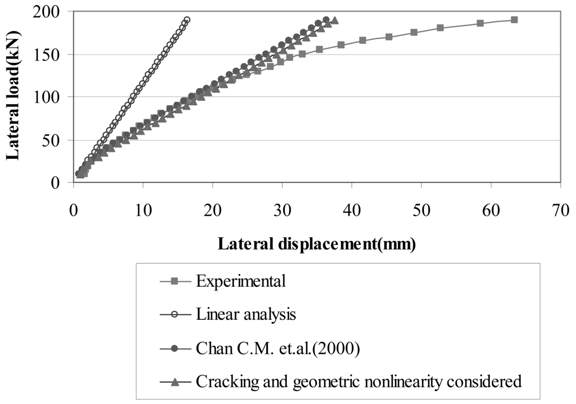


Fig. 12 Comparison between experimental and analytical results of the lateral deflection of joint 6

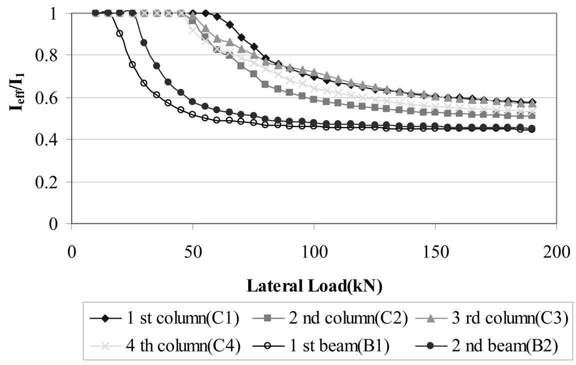


Fig. 13 Flexural stiffness reduction of beams and columns with respect to increasing lateral loads

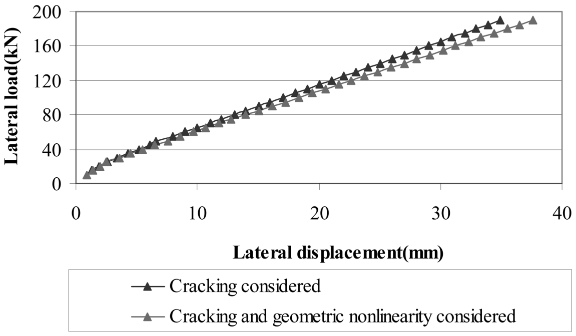


Fig. 14 Theoretical influence of geometric non-linearity on the lateral deflection of frame

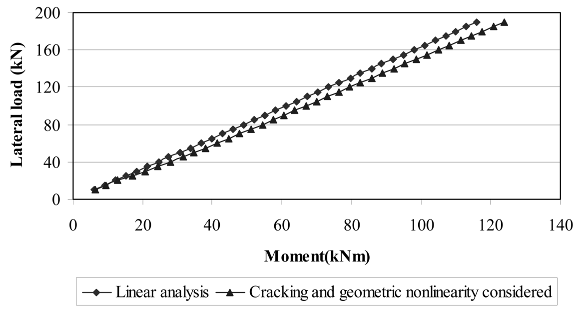


Fig. 15 The variation of the value of the moment of 4 th column with the lateral load

achieved. This frame is modeled by four columns of 250 × 375 mm and two beam elements of 250 × 350 mm cross sections. The concrete had a compressive strength of 29 MPa, and the reinforcing steel had a modulus of elasticity of 200 000 MPa. The reinforcing steel in the beams and columns, the span and the loads are also shown in the figure.

The comparison between the experimental and theoretical results of the lateral deflection of joint 6 is presented in Fig. 12. The numerical results obtained from the present study are in good agreement with the test results for applied loads within the 155 kN load level with maximum discrepancies of 13%. The proposed analytical method also predicts the lateral deflection in the serviceability loading range with a high degree of accuracy, and beyond this range the differences between the numerical and test results increase with the increase in the lateral loads.

The variation of the flexural stiffness of beams and columns with respect to the lateral applied load is also shown in Fig. 13. It can be seen from the figure that the beams at the first and second stories, B1 and B2, crack first and then two columns on the lateral loading side, C4 and C2, crack respectively. Finally the two columns on the opposite loading side, C3 and C1, start to crack, respectively. From Fig. 13 it is shown that at 82% of the ultimate lateral load, the two columns at the first story have 52 and 60%, respectively, of their gross moment of inertia, and two beams at the first and second stories have 45 and 46% of their uncracked values. The columns of the second story have also 55 and 60% of the gross moment of inertia.

Fig. 14 also shows the theoretical influence of geometric nonlinearity on the total lateral deflection (joint 6) of the R/C frame. It is seen that the contribution of the geometric nonlinearity to the total lateral deflection of the frame increases with increasing lateral loads, such as 10% for the ultimate lateral load.

Fig. 15 presents the influence of geometric nonlinearity on the value of flexural moment at j end of 4. th column (C4). It is seen that the contribution of the flexural moment due to P-Δ effect to the total flexural moment increases with increasing lateral loads.

6.3 Example 3

In the last example, the four-story reinforced concrete frame shown in Fig. 16 has been analyzed by the developed computer program. This three-dimensional R/C frame is subjected to lateral point loads at the floor levels and uniformly distributed loads on the beams. The dimensions and the reinforcement ratios (ρ) of the members, and the loads are given in Table 1. The design strength of concrete is assumed to be 32 MPa and the modulus of the elasticity of the steel is assumed to be 200000 MPa. In this example, the lateral loads acting at the floor levels, which are expressed in terms of the values of Q, are increased while the intensity of uniform loads (q = 30 kN/m) remain constant.

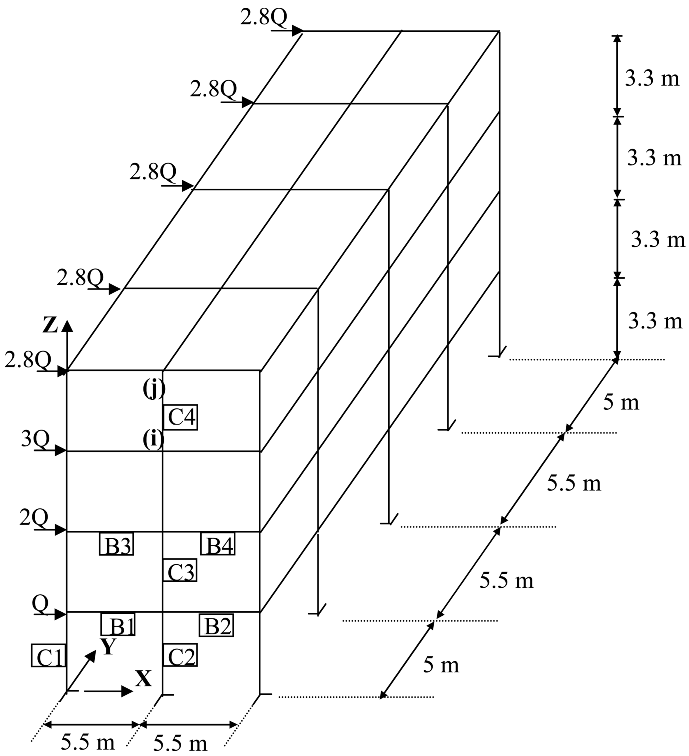


Fig. 16 Four-story reinforced concrete frame

Table 1 Dimensions of the members and loads applied to the frame

	First-Story	Second-Story	Third-Story	Fourth-Story
Dimensions of beam	300×500 mm	300×500 mm	300×500 mm	300×500 mm
ρ (%) (top)	0.6	0.6	0.6	0.6
ρ (%) (bottom)	1.1	1.1	1.1	0.9
Dimensions of column	500×500 mm	500×500 mm	500×500 mm	400×400 mm
ρ(%)	1.8	1.8	1.8	1.6
Uniformly distributed loads (kN/m)	q	q	q	0.8 q

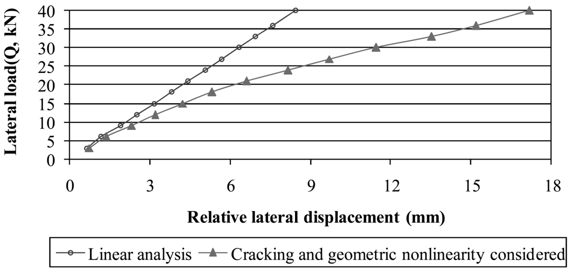


Fig. 17 The variation of the maximum relative lateral displacement of the second floor with respect to the lateral loads

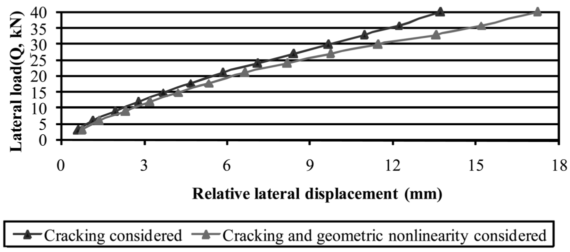


Fig. 18 Theoretical influence of the geometric nonlinearity on the lateral deflection of a three dimensional reinforced concrete frame

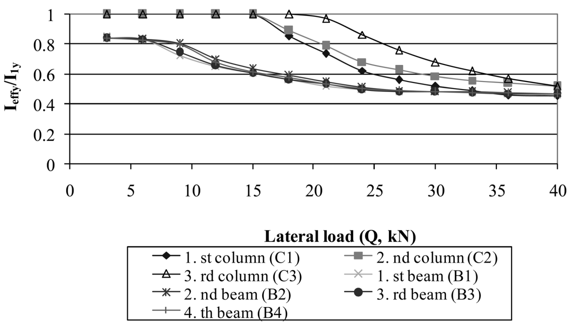


Fig. 19 Flexural stiffness reductions of beams and columns with increasing lateral load

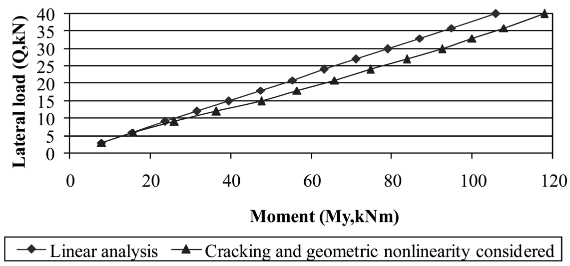


Fig. 20 Theoretical influence of the geometric nonlinearity on the value of the moment of 4 th column in a three dimensional reinforced concrete frame

The variation of the maximum relative lateral displacement of the second floor with lateral load, when cracking effect and geometric nonlinearity considered and not considered in the analysis, are shown in Fig. 17. As seen form the figure, the differences in the maximum relative lateral displacement of the second floor between the two cases increase with increasing lateral loads. The difference becomes significant at higher loads such as 95% for Q = 40 kN.

Fig. 18 also shows the theoretical influence of geometric nonlinearity on the maximum relative lateral displacement of the second floor. As seen from figure, the contribution of the geometric nonlinearity to the total lateral deflection of the frame increases with increasing lateral loads, such as 20% for Q = 40 kN.

The flexural stiffness reductions of various members with respect to the lateral applied load are shown in Fig. 19. This figure indicates that when the stiffness of beams is reduced to 50% of their uncracked stiffness, the two first-story columns have reduced to 57-64% of their uncracked stiffness.

Fig. 20 also presents the influence of geometric nonlinearity on the value of flexural moment (M_y) at j end of 4 th column (C4). It is seen that the influence of P-Δ effect on the flexural moment value increases with increasing lateral loads.

7. Conclusions

A computer program based on the iterative procedure has been developed for the three dimensional analysis of R/C frames with beams and columns in cracked state. In the analysis geometric nonlinearity due to the interaction of axial force and bending moment and the displacements of joints are also taken into account. The load deflection history of R/C frames can be determined by the proposed iterative procedure.

The effective flexural stiffness of a cracked member has been evaluated by the probability-based effective stiffness model which determines the relationship between the flexural stiffness reduction and various moments due to loads applied to the member.

The capability and the reliability of the proposed procedure have been tested by means of comparisons with the theoretical and experimental results. The numerical results of the analytical procedure agree well with the test results for applied loads up to approximately 80% of the ultimate load capacity of the frame. The analytical procedure not only predicts the deflections with a high degree of accuracy at a value of load equal to approximately 80% of the ultimate load, but also gives an estimation of behavior at approximately 85% of the ultimate load with an acceptable degree of accuracy. However beyond this load level, there is a large discrepancy between the theoretical and experimental results. Such differences are primarily due to the fact that steel and concrete materials become nonlinear when the load is beyond the serviceability loading range. Also the plastic hinges are forming at the ends of the members, and cracking occurs at the joints of the beam column near the ultimate limit state.

The analytical procedure developed in the present study can predict the extent of cracking of each member under any specified load level. Cracking sequence and flexural stiffness reductions of beams and columns with respect to the applied load can be obtained by the developed computer program. This is the major advantage of the proposed procedure, and the variations in the flexural stiffness reductions of beams and columns in the R/C frames can be observed explicitly. This feature can minimize the uncertainty of flexural stiffness of members and therefore provide design engineers with significant information on the consequences of cracking in members.

Direct stiffness matrix method has been employed to obtain the numerical solutions of the proposed procedure. The proposed iterative analytical procedure is efficient from the viewpoints of computational effort and convergence rate.

Cracks occurring in the beams of tall R/C frames have been found to be responsible for the initial reduction of lateral stiffness at a very low level of lateral load.

The analytical results show that the influence of geometric nonlinearity on the lateral deflection increases with increasing lateral load levels. The implementation of geometric nonlinearity in the analysis can therefore be significant for obtaining more accurate results particularly in the case of tall R/C structures under lateral loads.

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