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Elastic analysis for a strip weakened by periodic holes

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1. Introduction

Many researchers studied the problem for the elastic plane medium containing the holes (Savin 1961, Atsumi 1956, Chen 1983, Chen and Lee 2002a, 2002b, Wang and Chen 1989, Dong 2006, Dong and Lee 2006, Wang *et al*. 2003, Legros *et al*. 2004, Dejoie *et al*. 2006). In this paper, elastic analysis for a tension strip weakened by the periodic holes is studied. The problem is solved by analyzing a rectangular cell with hole from the strip. For the rectangular cell, a solution is obtained by using the eigenfunction expansion variational method (EEVM). Furthermore, from the elastic response in the solution, the notched strip can be equivalent to an orthotropic strip without holes.

2. Analysis and the numerical results

The complex variable function method in plane elasticity is used in the present study

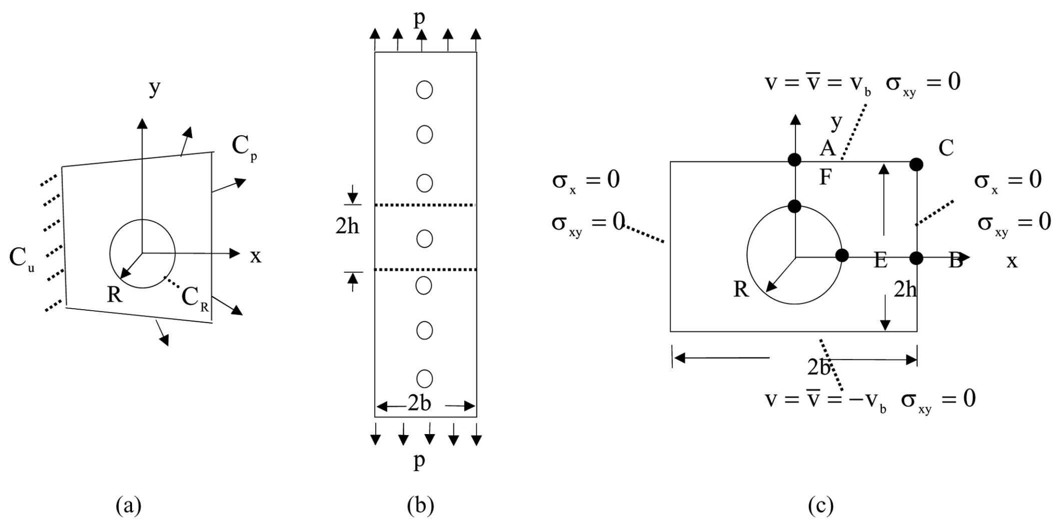

Fig. 1 (a) A finite plate with a circular hole, (b) Loading condition for a strip with the periodic holes, (c) Boundary value condition for the rectangular cell.

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(Muskhelishvili 1953). The traction free condition along the circular hole may be expressed as

$$\phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)} = 0, \qquad (z \in C_R) \tag{1}$$

where C_R denotes the circular hole boundary (Fig. 1(a)). The definition for the functions $\phi(z)$ and $\psi(z)$ can be referred to Muskhelishvili (1953).

For a cell cut from a weakened strip shown in Fig. 1(b), (c), from condition (1) one obtains the following eigenfunction expansion form

$$\phi(z) = \sum_{k=1}^{2M} X_k \phi^{(k)}(z), \qquad \psi(z) = \sum_{k=1}^{2M} X_k \psi^{(k)}(z) \tag{2}$$

where X_k are some undetermined coefficients and

$$\phi^{(k)}(z) = z^{2k-2M-1}, \quad \psi^{(k)}(z) = -R^{4k-4M-2} z^{-2k+2M+1} - (2k-2M-1)R^2 z^{2k-2M-3}, \quad (\text{k=1,2,..2M}) \tag{3}$$

The mixed boundary conditions for a notched region can be expressed as (Fig. 1(a))

$$\sigma_{ij} n_j = \bar{p}_i \ (\text{on } C_p) \qquad u_i = \bar{u}_i \ (\text{on } C_u) \tag{4}$$

where C_p is the portion of boundary where the tractions "$\bar{p}_i$" are given, and C_u is the portion of boundary where the displacements "$\bar{u}_i$" are assumed.

It is proved that (Chen 1983, Chen and Lee 2002a, 2002b, Hu 1954, Washizu 1975), under the condition (4), the actual solution of the boundary value problem can be obtained from the stationary condition of a functional Π defined as

$$\Pi = \iint_{\Sigma} A(e_{ij})\mathrm{dF} - \int_{C_p} \bar{p}_i \bar{u}_i \mathrm{ds} - \int_{C_u} \sigma_{ij} n_j (u_i - \bar{u}_i)\mathrm{ds} \tag{5}$$

where $A(e_{ij})$ is the strain energy, and Σ is the region of integration.

In this case, one needs to cut a rectangular cell with hole from the strip (Fig. 1(c)). Clearly, the boundary condition for the cell can be written as

$$\sigma_{xy} = 0, \qquad v = \bar{v} = \pm v_b \qquad (-b \le x \le b, \ \ y = \pm h) \tag{6a}$$

$$\sigma_x = 0, \qquad \sigma_{xy} = 0 \qquad (x = \pm b, \ \ -h \le y \le h) \tag{6b}$$

In Eq. (6a) v_b is an undetermined value, which will be determined by

$$\int_0^b \sigma_y(x, h)\mathrm{dx} = bp \tag{7}$$

Clearly, the boundary value condition (6a) is the complex mixed one. It was proved that the EEVM could also be used to the case that the boundary condition is complex mixed one (Chen and Lee 2002a, 2002b).

After letting the functional Π take a stationary value, the undetermined coefficients x_k (k=1,2,3..2M) can be obtained and the stress field for the notched strip is finally obtained (Chen and Lee, 2002a, 2002b).

Clearly, from the deformation response of the strip weakened by holes, an orthotropic strip without holes can model the weakened strip. It is known that the constitutive equation in the orthotropic medium takes the form (Lekhnitsky 1963)

$$\varepsilon_x = \frac{1}{E_1}\sigma_x - \frac{v_{21}}{E_2}\sigma_y, \qquad \varepsilon_y = -\frac{v_{12}}{E_1}\sigma_x + \frac{1}{E_2}\sigma_y, \qquad \gamma_{xy} = \frac{1}{G_{12}}\sigma_{xy} \tag{8}$$

In Eq. (8) there is a relation as follows

$$(E_1 v_{21})/(E_2 v_{12}) = 1 \tag{9}$$

In the condition of using $2M = 16$ in Eq. (2), computation is performed. The calculated elastic constants are expressed as

$$E_1 = f_1(h/b, R/c) E_o \qquad \text{(where } c = \min(b,h)\text{)} \tag{10}$$

$$E_2 = f_2(h/b, R/c) E_o \qquad \text{(where } c = \min(b,h)\text{)} \tag{11}$$

$$v_{21}/E_2 = v_{12}/E_1 = f_3(h/b, R/c)(v_o/E_o) \qquad \text{(where } c = \min(b,h)\text{)} \tag{12}$$

where v_o, G_o, E_o denote the elastic constant for the strip. The calculated $f_1(h/b,R/c)$, $f_2(h/b,R/c)$ and

Table 1 Normalized elastic constant $f_1(h/b,R/c)$ for a strip weakened by periodic holes (for E_1, see Fig. 1(c) and Eq. (10))

R/c=	0.1	0.2	0.3	0.4	0.5	0.6
h/b=0.2	.9953	.9814	.9585	.9269	.8851	.8309
h/b=0.4	.9907	.9635	.9204	.8635	.7940	.7113
h/b=0.6	.9860	.9460	.8845	.8069	.7173	.6180
h/b=0.8	.9814	.9284	.8484	.7503	.6418	.5281
h/b=1.0	.9767	.9099	.8084	.6851	.5526	.4199
h/b=1.5	.9843	.9367	.8570	.7478	.6164	.4754
h/b=2.0	.9881	.9516	.8880	.7955	.6756	.5357

Table 2 Normalized elastic constant $f_2(h/b,R/c)$ for a strip weakened by periodic holes (for E_2, see Fig. 1(c) and Eq. (11))

R/c=	0.1	0.2	0.3	0.4	0.5	0.6
h/b=0.2	0.9954	0.9826	0.9642	0.9431	0.9212	0.8994
h/b=0.4	0.9908	0.9653	0.9288	0.8869	0.8435	0.8005
h/b=0.6	0.9862	0.9479	0.8931	0.8300	0.7646	0.6996
h/b=0.8	0.9815	0.9302	0.8561	0.7704	0.6814	0.5929
h/b=1.0	0.9768	0.9119	0.8169	0.7061	0.5912	0.4773
h/b=1.5	0.9844	0.9382	0.8641	0.7668	0.6532	0.5313
h/b=2.0	0.9882	0.9529	0.8943	0.8135	0.7130	0.5971

Table 3 Normalized elastic constant $f_3(h/b,R/c)$ for a strip weakened by periodic holes (for v_{21}/E_2, see Fig. 1(c) and Eq. (12))

R/c=	0.1	0.2	0.3	0.4	0.5	0.6
h/b=0.2	1.0051	1.0189	1.0377	1.0568	1.0735	1.0876
h/b=0.4	1.0102	1.0382	1.0771	1.1187	1.1563	1.1868
h/b=0.6	1.0154	1.0586	1.1213	1.1930	1.2647	1.3326
h/b=0.8	1.0208	1.0815	1.1774	1.3018	1.4507	1.6324
h/b=1.0	1.0265	1.1092	1.2575	1.4859	1.8234	2.3572
h/b=1.5	1.0180	1.0784	1.2023	1.4302	1.8331	2.5401
h/b=2.0	1.0135	1.0591	1.1537	1.3311	1.6529	2.2371

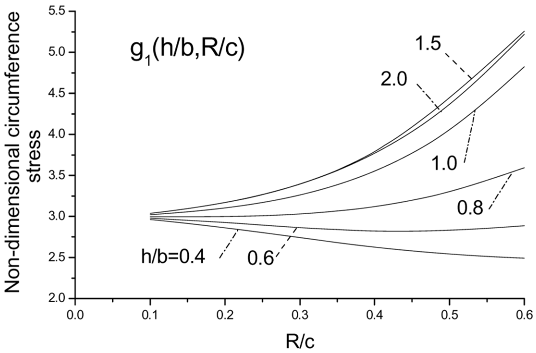

Fig. 2 Normalized circumference stress $g_1(h/b,R/c)$ $(=\sigma_{t,E}/p)$

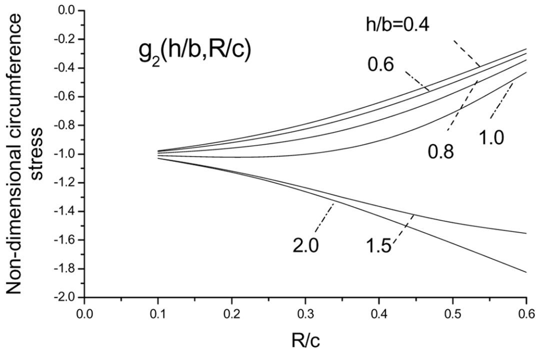

Fig. 3 Normalized circumference stress $g_2(h/b,R/c)$ $(=\sigma_{t,F}/p)$

$f_3(h/b,R/c)$ values are listed in Tables 1, 2 and 3, respectively. The relevant circumference stresses at the points "E" ($x = R$, $y = 0$) and "F" ($x = 0$, $y = R$), are expressed as

$$\sigma_{t,E} = g_1(h/b,R/c)p, \quad \text{(where } c = \min\,(b,h)) \tag{13}$$

$$\sigma_{t,F} = g_2(h/b,R/c)p, \quad \text{(where } c = \min\,(b,h)) \tag{14}$$

The calculated $g_1(h/b,R/c)$ and $g_2(h/b,R/c)$ values are plotted in Figs. 2 and 3, respectively.

References

Atsumi, A. (1956), "On the stresses in a strip under tension and containing two equal circular holes placed longitudinally", *ASME J. Appl. Mech.*, **23**, 555-562.

Chen, Y.Z. (1983), "An investigation of the stress intensity factor for a finite internally cracked plate by using variational method", *Eng. Fract. Mech.*, **17**, 387-394.

Chen, Y.Z. and Lee, K.Y. (2002a), "An infinite plate weakened by periodic cracks", *ASME J. Appl. Mech.*, **69**, 552-555.

Chen, Y.Z. and Lee, K.Y. (2002b), "Two-dimensional elastic analysis of doubly periodic circular holes in infinite plate", *KSME Int. J.*, **16**, 655-665.

Dejoie, A. Mogilevskaya, S.G. and Crouch, S.L. (2006), "A boundary integral method for multiple circular holes in an elastic half-plane", *Eng Anal. Bound. Elem.,* **30**, 450-464.

Dong, C.Y. (2006), "Effective elastic properties of doubly periodic array of inclusions of various shapes by the boundary element method", *Int. J. Solids Struct.*, **43**, 7919-7938.

Dong, C.Y. and Lee, K.Y. (2006), "Boundary element implementation of doubly inclusion problems", *Engng. Anal. Bound. Elem.*, **30**, 662-670.

Hu, H.C. (1954), "On some variational principles in the theory of elasticity and plasticit", *Scintia Sinica*, **4**,33-54.

Legros, B. Mogilevskaya, S.G. and Crouch, S.L. (2004), "A boundary integral method for multiple circular inclusions in an elastic half-plane", *Eng. Anal. Bound. Elem.*, **28**, 1083-1098.

Lekhnitsky, S.G. (1963), Theory of Elasticity of Anisotropic Elastic Body. San Francisco: Holden-Day.

Muskhelishvili, N.I. (1953), Some Basic Problems of the Mathematical Theory of Elasticity. Groningen: Noordhoff.

Savin, G.N. (1961), Stress Concentration Around Holes. New York: Pergamon.

Wang, J.L. Crouch, S.L. and Mogilevskaya, S.G. (2003), "A complex boundary integral method for multiple circular holes in an infinite plane", *Eng Anal. Bound. Elem.*, **27**, 789-802.

Wang, W.C. and Chen, J.T. (1989), "Stress analysis of finite interfacially bimaterial plates by using the variational method", *Comput. Meth. Appl. Mech. Eng.*, **73**, 153-171.

Washizu, K. (1975), Variational Methods in Elasticity and Plasticity. Oxford: Pergamon.