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Structural damage detection in continuum structures using successive zooming genetic algorithm

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Abstract. This study utilizes the fine-tuning and small-digit characteristics of the successive zooming genetic algorithm (SZGA) to propose a method of structural damage detection in a continuum structure, where the differences in the natural frequencies of a structure obtained by experiment and FEM are compared and minimized using an assumed location and extent of structural damage. The final methodology applied to the structural damage detection is a kind of pseudo-discrete-variable-algorithm that counts the soundness variables as one (perfectly sound) if they are above a certain standard, such as 0.99. This methodology is based on the fact that most well-designed structures exhibit failures at some critical point due to manufacturing error, while the remaining region is free of damage. Thus, damage of 1% (depending on the given standard) or less can be neglected, and the search concentrated on finding more serious failures. It is shown that the proposed method can find out the exact structural damage of the monitored structure and reduce the time and amount of computation.

Keywords: damage detection; structure; successive zooming genetic algorithm (SZGA); natural frequency; optimization.

1. Introduction

Most modern architectural structures are designed using sophisticated structural analysis packages

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(ABAQUS 2002, MARC 2005) and based on extensive empirical data and authentic codes.

FEM has already been successfully applied to many engineering fields, including stress analysis (NASTRAN 2003), thermal analysis, fluid flow, electro-magnetic fields (ANSYS 1989), plastic deformation, rubber analysis, and mechanical vibration (Bathe 1982). Thus, many structural analysis packages use FEM for processing solid elements and structural elements, truss elements, beam/frame elements, and plate/shell elements (Zienckiewicz and Taylor 2005). Plus, some analyses require solutions to locking problems, such as reduced/selected integration (Huang 1989), the addition of artificial modes of deformation (Choi *et al*. 1998), or adjusting the sampling points of a Gauss quadrature (Kwon *et al*. 1999).

Nonetheless, structures can sometimes experience failures far earlier than expected, due to fabrication errors, material imperfections, fatigue, or design mistakes, where fatigue failure is perhaps the most common of these. Therefore, to protect a structure from any catastrophic failure, regular inspections are conducted, including knocking, visual searches, and other nondestructive methods. However, these methods are all localized and depend strongly on the skill and experience of the inspector (Hajela and Soeiro1990, Kim *et al*. 1993, Oreta and Tanabe 1994, Zhao and DeWolf 1999, Oh and Jung 1998).

Consequently, smart and global ways of searching for damages in structures have recently been investigated using rational algorithms, powerful computers, and FEM (Fanning and Carden 2003, Yeo *et al*. 2000, El-Borgi *et al*. 2005, Cerria and Rutab 2004, Wang *et al*. 2001). The algorithms are variations of the Genetic Algorithm introduced in 1975 by Holland (Holland 1975), then further developed. For example, Krishnakumar (Krishnakumar 1989) proposed the Micro GA (MGA), which utilizes certain characteristics, while the current authors proposed the successive zooming genetic algorithm (SZGA) (Kwon *et al*. 2003), which successively zooms the search domain, maintains the convergence rate in the later stage of optimization, and predicts the upper bound of accuracy and reliability of the obtained optimum. Optimal parameters were also suggested by optimizing the SZGA itself to obtain the optimal zooming factor and population number in each generation (Kwon *et al*. 2006).

Monitoring possible damages in a structure involves finding their locations and intensities by minimizing the objective function of the difference between measured data and computed data as a function of the assumed structural damage (Chou 2001, Dieterle 2003, Juang 2004, Rao 2004, Koh 2007, Yu 2007). The measured data can be the displacement of certain points or the natural frequencies of the structure, while the computed data is obtained by FEM using the assumed structural damage, where the assumed damage represents the extent of damage based on a number between 0 and 1. For example, Chou *et al.* used static displacements at a few locations in a discrete structure composed of truss members, and adopted a kind of mixed string scheme as an implicit redundant representation. Meanwhile, Rao adopted a residual force method, where the fitness is the inverse of an objective function that is the vector sum of the residual forces, and Koh adopted a stacked mode shape correlation that can locate multiple damages without incorporating sensitivity information.

Yet, a typical structure can be sub-divided into many finite elements and has many d.o.f. Thus, FEM takes a long time for a static analysis, as well as a frequency analysis. For a GA, the analysis time is related to the number of functions used for evaluating fitness, which can become uncontrollable when monitoring a full structure, plus the RAM space required becomes too large and the access time too slow when handling so much data.

Accordingly, the proposed SZGA is more effective in this case, as it does not require many chromosomes, as few as 4, thereby solving the slow-down of the convergence rate with the conventional GA that makes it hard to pinpoint the extent of the damage. The final issue of many d.o.f. can also be solved by sub-dividing the monitoring problem into smaller sub-problems based on the fact that the number of damages will not likely be higher than 1~4, as long as the structure was designed properly. Advantage can also be taken of the fact that cracks usually initiate at the outer and tensile stressed locations of a structure. As a result, the number of sub-problems becomes manageable and the required time more reasonable.

Several tests were performed to determine the effectiveness of the SZGA for structure monitoring, where regional zooming was found to be ineffective. The procedure used to sub-divide the monitoring problem is also presented, along with a comparison of the amount of computation required for a full-scale monitoring analysis and sub-problem analysis as a function of the number of probable damage sites.

2. Fine-tuning and small-digit characteristics of SZGA

2.1 Fine-tuning characteristics of SZGA

The main idea of the successive zooming genetic algorithm (Kwon *et al.* 2006) is the smart reduction of the search space around the candidate optimum point. The operating procedure of the SZGA is as follows. First, the initial population is generated and the MGA applied. Thereafter, 100 generations are progressed, and the optimum point with the highest fitness identified. Second, the search domain is reduced to $(X_{kopt}-\alpha^k/2,\ X_{kopt}+\alpha^k/2)$, then the optimum procedure continues based on the reduced domain. This reduction of the search domain raises the resolution of the solution, and the procedure is repeated until the identified solution is satisfactory.

The SZGA can suggest the reliability of the obtained optimal solution, where the reliability (or possibility of the optimal solution) equation of the SZGA is expressed using three parameters (α, N_{SP}, N_{ZOOM}) and the dimension of the solution N_{VAR}.

$$R_{SZGA} = [1-(1-(\alpha/2)^{N_{VAR}} \times \beta_{AVG})^{N_{SP}}]^{N_{ZOOM}-1} \tag{1}$$

where, α : zooming factor
β_{AVG} : average improvement factor
N_{SP} : total number of individuals during the sub-iteration ($N_{SP} = N_{SUB} \times N_{POP}$)
N_{SUB} : number of sub-iteration
N_{POP} : number of population
N_{ZOOM} : number of zoom
N_{VAR} : dimension of the solution

There are three parameters that control the performance of the SZGA: the zooming factor (α), number of zooming operations (N_Z), and sub-iteration population number (N_{SP}). According to previous research (Kwon *et al.* 2006), the optimal parameters for the SZGA, such as the zooming factor, number of zooming operations, and sub-iteration population number, were found to vary according to the number of variables used in the optimization problem.

In the present study, three or six variables were used to solve the optimization problem for various cases of structural damage detection, 0.2 or 0.3 chosen for the zooming factor, several numbers of zooming operations used, and 100,000 or 150,000 used for the sub-iteration population number, respectively. The sub-iteration population number means the total population number in a sub-generation of one zooming. Thus, in the case of zooming, the function calculation was implemented

Table 1 Optimal parameters in SZGA according to the number of design variables

No. of variables	2	4	8	16
α	.02573	.1303	.4216	.5176
N_{ZOOM}	5	8	17	22
N_{SP}	1,000	2,000	9,510	1,479,230
No. of func. calculation	5,000	16,000	161,670	32,543,060

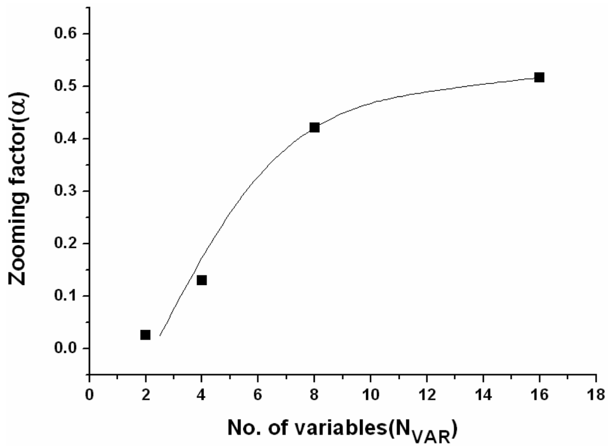

Fig. 1 Regression of data points, zooming factor with respect to the number of variables ($\alpha = a + bx + cx^2 + dx^3$ ($x = N_{VAR}$); a = −0.35769, b = 0.18152, c = −0.0016, d = 0.00033)

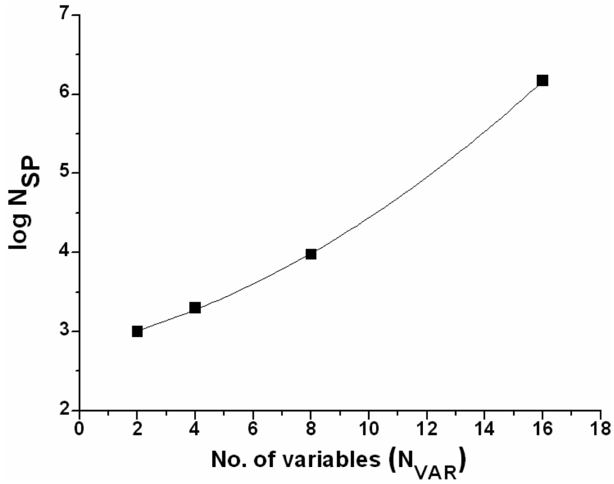

Fig. 2 Regression of data points, total number of individuals with respect to the number of variables in a sub-iteration ($\log N_{sp} = a + bx + cx^2$ ($x = N_{VAR}$); a = 2.814, b = 0.0845, c = 0.0078)

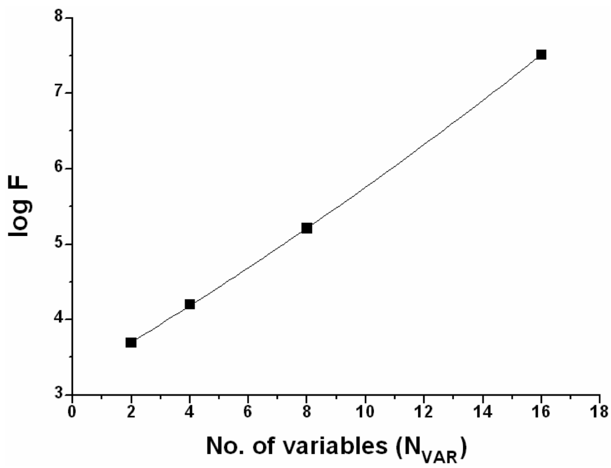

Fig. 3 Regression of data points, number of function calculations with respect to the number of variables ($\log F = a + bx + cx^2$ ($x = N_{VAR}$); a = 3.232, b = 0.2285, c = 0.0024)

100,000(1000 × 100) or 150,000(1500 × 100) times when 1,000 or 1,500 was the population number per generation and 100 was the number of generations per zooming, as the zooming was only implemented after progressing 100 generations. When the zooming factor, number of zooming operations and sub-iteration population number were 0.3, 15, and 100,000, respectively, the resolution of the final solution was $Z_{RANGE} = \alpha^{N_{ZOOM}-1} = 0.3^{15-1} = 4.78 \times 10^{-8}$. Table 1 shows the optimized values of SZGA parameters according to the number of design variables. Fig. 1, Fig. 2 and Fig. 3 are the fitting curves of 'N_{VAR}-α', 'N_{VAR}-N_{SP}' and 'N_{VAR}-Number of function calculation' relationship data, respectively.

2.2 Small-digit characteristics of SZGA (Kwon et al. 2007)

The SZGA has already been shown to be capable of pin pointing an optimal solution by searching a successively zoomed domain. Yet, in addition to its fine-tuning capability, the SZGA only requires several chromosomes for each zoomed domain, which is a very useful characteristic for structural damage detection in a large structure that has a great number of solution variables. In the present study, just four or eight digits of chromosomes were used.

3. Proposed method for structural damage detection in continuum structure using SZGA

A method of structural damage detection in a continuum structure is proposed that utilizes the fine-tuning and small-digit characteristics of the SZGA, thereby allowing the difference in the natural frequencies of a structure obtained by experiment and FEM to be compared and minimized using an assumed structural damage. This is accomplished by encoding the unmeasured damages, along with the material properties, in a GA and letting the correct values of the unmeasured natural frequencies evolve during the GA process.

The final methodology applied to the structural damage detection is a kind of pseudo-discrete-variable-algorithm (PDVA) that counts the soundness variables as one (perfectly sound) if they are above a certain standard, such as 0.99(Kwon *et al.* 2007). This methodology is based on the fact that most well-designed structures exhibit failures at certain critical points due to manufacturing errors, while the remaining regions are free from damage. Thus, if the damage is 1% (depending on the given standard) or less, this can be neglected, and the search effort concentrated on finding much larger damages.

3.1 Formulation of structural damage detection as an optimization problem

The object function used in the SZGA is Eq.(2), and the structural damage of each element is obtained by minimizing the object function, i.e. minimizing the difference (S) in the natural frequencies of a structure obtained by experiment and FEM using an assumed structural damage.

$$S = \sum_{k=1}^{n} [F_k - F_k^{'}] \tag{2}$$

where, F_k : experimental natural frequency

F_k' : natural frequency obtained by FEM with supposed structural damage

The zooming factor must be appropriately determined to identify the optimal string for the search. If the zooming factor is small, this decreases the reliability of the solution, whereas if the zooming factor is large, an accurate solution can only be obtained when increasing the incidence of zooming.

3.2 Proposed searching method for structural damage detection using combination theory

Most structures have few cracks, which may exist in different locations. Therefore, a combinational search method is suggested to search for separate cracks, and this can be performed by choosing probable damage site as nCk. n denotes number of total elements and k denotes the number of possible crack site (1~4). Thus, up to four cracks (k) were considered in a continuum structure modeled with *n*(=20) elements, followed by a comparison of the number of function calculations.

$$nCk = \frac{n!}{k!(n-k)!} \tag{3}$$

When monitoring the entire structure, the number of function calculations became six hundred million according to the relation of the number of variables and the number of function calculations. However, when the combinational searching method was used, the number of function calculations was reduced about 10^{-1}~10^{-4} times when compared to the full-scale monitoring case, as shown in Table 2.

Table 2 Result of combinational searching method to reduce amount of calculation in SZGA

No. of cracks	nCk	No. of function calculation		Ratio (Combi./Full)
		Combinational search	Full scale search	
1	20	0.580671×10^5	0.578096×10^9	0.100445×10^{-3}
2	190	0.950000×10^6	0.578096×10^9	0.164332×10^{-2}
3	1140	0.990843×10^7	0.578096×10^9	0.171398×10^{-1}
4	4845	0.740788×10^8	0.578096×10^9	0.128143

4. Structural damage detection in continuum structures using SZGA

The proposed methodology was tested using various cases of a continuum steel structure. The material properties for the steel structures are as follows: Young's modulus 207,000 MPa, Poisson's ratio of 0.3 and density 7.8×10^3 kg/m^3. The structures were modeled with two-dimensional finite elements with unit thickness. The damage was modeled as a reduction of stiffness in the damaged element, where the soundness factor was defined as the ratio of the damaged to the undamaged elements. Thus, the soundness factor of the undamaged section was defined as 1.0, and the soundness factor of the damaged element defined as 0.5. All the natural frequencies of the structure were measured (computed numerically using a finite element simulation).

4.1 Geometric zooming search method for structural damage detection

As a probable candidate for the searching method of structural damage, the geometric zooming approach method was tested. Three kinds of different mesh models for cantilever beam were tested.

First, the continuum structure was modeled with 28 nodes and 5 elements, as shown in Fig. 4. A specific load (10N) is applied upwardly at the tip. The experimental natural frequencies of the structure which were used in the optimization problem for structural damage detection are shown in Table 3. This rough mesh model identified the exact structural damage, as shown in Table 4, where the third element was assumed to be damaged (0.5) among the five elements under the upper nodal load. In Table 4, a comparison between the actual soundness factor and the result computed by the SZGA showed that the SZGA accurately identified the structural damage.

Second, the structure, damaged at element 25 in Fig. 6, was modeled with the 54-element model of locally-refined mesh and the 10-element model of rough mesh, as shown in Fig. 5. A specific

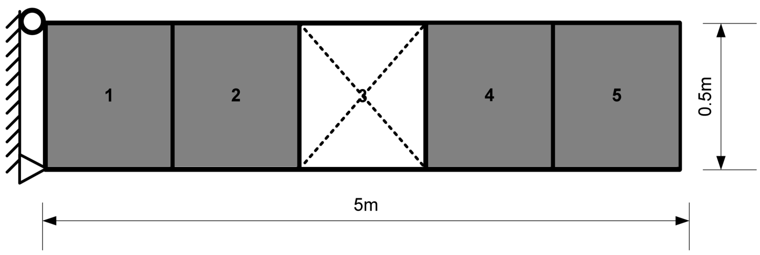

Fig. 4 Configuration of continuum structure for case I (5-element model)

Table 3 Experimental natural frequencies of the structure for case I

Mode	I	II	III	IV	V
Experimental natural frequency (F_k)	1583.5815	8913.15	23679.5	26958.4	51572.2

Table 4 Result of structural damage detection for case I

No. of element	1	2	3	4	5
Actual soundness factor	1	1	0.5	1	1
Damage detection result	1.0	1.0	0.500000	1.0	1.0

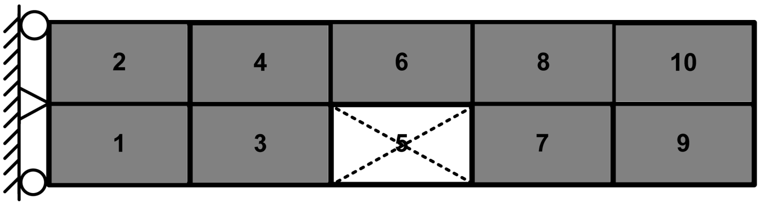

Fig. 5 Configuration of continuum structure for case II (10-element model)

Table 5 Experimental natural frequencies of the structure for case II

Mode	I	II	III	IV	V
Experimental natural requency (F_k)	1656.18	10133.6	25961.8	27818.6	53521.5

Table 6 Result of structural damage detection for case II

No. of element	1	2	3	4	5	6	7	8	9	10
Actual soundness factor	1	1	1	1	0~1	1	1	1	1	1
Damage detection result	1.0	1.0	1.0	1.0	0.878682	1.0	1.0	1.0	0.379975	0.404101

load (10 N) is applied upwardly at the tip of the each model. The experimental natural frequencies of the structure which were used in the optimization problem for structural damage detection are shown in Table 5. But, this kind of mesh model did not identify the exact structural damage, as shown in Table 6, since the sixth element was assumed to be damaged, but too big compared with the actual size of damage. Therefore, this kind of model was unable to simulate the actual continuum structure even roughly.

Thirdly, the structure was modeled with 193 nodes and 54 elements, as shown in Fig. 6. The experimental natural frequencies of the structure which were used in the optimization problem for structural damage detection are shown in Table 7. This locally-refined mesh model did identify the exact structural damage, as shown in Table 8. However, it could not specify the region to be zoomed (element 5 in Fig. 5). This kind of analysis ('54-element model' zoomed from '10-element

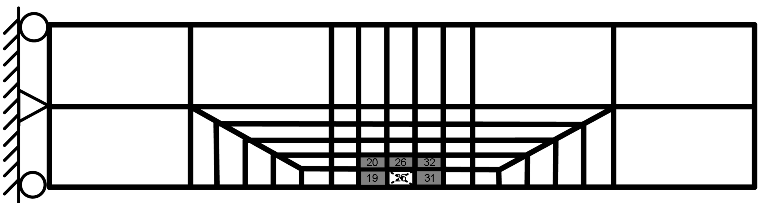

Fig. 6 Configuration of continuum structure for case III (54-element model)

Table 7 Experimental natural frequencies of the structure for case III

Mode	I	II	III	IV	V
Experimental natural frequency (F_k)	1655.4	9912.13	25899.8	27102.9	49917.4

Table 8 Result of structural damage detection for case III

No. of element	19	20	25	26	31	32
Actual soundness factor	1	1	0.5	1	1	1
Damage detection result	1.0	1.0	0.499999	1.0	1.0	1.0

model') is impossible.

Therefore, the results revealed that the geometric zooming approach method resulted in an unreliable conclusion, which was quite different to the actual state.

4.2 Combinational searching method for structural damage detection – Cantilever beam

When using the combinational searching method for structural damage detection, the cantilever continuum structure was modeled with 388 nodes and 120 elements, as shown in Fig. 7 and Fig. 8. The experimental natural frequencies of the structure which were used in the optimization problem for structural damage detection are shown in Table 9. The bottom three elements combination were selected out of twenty bottom elements on the assumption that a specific load (10 N) is applied upwardly at the tip. The exact structural damage was identified, as shown in Table 10.

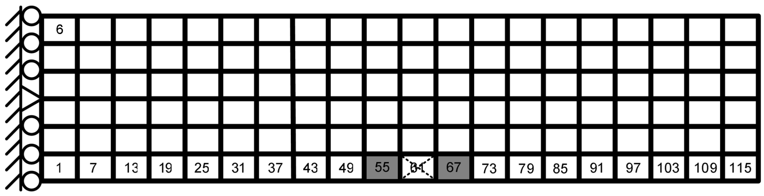

Fig. 7 Configuration of continuum structure for case IV

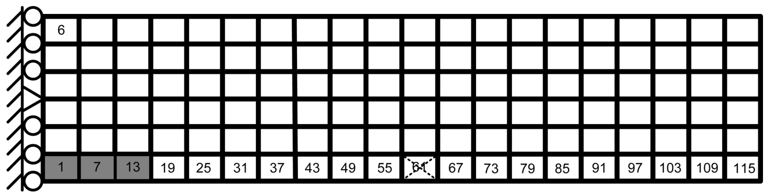

Fig. 8 Configuration of continuum structure for case IV

Table 9 Experimental natural frequencies of the structure for case IV

Mode	I	II	III	IV	V
Experimental natural frequency (F_k)	1653.86	9791.34	25807.1	25996.5	46739.1

Table 10 Result of structural damage detection for case IV

No. of element	55	61	67
Actual soundness factor	1	0.5	1
Damage detection result	1.0	0.499999	1.0
Fitness		1.000000	

Table 11 Result of structural damage detection for case IV

No. of element	1	7	13
Actual soundness factor	1	1	1
Damage detection result	0.873297	1.0	1.0
Fitness		0.277998	

Wrongly selected combination of probable damage was also evaluated and rejected by checking the fitness of the optimum solution, as shown in Table 11.

Thus, the combinational searching method can identify the exact structural damage and it can also reduce the amount of calculation in the case of multiple cracks as shown in section 3.1.

4.3 Combinational searching method for structural damage detection – Simple support beam

When using the combinational searching method for structural damage detection, the simple support continuum structure was modeled with 388 nodes and 120 elements, as shown in Fig. 9. A specific load (10 N) is applied downwardly at the center of the model. The soundness factor of the undamaged section was defined as 1.0, and the soundness factor of the damaged element defined as 0.3. The dimension of the structure was the same as the cantilever. The experimental natural frequencies of the structure which were used in the optimization problem for structural damage detection are shown in Table 12. The bottom three elements combination were selected out of twenty bottom elements on the assumption that the load is applied upwardly at the center. The exact structural damage was identified, as shown in Table 13.

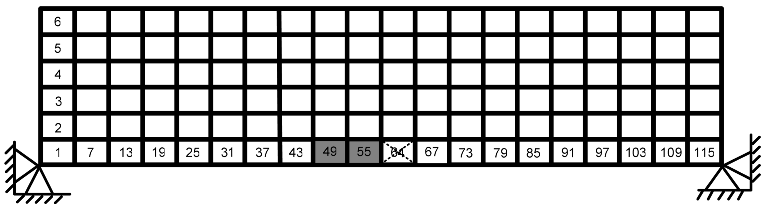

Fig. 9 Configuration of continuum structure for case V

Table 12 Experimental natural frequencies of the structure for case V

Mode	I	II	III	IV	V
Experimental natural frequency (F_k)	6682.16	15204.6	31284.1	36398.0	60854.6

Table 13 Result of structural damage detection for case V

No. of element	49	55	61
Actual soundness factor	1	1	0.3
Damage detection result	1.0	1.0	0.3
Fitness		1.000000	

5. Conclusions

This study used the SZGA to monitor structures sub-divided into many continuum elements. Hypothetical experiments were performed by analyzing the natural frequencies of damaged structures, then the difference between the hypothetical experimental data and data obtained by FEM was minimized assuming a certain structural damage. By searching for the best fitness, that is, the inverse of the above difference, the SZGA was able to identify the exact structural damage. As a solution vector, only a small subdivision of the d.o.f. was used, rather than the whole structure, thereby significantly reducing the d.o.f. and amount of computation, where each function requires a cycle of FEM analysis. Therefore, the proposed approach and verification examples can be summarized as follows:

1. The SZGA provided the exact structural damage of the monitored structure, where the frequencies of the test example were analyzed using continuum finite elements.
2. The geometric zooming approach resulted in an unreliable conclusion, which differed significantly from the actual state.
3. Sub-dividing the d.o.f. of the structural damage variables significantly reduced the time and amount of computation compared with the full-scale approach.
4. The cutting-up and small-chromosome characteristics of the SZGA as regards adopting the displacements of discrete structures worked effectively in this study when using the frequencies of continuum structures, especially when the d.o.f. was large.
5. Accuracy and reliability can be expected of the obtained optimal solution of the structural damage of the monitored structure, as in common optimization.

Acknowledgements

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