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Dynamic characteristics of cable vibrations in a steel cable-stayed bridge using nonlinear enhanced MECS approach

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Abstract. This paper focuses on the nonlinear vibrations of stay cables and evaluates the dynamic characteristics of stay cables by using the nonlinear enhanced MECS approach and the approximate approach. The nonlinear enhanced MECS approach is that both the girder-tower vibrations and the cable vibrations including parametric cable vibrations are simultaneously considered in the numerical analysis of cable-stayed bridges. Cable finite element method is used to simulate the responses including the parametric vibrations of stay cables. The approximate approach is based on the assumption that cable vibrations have a small effect on girder-tower vibrations, and analyzes the local cable vibrations after obtaining the girder-tower responses. Under the periodic excitations or the moderate ground motion, the differences of the responses of stay cables between these two approaches are evaluated in detail. The effect of cable vibrations on the girder and towers are also discussed. As a result, the dynamic characteristics of the parametric vibrations in stay cables can be evaluated by using the approximate approach or the nonlinear enhanced MECS approach. Since the different axial force fluctuant of stay cables in both ends of one girder causes the difference response values between two approach, it had better use the nonlinear enhanced MECS approach to perform the dynamic analyses of cable-stayed bridges.

Keywords: stay cable; parametric vibrations; cable-stayed bridge; nonlinear enhanced MECS approach; approximate approach; finite element method.

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1. Introduction

Recent problems provide evidence that the stay cables of cable-stayed bridges undergo large amplitude vibrations (Abdel-Ghaffar *et al.* 1991). In the forced excitation experiment of Tatara Bridge, which has the longest center span of 896m in the world, the large-amplitude vibrations in some stay cables were found (Fujino *et al.* 1997). When the bridge was excited at the frequency of the first torsional antisymmetric mode, some stay cables having twice of the frequency of the excitation vibrated greatly after about fifteen minutes while the responses of the girder were small. This phenomenon has been observed in Hitsuishijima and Yohkura bridges in Japan (Fujino *et al.* 1997). Some researchers described the local vibrations of stay cables in a few cable-stayed bridges in Europe and USA (Royer-Carfagni 2003, Smith *et al.* 2001, Campbell *et al.* 2003, Caetano *et al.* 2008).

Although the mechanism of these cable vibrations is not yet fully explained, there are two different explanations for the causes of the cable vibrations (Royer-Carfagni 2003). One is that vibrations are due to external environmental actions acting directly on the stay cables such as wind and rain, which leads to aerodynamic instability of stay cables (Hikami *et al.* 1988). The second explanation is that an interaction between the vibrations of the stays and those of the girder and towers, which leads to dynamic instability of stay cables (i.e., parametric vibrations). That is to say, when the natural frequency of the global modes in a cable-stayed bridge is close to one or two times that of the stay cable, the large-amplitude vibration of the stay cable may be induced (Kovacs *et al.* 1982). Parametric vibrations in cables have been analyzed and verified using analytical models based on single cable (Takahashi 1991, Fujino *et al.* 1993, Lilien *et al.* 1994, Pinto Da Costa *et al.* 1996). Fujino *et al.* treated with the linear and nonlinear internal resonance in a stay cable of cable-stayed bridges and used a physical 3-DOF model of a cable-stayed, cantilevered beam to study the influence of parametric vibrations (Fujino *et al.* 1993, Warnitchai *et al.* 1995). This cable-stayed beam model is from the simplification of cable-stayed bridges, and can consider the nonlinear cable-deck interaction. In their experiment and analysis, the large cable transverse oscillations due to a two-to-one parametric resonance, in which the low-frequency local mode is quadratically coupled with the high-frequency global mode externally excited at its primary resonance, have been observed. The researches of the parametric vibration have been stated in detail in our reference (Wu *et al*. 2006). Recently, Gattulli *et al.* discussed the 1:2 nonlinear vibrations using the cable-deck model (Gattulli and Lepidi 2003, Gattulli *et al*. 2005). This cable vibration is induced by a global beam motion oscillating at half of the first cable frequency, and it is called as the second super-harmonic resonance in this paper (Nayfeh *et al*. 1979, Nayfeh and Mook 1970, Haddow *et al*. 1984, Nayfeh and Balachandran 1990). Since multi-cable systems are widely used in cable-stayed bridges, the natural frequencies of the global modes easily approach the natural frequencies of the stay cables (parametric in second unstable region), or the doubled natural frequencies of the stay cables (parametric in principle unstable region), or half of the natural frequencies of the stay cables (second super-harmonic resonance) (Takahashi 1991, Wu *et al*. 2003). This tends to produce large-amplitude vibrations in the cables, which is a result of the non-linearity of the cables.

The interaction between cable vibrations and girder-tower vibrations in real cable-stayed bridges has been investigated by many researches. Abdel-Ghaffar and Khalifa modeled the stay cables using multiple elements and discussed the effect of stay cable vibrations on the overall natural characteristics of cable-stayed bridges (Abdel-Ghaffar *et al.* 1991). Tuladhar *et al.* and Caetano *et* *al.* discussed the influence of cable vibrations on the whole cable-stayed bridge in terms of the responses to seismic excitations using the numerical method and the physical model (Tuladhar *et al.* 1995, Caetano *et al.* 2000a, Caetano *et al.* 2000b). As a result, they thought that the MECS approach, which simultaneously takes into account the vibrations of the girder, towers and cables, should be used in the numerical analysis of cable-stayed bridges. Although the importance of considering the interaction between cable vibrations and girder-tower vibrations in cable-stayed bridges was emphasized, the parametric vibrations of stay cables couldn't be included because the truss elements were used to model the stay cables in the previous researches (Wu *et al.* 2006).

In this paper, the authors propose using the cable finite element to consider the parametric vibrations of stay cables (Wu *et al.* 2006). In the finite element model of a cable-stayed bridge, each stay cable is represented by some elements using cable finite element so that the parametric vibrations of stay cables can be included in the cable vibrations. The overall responses of the girder, towers and stay cables can be simultaneously obtained by using the "nonlinear enhanced MECS approach".

The dynamic characteristics of cable parametric vibrations in a steel cable-stayed bridge are discussed by using this proposed MECS approach. Because the parametric vibrations of stay cables may occur under the periodic excitations (Wu *et al.* 2003), the dynamic analysis is firstly performed when the cable-stayed bridge is subjected to the periodic sinusoidal excitations. The moderate ground motion is used as the excitation and the seismic characteristics of the cable-stayed bridge are also discussed. Comparing the responses obtained by the "nonlinear enhanced MECS approach" with those obtained by the "approximate approach", the effect of cable vibrations including parametric vibrations on the overall dynamic characteristics of the steel cable-stayed bridge is discussed in detail. Furthermore, the possibility of using the chord element to simulate parametric of stay cable is stated.

2. Modeling of a cable-stayed bridge

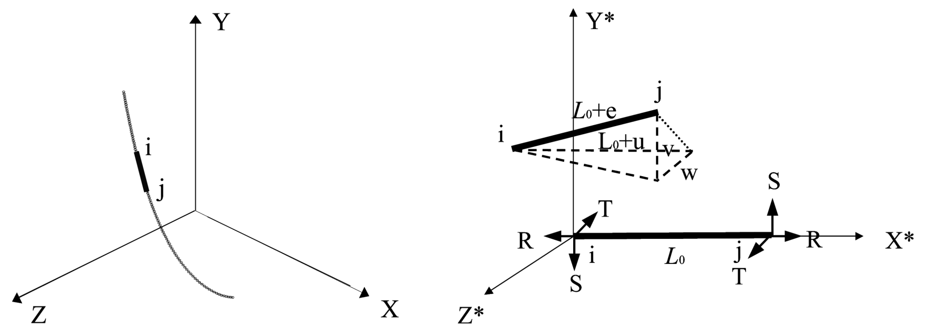
2.1 Nonlinear enhanced MECS approach

The nonlinear enhanced MECS approach attempts to consider the overall vibrations of the girder, towers and stay cables, so each stay cable is discretized into a total number of some elements in the overall finite element bridge model, which is called as the MECS (Multi-Element-Cable-System) model proposed by Abdel-Ghaffar and Khalifa (Abdel-Ghaffar *et al.* 1991). The overall vibrations of the girder, towers and stay cables can be simultaneously taken into account by using the nonlinear enhanced MECS approach.

In order to consider the parametric vibrations of stay cables, the cable finite element firstly presented by Broughton and Ndumbaro is used for considering the non-linearity and parametric vibrations of the sagged cable (Broughton *et al.* 1994).

The authors proposed the method of removing the self-weight term of the cable element in the dynamic analysis, and the precision of this method is verified (Wu *et al.* 2006). The detail of using this cable finite element to analyze parametric vibrations of stay cables in cable-stayed bridges is discussed in this paper, and the simple explanation is shown here.

In the local coordinate system of the cable element, shown in Fig. 1, the original length of the element is L_0, the initial basic force is P_0, and the displacements of three directions (x^*, y^*, z^*) are



(a) Global coordinate system (b) Local coordinate system

Fig. 1 Cable finite elements

(u_i, v_i, w_i) in the node-i and (u_j, v_j, w_j) in the node-j. The restoring force of cable element removing the self-weight term in the local coordinate system is shown as follows

$$R^e = \left[-P\frac{L_0+u}{L_0+e}+P_0 \quad -P\frac{v}{L_0+e} \quad -P\frac{w}{L_0+e} \quad P\frac{L_0+u}{L_0+e}-P_0 \quad P\frac{v}{L_0+e} \quad P\frac{w}{L_0+e} \right]^T \tag{1}$$

where $P = P_0 + E_cA_c/L_0 \times e$ is the updated element basic tension, $e = \sqrt{(L_0+u)^2+v^2+w^2} - L_0$ is the element extension, $u = u_j - u_i$, $v = v_j - v_i$ and $w = w_j - w_i$ are the relative displacements from (u_i, v_i, w_i) in node-i and (u_j, v_j, w_j) in node-j.

The incremental stiffness matrix of cable element in the local coordinate system is

$$[K]^e_{cable} = \frac{E_cA_c}{L_0(L_0+e)^2}\begin{bmatrix} (L_0+u)^2 & (L_0+u)v & (L_0+u)w & -(L_0+u)^2 & -(L_0+u)v & -(L_0+u)w \\ & v^2 & vw & -(L_0+u)v & -v^2 & -vw \\ & & w^2 & -(L_0+u)w & -vw & -w^2 \\ & & & (L_0+u)^2 & (L_0+u)v & (L_0+u)w \\ & sym. & & & v^2 & vw \\ & & & & & w^2 \end{bmatrix}$$

$$+\frac{P}{(L_0+e)^3}\begin{bmatrix} v^2+w^2 & -(L_0+u)v & -(L_0+u)w & -v^2-w^2 & (L_0+u)v & (L_0+u)w \\ & (L_0+u)^2+w^2 & -vw & (L_0+u)v & -(L_0+u)^2-w^2 & vw \\ & & (L_0+u)^2+v^2 & (L_0+u)w & vw & -(L_0+u)^2-v^2 \\ & & & v^2+w^2 & -(L_0+u)v & -(L_0+u)w \\ & sym. & & & (L_0+u)^2+w^2 & -vw \\ & & & & & (L_0+u)^2+v^2 \end{bmatrix} \tag{2}$$

where, E_c and A_c are the Young's modulus and cross-sectional area of every cable element.

Since the initial shapes of cables with sag are taken into account, there is no need to consider the

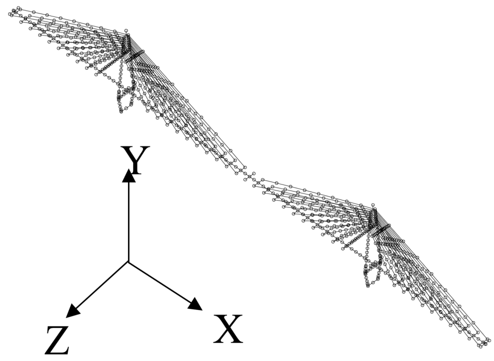


Fig. 2 Nonlinear enhanced MECS approach for cable-stayed bridge

equivalent modulus that allows for sagging (Walther *et al.* 1999).

The girder is treated as a single central spine with offset links to the cable anchor points. Three-dimensional beam elements are used for modeling the girder and towers. Fig. 2 shows the nonlinear enhanced MECS approach for cable-stayed bridge.

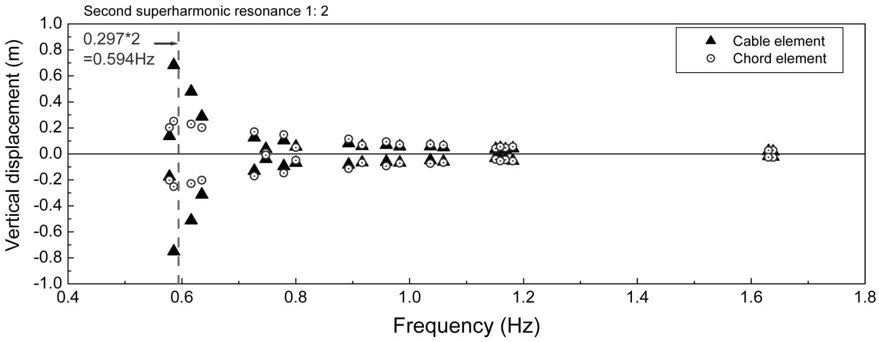
Abdel-Ghaffar *et al.*, Tuladhar *et al.*, Caetano *et al.* used the MECS approach to simulate the overall vibrations of cable-stayed bridge within the context of a linear spectral analysis (Abdel-Ghaffar *et al.* 1991, Tuladhar *et al.* 1995, Caetano *et al.* 2000a, Caetano *et al.* 2000b). Since the finite element bridge model is used for a linear study, the MECS term is used to signify only that each cable is meshed in several elements, whose nodal displacements describe the cable transversal response.

The difference between those references and this paper is the element type for stay cables. This paper uses the cable finite element and those references used the chord/truss element. The stiffness matrix of the chord element considering the geometry non-linearity is shown as follows (Wu *et al.* 2006)

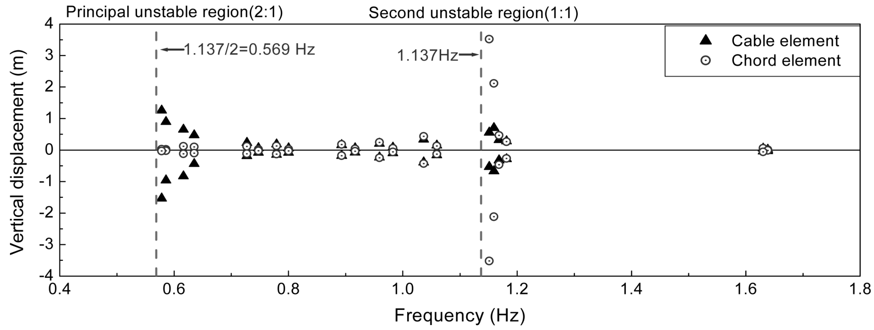
$$[K]^e_{chord} = \frac{E_c A_c}{L_0}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ & sym. & & & 0 & 0 \\ & & & & & 0 \end{bmatrix} + \frac{P}{L_0}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & -1 & 0 \\ & & 1 & 0 & 0 & -1 \\ & & & 0 & 0 & 0 \\ & sym. & & & 1 & 0 \\ & & & & & 1 \end{bmatrix} \tag{3}$$

The stiffness matrix in Eq. (1) of the chord element doesn't change when the displacements in three directions u, v and w change, so the chord element cannot be used to evaluate the nonlinear parametric vibrations of cables. It has been confirmed that the chord element cannot simulate the parametric vibrations of single cable in our reference (Wu *et al.* 2006).

Moreover, Fig. 3 shows the maximum responses of stay cables by the nonlinear enhanced MECS approach using the chord elements in a cable-stayed bridge, which will be stated in the next chapter. The corresponding values using the cable elements are also shown in this figure. The ordinate is the frequencies of stay cables, and the abscissa is the maximum vertical displacements of stay cables. The maximum displacements of the stay cables using the chord elements are very small than those using cable element, and the difference between chord element and cable element is very clear.



(a) Under vertical excitation



(b) Under torsional excitation

Fig. 3 Maximum responses of stay cables under vertical excitation (cable element vs. truss element)

Therefore, the nonlinear parametric vibration of cables cannot be evaluated by using the chord element, and it had better use the cable finite elements for stay cables in the nonlinear enhanced MECS approach in order to correctly evaluate the dynamic characteristics of cable vibrations including parametric vibrations.

The MECS approach can evaluate the overall vibrations of the girder, towers and stay cables. The non-linear vibrations of stay cables including parametric vibration can be evaluated accurately using the cable element presented. In this paper, the cable finite element presented has been implemented by the authors in the own-made software, NL_Beam3D (Wu *et al.* 2007). On this respect, because the proposed approach can be considered an enhancement of the MECS method, which utilizes linearized chord/truss element, the method name is not "MECS", but "Nonlinear enhanced MECS approach".

2.2 Approximate approach

Because the vibrations of a stay cable refer to the motions of a cable due to the excitations at the supports and the external excitation forces, the cable vibrations is a combined parametric and forced vibrations, and not simple forced vibration. Cable vibrations including parametric vibrations cannot be calculated by using some commercial finite codes for structural analysis, such as TDAP in Japan (ARK 2003). This disadvantage leads to the application of the approximate approach, which analyzes the cable vibrations including parametric vibrations by using the single cable model after obtaining the girder-tower responses.

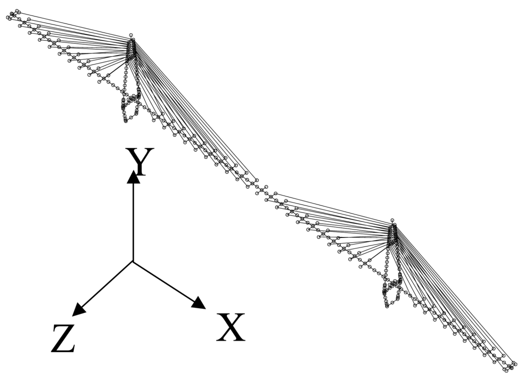
Assumed that the cable vibrations have a small effect on the girder-tower vibrations, the approximate approach includes two steps for obtaining the responses of stay cables:

Step-1 performs the dynamic analysis using the OECS (One-Element-Cable-System) cable-stayed bridge model [1]. The modeling of the girder and towers in the OECS model are same as those in the nonlinear enhanced MECS approach. The difference is that every cable is modeled by single cable element, so the equivalent modulus proposed by Ernst should be used for considering the non-linear behavior of stay cables due to their sags. Fig. 4(a) shows the OECS model. The responses of the girder and towers are obtained when the bridge is subjected to external excitation.

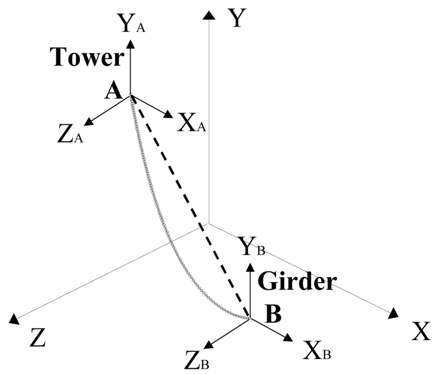
Step-2 performs the dynamic analysis of stay cables using the single cable model shown in Fig. 4(b). The cable has time-varying displacements (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B) at both ends. It is assumed that there is no restraint against rotation at the anchorage that is independent of the amplitude of the cable vibration. The excitations (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B) are the displacements of the girder and tower anchored to the stay cable, which is obtained from Step-1.

The authors have used this approximate approach to discuss the dynamic characteristics of parametric vibrations in stay cables (Wu *et al.* 2003). Compared with the in-plane cable model using analytical method in this paper, the renewal three-dimensional model of a single cable using cable finite elements is presented in our new reference (Wu *et al.* 2006). The in-plane (longitudinal and transverse) and out-of-plane responses of cables can be considered, and the loading on stay cables such as earthquake can be taken into account in this renewal cable model.

Some researches compared the axial force of single cable element in OECS model with those in MECS model, in which the displacements of stay cables in the OECS model were not obtained precisely and the non-linear local vibration of stay cables in OECS model cannot be considered (Caetano *et al.* 2000b, Caetano *et al.* 2008, Gattulli and Lepidi 2007). That is to be said that, the OECS model is to the global girder-tower vibration, and don't include the non-linear local vibration of stay cables. Therefore, the approach shown in Fig. 4 is approximation for stating the overall of the girder, towers and stay cables, so the method name is not "OECS", but "Approximate approach", to emphasize the non-linear vibrations of stay cables including parametric vibration.



(a) OECS model



(b) Single cable model

Fig. 4 Approximate approach for cable-stayed bridge

3. Analytical model and dynamic analysis procedure

The object of this paper is a steel cable-stayed bridge with three spans that was constructed in Japan. As shown in Fig. 5, the center span is 350 m and two side spans are 160 m each. The steel towers are A-shaped, and the 10×4 cables are arranged in a two-plane, multi-cable system. The cables are numbered sequentially from the side span to the center span.

The three-dimensional finite element models of this bridge are shown in Figs. 2 and 4.

In the nonlinear enhanced MECS approach and the single cable model in approximate approach, each stay cable is divided into eight cable elements. Table 1 shows the natural frequencies of stay cables using finite element method and using numeral method, including the first out-of-plane and in-plane frequencies. The difference between two methods is no more than 1.0%, then the precision of cable finite element is verified.

The following boundary conditions of this cable-stayed bridge are assumed: (i) the longitudinal displacement at the supports of the girder is free, and the vertical and out-of-plane displacements at the supports are restrained; (ii) all displacements at the end of piers are fixed. Two horizontal and vertical linear springs are provided at the connections of the girder and each tower, which follow Abdel-Ghaffar and Khalifa 1991, then lead to small difference with our previous reference (Wu *et al*. 2003).

A subspace iteration algorithm is used to extract the first 400 modes of the MECS model. Fig. 6 shows the natural frequencies of the first 400 modes using nonlinear enhanced MECS approach. These 400 modes range from about 0.0 Hz to 2.5 Hz. As with the common natural vibration analysis of cable-stayed bridges, the modes of the girder/tower vibrations are very important for understanding the properties of the bridge. If single elements are used to model the cables of the cable-stayed bridge, the modes of the girder-tower vibrations can be easily obtained. An eigenvalue analysis using appreciate approach is performed to determine the number of natural modes of the girder/tower vibrations. 23 natural modes in the range of 0.0-2.5 Hz are obtained.

The eigenvalue analysis process using dynamic condensation method proposed in our reference (Wu *et al*. 2006) is used for sorting out the natural modes of the girder and tower vibrations in the nonlinear enhanced MECS approach. Table 2 shows the some modes of the global girder-tower vibration. As shown in Fig. 7, the maximum differences in the first in-plane, out-of-plane and torsional modes between two approach are no more than 4%.

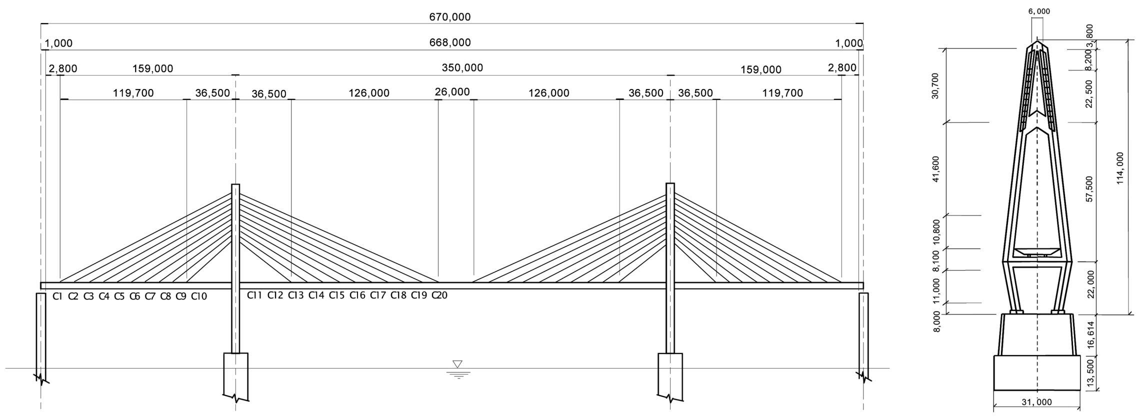
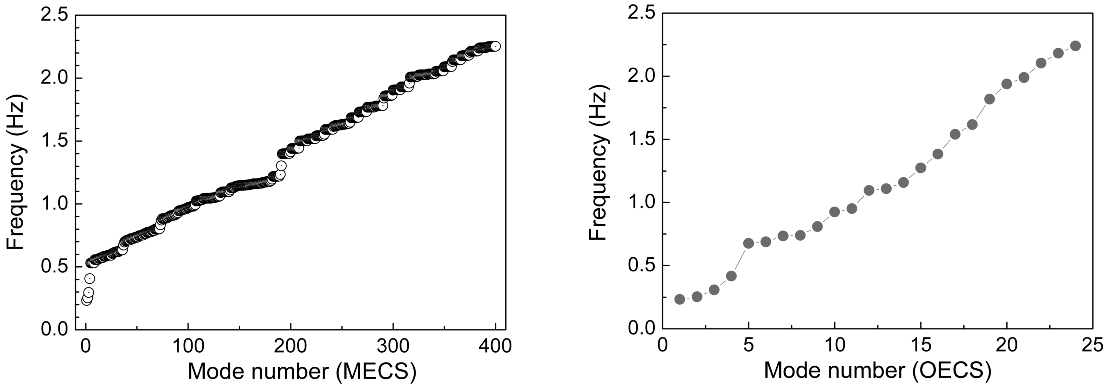


Fig. 5 General view of a steel cable-stayed bridge (unit: mm)

Table 1 Natural frequencies of stay cables fixed at both supports

Mode	Cable no.	(1) Finite element method				(2) Analytic value	(1)/(2)-1
		Frequency (Hz)	Participation factor			Frequency (Hz)	(%)
			X dir.	Y dir.	Z dir.		
First in-plane mode	C1	0.586	−0.42	0.85	0.01	0.585	0.15%
	C2	0.578	−0.42	0.81	0.01	0.573	0.86%
	C3	0.747	−0.31	0.57	0.01	0.749	−0.26%
	C4	0.800	−0.32	0.54	0.01	0.802	−0.23%
	C5	0.917	−0.29	0.46	0.01	0.920	−0.35%
	C6	0.983	−0.30	0.42	0.01	0.986	−0.30%
	C7	1.060	−0.30	0.38	0.01	1.062	−0.23%
	C8	1.181	−0.31	0.34	0.01	1.184	−0.22%
	C9	1.168	0.32	−0.28	−0.01	1.164	0.36%
	C10	1.638	0.35	−0.24	−0.01	1.644	−0.36%
	C11	1.630	0.35	0.24	0.01	1.636	−0.35%
	C12	1.151	0.32	0.29	0.01	1.146	0.43%
	C13	1.159	0.31	0.34	0.01	1.162	−0.20%
	C14	1.037	0.30	0.39	0.01	1.039	−0.22%
	C15	0.959	−0.29	−0.43	−0.01	0.962	−0.29%
	C16	0.893	0.29	0.47	0.01	0.896	−0.35%
	C17	0.779	0.31	0.55	0.01	0.781	−0.24%
	C18	0.727	0.31	0.58	0.01	0.729	−0.27%
	C19	0.616	0.36	0.72	0.01	0.616	0.06%
	C20	0.635	0.35	0.76	0.01	0.637	−0.29%
First out-of-plane mode	C1	0.559	0.02	0.00	0.95	0.563	−0.65%
	C2	0.533	0.02	0.00	0.91	0.536	−0.65%
	C3	0.734	0.01	0.00	0.65	0.739	−0.64%
	C4	0.787	0.01	0.00	0.62	0.792	−0.64%
	C5	0.907	0.01	0.00	0.54	0.913	−0.64%
	C6	0.972	0.01	0.00	0.52	0.979	−0.64%
	C7	1.048	0.01	0.00	0.49	1.055	−0.64%
	C8	1.170	0.01	0.00	0.46	1.178	−0.64%
	C9	1.147	0.01	0.00	0.43	1.155	−0.64%
	C10	1.632	0.01	0.00	0.43	1.642	−0.64%
	C11	1.623	−0.01	0.00	0.43	1.634	−0.64%
	C12	1.129	−0.01	0.00	0.43	1.136	−0.64%
	C13	1.148	−0.01	0.00	0.46	1.155	−0.64%
	C14	1.024	−0.01	0.00	0.49	1.031	−0.64%
	C15	0.948	−0.01	0.00	0.52	0.954	−0.64%
	C16	0.883	−0.01	0.00	0.55	0.889	−0.64%
	C17	0.765	−0.01	0.00	0.63	0.770	−0.64%
	C18	0.714	−0.01	0.00	0.66	0.718	−0.64%
	C19	0.592	−0.02	0.00	0.81	0.596	−0.65%
	C20	0.621	−0.02	0.00	0.84	0.625	−0.65%



(a) Nonlinear enhanced MECS approach (b) Girder-tower modes

Fig. 6 Natural frequencies obtained by whole-bridge and approximate approach

Table 2 Natural girder-tower vibrations

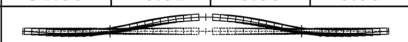
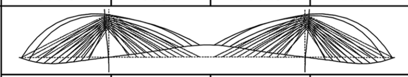
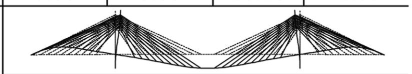
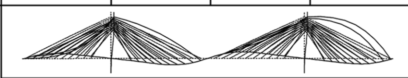
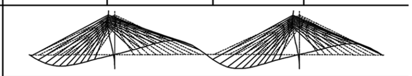
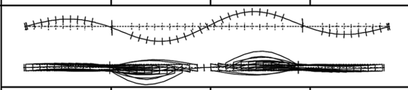
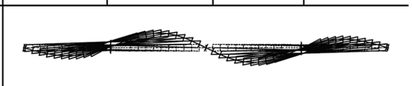
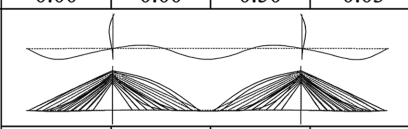
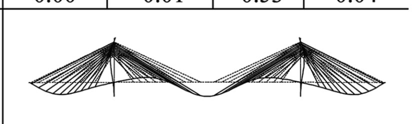
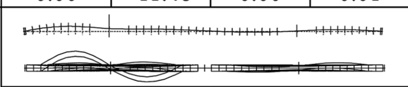
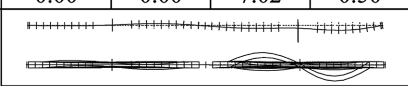
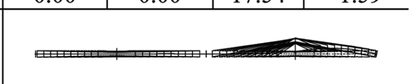
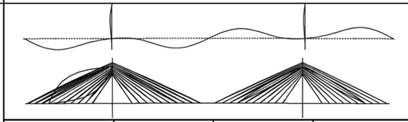
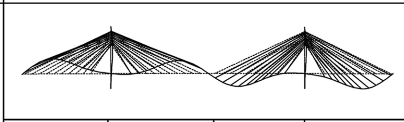
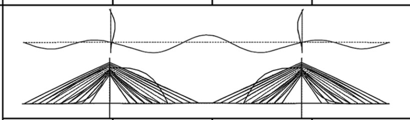
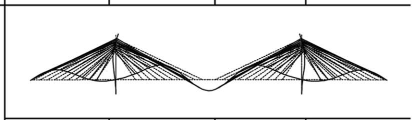
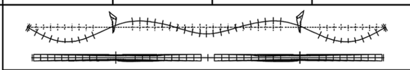
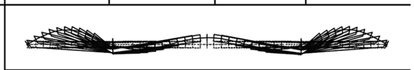
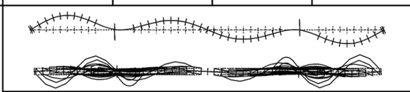
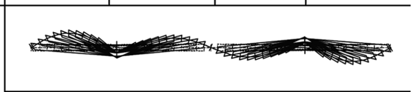
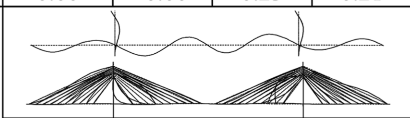
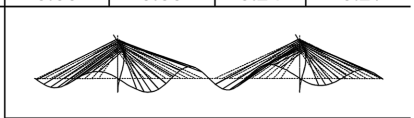
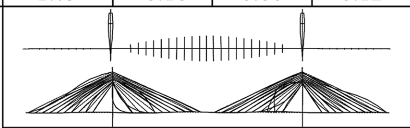
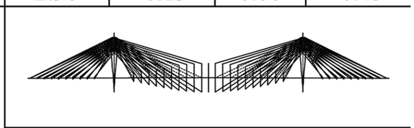
Nonlinear enhanced MECS approach						Approximate approach					
No.	Freq. (Hz)	Modal shape and participation factor				No.	Freq. (Hz)	Modal shape and participation factor			
		X dir.	Y dir.	Z dir.	Rx dir.			X dir.	Y dir.	Z dir.	Rx dir.
1	0.233					1	0.233				
		31.06	-0.01	0.00	0.00			31.05	-0.01	0.00	0.00
2	0.254					2	0.253				
		0.00	0.00	16.29	1.71			0.00	0.00	16.18	-1.73
3	0.297					3	0.308				
		0.06	8.19	0.00	0.00			0.06	7.55	0.00	0.00
4	0.407					4	0.417				
		6.85	-0.06	0.00	0.00			-7.18	0.06	0.00	0.01
37	0.674					5	0.676				
		0.00	0.00	0.50	-0.03			0.00	-0.01	-0.53	0.04
38	0.697					6	0.688				
		0.00	-11.48	0.00	0.01			0.01	15.61	0.00	0.01
45	0.764					7	0.734				
		0.00	0.00	7.02	0.50			0.00	0.00	17.54	-1.59
46	0.770					8	0.740				
		0.00	0.00	-4.98	-0.38			0.00	0.00	13.31	-1.24

Table 2 Continued

Nonlinear enhanced MECS approach						Approximate approach					
No.	Freq. (Hz)	Modal shape and participation factor				No.	Freq. (Hz)	Modal shape and participation factor			
		X dir.	Y dir.	Z dir.	Rx dir.			X dir.	Y dir.	Z dir.	Rx dir.
71	0.800					9	0.809				
		6.85	0.65	0.00	0.00			-1.00	0.08	0.00	0.01
82	0.902					10	0.925				
		0.00	-5.55	0.00	-0.01			0.03	17.04	0.00	-0.03
95	0.956					11	0.952				
		0.00	0.00	0.13	-7.58			0.00	-0.01	-10.55	2.03
141	1.111					12	1.096				
		0.00	-0.00	0.23	0.21			0.00	0.00	0.24	-0.27
132	1.091					13	1.109				
		1.79	-0.16	0.00	0.12			2.90	-0.13	0.00	-0.45
146	1.137					14	1.158				
		0.00	0.00	0.53	-32.63			0.01	0.01	1.20	74.25

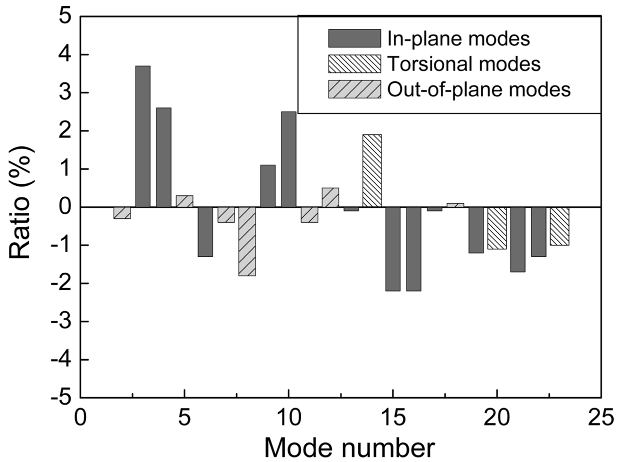


Fig. 7 Difference between Nonlinear enhanced MECS and approximate approach

Newmark b method for direct integration is adopted in the step-by-step dynamic analysis. Newton-Raphson method is used for iterative procedure.

In order to take into account the different damping constants of the stay cables, which have small structural damping, and the girder/towers, which have relatively greater damping, the Rayleigh damping that takes into account element damping proposed in our reference (Wu *et al.* 2006) is used. The damping matrix $[C]$ has the following formula

$$[C] = \sum_{k=1}^{N} (\alpha_k [M]_k + \beta_k [K]_k) \tag{3}$$

where $[M]_k$ and $[K]_k$ are the mass and stiffness matrix of element-$\boldsymbol{k}$, α_k and β_k are the arbitrary proportional factors of element-$\boldsymbol{k}$ and N is the number of elements. The α_k and β_k of element-$\boldsymbol{k}$ can be obtained as follows

$$\alpha_k = \frac{4\pi \cdot f_i^k f_j^k (h_i^k f_j^k - h_j^k f_i^k)}{(f_j^k)^2 - (f_i^k)^2}, \quad \beta_k = \frac{h_j^k f_j^k - h_i^k f_i^k}{\pi[(f_j^k)^2 - (f_i^k)^2]} \tag{4}$$

where f_i^k, f_j^k and h_i, h_j ($i < j$) are two different frequencies and corresponding damping constants of element-$\boldsymbol{k}$.

Since the girder and tower of this bridge are steel, the damping constants for the girder and towers are set to 0.02 and the stay cables are set to 0.001 (Wu *et al.* 2003).

4. Dynamic characteristics subjected to periodic excitation

This section discusses the dynamic properties of the cable-stayed bridge under the sinusoidal excitation, which may be induced by an exciter during a vibration test. The authors discussed the dynamic characteristics of stay cable by using the approximate approach (Wu *et al.* 2003). In this paper, the dynamic properties by using the nonlinear enhanced MECS approach are shown, and the difference between the nonlinear enhanced MECS approach and approximate approach is discussed.

Furthermore, the chord element has been always used to model the stay cable in certain types of numerical analyses (Tuladhar *et al.* 1995, Caetano *et al.* 2000a, Caetano *et al.* 2000b). In case of a cable subjected to periodic support excitation, the difference between chord element and cable element was discussed in our reference (Wu *et al.* 2006). In this paper, the difference between chord element and cable element in a cable-stayed bridge subjected to excitation is also discussed.

4.1 Vertical periodic excitation

This photograph discusses the dynamic properties of the cable-stayed bridge under the vertical sinusoidal sinusoidal loading $P = A\sin\omega t$. The frequency of the first vertical mode, which is 0.297 Hz in the nonlinear enhanced MECS approach and 0.308 Hz in the approximate approach (Table 2), is used as the frequency of the excitation. Since the mode has symmetric shape, the excitation point is set to the center of the main span. The excited direction is vertical, and the amplitude of the sinusoidal excitation is 50 kN (Wu *et al.* 2003).

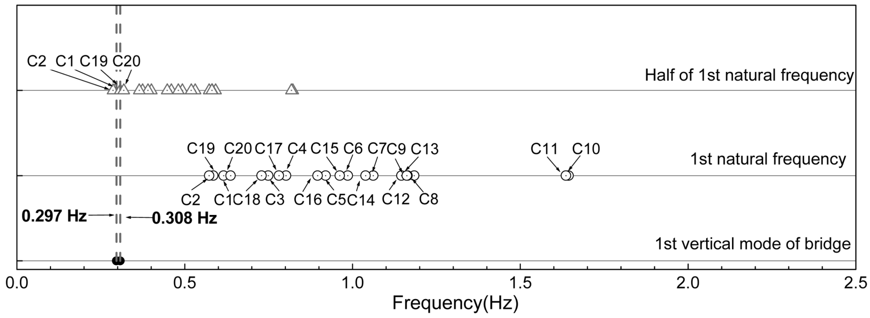


Fig. 8 Relationship between the first in-plane frequencies of stay cables and the first vertical mode of girder-tower vibrations

4.1.1 Cables in which parametric vibrations may occur

With the possibility of the cable excitation through deck motion as explored in Wu *et al.* (2003), the cable frequency results are compared to the girder-tower frequencies obtained from finite element method. As shown in Fig. 8, no cable frequency is in the vicinity of the first vertical frequency of girder-tower vibration, then no cable generates the parametric vibration in the second unstable region.

It is thought that the second super-harmonic resonance of non-linear vibration (Nayfeh *et al.* 1979) may occur in C1, C2, C19, and C20, for the frequencies of those cables vibration is about the twice that of the first vertical frequency of girder-tower vibration.

4.1.2 Dynamic characteristics of parametric vibrations in stay cables

Using the nonlinear enhanced MECS approach, the maximum displacements of stay cables are shown in Fig. 9. The ordinate is the coordinate along the girder direction, and the abscissa is the maximum vertical responses of stay cables and anchored girder. The indicated displacement of stay cable is at the center point.

The large-amplitude vibrations occur in Cables C1, C2, C19, and C20. The displacements of other cables are almost same as those of girder.

The time histories of Cable C1 and the anchored girder are shown in Fig. 10. The in-plane frequency of C1, which is 0.586 Hz, is close to the twice of the predominant frequency of the girder, which is 0.297 Hz. So the second super-harmonic resonance is generated in this cable. Large amplitude is induced, while the responses in the girder are small. The first natural frequency of

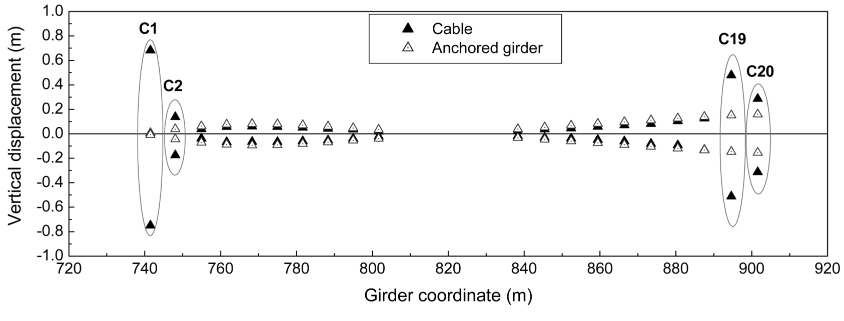
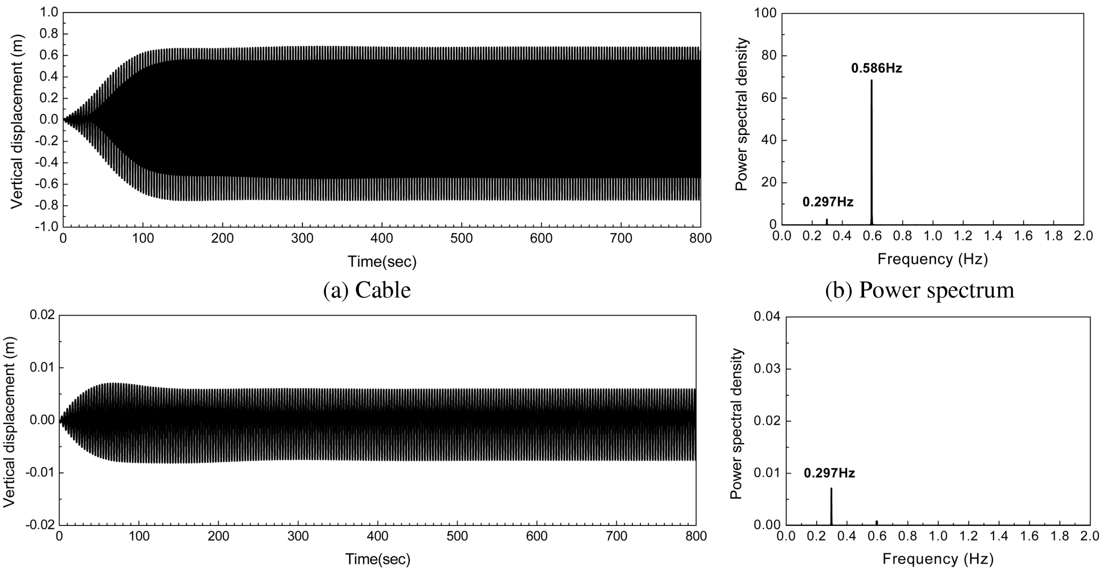


Fig. 9 Maximum displacements of stay cables under vertical excitation (nonlinear enhanced MECS approach)



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 10 Responses of Cable C1 under vertical excitation (nonlinear enhanced MECS approach)

cable C1 is close to that of cable C2, C19, C20, and the maximum displacement has the same order of cable C1. So the second super-harmonic resonance occurs in these four cables, which is same as the discussion in 4.1.1. This nonlinear vibration is strongly dependent on the cable-deck interaction at the boundary (Gattulli *et al.* 2003, Gattulli *et al.* 2005).

4.1.3 Difference between nonlinear enhanced MECS approach and approximate approach

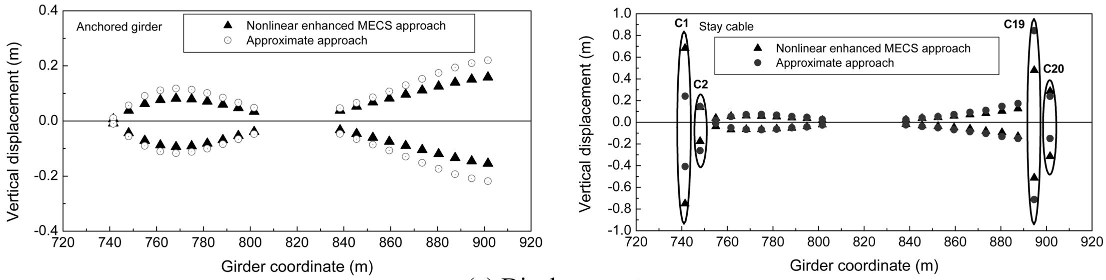
The approximate approach is based on the assumption that the cable vibrations have a small effect on the overall vibrations of the girder and towers. Whether this assumption is correct and how much the nonlinear enhanced MECS approach and approximate approach are different is discussed by comparing the responses using the nonlinear enhanced MECS approach with those using the approximate approach.

Fig. 11 shows the maximum responses of stay cables at the center points using the nonlinear enhanced MECS approach and the approximate approach.

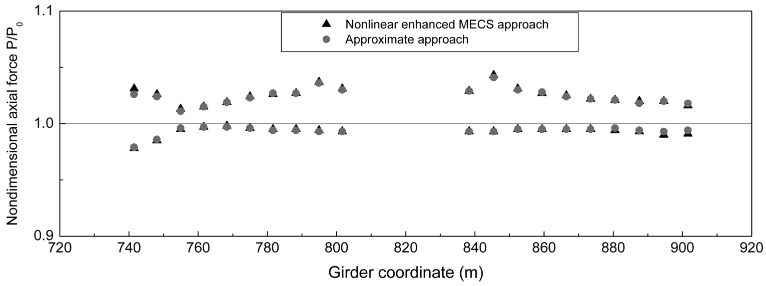
Using the approximate approach, the relatively large-amplitude vibrations are generated in cables C1, C2, C19 and C20. The time histories of cable C1 and the anchored girder using the approximate approach are shown in Fig. 12. From this figure, it is confirmed that the second super-harmonic resonance is generated in this cable. Then, the dynamic characteristics of those cables obtained by the approximate are same as those obtained by the nonlinear enhanced MECS approach.

Fig. 11(b) shows the maximum axial forces of stay cables. The abscissa is the non-dimensional axial force P/P_0 of stay cables. The fluctuations of stay cables are less than 10% of the initial forces P_0, then the cable loosening don't occur in this case of the vertical periodic excitation (Wu *et al.* 2007). The axial force of those cables obtained by the approximate are almost same as those obtained by the nonlinear enhanced MECS approach.

But, the displacements of stay cable using the approximate approach is different from those using the nonlinear enhanced MECS approach. Regarding the stay cable C1, the maximum displacement

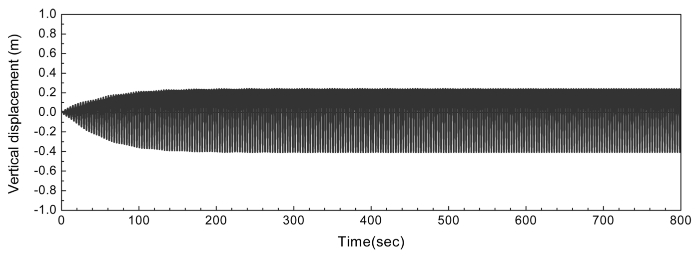


(a) Displacement

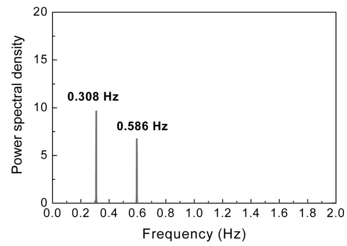


(b) Axial force

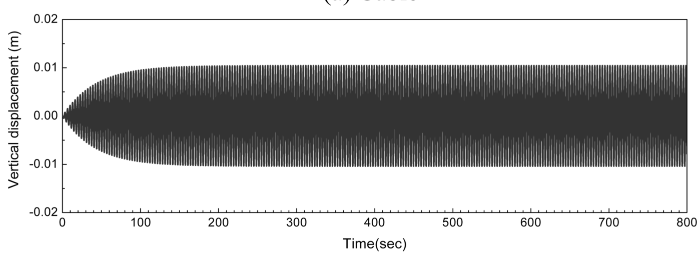
Fig. 11 Maximum responses of stay cables under vertical excitation (nonlinear enhanced MECS approach vs. approximate approach)



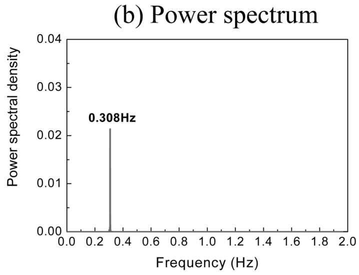
(a) Cable



(b) Power spectrum



(c) Girder

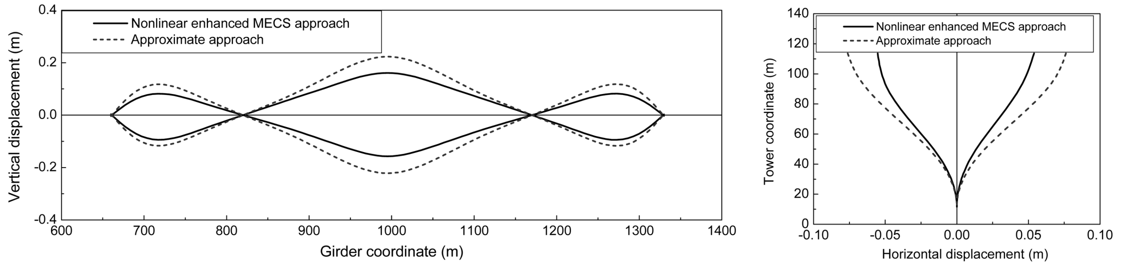


(d) Power spectrum

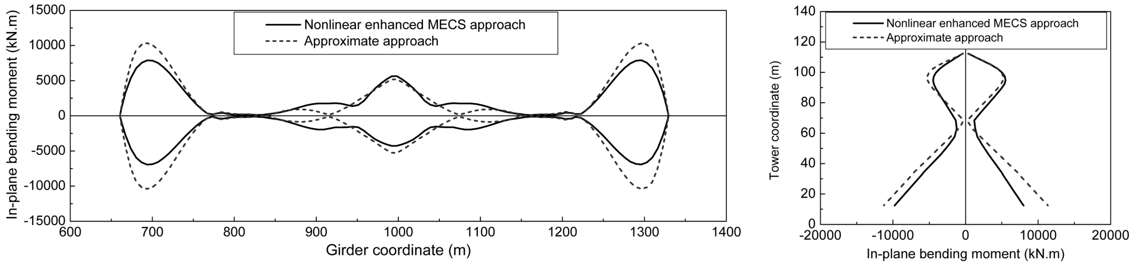
Fig. 12 Responses of Cable C1 under vertical excitation (approximate approach)

using the nonlinear enhanced MECS approach are about 0.7 m, while that using the approximate approach is 0.4 m.

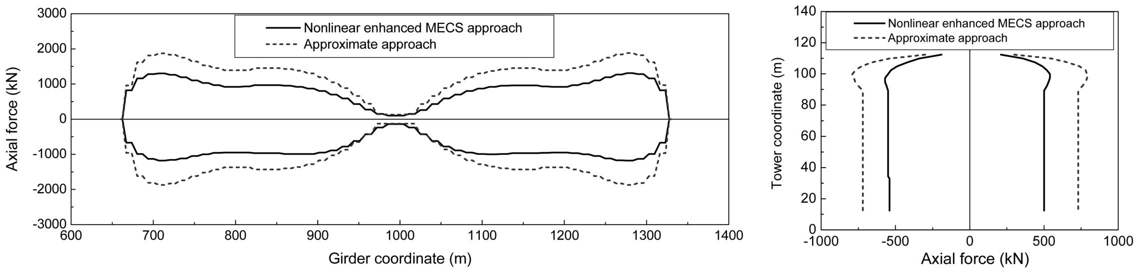
Fig. 13 shows the maximum responses of the girder and one tower. The displacements, bending moments and axial force in the girder and towers using the nonlinear enhanced MECS approach are 30% smaller than those using the approximate approach. Those differences are from the fact that the



(a) Displacement



(b) Bending moment



(c) Axial force

Fig. 13 Maximum responses of the girder and one tower under vertical excitation (nonlinear enhanced MECS approach vs. approximate model)

interference of cable vibrations with the girder-tower vibrations cannot be considered using the approximate approach.

Therefore, the differences of dynamic responses between the two methods are from the fact that the interference of cable vibrations with the girder-tower vibrations cannot be considered using the approximate approach.

4.2 Torsional periodic excitation

The parametric vibrations in the second or principal region may be generated in stay cables when the motion of the anchorage was either one or two times the first natural frequency of the stay cables (Takahashi 1991, Wu *et al.* 2003). Especially, the parametric vibrations in the principal region cause the large-amplitude responses in the stay cable. Because the parametric vibrations in the principal region may be generated under the torsional periodic excitation (Wu *et al.* 2003), the dynamic characteristics of stay cables are discussed when the cable-stayed bridge is subjected to the torsional periodic excitations.

The frequency of the first torsional mode, which is 1.137 Hz in the nonlinear enhanced MECS

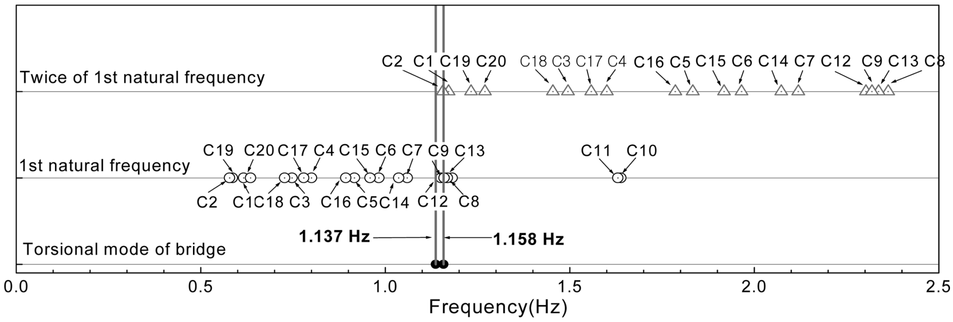


Fig. 14 Relationship between the first in-plane frequencies of stay cables and the first torsional mode of girder-tower vibrations

approach and 1.158 Hz in the approximate approach (Table 2), is used as the frequency of the excitation. Since the mode has symmetric shape, the excitation point is set to the center of the main span. The freedom of the excitation point is the rotation along the longitudinal direction, and the amplitude of the sinusoidal excitation is 500 kN-m (Wu *et al*. 2003).

4.2.1 Cables in which parametric vibrations may occur

Fig. 14 shows the relationship between the first in-plane frequencies of stay cables and the first torsional frequency of the girder-tower vibrations.

Since the first natural frequencies of cables C8, C9, C12 and C13 are in the vicinity of the natural torsional frequency, the parametric vibration in the second unstable region may occur under period loading with this frequency.

The natural frequencies of the first torsional mode are close to twice of the first natural frequencies of cable C1, C2, and C19. So the parametric vibrations in the principal unstable region in those two cables may be expected under the periodic loading with the first torsional frequency.

4.2.2 Dynamic characteristics of parametric vibrations in stay cables

Fig. 15 shows the maximum displacements and axial forces at the center points of stay cables using the nonlinear enhanced MECS approach. The ordinate is the coordinate along the girder direction, and the abscissa is the vertical responses of stay cables.

The large-amplitude vibrations occur in Cables C1, C2, C8, C9, C12, C13, and C19.

The time histories of Cable C2 and the anchored girder are shown in Fig. 16. The parametric

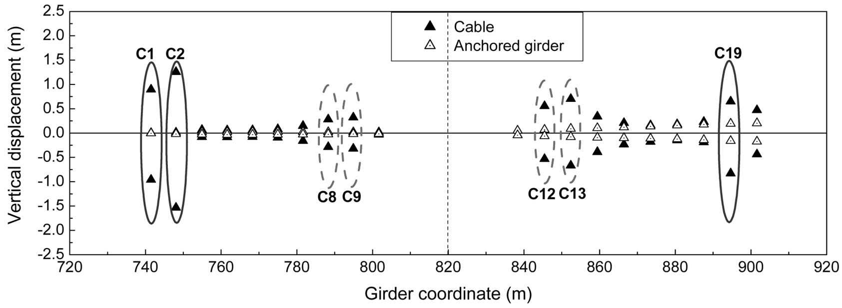
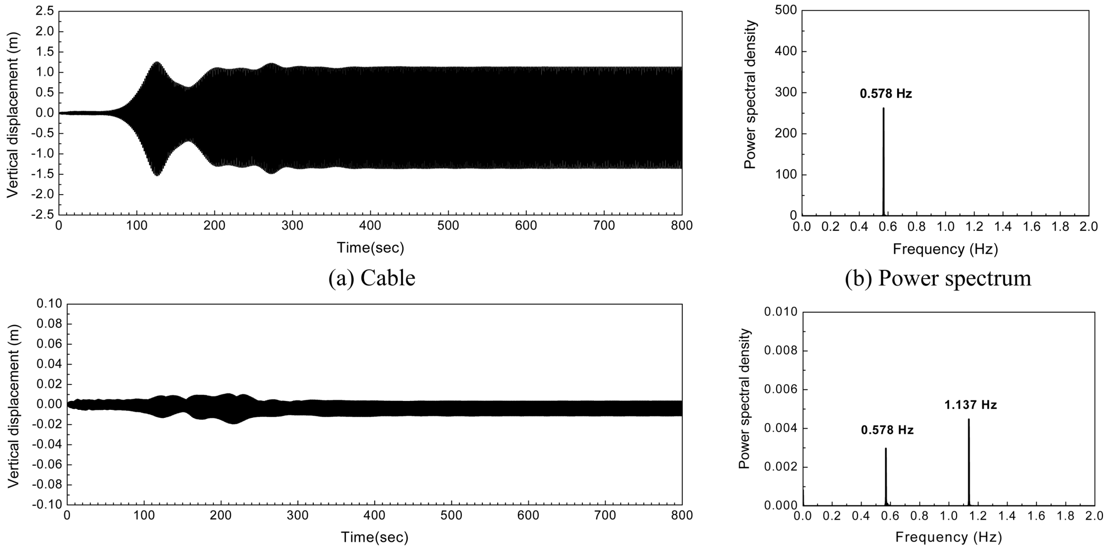


Fig. 15 Maximum displacements of stay cables under torsional excitation (nonlinear enhanced MECS approach)

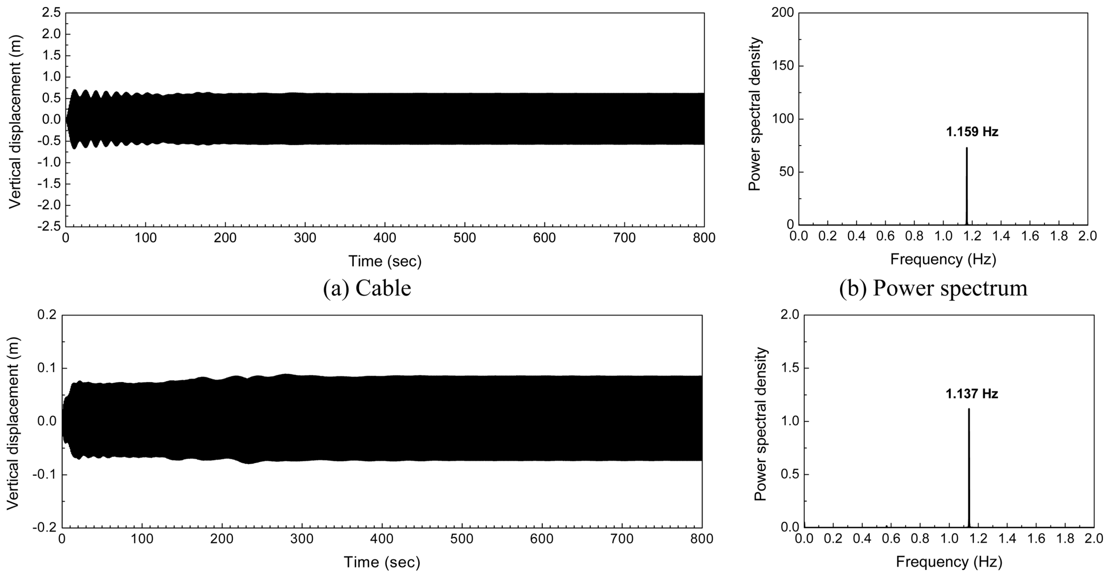
vibrations in the principal unstable region is generated in this cable, since the frequency of the cable vibration, which is 0.578 Hz, is close to the half of the predominant frequency of the girder, which is 1.137 Hz. Large-amplitude parametric vibrations in the principal region are induced, while the responses in the girder are very small. The first natural frequency of cable C1 is close to that of cable C2 and the maximum displacement has the same order of cable C2 and C19. So the



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 16 Responses of Cable C2 at the center point under torsional excitation (nonlinear enhanced MECS approach)



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 17 Responses of Cable C13 at the center point under torsional excitation (nonlinear enhanced MECS approach) parametric vibrations in the principal unstable region occur in Cables C1, C2, and C19.

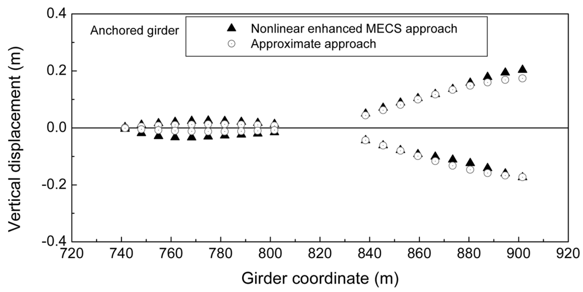
The natural frequencies of the first in-plane modes in Cables C8, C9, C12 and C13 are in the range of 1.04~1.18 Hz, and the maximum displacements in those cables are almost of the same order. The response of cable C13 is shown in Fig. 17. The parametric vibration in the second unstable region is generated in cable C13, since the frequencies of cables, which is 1.159 Hz, is close to the predominant frequencies of the girder, which is 1.137 Hz.

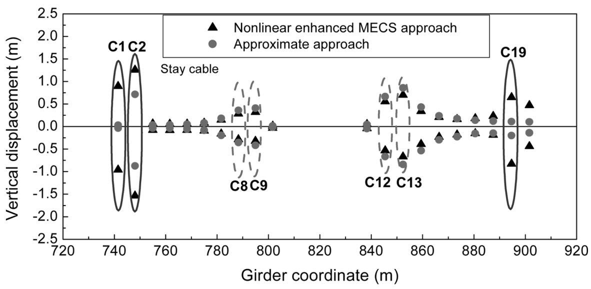
The maximum responses of the cables in which the parametric vibrations in the principal unstable region occur are much greater than those in the second unstable region. The transient time of reaching the maximum responses in the principal unstable region is much longer than that in the second unstable region. Those dynamic properties of stay cable are same as that shown in our reference (Wu *et al*. 2003).

4.2.3 Difference between nonlinear enhanced MECS approach and approximate approach

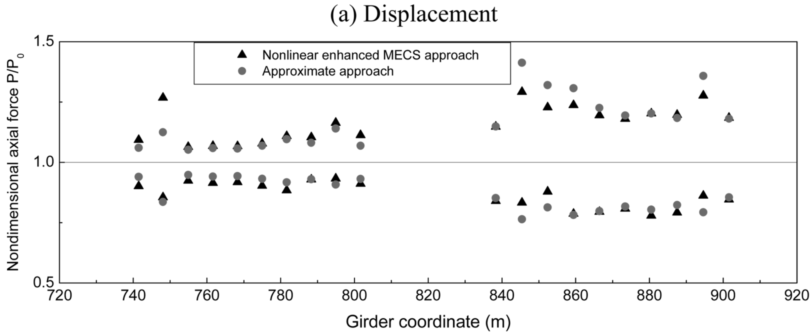
Fig. 18 shows the maximum responses of stay cables at the center points using the nonlinear enhanced MECS approach and the approximate approach. Fig. 19 shows the responses of cable C2 and the anchored girder using the approximate approach. Compared Fig. 19 with Fig. 13, the dynamic characteristic of this cable obtained by the approximate approach is similar to that obtained by the nonlinear enhanced MECS approach. The parametric vibration in the principal unstable region occurs in cable C2 by using the approximate approach, while the phenomenon has found by using the nonlinear enhanced MECS approach in cables C1 besides cable C2.

The time histories of cable C13 and the anchored girder using the approximate approach are shown in Fig. 20. From this figure, it is confirmed that the parametric vibrations in the second unstable region is generated in this cable. Using the approximate approach, it is obtained that the



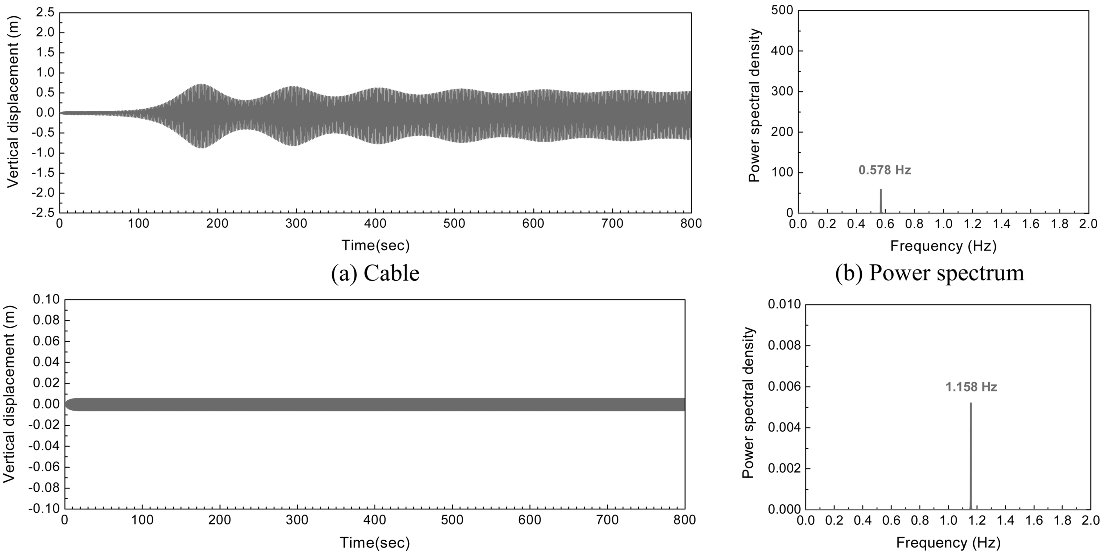


(a) Displacement



(b) Axial force

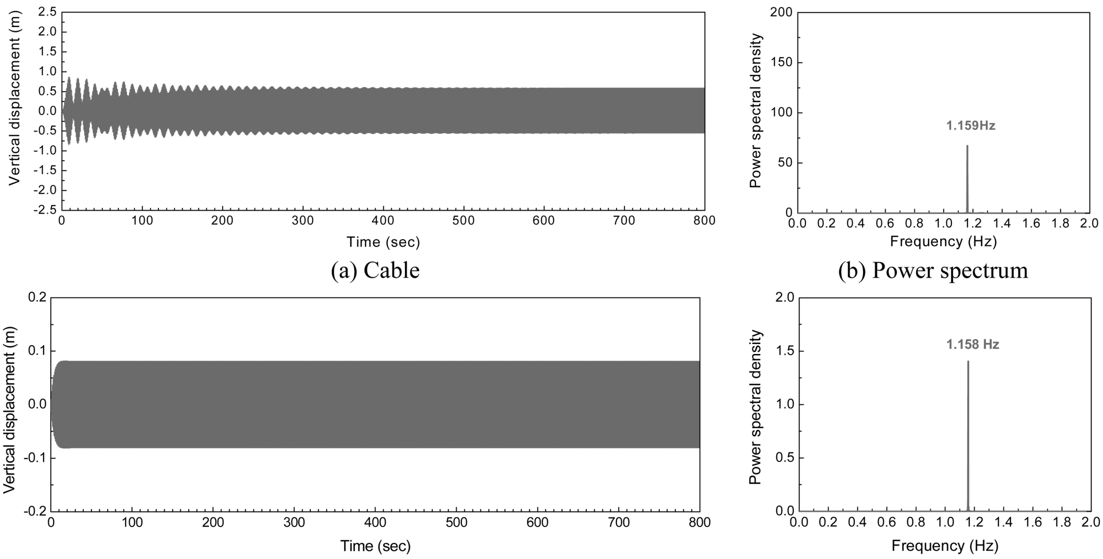
Fig. 18 Maximum responses of stay cables under torsional excitation (nonlinear enhanced MECS approach vs. approximate approach)



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 19 Responses of Cable C2 under torsional excitation (approximate approach)



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 20 Responses of Cable C13 under torsional excitation (approximate approach)

parametric vibration in the second unstable region occurs in cable C8, C9, C13, and C13, which is same as those using the nonlinear enhanced MECS approach in the previous photograph.

Therefore, totaling the analysis of cable-stayed bridge under the vertical and torsional excitation, the dynamic characteristics of those cables using the approximate are almost same as those using the nonlinear enhanced MECS approach. It can be said that the dynamic characteristics of stay cables can be evaluated either using the nonlinear enhanced MECS approach or approximate approach.

Fig. 18(b) shows the maximum axial forces of stay cables. The fluctuations of stay cables are less

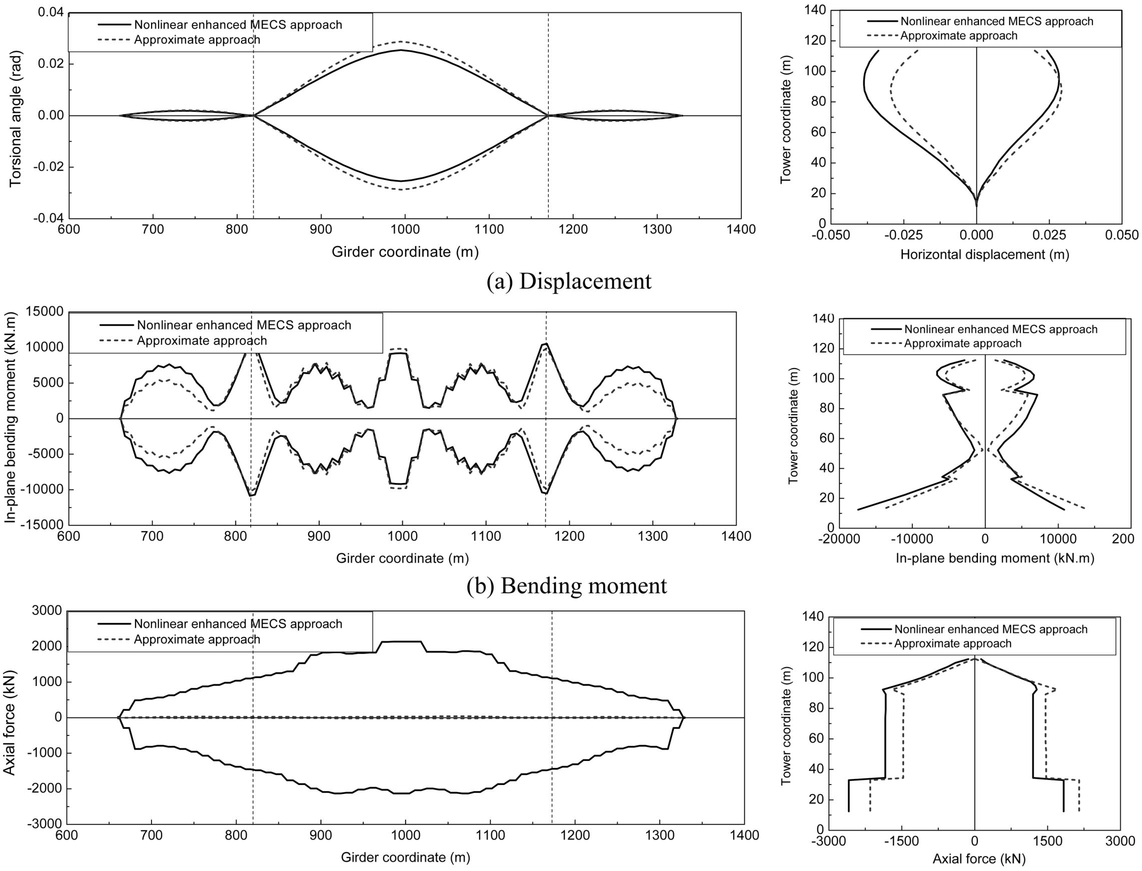


Fig. 21 Maximum responses of the girder and one tower under torsional excitation (nonlinear enhanced MECS approach vs. approximate model)

than 50% of the initial forces P_0, then the cable loosening don't occur when cable parametric vibration in the second or principal region occurs (Wu *et al.* 2007).

The maximum responses of the girder and one tower are shown in Fig. 21.

Similarly as the difference under vertical excitation, the displacements in the girder using the nonlinear enhanced MECS approach are 20% smaller than those using the approximate approach. But the bending moments in the girder and towers using the nonlinear enhanced MECS approach have a great difference with those in the approximate approach. Especially, the axial force of the girder using the nonlinear enhanced MECS approach is great different from those using approximate approach. Moreover, there isn't the regularity of the curves that represent the maximum responses in the girder and towers obtained by the nonlinear enhanced MECS approach, while there is this regularity in approximate approach.

The maximum axial forces of the girder using the nonlinear enhanced MECS approach is about 2500 kN, while the axial force of girder using the approximate approach are almost zero. When one-end anchored girder moves one unit displacement in the upper direction and the other-end moves the same displacement but in the opposite direction, the fluctuant of axial force of stay cable is defined as F_u and F_d, shown in Fig. 22. For the case of the approximate approach, one straight cable element without sag is used to simulate the sagged stay cable, and the relationship of the cable axial forces are $F_u + F_d = 0$, subsequently the axial force affected the girder is almost zero. For the case of the accurate nonlinear enhanced MECS approach, the cable with sag is simulated using some cable element, and the relationship of the cable axial forces are $F_u + F_d \neq 0$, subsequently the axial force affected the girder is $F_u + F_d$, which is not zero.

Due to the difference between both stay cable at both ends, the significant cable responses associated with the parametric vibrations in the principal region may be responsible for the certain loss of regularity of the curves that represent the maximum responses in the girder and towers.

Fig. 23 shows the axial forces of girder under the self-weight loading, and Fig. 24 shows the ratio

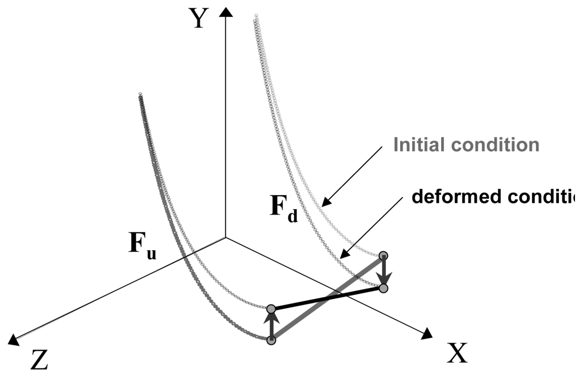


Fig. 22 Axial force of both stay cable in one girder

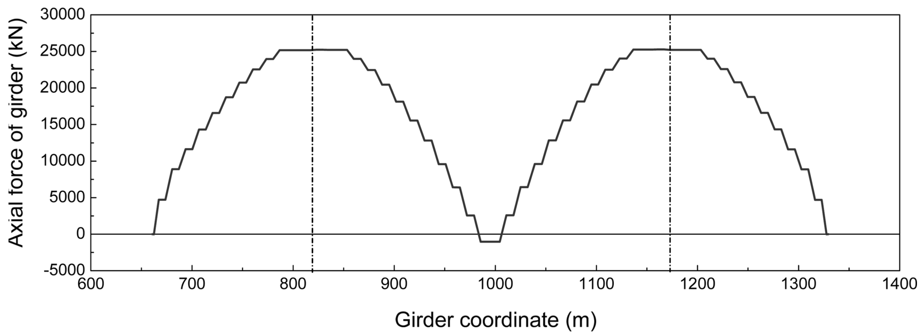


Fig. 23 Axial forces of the girder under the self-weight load

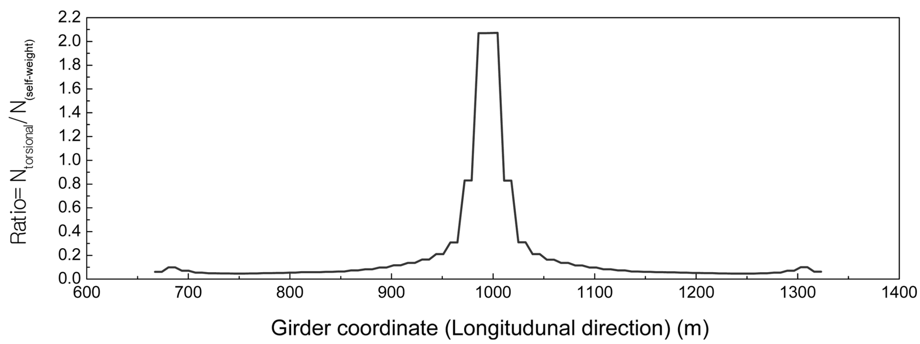


Fig. 24 Ratio of axial forces of the girder under torsional excitation

of axial forces of the girder under the torsional excitation to those under the self-weight load. At the region of one tower, the axial force of girder under the self-weight load is about 25160 kN, and the maximum axial force under the torsional excitation is about 1380 kN. The ratio of axial force under the torsional excitation to that under the self-weight load is about 1380/25160 = 5%. At the quarter of main span, the ratio is about 2090/18120 = 12%. At the center of main span, the ratio is about 2130/1030 = 2.07%. Therefore, the produced axial forces of the girder under the torsional excitation using the nonlinear enhanced MECS approach, which are neglected in the approximate approach, should be considered correctly.

5. Seismic properties under earthquake

The seismic properties of cable-stayed bridges are important for the design of cable-stayed bridges. Using the cable finite elements that can include the parametric vibrations of stay cables, the effect of the cable vibrations on the overall responses of the girder, towers and stay cables are discussed by comparing the responses obtained by the nonlinear enhanced MECS approach with those obtained by the approximate approach. Two cases are taken into account, the longitudinal and out-of-plane seismic loading.

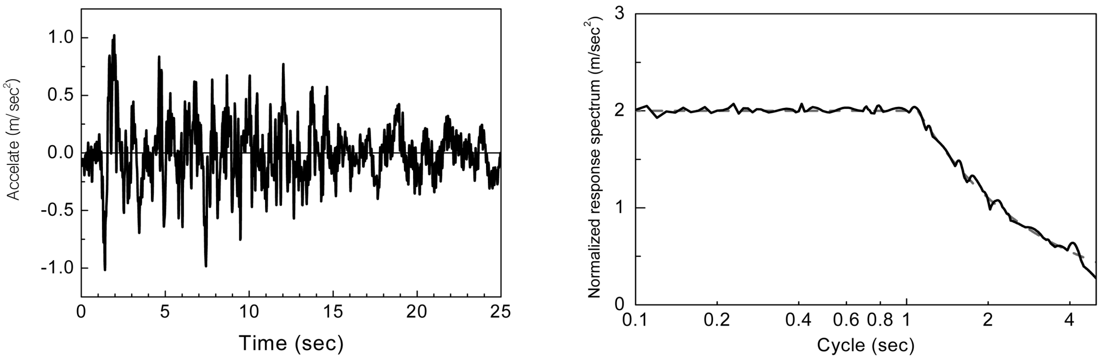
The moderate ground motion according to the Design Specifications for Highway Bridges in Japan is applied (Earthquake Engineering Committee of Japan Society of Civil Engineers 2000). The time history and normalized response spectrum of this ground motion are shown in Fig. 25. The maximum acceleration is about 1.0 m/sec^2.

In the approximate approach, the external earthquake is considered in the analysis of the single cable model.

5.1 Subjected to longitudinal seismic loading

When the earthquake applies in the longitudinal direction, the maximum responses of stay cables are shown in Fig. 26.

Compared with the responses of the anchored girder, the amplitudes of some stay cables is a little larger than those of anchored girder. The maximum displacements and axial forces of stay cables



(a) Time history (b) Response Spectrum

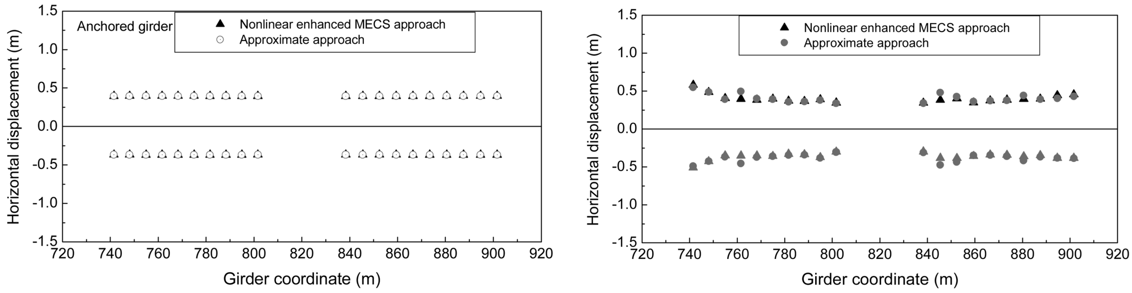
Fig. 25 Used ground motion

obtained by the approximate approach are almost same as those obtained by the nonlinear enhanced MECS approach.

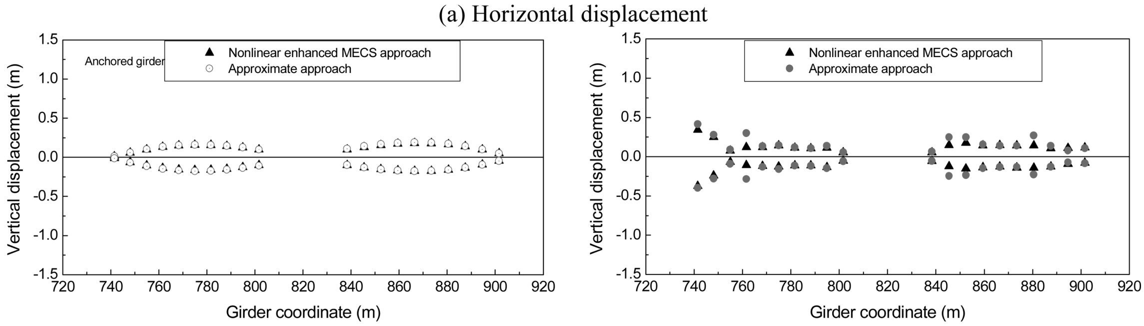
Fig. 27 shows the time responses and spectra of the girder and cable C1, the response of which is relatively large. The predominant frequencies of the girder response are the natural frequencies of the global girder-tower mode, while that of cable C1 is also the natural frequency of itself. The waveform of cable C1 under parametric vibration is not accompanied by beating. Therefore, it can be concluded that parametric vibration of the cables when bridge is subjected to the longitudinal earthquake does not occur, which conclusion is same as that drawn in our reference (Wu *et al*. 2003).

The dynamic characteristics of stay cables obtained by those two approaches are similar, and the responses of stay cables are also same.

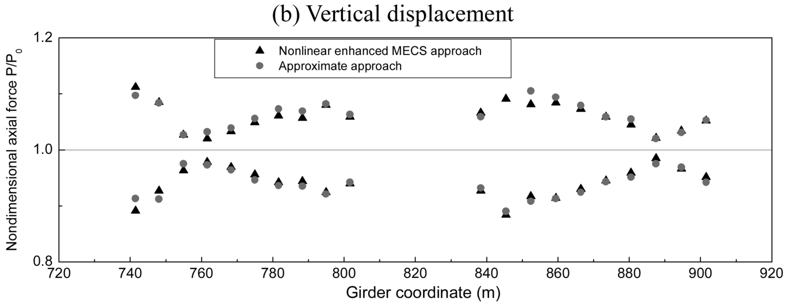
Fig. 28 shows the maximum responses of the girder and one tower. The displacements, bending moments and axial force of the girder and tower obtained from the nonlinear enhanced MECS approach are relatively smaller than the corresponding responses obtained by the appreciate approach. The responses of girder and tower are almost same.



(a) Horizontal displacement

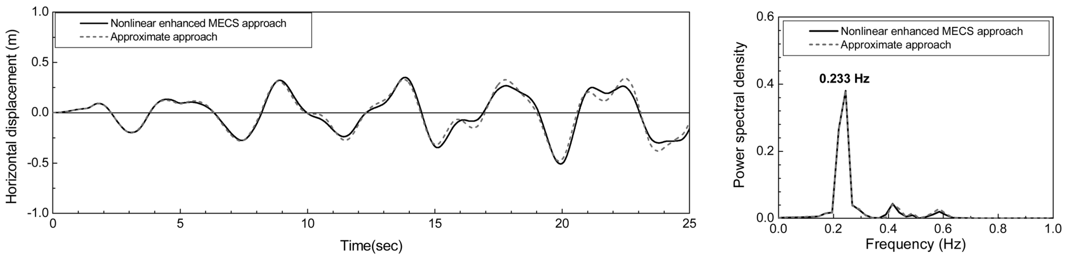


(b) Vertical displacement

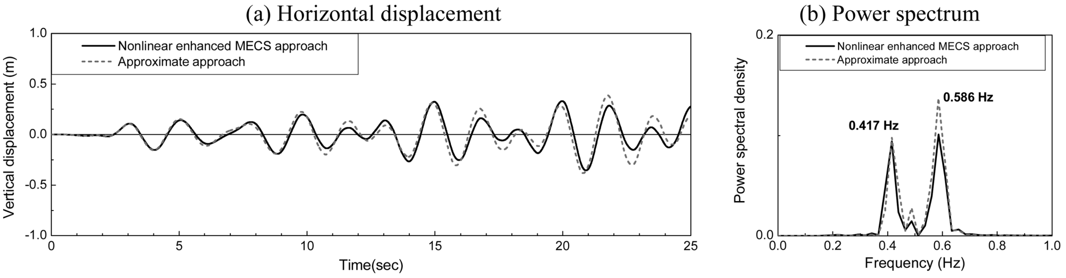


(c) Axial force

Fig. 26 Maximum responses of stay cables under longitudinal earthquake (nonlinear enhanced MECS approach vs. approximate approach)

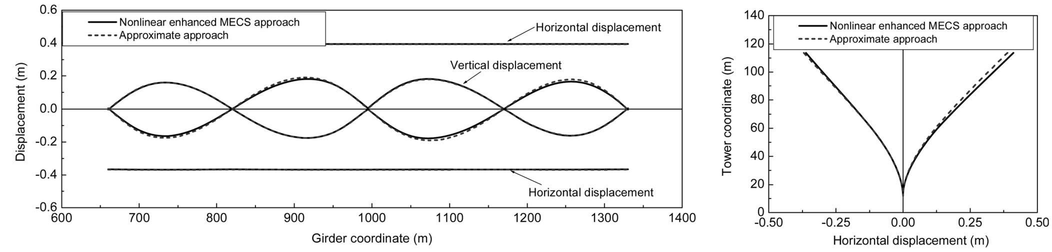


(a) Horizontal displacement (b) Power spectrum

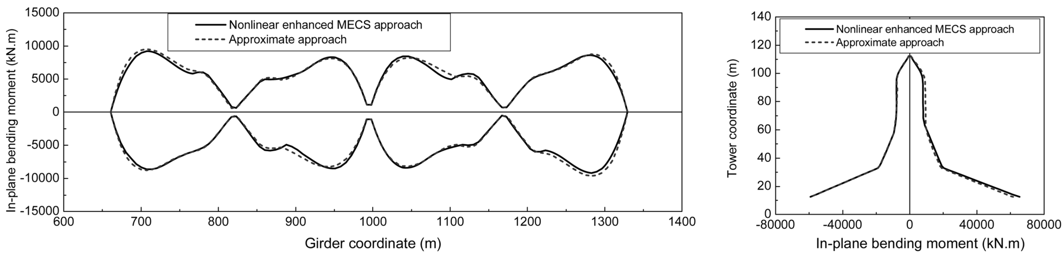


(c) Vertical displacement (d) Power spectrum

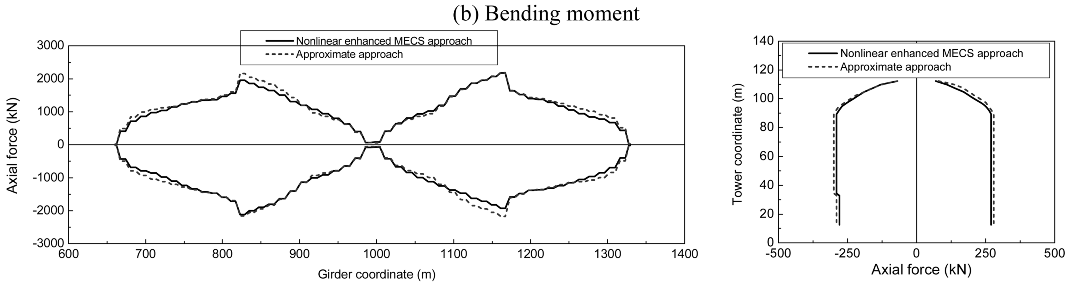
Fig. 27 Responses of Cable C1 under longitudinal earthquake (nonlinear enhanced MECS approach vs. approximate approach)



(a) Displacement



(b) Bending moment



(c) Axial force

Fig. 28 Maximum responses of girder and tower under longitudinal earthquake (nonlinear enhanced MECS approach vs. approximate approach)

5.2 Subjected to out-of-plane seismic loading

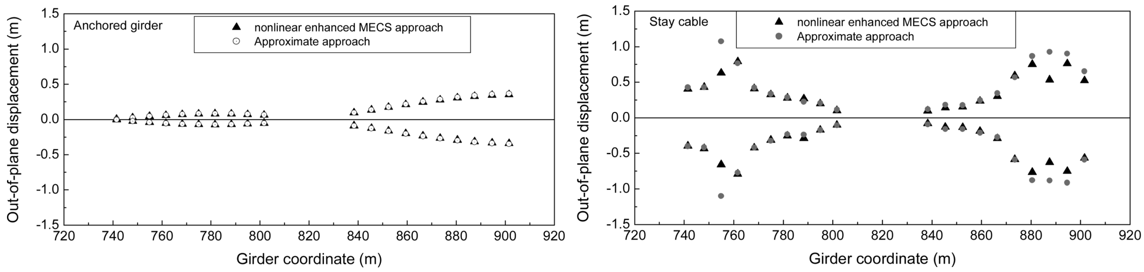
When the earthquake applies in the out-of-plane direction, the maximum responses of stay cables are shown in Fig. 29. Compared with the responses of the anchored girder, some stay cables undergo the large-amplitude vibrations when the cable-stayed bridge is subjected to the out-of-plane earthquake excitation. From the time-domain and frequency-domain responses of cable C19 shown in Fig. 27, it can be concluded that parametric vibration of the cables when bridge is subjected to the out-of-plane earthquake does not occur.

The maximum displacements and axial forces of stay cables obtained by the approximate approach are almost same as those obtained by the nonlinear enhanced MECS approach. And the dynamic characteristics of stay cables obtained by those two approaches are similar.

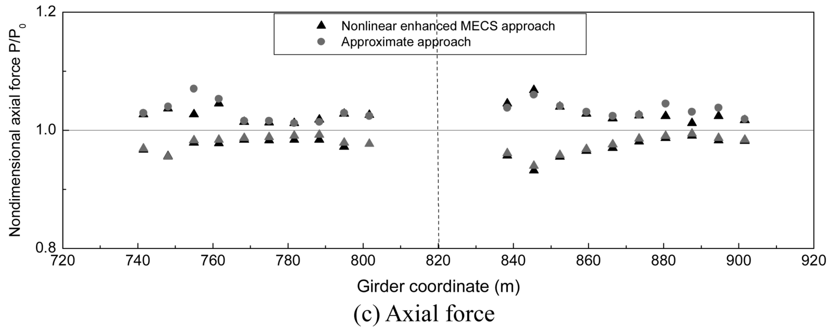
Fig. 31 shows the maximum responses of the girder and one tower. The axial forces of the girder obtained by the approximate approach are almost zero, but the axial forces of the girder using the nonlinear enhanced MECS approach are generated. The axial force in tower using the nonlinear enhanced MECS approach is smaller than those obtained by the approximate approach. This is because the axial force fluctuant of stay cables in both ends of one girder are different, which is stated in detail in chapter 4.2.4.

The displacements and bending moments of the girder and tower obtained from the nonlinear enhanced MECS approach are relatively smaller than the corresponding responses obtained by the approximate approach. The interference of cable vibrations leads to the slight decrease of the responses in the girder and towers, about 10%.

Fig. 32 shows the ratio of axial forces of the girder under the out-of-plane earthquake to those under the self-weight load. At the region of one tower, the ratio of axial force under the out-of-plane earthquake to that under the self-weight load is about 130/25160 = 0.5%. At the quarter of

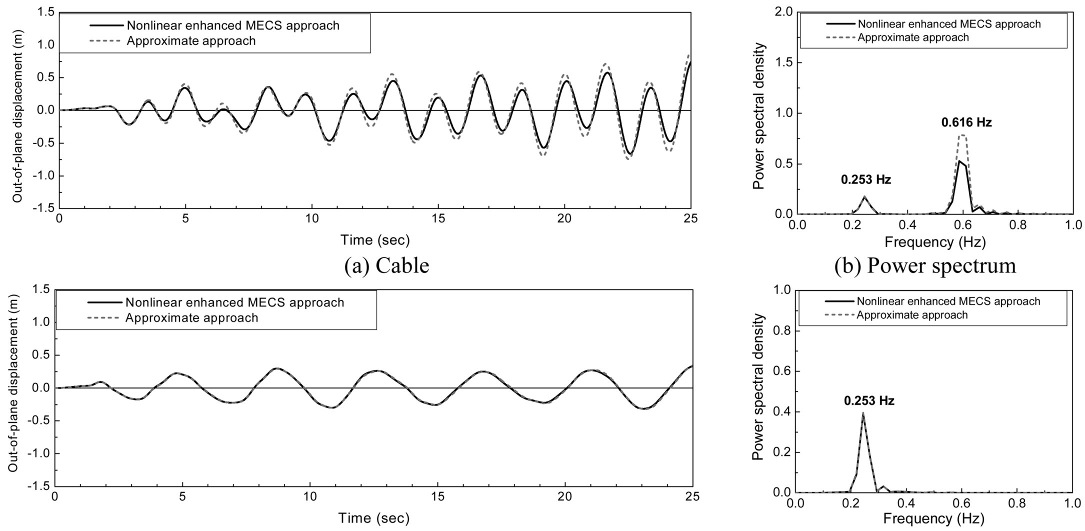


(a) Out-of-plane displacement



(c) Axial force

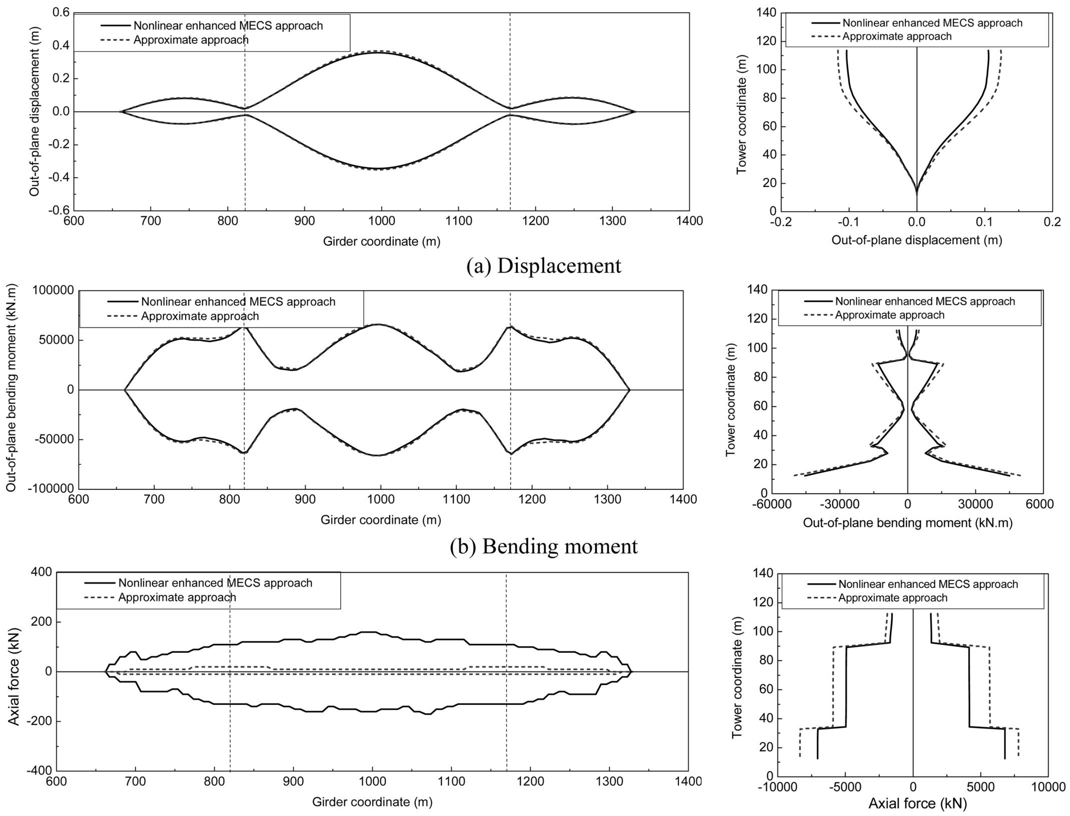
Fig. 29 Maximum responses of stay cables under out-of-plane earthquake (nonlinear enhanced MECS approach vs. approximate approach)



(a) Cable (b) Power spectrum

(c) Girder (d) Power spectrum

Fig. 30 Responses of Cable C19 under out-of-plane earthquake (nonlinear enhanced MECS approach vs. approximate approach)



(a) Displacement

(b) Bending moment

(c) Axial force

Fig. 31 Maximum responses of girder and tower under out-of-plane earthquake (nonlinear enhanced MECS approach vs. approximate approach)

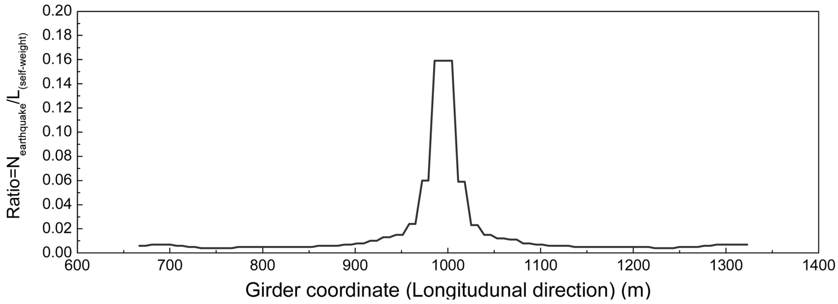


Fig. 32 Ratio of axial forces of the girder under out-of-plane earthquake

main span, the ratio is about 150/18120 = 0.8%. At the center of main span, the ratio is about 160/ 1030 = 15%. Because the moderate earthquake is selected, the produced axial forces of the girder under the out-of-plane earthquake are 1/10 time of those under the torsional excitation, so the influence of local cable vibrations is not very larger in this case. If using the strong ground motions like the 1995 Great Hanshin Earthquake (Japan) (Earthquake Engineering Committee of Japan Society of Civil Engineers 2000), whose peak ground acceleration are about 8 m/sec^2, the produced axial forces are 8 time of those under the moderate ground motion, and the influence of local cable vibration may not be neglected. Therefore, the dynamic analysis of this cable-stayed bridge subjected to out-of-plane earthquake should adopt the nonlinear enhanced MECS approach to consider the interaction of cable vibrations and the girder-tower vibrations.

6. Conclusions

This paper focuses on the cable vibrations including the parametric vibrations and evaluates the dynamic characteristics of a steel cable-stayed bridge using the nonlinear enhanced MECS approach. Comparing the cable element with the chord element, it is confirmed that it had better use the proposed cable finite elements for stay cables in the nonlinear enhanced MECS approach in order to correctly evaluate the non-linear cable vibrations including parametric vibrations.

Compared the responses obtained by the nonlinear enhanced MECS approach with those using the approximate approach, the following conclusions for the overall vibrations of the girder, towers and stay cables in this steel cable-stayed bridge can be reached:

1. The second super-harmonic resonances are generated in some cables under the first vertical frequency of the girder-tower vibration. The parametric vibrations in the second unstable region or in the principal unstable region are generated in some cables under the torsional excitation. The maximum displacements of those cables are larger than those of the anchored girder. The responses of those cables are much greater than those in the second unstable region, and the transient time of reaching the maximum responses in the principal unstable region is much longer than that in the second unstable region. Those dynamic characteristics of stay cables can be evaluated using the nonlinear enhanced MECS approach or the approximate approach. However, the response values of the generated cables are different. This difference may be responsible for the different axial force fluctuant of stay cables in both ends of one girder.
2. When the bridge is subjected to the longitudinal or out-of-plane earthquake, stay cables undergo the large-amplitude vibrations; however, the parametric vibrations of stay cables don't occur.

Those dynamic characteristics of stay cables using the nonlinear enhanced MECS approach are almost same as those using the approximate approach. The interference of cable vibrations may cause a slight decrease of the responses in the girder and towers except of the axial forces.

In view of these results, the authors believe that, the conclusions for the dynamic characteristics of the parametric vibrations in stay cables using the approximate approach in the reference (Wu *et al.* 2003) are valid. In the other side, when the local frequencies of stay cables are close to the deck-tower global frequencies, the approximate approach has been demonstrated to completely miss the description of the bridge dynamic response, regarding both the spectrum properties and the forced response to external loads. According to the author's opinion, the nonlinear enhanced MECS approach using the presented cable element should be used for modern cable-stayed bridges to correctly evaluate the interaction of cable vibrations and the girder-tower vibrations, since modern long span cable-stayed bridges built today or proposed for future bridges are subjected to large displacements.

On the other hand, the MECS approach has been already implemented in a classical nonlinear finite element model to describe the nonlinear response of a suspended cable (in the framework of an own-made code) by Gattulli (Gattulli *et al.* 2004), and also to describe the nonlinear response of a cable-stayed system in the nonlinear regime (with the commercial code ADINA) (Gattulli *et al.* 2005). However, when using commercial finite element codes for analysis, the drawback is that only few commercial codes, e.g., ABAQUS, enable the users to define their own elements. Therefore, the own-made software was used in this paper. The next subjected for study is to use ABAQUS defining the presented cable element, and implement this modeling approach in commercial finite element code.

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