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Design curves for prestressed concrete rectangular beam sections based on BS 8110

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Abstract. Design curves have been prepared for prestressed rectangular beam section based on BS 8110, for determining area of steel for any given cross section, for stresses in concrete and steel and for the design moment. The design moment and the area of steel have been expressed in dimensionless form in terms of cross sectional dimensions and the characteristic strength of concrete. The choice and combination of design parameters result in considerably less number of curves as aid for design of rectangular prestressed beam sections, than those reported in CP 110 (Part 3).

Key words: prestressed; concrete; beam; rectangular; design curves.

1. Introduction

The ultimate strength design method is widely used for the design of beam sections (British Standards Institution 1972a, 1985, Kong and Evans 1989). Several design handbooks (British Standards Institution 1972b, Indian Standards Institution 1978, Reynolds and Steadman 1974, American Concrete Institute 1973) have been published to reduce the design efforts. The design aid for prestressed rectangular pretensioned or bonded post tensioned beam section (British Standards Institution 1972b) covers design charts for tendons of normal and low relaxation products and tendons of as 'drawn' wire or 'as spun' strands. These charts have been prepared for determining the area of tendon A_{ps}, for any given cross sectional dimensions b, d; for stresses in concrete f_{cu} and steel f_{pu}; and for the design moment M_u. The curves have been obtained by plotting $M_u/(b \cdot d^2)$ against 100 $A_{ps}/(b \cdot d)$ for different values of f_{pe}/f_{pu} while keeping f_{cu} and f_{pu} constant. The number of such curves are twenty, one for each combination of f_{pu} and f_{cu}.

In this study, design curves for pretensioned or bonded post tensioned prestressed concrete rectangular beam sections using tendons of normal or low relaxation products and tendons of 'as drawn' wire or 'as spun' strand have been presented for determining the area of steel for given cross sectional dimensions; for strength of concrete and steel; effective prestress in tendons; and for the given design moment. The study is based on BS 8110 Part 1 and Part 3. Generally the presentation of design curves on two orthogonal axes incorporates the effect of three variables, by presenting several curves for different values of third variable. In case of more than three variables, when the presentation of curves on two orthogonal axes requires a number of curves for different values of variables in excess of three, the proposed presentation combines the effect of all the variables in a single curve. Thus the number of curves is reduced

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to one for the prestressed rectangular beam section.

Two different approaches for obtaining design curves have been highlighted in this presentation. The first approach for drawing unified curves is based on the values of x/d for differrent values of $f_{pu}\cdot A_{ps}/(f_{cu}\cdot b\cdot d)$ and f_{pe}/f_{pu}, as given in BS 8110. The second approach is based on actual computation of x/d from the first principles for various variables *i.e.*, f_{cu}, f_{pu} and f_{pe}.

2. Design curves

Having designed for service condition and checked the stresses at transfer; it is still necessary to check that the ultimate limit state requirements are satisfied. To facilitate this, design curves based on two approaches have been developed as described below.

2.1. Design curves based on approach I

The ultimate moment of resistance of the rectangular beam section is given by,

$$M_u = K_1\cdot f_{cu}\cdot b\cdot x\cdot(d-K_2\cdot x)$$

$$\therefore \quad \frac{M_u}{f_{cu}\cdot b\cdot d^2} = K_1\cdot\frac{x}{d}\left(1-\frac{x}{d}\right) \tag{1}$$

where

$$K_1 = (0.0035-\varepsilon_o/3)\ 0.45/0.0035 \tag{2}$$

$$K_2 = ((2-\varepsilon_o/0.0035)^2+2)/(4\cdot(3-\varepsilon_o/0.0035)) \tag{3}$$

$$\varepsilon_o = \sqrt{f_{cu}}/5000 \tag{4}$$

The moment Eq. (1) gives the ultimate moment of resistance in dimensionless form which can also be applied for flanged beams in which neutral axis lies within the flange. The values of x/d as recommended by BS 8110 may be taken from Table 1 as given below.

To facilitate the design of rectangular prestressed beam sections, curves have been prepared for determining the area of steel for given cross sectional dimensions of beam b,d; strength

Table 1 Values of x/d for pretensioned beams or bonded post tensioned beams

$f_{pu}\cdot A_{ps}/(f_{cu}\cdot b\cdot d)$	Ratio of depth of neutral axis to that of the centroid of the tendons for different values of f_{pe}/f_{pu}		
	0.4	0.5	0.6
0.05	0.11	0.11	0.11
0.1	0.22	0.22	0.22
0.15	0.31	0.32	0.32
0.20	0.38	0.39	0.40
0.25	0.46	0.47	0.48
0.30	0.52	0.54	0.55
0.35	0.58	0.60	0.63
0.40	0.62	0.67	0.70
0.45	0.66	0.72	0.77
0.50	0.69	0.77	0.83

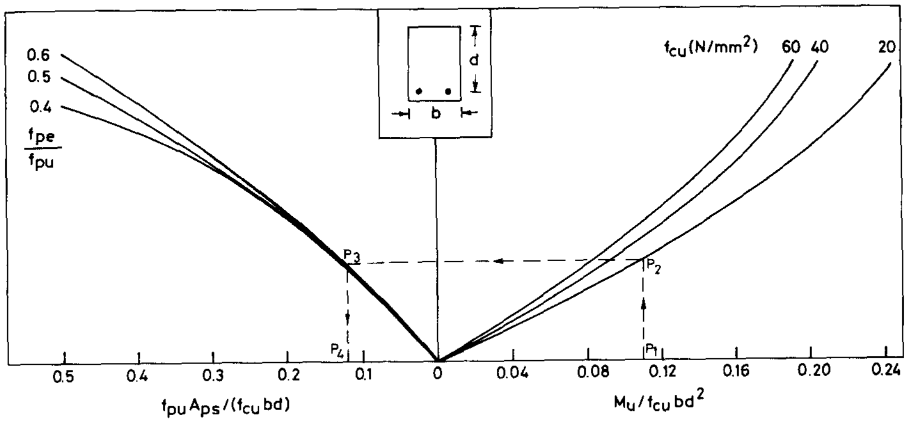

Fig. 1 Unified curves based on Approach I.

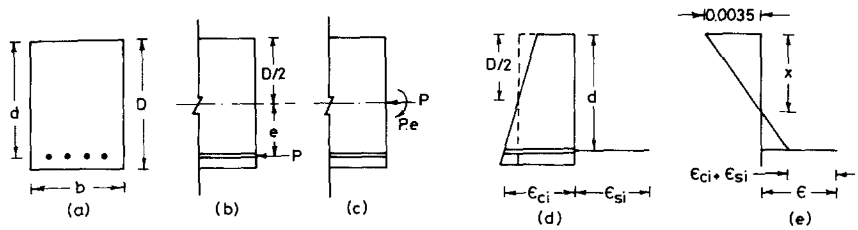

Fig. 2 Prestressed beam section
(a) Beam section, (b) Prestress force, (c) Equivalent force diagram,
(d) Strain profile under prestress, (e) Strain profile at ultimate load.

of concrete f_{cu}; strength of steel f_{pu}; and the given design moment M_u, as shown in Fig. 1. The curves have been obtained by plotting $f_{pu}\cdot A_{ps}/(f_{cu}\cdot b\cdot d)$ on the horizontal axis against x/d on the vertical axis for different values of f_{pe}/f_{pu}. The variations of $M_u/(f_{cu}\cdot b\cdot d^2)$ with x/d for different values of f_{cu} are obtained by plotting $M_u/(f_{cu}\cdot b\cdot d^2)$ on the horizontal axis against x/d on the vertical axis for different values of f_{cu}. The two graphs are then combined by their common axis of x/d to give a unified graph, giving a relation between $M_u/(f_{cu}\cdot b\cdot d^2)$ and $f_{pu}\cdot A_{ps}/(f_{cu}\cdot b\cdot d)$, for different values of f_{pe}/f_{pu} and f_{cu}. The value of x/d is no more required and hence has been eliminated from the vertical axis. The use of the graph is simple and is shown with the help of directed arrows p_1, p_2, p_3 and p_4.

2.2. Design curves based on approach II

Fig. 2 shows the details of prestressed rectangular beam. Considering initial stage of prestress only,

$$\varepsilon_{ci}=\frac{1}{E_c}\cdot\left[\frac{P}{A}+\frac{P\cdot e}{z}\right]=\frac{1}{E_c}\left[f_{pe}\frac{A_{ps}}{b\cdot D}+f_{pe}\cdot A_{ps}\cdot\frac{e}{z}\right] \tag{5}$$

$$e=0.5D-(D-d) \tag{6}$$

$$z=\frac{1}{12}\cdot\frac{b\cdot D^3}{0.5D-(D-d)} \tag{7}$$

$$\therefore \qquad \varepsilon_{ci}=\frac{1}{E_c}\cdot\left[f_{pe}\cdot\frac{A_{ps}}{b\cdot D}+12f_{pe}\cdot A_{ps}\cdot\frac{(0.5D-(D-d))^2}{b\cdot D^3}\right]$$

$$=\frac{f_{pe}A_{ps}}{E_c\cdot b\cdot D}\cdot\left[1+\frac{12(d-0.5D)^2}{D^2}\right] \tag{8}$$

$$\varepsilon_{si}=f_{pe}/E_s \tag{9}$$

Therefore, strain lag between steel and concrete at the level of tendon is given by,

$$\varepsilon_{ci}+\varepsilon_{si}=\frac{f_{pe}}{E_s}+\frac{f_{pe}\cdot A_{ps}}{E_c\cdot b\cdot D}\cdot\left[1+\frac{12(d-0.5D)^2}{D^2}\right] \tag{10}$$

It can be safely assumed that the strain lag between steel and concrete remains same at all stages of loading.

Considering the strain diagram at failure,

$$\frac{x}{d}=\frac{0.0035}{0.0035+\varepsilon-(\varepsilon_{ci}+\varepsilon_{si})} \tag{11}$$

As the value of ε_{ci} is very small as compared to ε_{si}, it may be neglected in the above expression for x/d. This assumption simplifies the design of prestressed beam considerably. The validity of this assumption can be observed by carrying out order of magnitude analysis for x/d as follows.

$$\frac{x}{d}=\frac{0.0035}{0.0035+\varepsilon-(\varepsilon_{ci}+\varepsilon_{si})} \tag{12}$$

where

$$\varepsilon_{ci}=\frac{f_{pe}\cdot A_{ps}}{E_c\cdot b\cdot D}\cdot\left[1+\frac{12(d-0.5D)^2}{D^2}\right] \tag{13}$$

Typically, consider

$$\frac{A_{ps}}{b\cdot D}\approx 10^{-3}$$

$$1+\frac{12(d-0.5D)^2}{D^2}\approx 4$$

$$E_c=4500\sqrt{f_{cu}}$$

$$\varepsilon_{ci}=f_{pe}\cdot 10^{-3}\cdot 4/(4500\sqrt{f_{cu}})$$

considering

$$f_{cu}=30\ \text{N/mm}^2$$

$$\varepsilon_{ci}=1.623\cdot f_{pe}/10^7$$

$$\varepsilon_{si}=f_{pe}/E_s=f_{pe}/175000=57.143\ f_{pe}/10^7$$

As $\varepsilon_{ci}\ll\varepsilon_{si}$, it may be neglected.

$$\frac{x}{d}=\frac{0.0035}{0.0035+\varepsilon-\varepsilon_{si}}$$

The insignificant effect of ε_{ci}, on the value of x/d can also be observed by the following example.

Consider

$$f_{pu}=1550\ \text{N/mm}^2,\ f_{cu}=40\ \text{N/mm}^2,\ f_{pe}=0.4\ f_{pu},\ d/D=0.9$$

Compute

$$\frac{x_{u,l}}{d}=\frac{0.0035}{0.0085+0.87\ f_{pu}/E_s-f_{pe}/E_s}=0.2764$$

For balanced section,

$$\frac{A_{ps}}{bd}=\frac{x_{u,l}}{d}\ \frac{K_1\cdot f_{cu}}{0.87f_{pu}}=3.245\times10^{-3}$$

$$\therefore \quad \varepsilon_{si}=f_{pe}/E_s=0.4\times1550/175000=3.543\times10^{-3}$$

$$\varepsilon_{ci}=\frac{f_{pe}\cdot A_{ps}}{E_c\cdot b\cdot D}\cdot\left[1+\frac{12(d-D/2)^2}{D^2}\right]$$

$$=0.4\times1550\times0.9\times3.247\times10^{-3}\times(1+12(0.4)^2)/4500\sqrt{40}$$

$$=3.1035\times10^{-5}$$

By considering ε_{ci},

$$x_{u,l}/d=0.2805$$

By ignoring ε_{ci},

$$x_{u,l}/d=0.2764$$

The moment of resistance of the balanced section, under-reinforced and over-reinforced sections can be obtained as follows.

(1) Balanced bonded beam section: For a balanced section the value ε is given by,

$$\varepsilon=0.87f_{pu}/E_s+0.005 \tag{14}$$

$$\frac{x_{u,l}}{d}=\frac{0.0035}{0.0085+0.87f_{pu}/E_s-f_{pe}/E_s} \tag{15}$$

$$\frac{M_u}{f_{cu}\cdot b\cdot d^2}=K_1\cdot\frac{x_{u,l}}{d}\left(1-K_2\cdot\frac{x_{u,l}}{d}\right) \tag{16}$$

(2) Under-reinforced bonded beam section: For under-reinforced section, the value of x/d is determined by equilibrium of compressive and tensile forces as follows,

$$(0.87\cdot f_{pu})\cdot A_{ps}=K_1\cdot f_{cu}\cdot b\cdot x; \tag{17}$$

$$\therefore \quad \frac{x}{d}=\frac{(0.87f_{pu}\cdot A_{ps})}{K_1\cdot f_{cu}\cdot b\cdot d}$$

$$\frac{M_u}{f_{cu}\cdot b\cdot d^2}=K_1\cdot\frac{x}{\mathrm{d}}\left(1-K_2\cdot\frac{x}{\mathrm{d}}\right) \tag{18}$$

(3) Over-reinforced bonded beam section: For over reinforced section, the value of x/d is obtained as follows.

x/d as obtained from balance of compressive and tensile force is, x/d as obtained from strain diagram is,

$$\frac{x}{d}=\frac{f_{pb}A_{ps}}{k_1 f_{cu} bd} \tag{19}$$

x/d as obtained form strain diagram is,

$$\frac{x}{d}=\frac{0.0035}{0.0035+\varepsilon-f_{pe}/E_s} \tag{20}$$

In an over reinforced section, $\varepsilon<0.005+0.87f_{pu}/E_s$. The value of x/d is obtained in an iterative process which is as follows,

From Eqs. (18) and (19) f_{pb} can be determined as,

$$f_{pb}=\frac{0.0035}{0.0035+\varepsilon-f_{pe}/E_s}\cdot K_1\cdot f_{cu}\cdot\frac{b\cdot d}{A_{ps}} \tag{21}$$

The iteration is performed by assuming different values of ε till the value of f_{pb} as given by Eq. (21) and the value of f_{pb} corresponding to the value of ε from the stress strain graph compare. Then the moment of resistance can be obtained by,

$$\frac{M_u}{f_{cu}\cdot b\cdot d^2}=K_1\cdot\frac{x}{d}\left(1-K_2\cdot\frac{x}{d}\right) \tag{22}$$

To facilitate the design of rectangular prestressed beam sections using tendons of normal and low relaxation products, curves have been prepared for different values of strength of concrete, f_{cu} equal to 30, 40, 50 60 N/mm^2 (Figs. 3 to 6). These curves can be used to determine the area of steel for the given cross sectional dimensions of the beam b, d; strength of steel f_{pu}; the given moment M_u and effective prestress f_{pe}. The curves for a given value of f_{cu} are obtained by plotting $M_u/(f_{cu}\cdot b\cdot d^2)$ against $100\cdot A_{ps}/(b\cdot d)$ for a chosen value f_{pu} equal to 1550 N/mm^2, by considering different values of effective prestress in the form of the dimensionless parameter

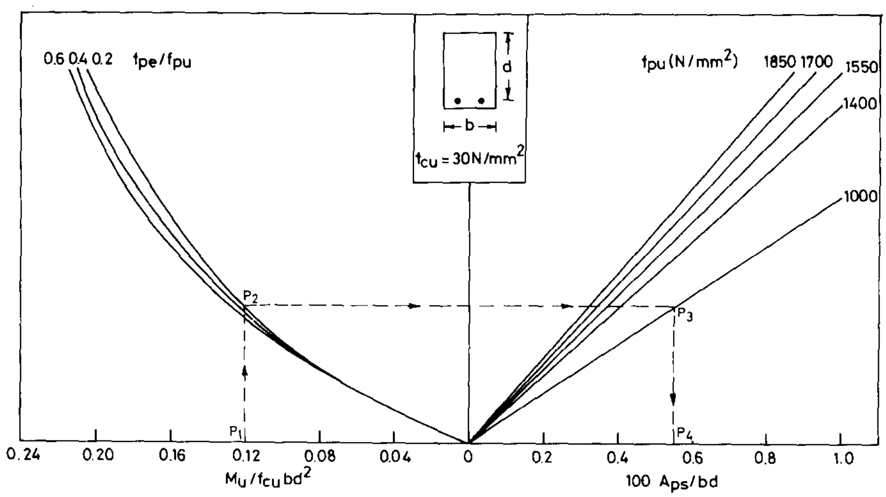

Fig. 3 Design curves for different values of f_{pu} and f_{pe}/f_{pu} for $f_{cu}=30$ N/mm^2: Tendons of normal and low relaxation products.

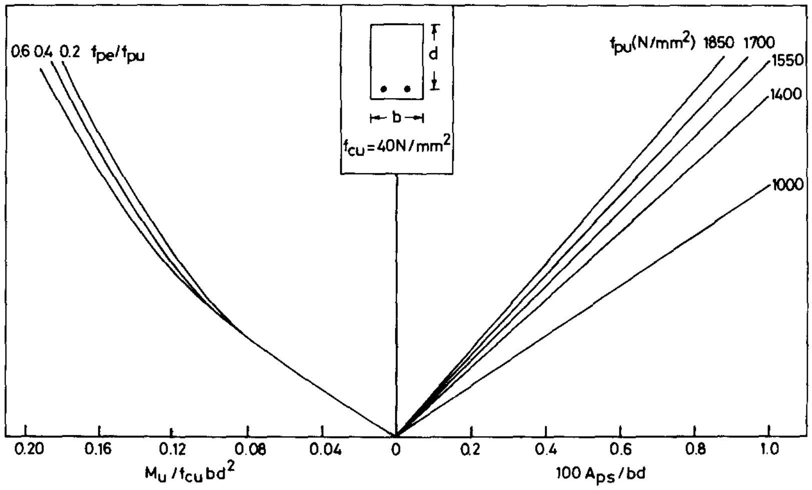

Fig. 4 Design curves for different values of f_{pu} and f_{pe}/f_{pu} for $f_{cu}=40$ N/mm^2: Tendons of normal and low relaxation products.

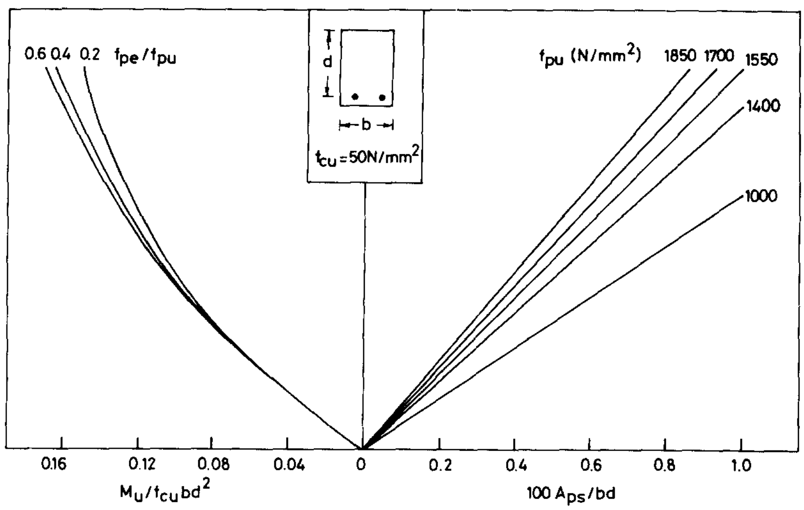

Fig. 5 Design curves for different values of f_{pu} and f_{pe}/f_{pu} for $f_{cu}=50$ N/mm^2: Tendons of normal and low relaxation products.

f_{pe}/f_{pu} equal to 0.2, 0.4 and 0.6. Thus a set of curves is obtained for the chosen value of f_{pu} and different values of f_{pe}/f_{pu}. The curves for different values of f_{pu} are obtained by transferring the axis for $100 \cdot A_{ps}/(b \cdot d)$ from vertical to horizontal position. Different points from f_{pe}/f_{pu} curves are projected horizontally. Each point corresponds to a particular value of $M_u/(f_{cu} \cdot b \cdot d^2)$ and f_{pe}/f_{pu}, for the given value of f_{cu}. For these values of $M_u/(f_{cu} \cdot b \cdot d^2)$ and f_{pe}/f_{pu}, the values of $100 \cdot A_{ps}/(b \cdot d)$ for a chosen value of f_{pu} are plotted on horizontal axis. Vertical lines are drawn from these points to intersect the corresponding horizontal lines from f_{pe}/f_{pu} curves. It is observed that these points lie on a straight line and correspond to the chosen value of f_{pu}. Similarly curves for other values of f_{pu} are plotted. Using the same procedure, design curves for different values of f_{cu} equal to 30, 40, 50 and 60 N/mm^2 have been plotted as shown in Figs. 3 to 6

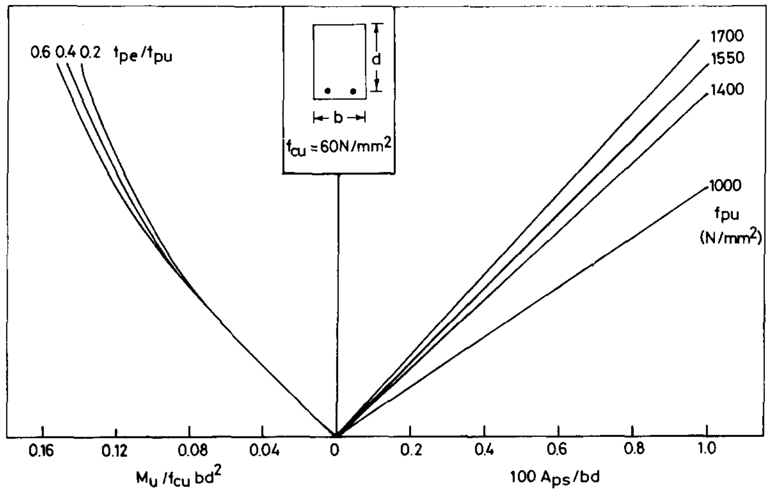

Fig. 6 Design curves for different values of f_{pu} and f_{pe}/f_{pu} for $f_{cu}=60$ N/mm^2: Tendons of normal and low relaxation products.

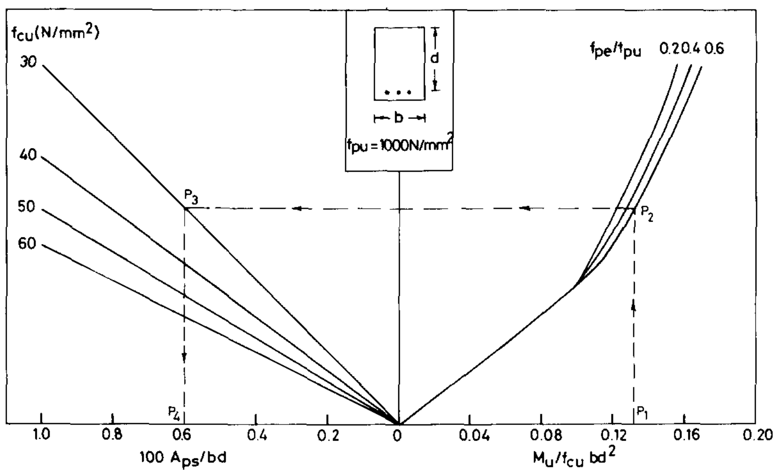

Fig. 7 Design curves for different values of f_{cu} and f_{pe}/f_{pu} for $f_{pu}=1000$ N/mm^2: Tendons of 'as drawn' wires or 'as spun' strands.

respectively. The use of the curves is simple as shown with the help of directed arrows P_1, P_2, P_3 and P_4.

Design curves for tendons of 'as drawn' wires and 'as spun' strand are shown in Figs. 7 to 11. These curves have been prepared for different values of strength of steel, f_{pu} equal to 1000, 1400, 1550, 1700 and 1850 N/mm^2. The procedure adopted for obtaining these curves is same as described above.

An attempt has been also made to obtain a single curve by incorporating the affect of all the variables. Figs. 12 and 13 show such unified graphs for tendons of normal and low relaxation products and for tendons of 'as drawn' wires and 'as spun' strands respectively, which incorporates all the variables in a single graph. These curves have been obtained by plotting $M_u/(f_{cu}\cdot b\cdot d^2)$

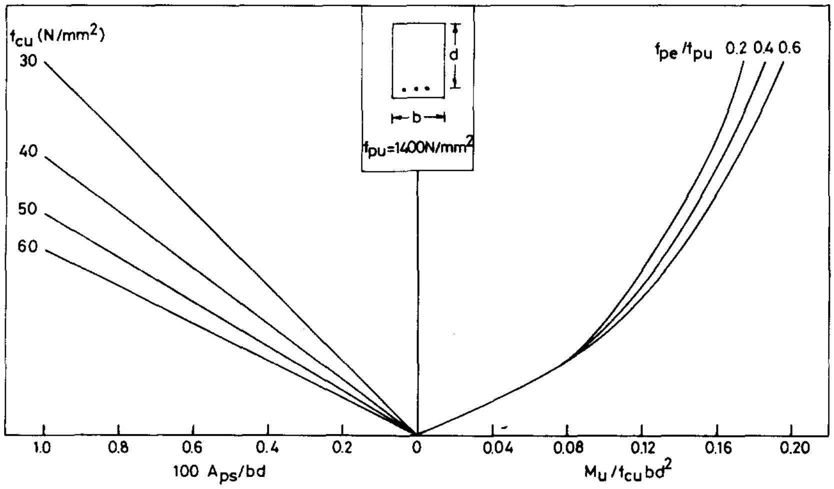

Fig. 8 Design curves for different values of f_{cu} and f_{pe}/f_{pu} for $f_{pu}=1400$ N/mm^2: Tendons of 'as drawn' wires or 'as spun' strands.

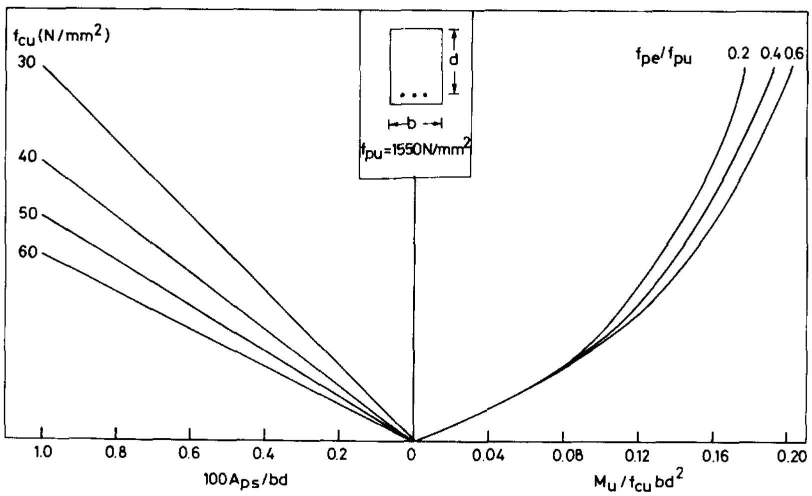

Fig. 9 Design curves for different values of f_{cu} and f_{pe}/f_{pu} for $f_{pu}=1550$ N/mm^2: Tendons of 'as drawn' wires or 'as spun' strands.

against $100 \cdot A_{ps}/(b \cdot d)$ on two orthogonal axes for a chosen value of f_{pu} equal to 1700 N/mm^2 and different values of the effective stress taken as dimensionless parameter f_{pe}/f_{pu} equal to 0.2, 0.4 and 0.6; for a given value of f_{cu} equal to 30 N/mm^2. The curves for different values of f_{pu} equal to 1000, 1400, 1550 and 1850 N/mm^2 and the given value of f_{cu} equal to 30 N/mm^2 are obtained by transferring the $M_u/(f_{cu} \cdot b \cdot d^2)$ axis from the horizontal to vertical position. The dashed lines have been employed to vary the scale of $M_u/(f_{cu} \cdot b \cdot d^2)$ for different sets of curves so that sufficient spacing is obtained between the curves. The curves for the other values of f_{cu} are drawn by transferring the $100 \cdot A_{ps}/(b \cdot d)$ axis from vertical to horizontal position. Different points from the f_{pu} curves are projected horizontally. Each point corresponds to a particular value of $M_u/(f_{cu} \cdot b \cdot d^2)$ for different values of f_{pu} and f_{pe}/f_{pu}. For these values of $M_u/(f_{cu} \cdot b \cdot d^2)$,

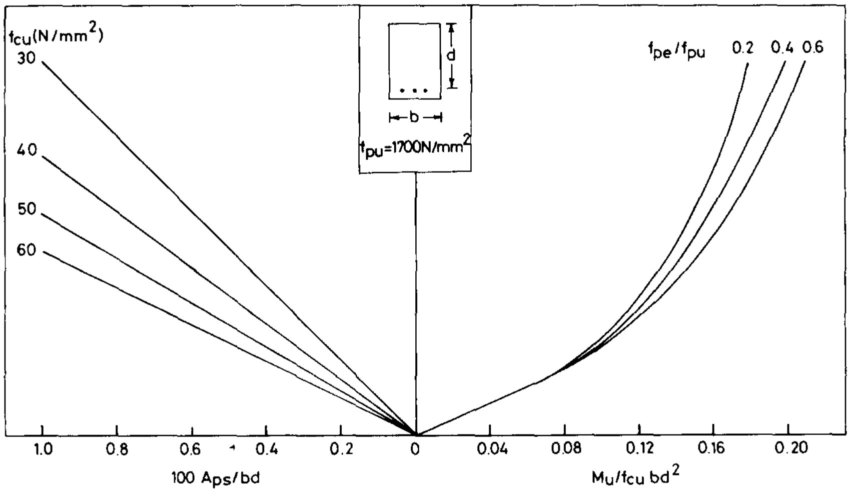

Fig. 10 Design curves for different values of f_{cu} and f_{pe}/f_{pu} for f_{pu}=1700 N/mm^2: Tendons of 'as drawn' wires or 'as spun' strands.

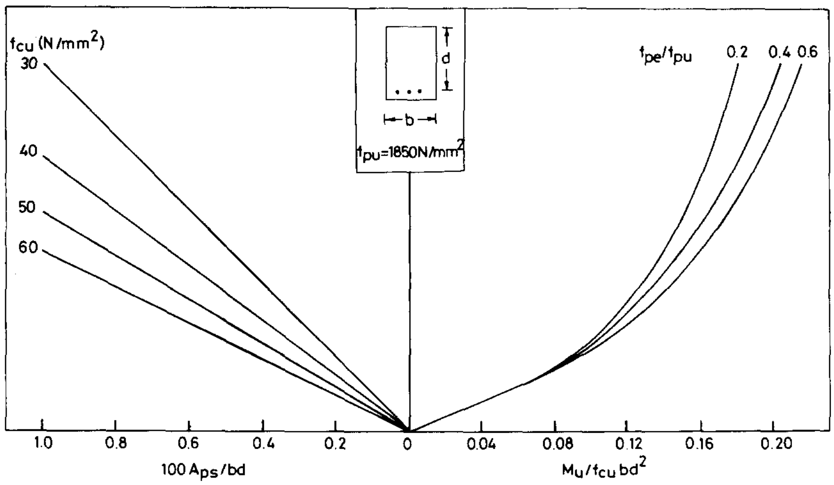

Fig. 11 Design curves for different values of f_{cu} and f_{pe}/f_{pu} for f_{pu}=1800 N/mm^2: Tendons of 'as drawn' wires 'as spun' strands.

f_{pu} and f_{pe}/f_{pu}, the values of $100 \cdot A_{ps}/(b \cdot d)$ for the chosen value of f_{cu} are plotted on the horizontal axis. Vertical lines are drawn from these points to intersect the corresponding horizontal lines from the f_{pu} curves. It is observed that all such points corresponding to a particular value of f_{cu} lie on a straight line, with very little scatter. The same procedure has been adopted for drawing the curves for different values of f_{cu} equal to 40, 50 and 60 N/mm^2. The use of the curves is simple and is demonstrated with the help of directed arrows P_1, P_2, P_3, P_4 and P_5.

3. Example

Calculate the area of steel for tendons of normal and low relaxation products required for

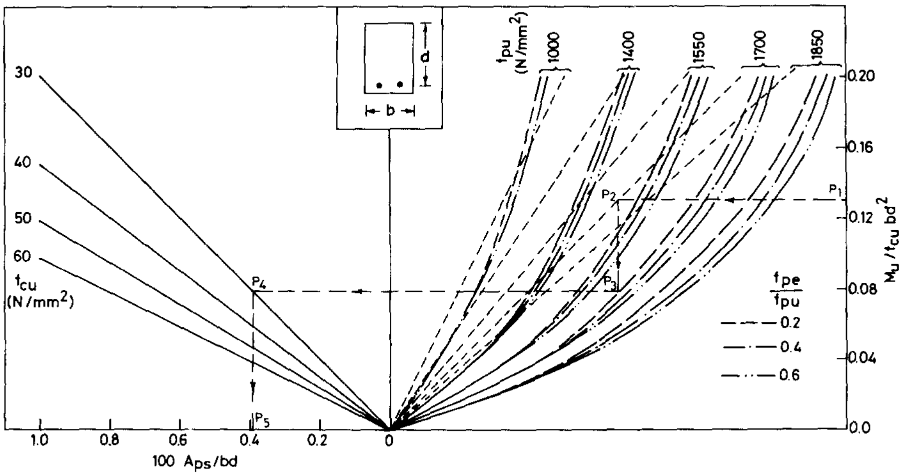

Fig. 12 Unified curves for all values of f_{cu}, f_{pu}, f_{pe}/f_{pu}: Tendons of normal and low relaxation products.

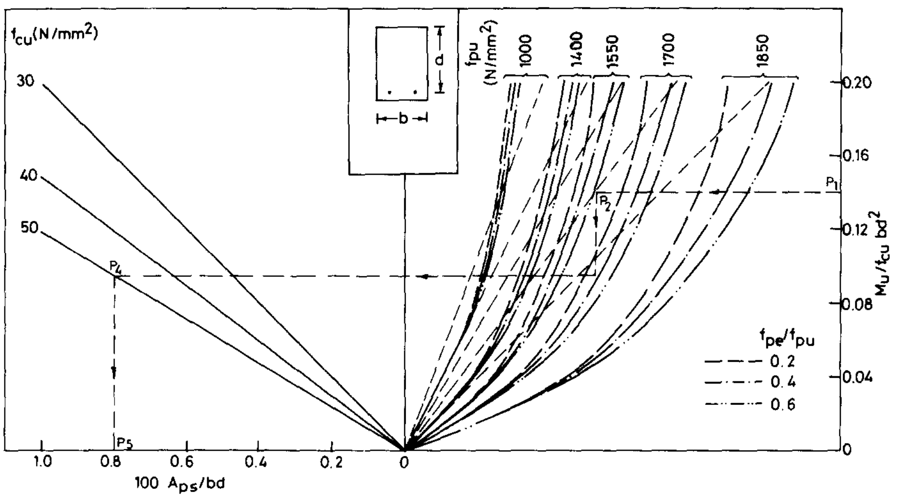

Fig. 13 Unified curves for all values of f_{cu}, f_{pu}, f_{pe}/f_{pu}: Tendons of 'as drawn' wires or 'as spun' strands.

rectangular beam section of width 400 mm, depth 1200mm, effective depth 870 mm, subjected to ultimate moment of 2500 kNm.

Consider $f_{cu}=60$ N/mm^2, $f_{pu}=1700$ N/mm^2 and $f_{pe}=0.4f_{pu}$ and $M_u=2500$ kN·m.

Compute,

$$M_u/(f_{cu}\cdot b\cdot d^2)=0.1375$$

$$\therefore \quad M_u/(b\cdot d^2)=8.25$$

Using design curves in CP 110,

$$100\cdot A_{ps}/(b\cdot d)=0.825$$

Using design curves given in Approach I (Fig. 1),

$$f_{pu} \cdot A_{ps}/(f_{cu} \cdot b \cdot d) = 0.275$$

$$\therefore \quad 100 \cdot A_{ps}/(b \cdot d) = 0.97$$

Using design curves given in Approach II (Fig. 6),

$$100 \cdot A_{ps}/(b \cdot d) = 0.825$$

Using unified design curve (Fig. 12),

$$100 \cdot A_{ps}/(b \cdot d) = 0.83$$

It is observed that the value of A_{ps} computed using design curves given in Approach I is significantly conservative than the values obtained by using design curves in CP 110 and the design curves based on Approach II. This is because of the approximation made in deriving the values of x/d in BS 8110. Therefore the use of design curves based on Approach II is recommended.

4. Conclusions

The use of design curves as presented for pretensioned or bonded post tensioned prestressed concrete rectangular section is simple. The number of curves is considerably less due to the presentation of a large number of variables in a single graph. It is observed that curves given in Approach I give conservative values of A_{ps}, than the values of A_{ps} obtained from CP 110 or the curves based on Approach II. Therefore the use of curves based on Approach II is recommended.

Notations

b	breadth of the beam
d	effective depth of the beam
x	depth of neutral axis from extreme compression fibre of concrete
A_{ps}	area of prestressing tendons
f_{cu}	characteristic strength of concrete
f_{pu}	characteristic tensile strength of steel
M_u	ultimate design moment
$M_{u,l}$	ultimate moment of balanced section
$x_{u,l}$	depth of neutral axis from extreme compression fibre of concrete for a balanced section
f_{pe}	effective prestress in tendon after all losses
ε_{si}	strain in tendon due to prestress after all losses
ε_{ci}	strain in concrete at level of tendon due to prestress after all losses
ε	strain in tendon at the time of failure
f_{pb}	stress in tendon at failure corresponding to strain ε
z	section modulus of the concrete section

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