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Weighting objectives strategy in multicriterion fuzzy mechanical and structural optimization

C.J. Shih†

Department of Mechanical Engineering, Tamkang University, Tamsui, Taiwan 25137, China

K. C. Yu‡

Center of Vehicle Design and Development,
Yeu Tyan Machinery Manufacturing Company, Chang-Hwa, Taiwan 51500, China

Abstract. The weighting strategy has received a great attention and has been widely applied to multicriterion optimization. This paper examines a global criterion method (GCM) with the weighting objectives strategy in fuzzy structural engineering problems. Fuzziness of those problems are in their design goals, constraints and variables. Most of the constraints are originated from analysis of engineering mechanics. The GCM is verified to be equivalent to fuzzy goal programming via a truss design. Continued and mixed discrete variable spaces are presented and examined using a fuzzy global criterion method (FGCM). In the design process a weighting parameter with fuzzy information is introduced into the design and decision making. We use a uniform machine-tool spindle as an illustrative example in continuous design space. Fuzzy multicriterion optimization in mixed design space is illustrated by the design of mechanical spring stacks. Results show that weighting strategy in FGCM can generate both the best compromise solution and a set of Pareto solutions in fuzzy environment. Weighting technique with fuzziness provides a more relaxed design domain, which increases the satisfying degree of a compromise solution or improves the final design.

Key words: weighting objectives strategy; multicriterion fuzzy optimization; fuzzy goal programming; fuzzy global criterion method; fuzzy weighting technique; continuous design space; mixed-discrete design space; truss; machine-tool spindle; mechanical spring.

1. Introduction

A decision maker often encounters a design problem that attain more than one objective or goal. Those objectives are apparently non-commensurable and fuzzy. The design constraint, design environment and even design variable are frequently imprecise to define. A wonderful article by Hwang, Paidy and Yoon in 1980, had pointed out that the development of methods for "fuzzy" area which combines multicriterion optimization would be a topic for future research. Today, one can see many successful developments of method and application of fuzzy logic in optimization. Nevertheless, the combination of the fuzzy theory and multicriterion optimization has not been fully explored yet.

† Associate Professor
‡ Research Assistant

The father of fuzzy theory, Zadeh (1965), gave the first guidance in dealing with a model containing vague and imprecise nature. We may appreciate the earlier work of single objective structural optimization done by Brown and Yao (1993) who studied several civil engineering designs with application of the fuzzy theory. Wangs (1985) initiated the well-known level-cut method using fuzzy logic. Then it was improved by Xu (1989) with the bound search algorithm. Perhaps Rao's paper in 1987 was the first work that apply fuzzy logic to multiple design goals in structural engineering. A comparative study of the level-cut approach (Wang 1985) and the λ-formulation (Rao 1987) was investigated with the designs of a 3-bar and a 25-bar truss. The conclusion was that the level-cut approach provides a parametric form and the λ-formulation yields a compromise solution. Three models of fuzzy goal programming were presented (May 1992) by Rao, Sundararaju, Prakash and Balakrishna. They concluded that the simple additive model with or without fuzzy constraints will result the better or the more balanced compromise design.

The simple additive model described above is the first appearance that uses the weighting technique in fuzzy multicriterion structural design. Other weighting strategies developed by Shih and Lai (1994) were discussed in an environment of fuzzy and multiple goals. However if the design space contains a mix of integer, discrete and real continuous variables, what is the formulation and algorithm for solving such fuzzy and multiple objectives problems. The algorithm of branch-and-bound has been widely recognized as a reliable method for obtaining a solution to the crisp optimization problem with mixed variables. Typical efforts are the works of Gupta & Ravindran (1983) and Sandgren (1990). The main disadvantage is that an unestimated number and a large amount of nonlinear subproblems are generated in this method. A technique called the maximum-partial derivative branching was developed by Shih and Chi (1992) to overcome this shortage. This paper introduces the combining algorithm of FGCM and modified branch-and-bound for dealing with multicriterion fuzzy problems in mixed design space. People are also interested in what will happen when the weighting parameters contain fuzzy information in continuous or mixed design space.

We first examine the design of a 3-bar truss with the simple additive method and the global criterion method (Osyczka 1984). Then a mechanical structural problem is studied which shows the effects of fuzzy objectives/constraints and also fuzzy relative priority of objectives. An optimum design of mechanical spring stacks in fuzzy mixed variables space with crisp or fuzzy weighting techniques is introduced at later section. All the above methods, algorithms and numerical results are presented and discussed in this paper subsequently.

2. Comparison of simple additive and global criterion method

A function describes the global criterion is a measure of "distance to the ideal vector of f^{id}". A preferred relative L_d-metric form defined as follows (Osyczka 1984):

$$L_d(f)=\left[\sum_{i=1}^{k}\left|\frac{f_i(X)-f_i^{id}(X)}{f_i^{id}(X)}\right|^d\right]^{1/d},\quad 1\leq d\leq\infty \tag{1}$$

The global criterion method with $L_\infty(f)$ metric is commonly called the min-max approach. Usually the min-max approach can be considered as a scalar optimization problem. By minimizing this function, one can obtain a Pareto optimal solution or a set of such solutions with weighting objectives method. The mathematical formulations in a continuous design space are:

$$\text{minimize a scalar } \beta \tag{2}$$

$$s.t.\ g_j(X) \le 0, \qquad j=1,2,\cdots,m \tag{3}$$

$$h_k(X) \le 0, \qquad k=1,2,\cdots,q \tag{4}$$

$$\omega_i \left| \frac{f_i(X)-f_i^{id}(X)}{f_i^{id}(X)} \right| -\beta \le 0, \qquad i=1,2,\cdots,p \tag{5}$$

$$\sum_{i=1}^{p} \omega_i = 1 \tag{6}$$

Where the vector of design variables is $X=[x_1, x_2, \cdots, x_N, \beta]^T$ contains $(N+1)$ real continuous parameters, and $\omega_i(\ge 0)$ are the weighting coefficients representing the relative importance of the design goals.

The additive formulations of the fuzzy goal programming with weighting objectives strategy are:

$$\text{maximize } F(X)=\sum_{i=1}^{p} \mu_{fi}(X) \tag{7}$$

subject to

$$g_j(X) \le 0, \qquad j=1,2,\cdots,m \tag{8}$$

$$h_k(X) \le 0, \qquad k=1,2,\cdots,q \tag{9}$$

$$0 \le \mu_{fi}(X) \le 1 \tag{10}$$

$$\omega_i \left| \frac{f_i(X)-f_i^{id}(X)}{f_i^{id}(X)} \right| = \omega_{i+1} \left| \frac{f_{i+1}(X)-f_{i+1}^{id}(X)}{f_{i+1}^{id}(X)} \right|, \qquad i=1,2,\cdots,p \tag{11}$$

$$\sum_{i=1}^{p} \omega_i = 1 \tag{12}$$

where $f^{id}(X)$ is the ideal solution of the objective function and $\mu_{fi}(X)$ represents the following form:

$$\mu_{fi}(X)=\frac{-f_i(X)+b_i+t_i}{d_i}, \qquad i=1,2,\cdots,p \tag{13}$$

where b_i, t_i and d_i are the ideal solution, the fuzzy zone of tolerance and the tolerance limit of ith design goal, respectively.

A 3-bar truss shown in Fig. 1 is considered as the example comparing the effect of both fuzzy goal and global criterion approaches. The definition and mechanics of this problem are the same as Rao's paper, 1987. The minimum structural weight f_1 and the displacement f_2 of a loading point are two design goals. The optimum results are summarized in Table 1, where α_i is calculated by following formula:

$$\alpha_i=\frac{f_i^{max}-f_i(X^*)}{f_i^{max}-f_i^{min}}, \qquad i=1,2,\cdots,p \tag{14}$$

Table 1 shows that both methods have the same results. The sum of α_1 and α_2 is the same as $(\mu_{f1}+\mu_{f2})$. Therefore, these two formulations are exchangeable. We prefer to use the global criterion method with weighting strategy because of its simplicity.

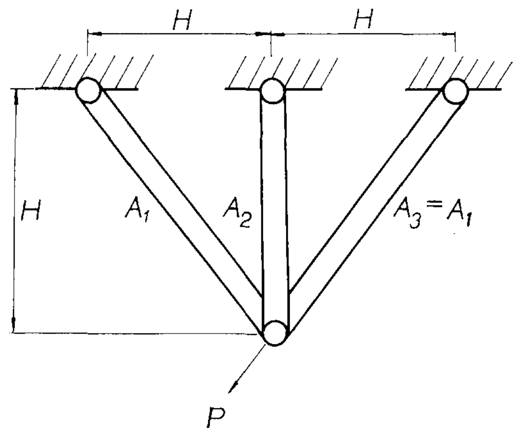

Fig. 1 A three-bar truss for multicriterion design.

Table 1 Results of a 3-bar truss by the additive fuzzy goal and the GCM with weighting objectives strategy

Weightings		Global criterion approach				Additive fuzzy goal		
ω_1	ω_2	f_1	f_2	α_1	α_2	f_1	f_2	$\mu_{f1}+\mu_{f2}$
0.0	1.0	19.142	1.657	0.0000	1.0000	19.142	1.657	1.0000
0.2	0.8	7.840	2.477	0.6846	0.9370	7.845	2.477	1.6213
0.3	0.7	6.217	2.871	0.7829	0.9067	6.217	2.871	1.6896
0.4	0.6	5.934	3.042	0.8001	0.8936	5.935	3.041	1.6937
0.5	0.5	5.418	3.409	0.8313	0.8653	5.419	3.409	1.6966
0.6	0.4	4.943	3.836	0.8601	0.8326	4.942	3.836	1.6927
0.7	0.3	4.710	4.082	0.8742	0.8137	4.708	4.084	1.6879
0.8	0.2	3.996	5.086	0.9175	0.7365	3.996	5.086	1.6540
1.0	0.0	2.634	14.672	1.000	0.000	2.634	14.672	1.000

3. Weighting strategy and fuzzy constraints in FGCM

For a problem containing fuzzy design and side constraints, we need to establish the membership functions, μ_β of scalar objective and μ_{gj} of constraint.

3.1. Crisp weighting objectives method

The formulations of FGCM combined with crisp weighting technique are written as:

$$\text{maximize } \lambda \tag{15}$$

$$s.t. \ \lambda \leq \mu_\beta(X) \tag{16}$$

$$\lambda \leq \mu_{gj}(X) \tag{17}$$

and other constraints which are the same as Eqs. (4)-(6). The vector of design variables is $X=[x_1, x_2,\cdots,x_N, \beta, \lambda]^T$.

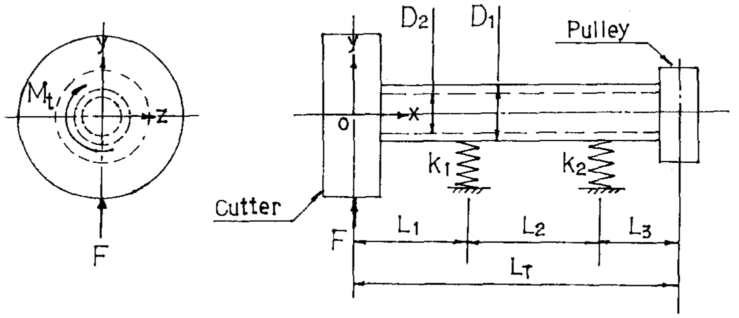

Fig. 2 Structural model of a uniform spindle for weighting strategy in real continuous design space.

Table 2 Fuzzy optimum results of a machine-tool spindle with crisp degree of importance

Weightings		Design variables		Fuzzy optimization			Crisp optimization	
ω_{st}	ω_W	D_1(m)	D_2(m)	f_{st}(rad)	W_T(kg)	λ	f_{st}(rad)	W_T(kg)
0.45	0.55	0.1018	0.0832	0.2126E−5	21.25	0.66	0.2158E−5	22.15
0.50	0.50	0.1020	0.0824	0.2074E−5	21.96	0.72	0.2146E−5	22.19
0.60	0.40	0.1031	0.0820	0.1872E−5	24.04	0.85	0.1983E−5	24.60
0.70	0.30	0.1047	0.0818	0.1679E−5	26.36	0.76	0.1795E−5	28.22

3.1.1. Design of a machine-tool spindle

A simple model of a uniform machine-tool spindle is shown in Fig. 2. The loading conditions are torsional moment M_t and normal cutting force F. Minimization of static torsional compliance f_{st} is the same as minimization of static bending compliance f_{sb} for design D_1 and D_2. For comprehensive description please refer to Yoshimura, *et al.* (1983). The present problem minimizing both the total weight W_T and f_{st} is formulated as follows:

$$\text{minimize } W_T = \frac{\pi \rho L_T (D_1^2 - D_2^2)}{4} \tag{18}$$

$$\text{minimize } f_{st} = \frac{32 L_T}{\pi G (D_1^4 - D_2^4)} \tag{19}$$

$$s.t. \;\; D_1 - D_{max} \le 0 \tag{20}$$

$$D_{min} - D_1 \le 0 \tag{21}$$

$$2t_{min} - (D_1 - D_2) \le 0 \tag{22}$$

Numerical data that were used are $L_T = 1.0$ m, $D_{min} = 0.05$ m, $D_{max} = 0.1$ m, $t_{min} = 0.01$ m, $\rho = 7.85(10^3)$ kg/m^3 and $G = 8.04(10^{10})$ N/m^2. The problem assumes a 20% fuzzy zone of constraints. Final designs of the fuzzy and crisp optimization are shown in Table 2. It is obvious that the two goals in fuzzy formulation are smaller than that in crisp formulation. A larger weighting of machine-tool spindle yields a smaller objective value.

3.2. Fuzzy weighting objective method

In this case, the weighting coefficients are taken as design variables with fuzzy tolerance.

Table 3 Fuzzy results of a machine-tool spindle with fuzzy degree of importance

Weightings		Design variables		Fuzzy optimization		
ω_{st}	ω_W	D_1(m)	D_2(m)	f_{st}(rad)	W_T(kg)	λ
0.448	0.552	0.1023	0.0839	0.2114E−5	21.24	0.63
0.513	0.487	0.1033	0.0843	0.2007E−5	21.90	0.74
0.610	0.390	0.1039	0.0831	0.1850E−5	23.85	0.80
0.715	0.285	0.1060	0.0839	0.1653E−5	25.87	0.70

It is nature to do so because the relative importance of objectives is frequently not precisely defined. If a linear membership is assumed, then it can be expressed as:

$$\mu_{\omega i}=\frac{\omega_{ci}+\delta_R-\omega_i}{\delta_R}, \qquad \text{if } \omega_{ci}\leq\omega_i\leq\omega_{ci}+\delta_R \tag{23}$$

$$\mu_{\omega i}=\frac{\omega_i-\omega_{ci}+\delta_L}{\delta_L}, \qquad \text{if } \omega_{ci}-\delta_L\leq\omega_i\leq\omega_{ci}, \qquad i=1,2,\cdots,p \tag{24}$$

where ω_{ci} represents the weighting value, and has the highest satisfaction. δ_L and δ_R represent the allowable fuzzy interval on the left and right-hand side, respectively. Thus, the formulation of fuzzy multicriterion with fuzzy weighting strategy can be written as Eqs. (15)-(17), (4)-(6), (23), (24). A $\pm 5\%$ of weighting range is assumed to be fuzzy, optimum results are listed in Table 3.

We see the results of f_{st} and W_T in Table 3 are smaller than that in Table 2. Since the weighting coefficients are relaxed it yields a better result.

4. Weighting strategy and FGCM in mixed design space

A nonlinear design/analysis space often contains a mixture of integer, discrete, continuous variables, or even simply zero or one. A task usually requires us to optimize two or more criteria simultaneously in such a mixed design domain. The mathematical formulation of this category is similar to Eqs. (15)-(17) and (4)-(6) except the set of design variables represented as:

$$X=[x_1,\ x_2,\cdots,x_L,\cdots,x_M,\cdots,x_N,\ \beta,\ \lambda]^T \qquad x_i^l\leq x_i\leq x_i^u \tag{25}$$

in which the design variable contain L nonnegative discrete variables, $M-L$ nonnegative integer variables and $N-M$ positive real variables. In this paper, a modified branch-and-bound algorithm (Shin and Chi 1993) is applied to FGCM to solve the optimization problem in a mixed design space.

4.1. Crisp weighting objective method

For the entire description and the crisp weighting objectives method please refer to the recent work of Shih, Lai and Chang (1994). We use an example to illustrate the function of this merging method.

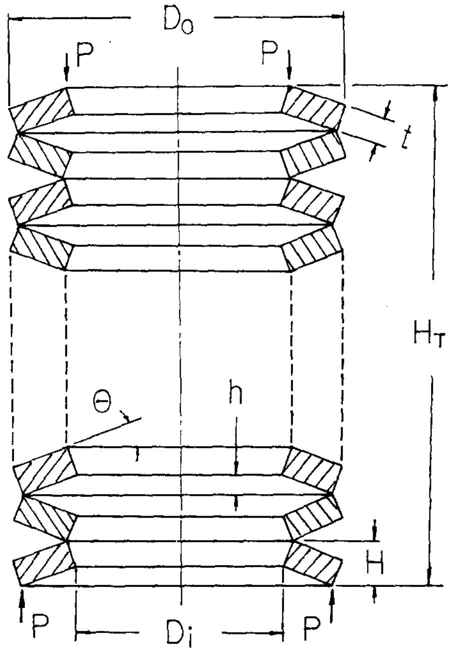

Fig. 3 Belleville spring stacks for weighting strategy in mixed design space.

4.1.1. Design of mechanical belleville spring stacks

A detail of a single disk spring under external loading and associated mechanics is available in the handbook of mechanical spring. The geometrical shape of a series belleville spring stacks is shown in Fig. 3. The applied load uniformly around the end of the stack for a prescribed deflection of 0.75h is to be maximized. Total weight W_T of this stack and the total height H_T of the stack have to be minimized. The thickness of a single disk spring t (mm), outside diameter D_o (mm) and internal diameter D_i (mm) are the variables of discrete type. The number of individual spring in this stack N is an integer variable. The cone angle θ (deg.) is a variable of the continuous type. The design problem is formulated by three design goals and six inequality constraints as follows:

$$\text{maximize} \quad P=\frac{Ey}{(1-v^2)MR^2}\left[(h-y/2)(h-y)t+t^3\right] \tag{26}$$

$$\text{minimize} \quad W_T=N\rho(V_1+V_2+V_3+V_4) \tag{27}$$

$$\text{minimize} \quad H_T=N[R\sin\theta+t\cos\theta-(D_o/150)\sin\theta\cos\theta] \tag{28}$$

subject to

$$g_1=1.5D_i/D_o-1\leq 0 \tag{29}$$

$$g_2=D_i/(2.5D_o)-1\leq 0 \tag{30}$$

$$g_3=40/Ny-1\leq 0 \tag{31}$$

$$g_4=S_1/S_{1all}-1\leq 0 \tag{32}$$

Table 4 The crisp optimization for belleville spring stacks with crisp weighting coefficients

Optimized OBJ.	Weighting Coef. (P, W_T, H_T)	Design variables t	D_o	D_i	N	θ^o
P=81702.8N W_T=113.28N H_T=114.36N	(0.33, 0.33, 0.33)	7.49	200	94	8	4.041
P=70149.09N W_T=110.69N H_T=114.36N	(0.30, 0.35, 0.35)	7.10	201	91	8	4.088
P=70209.4N W_T=105.49N H_T=110.45N	(0.25, 0.35, 0.40)	7.10	200	97	8	3.980
P=60610.6N W_T=91.84N H_T=98.81N	(0.20, 0.30, 0.50)	6.50	205	94	7	4.411

Table 5 The fuzzy optimization for belleville spring stacks with crisp weighting coefficients

Optimized OBJ.	Weighting Coef. (P, W_T, H_T)	Design variables t	D_o	D_i	N	θ^o	Design degree λ
P=89685.0N W_T=102.38N H_T=104.88N	(0.33, 0.33, 0.33)	7.49	202	93	7	4.409	0.319
P=88577.0N W_T=101.07N H_T=103.88N	(0.30, 0.35, 0.35)	7.49	201	93	7	4.347	0.438
P=79067.0N W_T=98.35N H_T=103.42N	(0.25, 0.35, 0.40)	7.10	203	93	7	4.492	0.380
P=76804.0N W_T=96.17N H_T=92.09N	(0.20, 0.30, 0.50)	7.10	214	93	6	4.580	0.400

$$g_5 = S_2/S_{2all} - 1 \leq 0 \tag{33}$$

$$g_6 = S_3/S_{3all} - 1 \leq 0 \tag{34}$$

Table 4 and 5 summarizes optimum designs obtained by using crisp and fuzzy design constraints, respectively. A 15% fuzzy transition zone is allowed for the constraints and it is expressed by a linear membership function.

Apparently the fuzzy optimum solutions result in a larger loading force, a smaller total weight and a smaller total height. This shows an obvious design improvement exists in the fuzzy optimization.

4.2. Fuzzy weighting objective method

A ± 5% of fuzzy transition zone on weighting coefficients is given in the problem formulation.

Table 6 The fuzzy optimization for belleville spring stacks with fuzzy weighting coefficients

Optimized OBJ.	Weighting Coef. (P, W_T, H_T)	Design variables t	D_o	D_i	N	Design degree θ^o	λ
P=87531.0N W_T=100.36N H_T=103.10N	(0.309, 0.345, 0.346)	7.49	200	92	7	4.302	0.511
P=87745.0N W_T=99.78N H_T=103.02N	(0.288, 0.338, 0.374)	7.49	200	93	7	4.295	0.503
P=77654.0N W_T=97.64N H_T=102.02N	(0.240, 0.352, 0.408)	7.10	202	92	7	4.428	0.518
P=83439.0N W_T=101.28N H_T=101.84N	(0.226, 0.305, 0.469)	7.49	204	99	7	4.110	0.380

The other formulations are the same as that in section 4.1. The results are presented in Table 6.

Comparing the results of Table 5 and Table 6, we see that there is some improvement on total weight and total height. The degree of satisfaction can be dramatically increased sometimes.

5. Observations and discussions

In the additive model of fuzzy goal programming, Eq. (11) restricts the relative degrees of importance of objectives. The relative degree of importance is defined as the closest distance to the individual ideal objective. This provides a best definition and meaning for weighting objectives method.

Comparing Table 2 and Table 3, we found out that fuzzy weighting strategy gives a better objective result than that in crisp weighting strategy. The relaxation of weighting parameters is positively beneficial to the final design in continuous design space. However, it is not positively efficient in a mixed design space. That is reasonable because a discrete space cannot provide a continuous variation of variables. Generally the degree of overall satisfaction λ in Table 3 is smaller than that in Table 2. Nevertheless, in Table 6 it is generally larger than that in Table 5.

6. Conclusions

We have introduced the weighting objectives strategy into fuzzy goal programming for structural design. And the results are the same as the fuzzy global criterion method. The optimum design of a uniform machine-tool spindle depicts the fuzzy global criterion method with the fuzzy weighting information. The results suggest that fuzzy weighting strategy is superior to the crisp weighting strategy in continuous design space. However, the general degree of design level is not certainly increased.

The degree of design level in mixed design space of fuzzy weighting is larger than that in crisp weighting environment. In this situation, some design goals are improved and some are not. All of them show the advantage of fuzzy weighting strategy is to provide more alternatives in decision making.

This paper also shows the feasibility of the fuzzy global criterion method with weighting objectives technique in both the continuous design or mixed design space. If the weighting is not given, it is the case of equal important of objectives. Any assign weighting coefficient in FGCM always results in a solution within Pareto solutions. The proposed weighting strategy in fuzzy global criterion method can be applied to other engineering discipline and analysis.

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