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Structural damping for soil-structure interaction studies

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Abstract. A soil-structure interaction formulation is used here which is based on consideration of the dynamics of the structure with a free, rather than a fixed, base. This approach is shown to give a quite simple procedure for coupling the dynamic characteristics of the structure to those of the foundation and soil in order to obtain a matrix formulation for the complete system. In fixed-base studies it is common to presume that each natural mode of the structure has a given fraction of critical damping, and since the interaction formulation uses a free-base model, it seems natural for this situation to assign the equal modal damping values to free-base modes. It is shown, though, that this gives a structural model which is significantly different than the one having equal modal damping in the fixed-base modes. In particular, it is found that the damping matrix resulting in equal modal damping values for free-based modes will give a very significantly smaller damping value for the fundamental distortional mode of the fixed-base structure. Ignoring this fact could lead one to attribute dynamic effects to interaction which are actually due to the choice of damping.

Key words: damping; modal analysis; modeling; seismic analysis; soil-structure interaction; vibration.

1. Introduction

For any complex problem there are apt to be several reasonable ways to formulate the details of the mathematical analysis. Particularly for a linear system one can easily show that the different procedures are strictly equivalent if used properly, so that the choice of formulation is only significant inasmuch as it may affect the efficiency or accuracy of the computations. There are situations, though, in which one may be inclined to actually vary some details of the physical model being studied, based on the formulation of the mathematical analysis. The case which prompts the current study is the choice of damping in a structural model to be used in seismic analysis.

The mass and stiffness of a structural model are usually chosen based on logical estimation of the appropriate values for the physical prototype system. These estimates are based on consideration of the elements in the structure which will contribute to kinetic and potential energy, respectively. On the other hand, one can rarely identify with any confidence or accuracy the amount of energy dissipation which will occur in various elements of the structural prototype. Rather, past experience is used to justify the choice of some level of damping which is considered

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appropriate, and the precise assignment of this damping to various elements of the structural model is done in some ad hoc fashion (Clough and Penzien 1993). Often the primary consideration is to assign the damping in a way which will expedite the mathematical analysis. A prime example of this is the selection of modal damping values (often chosen as equal values in all modes) after doing a modal analysis of the undamped structure. This is certainly a logical pragmatic approach in the absence of evidence that this assignment is incorrect, but this paper will demonstrate that there are situations in which the method may lead to undesired results. In particular, it will be shown that an apparently small variation in the choice of the damping values can lead to a model with significantly different dynamic response behavior.

Structural analysis, including modal analysis and damping selection, has usually been done with models for which the base of the structure is "fixed", having no components of translation or rotation. For problems of soil-structure interaction, of course, this fixity condition is no longer true, and more general models must be analyzed (Wolf and Somaini 1986, Luco and Mita 1987, Veletsos and Prasad 1989). Furthermore, the selection of a particular formulation for the equations for such a generalized model might lead the analyst to choose one distribution of damping rather than another. In particular, in using an assignment of modal damping values, one might choose to use either the fixed-base or the free-base modes of the structure. It will be shown that this choice has important consequences, so should be seriously considered.

The approach will be first to present a fairly general procedure for obtaining a mathematical description of the dynamic behavior of a soil-foundation-structural system. This will be illustrated with one specific choice for the coordinates used in describing the motion. Techniques will be presented for using either the fixed-base or the free-base structural modes in the analysis. Finally, it will be shown that the assignment of equal modal damping values in these two formulations leads to significantly different structural models.

2. Soil-structure interaction

Let the vector $\boldsymbol{u}$ consist of the components of motion of the rigid body foundation of the structure. In the most general situation, then, $\boldsymbol{u}$ has six components, consisting of three translations and three rotations. We will let the soil be modeled as an elastic medium and replace the building and foundation by a massless rigid foundation. Let $\boldsymbol{u}_0(t)$ denote $\boldsymbol{u}(t)$ in the absence of any seismic motion, it is then conceptually possible to find a 6×6 matrix which relates the vector $\boldsymbol{u}_0(t)$ to the corresponding vector $\boldsymbol{f}_{soil}(t)$ giving the six components of force and moment applied to the massless foundation (Veletsos and Meek 1974, Luco and Mita 1987). This could be written as a convolution integral in the time domain, but it is more convenient here to write the frequency domain relationship as

$$\tilde{\boldsymbol{u}}_0(\omega)=\boldsymbol{H}_{soil}(\omega)\tilde{\boldsymbol{f}}_{soil}(\omega) \tag{1}$$

in which the symbol ~ over a quantity denotes a Fourier transform. Next let $\boldsymbol{u}_{seis}$ denote the seismic motion which the rigid massless foundation would experience in the absence of any structure or any forces or stresses applied to the top of the foundation. From superposition one can then say that the total foundation motion is described by

$$\tilde{\boldsymbol{u}}(\omega)=\tilde{\boldsymbol{u}}_0(\omega)+\tilde{\boldsymbol{u}}_{seis}(\omega)=\boldsymbol{H}_{soil}(\omega)\tilde{\boldsymbol{f}}_{soil}(\omega)+\tilde{\boldsymbol{u}}_{seis}(\omega) \tag{2}$$

Similarly, one can analyze the structure-foundation model (including the mass of the foundation), and relate the motion vector $\boldsymbol{u}(t)$ to the vector of forces and moments applied to this subsystem.

$$\tilde{\boldsymbol{u}}(\omega)=\boldsymbol{H}_{struc}(\omega)\tilde{\boldsymbol{f}}_{struc}(\omega) \tag{3}$$

Equilibrium then requires that $\boldsymbol{f}_{struc}=-\boldsymbol{f}_{soil}$ and this term can be eliminated from Eqs. (2) and (3) to give

$$\tilde{\boldsymbol{u}}(\omega)=\boldsymbol{H}_T(\omega)\tilde{\boldsymbol{u}}_{seis}(\omega) \tag{4}$$

with

$$\boldsymbol{H}_T(\omega)=\boldsymbol{H}_{struc}(\omega)[\boldsymbol{H}_{struc}(\omega)+\boldsymbol{H}_{soil}(\omega)]^{-1} \tag{5}$$

Thus, in principle, the problem is straightforward once the $\boldsymbol{H}_{soil}$ and $\boldsymbol{H}_{struc}$ matrices are found. Note that Eqs. (4) and (5) are descriptions of what is commonly called inertial coupling or kinetic coupling between the structure and the soil. The so-called kinematic coupling (Scanlan 1976, Bycroft 1980, Harichandran 1987, Veletsos and Prasad 1989) is already included in the determination of the $\boldsymbol{H}_{soil}$ matrix, and will not be discussed here. Stated another way, the components of $\boldsymbol{u}_{seis}$ are not the same as the free-field motion in the absence of a foundation, but rather represent the response of a massless rigid foundation to the free-field motion. The emphasis here is on the other side of the model, namely the determination of the $\boldsymbol{H}_{struc}$ matrix, and investigations of how that matrix is affected by the choice of structural damping.

It should be noted that typically it is possible to subdivide the matrices in Eqs. (1)-(5) so that one can solve several smaller problems, rather than one problem of dimension 6. In particular, if the structure is not eccentric then one can use two separate analyses to find the elements of $\boldsymbol{H}_{struc}$ corresponding to motion in two orthogonal principal planes. Each of these reduced problems involves a horizontal motion and a coupled rocking about an axis normal to that direction. The other two components of motion, vertical translation and torsion about a vertical axis, are typically uncoupled from each other, as well as from the horizontal and rocking motions. Homogeneity of the soil leads to a similar subdivision of $\boldsymbol{H}_{soil}$. Numerical data have further indicated that for many situations one can also neglect the coupling between translation and rocking and approximate $\boldsymbol{H}_{soil}$ as being diagonal (Veletsos and Meek 1974).

3. Formulation of structural equations

Let the vector $\boldsymbol{x}_r$ denote a reduced set of coordinates which would be adequate to describe the motion of the structure if it had a fixed base. The components of $\boldsymbol{x}_r$ may be taken as absolute motions, as relative motions, or as any combination thereof. It should be emphasized that the vector $\boldsymbol{x}_r$ will be used in describing the structure for any base condition of interest, but it must be supplemented with other coordinates to represent the base motion if the base is not fixed. In particular, the complete motion of the structure-foundation system can then be described by a vector $\boldsymbol{x}=\{\boldsymbol{x}_r^T, \boldsymbol{u}^T\}^T$ incorporating both the structural degrees of freedom and the foundation degrees of freedom, as indicated in Fig. 1. Probably the simplest formulation is now to identify mass, damping and stiffness matrices ($\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$) appropriate for use with $\boldsymbol{x}(t)$, so that the equation of motion for the system can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{F} \tag{6}$$

in which $\boldsymbol{F}=\{\boldsymbol{0}^T, \boldsymbol{f}^T_{struc}\}^T$ since there are considered to be no forces on the structure-foundation system except those from the soil. Using Fourier transforms gives

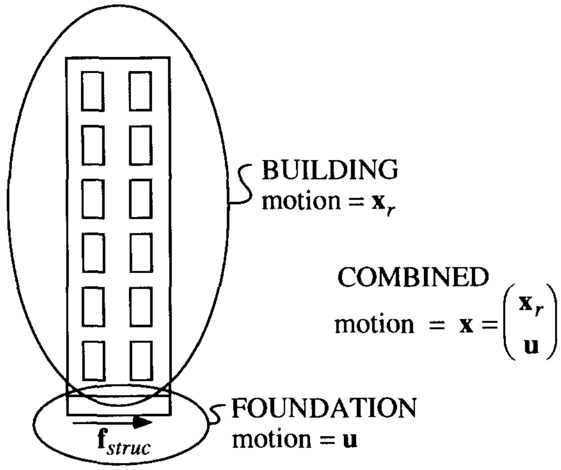

Fig. 1 Coordinates for the foundation and structure.

$$\tilde{x}(\omega)=H(\omega)\tilde{F}(\omega) \tag{7}$$

with

$$H(\omega)=[K+i\omega C-\omega^2 M]^{-1} \tag{8}$$

One can then subdivide the $H(\omega)$ matrix as

$$H(\omega)=\begin{bmatrix} H_{rr}(\omega) & H_{ru}(\omega) \\ H_{ur}(\omega) & H_{uu}(\omega) \end{bmatrix} \tag{9}$$

so that the structural matrix $H_{struc}(\omega)$ needed for the soil-structure interaction analysis is simply the H_{uu} subset of H.

4. Matrix inversion and damping choice

While Eqs. (8) and (9) are theoretically exact and complete, they do not necessarily represent the best approach for finding $H_{struc}(\omega)$, since for each ω of interest they involve inverting a matrix which can be of quite large dimension for a highrise building. In particular, even if consideration is limited to motion in one plane and the shear-beam building model is used, the dimension n of the matrix $H_{rr}(\omega)$ must be as great as the number of stories of the building. Other complications can make it considerably larger than this. Under certain conditions (namely, classical modal damping) the matrix inversion shown in Eq. (8) can be significantly simplified by using the eigen solution for the matrix. In particular, let Φ denote a matrix of eigenvectors of $M^{-1}K$, so that

$$K\Phi=M\Phi\Lambda \tag{10}$$

in which Λ is the diagonal matrix of eigenvalues. The symmetry of M and K then requires that

$$\Phi^T M\Phi=D \tag{11}$$

is also diagonal, as is

$$\Phi^T K \Phi = D\Lambda \tag{12}$$

If the system has what has come to be called classical modal damping or uncoupled modes, then the damping matrix can be similarly diagonalized and the result can be written in the form

$$\Phi^T C \Phi = D\Gamma \tag{13}$$

Under this condition the matrix $H(\omega)$ of Eq. (8) can be written as

$$H(\omega) = \Phi[D\Lambda + i\omega D\Gamma - \omega^2 D]^{-1}\Phi^T \tag{14}$$

This expression involves matrix multiplication, but the actual inversion shown is almost trivial, since the matrix is diagonal. Finding the eigen solution is not trivial, but one needs do this operation only once for a given structure, in contrast to the computation of the inverse in Eq. (8) for each frequency of interest.

As noted earlier, it is common to choose values of modal damping, rather than attempting to choose the matrix C directly from identified sources of energy dissipation (Clough and Penzien 1993). When this is done there is no question about the possibility of diagonalizing C, provided that the modes with assigned damping are those of the system of Eq. (6). In particular, note that the modes found from an eigen solution of $M^{-1}K$ will be those of a free-base structure, since the vector x includes the degrees of freedom of the foundation as well as those of the structural motion. Usual practice in dynamic analysis, on the other hand, is to assume values for the modal damping of the fixed-base structure. In particular, the modal damping values of the fixed-base structure are often chosen to all have the same value. The example in the following section will show that quite different results are obtained depending on whether one chooses damping values in the free-base or the fixed-base structure.

In some situations one may wish to use an eigen analysis of the fixed base structure and to avoid doing the same for the augmented system of Eq. (6). One can use this fixed-base eigen analysis to find a harmonic response matrix ignoring the foundation degrees of freedom. Specifically, if we write

$$B = K + i\omega C - \omega^2 M = \begin{bmatrix} B_{rr} & B_{ru} \\ B_{ur} & B_{uu} \end{bmatrix} \tag{15}$$

as a description of the full problem, then applying the analysis of Eqs. (10)-(14) to the reduced description of the fixed-base structure will give $B_{rr}^{-1}(\omega) = \Phi_r[D_r\Lambda_r + i\omega D_r\Gamma_r - \omega^2 D_r]^{-1}\Phi_r^T$. Solving $BH = I$ then gives

$$H_{struc} \equiv H_{uu} = [B_{uu} - B_{ur}B_{rr}^{-1}B_{ru}]^{-1} \tag{16}$$

Note that the dimension of the remaining matrix inversion in Eq. (16) is equal to the number of foundation degrees of freedom being considered, which is generally much less than the number of structural degrees of freedom, and often is only one or two. If one choose Γ_r on the basis of assigning modal damping values in this formulation, then it will be natural to assign those values to the modes of the fixed-base structure, since that is the system for which the eigen solutions have been found. Thus, the choice of the method of analysis may affect the choice of the damping matrix for the structure.

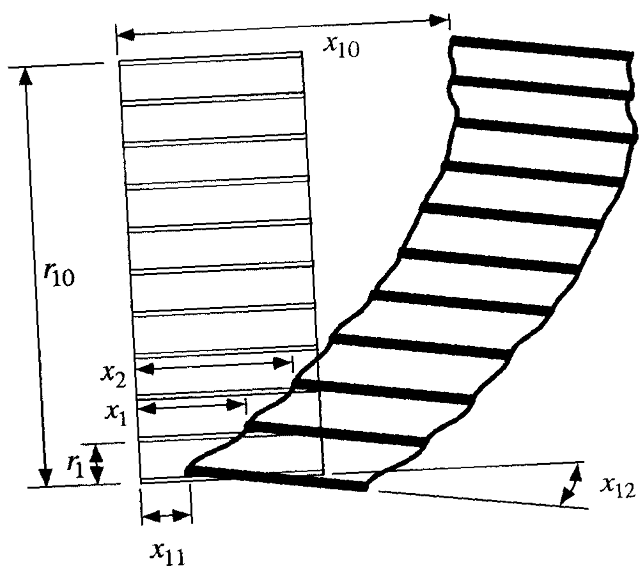

Fig. 2 Example coordinates.

5. Example problem

To illustrate the concepts, consider the dynamics of a ten-story shear-beam building model. Attention will be restricted to one component of horizontal motion and the coupled rocking motion. Thus, $\boldsymbol{x}_r$ will have dimension $n=10$ and $\boldsymbol{u}$ will have dimension 2. The choice of coordinates is shown in Fig. 2. Element $x_{11}=u_1$ has been chosen as the horizontal translation of the base of the structure and $x_{12}=u_2$ as the rocking angle in radians. The columns are considered to be inextensible, so that every floor mass rotates through the angle x_{12}. For $j=1$ to 10 the element x_j has been taken as the horizontal displacement of mass m_j away from its original location. This formulation leads to a diagonal mass matrix $\boldsymbol{M}$, since each component of $\boldsymbol{x}$ denotes absolute motion. Elements m_1 to m_{10} are exactly the lumped mass elements for the analysis of the fixed base structure, m_{11} is the lumped mass for the base of the structure and the foundation, and m_{12} is the sum of the rotational inertia terms for masses 1 to 11. The elements of $\boldsymbol{K}$ can be found from taking derivatives of the potential energy

$$PE=\frac{1}{2}\left[k_1(x_1-x_{11}-r_1x_{12})^2+\sum_{j=2}^{10}k_j[x_j-x_{j-1}-(r_j-r_{j-1})x_{12}]^2\right] \tag{17}$$

in which k_j is the spring immediately below m_j and r_j is the elevation of mass m_j above the foundation. For the first 10 coordinates these elements are identical to those in the usual tridiagonal $\boldsymbol{K}_{ss}$ matrix for a fixed-base structure. Row 12 is found to be full, in general, since imposition of a base rocking with no translation of the masses requires an external force on each mass. Row 11 is zeros except in columns 1, 11 and 12. The matrix is symmetric, of course.

It should be noted that the effect of gravity has been omitted from Eq. (17). Inclusion of this effect would not be difficult, but it would have little effect on the damping comparisons being studied here. Basically the effect of gravity is to reduce the modal frequencies.

The simplest way to choose the damping matrix $\boldsymbol{C}$ is on the basis of equal modal damping values. Doing the eigen analysis of $\boldsymbol{M}^{-1}\boldsymbol{K}$ described above, one can denote the undamped modal

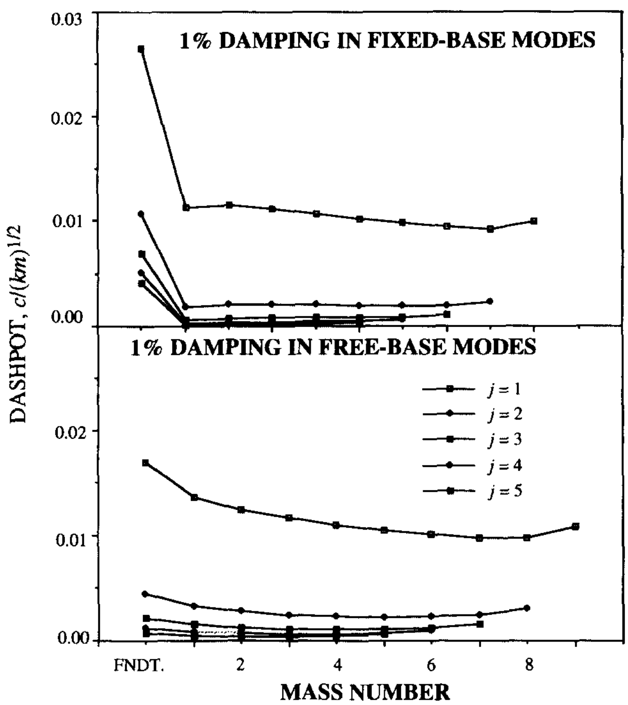

Fig. 3 Dashpot between a given mass and the mass j stories higher.

frequencies as $\omega_j = \lambda_j^{1/2}$ in which λ_j is the jth element of the diagonal Λ matrix. Taking Γ to be diagonal with element $\gamma_j = 2\zeta\omega_j$ then gives ζ as the fraction of critical damping in each mode. If needed, the actual damping matrix, which will be denoted by C_{free}, can be found by solving Eq. (13). Note that two of the modes of the free-base structure of Eq. (6) will have zero as their natural frequency, since there is no potential energy associated with rigid body translation or rotation of the structure and foundation. Furthermore, the equal modal damping approach will give these rigid body modes as also being undamped.

For comparison it is useful also to find the damping matrix which will give equal modal damping to all the fixed-base modes of the structure. One simple way to do this in a completely compatible way is to use Eq. (6) with the same stiffness matrix as for the free-base structure, but with a mass matrix M_{fix} which differs from M in that m_{11} and m_{12} are chosen to be very large. Adding the large inertia terms essentially fixes the base of the structure, so that this free-base structure has modes of vibration which are the same as the fixed-base modes being sought. One can then use the approach described above to choose a C_{fix} matrix which gives equal modal dampings in these fixed-base modes. One finds that the C_{fix} matrix is significantly different from the original C_{free} matrix found from analysis of the free-base structure. [In principle one can also effectively fix the base of the structure by choosing $K_{11,11}$ and $K_{12,12}$ to be very large. Appendix considers reasons why this is a less desirable approach.]

Numerical values have been obtained for a few particular situations. In particular, the story heights have all been taken to be the same, giving $r_j = jh$. Similary the masses have been taken as uniform: $m_j = m$ for $j = 1$ to 10, but the stiffness has been allowed to taper such that $k_j = (1+\beta)^{(10-j)}k$. The modal dampings have been arbitrarily chosen as 1% in both the free-base and fixed-base configurations. (It will be shown, though, that the trends observed here are independent of the magnitude chosen for the modal damping values). Fig. 3 shows one type of direct comparison between C_{free} and C_{fix} for the situation with $m_{11}=m$, $m_{12}=20\ mh^2$ and $\beta=10\%$. In particular, a set of physical dashpots has been identified which corresponds to each of the C matrices. The magnitudes of the most significant of these dashpots are shown in Fig. 3. Results are given for dashpots that connect masses up to five stories apart, and it is observed that the major difference between the two situations is that C_{fix} gives considerably larger dashpots between the story masses and the foundation than does C_{free}. Note that the notation in Fig. 3 treats the foundation mass as m_0, since it is j stories below floor mass j.

It should be noted that the dashpot comparisons shown in Fig. 3 would be unaffected by selection of a different (but constant) value for the modal damping ζ in the two structures. This is because the dashpot values shown in Fig. 3 are all directly proportional to ζ. Thus, one can use the data in Fig. 3 to calculate the ratio between the value of a given dashpot for C_{free} damping and the value of the same dashpot for C_{fix} damping, and this ratio would be unchanged by any other choice of ζ. For example, the dashpot between the foundation and mass m_1 will always be over 50% larger for C_{fix} than for C_{free} damping, and the corresponding C_{fix}/C_{free} ratio for the foundation to m_2 dashpot will always be greater than two.

Although the results in Fig. 3 do show that the two C matrices differ, they do not readily show the extent to which this difference is significant to the study of dynamics. The following section presents information which is thought to be more useful for this purpose. Also, some of the results in the following section may be somewhat easier to interpret inasmuch as they are presented in a form normalized by the modal damping factor ζ.

It should be noted that the choice used for the $\boldsymbol{x}_r$ coordinates was rather arbitrary. Another common choice could be represented by a vector z_r in which $z_j=x_j-x_{11}-r_j x_{12}$ for $j=1$ to 10, so that z_j denotes the movement of m_j relative to an axis system attached to the foundation mass. This choice, of course, simplifies the $\boldsymbol{K}$ matrix, but complicates $\boldsymbol{M}$. Using this or some other equivalent coordinate system can be represented as $\boldsymbol{z}=\boldsymbol{\Psi x}$ using a square matrix $\boldsymbol{\Psi}$. One can substitute $\boldsymbol{x}=\boldsymbol{\Psi}^{-1}\boldsymbol{z}$ into Eq. (6) to obtain the equation of motion in the new coordinates, and the eigen analysis for the matrix $\boldsymbol{M}_z^{-1}\boldsymbol{K}_z=\boldsymbol{\Psi M}_x^{-1}\boldsymbol{K}_x\boldsymbol{\Psi}^{-1}$ shows that the eigenvalues for this formulation are identical to those in Eq. (10) while the eigenvector matrix is given by $\boldsymbol{\Phi}_z=\boldsymbol{\Psi\Phi}_x$. Thus, no essential difference will be introduced by the choice of different coordinates for describing the structural motion.

6. Effect of C matrix on modal dampings

Since dynamic analysis is usually performed for the fixed base structure, it is useful to focus on the structure with the mass chosen to give this base constraint, and to see how its characteristics would differ if it had the damping matrix C_{free} instead of the usual C_{fix}. One complication of this analysis is the fact that the $\boldsymbol{\Phi}$ matrix which diagonalizes $\boldsymbol{M}$ and $\boldsymbol{K}$ for the fixed-base structure according to Eqs. (11) and (12), will not diagonalize C_{free} when Eq. (13) is used. That is, the system described by $(\boldsymbol{M}, \boldsymbol{K})_{fix}$ and C_{free} does not have classical normal modes. This simply means that in any mode of free vibration, the motions of the various masses do not all have perfect 0° or 180° phase relationships. A convenient way to study the modal dampings of this system is to rewrite Eq. (6) as

$$\dot{\boldsymbol{y}}+\boldsymbol{A y}=\boldsymbol{G}(t) \tag{18}$$

in which $\boldsymbol{y}=(\boldsymbol{x}^T, \dot{\boldsymbol{x}}^T)^T$, $\boldsymbol{G}=(\boldsymbol{0}^T, (\boldsymbol{M}^{-1}\boldsymbol{F})^T)^T$ and

$$A=\begin{bmatrix} 0 & -1 \\ M^{-1}K & M^{-1}C_{free} \end{bmatrix} \tag{19}$$

The eigenvalues α_j for the matrix A come in complex conjugate pairs. For each pair, the imaginary part of α_j corresponds to the damped frequency of the modal pair, $p_j=\omega_j(1-\zeta_j^2)^{1/2}$, while the real part gives the rate of decay of the mode. The fraction of critical damping can be found from

$$\zeta_j=\frac{Re[\alpha_j]}{\left[\left(Re[\alpha_j]^2\right)+\left(Im[\alpha_j]\right)^2\right]^{1/2}} \tag{20}$$

in which the denominator of Eq. (20) gives the undamped modal frequency ω_j. Even though the phase relationships of the non-classical modes are more complicated than for classical damping, the damping factors have exactly the same meaning in the two situations. In particular, at any instant of time the free vibration motion in a pure mode j is exactly exp $[-2\pi\zeta_j\omega_j/p_j]$ times what it was at the time $2\pi/p_j$ earlier.

The values of ζ_j have been obtained for the fixed-base structure considered in Fig. 3, but including the C_{free} which gives each of the modes of the free-base structure a common damping value of ζ_{free}. These values, along with the modal frequencies, are shown in the left-hand portion of Table 1. The result which stands out from these values is the fact that the damping in the fundamental mode of the fixed-base structure is only about 21% of ζ_{free}. A similar, but less pronounced, damping reduction exists for the other lower modes. Note that the damping values in Table 1 are presented in the normalized form of ζ_j/ζ_{free}. A variety of situations have been investigated, and the ζ_j/ζ_{free} values have been found to be almost independent of the particular values chosen for ζ_{free} and β.

Two parameters which can be expected to affect the damping results are m_{11} and m_{12}. These two inertia components tend to restrict the base motion in the free-base model, so the modal damping values obtained using C_{free} should be closer to ζ if m_{11} and m_{12} are large than if they are small. The magnitude of m_{12} depends on the slenderness of the building. In particular, if b is the dimension of the building in the direction of motion, then the rotational inertia term for a rigid floor building can be written as

$$m_{12}=\frac{b^2}{12}\sum_{j=1}^{11} m_j \tag{21}$$

so that a given set of mass values gives a larger m_{12} values if the building is less slender. Fig. 4 shows this slenderness dependence using $m_{12}/(mh^2)$ as the abscissa and the normalized modal damping as the ordinate. An exaggerated range of the abscissa is shown in order better to illustrate the effect of rotational inertia, but reasonable values of b/h would be much more restricted. For example, varying b from 25% to 50% of the building height ($10h$) would give a range of abscissa values from 5.7 to 23. The effect of varying m_{11} is also shown in Fig. 4, but this time both m_{11} and m_{12} vary along the abscissa, since there is no obvious physically meaningful way to vary m_{11} while holding m_{12} constant. Rather the plot has been prepared using Eq. (21) with $b=5h$ for m_{12} while varying m_{11} over a very wide range. As with m_{12}, the more meaningful values are toward the left-hand-side of the picture, with the more extreme values included only to illustrate the effect of large inertia terms. Both portions of this figure are for a building with $\beta=10\%$.

The curves versus m_{11}/m in Fig. 4 ($b=5h$) do all tend to unity as m_{11} tends to infinity. That

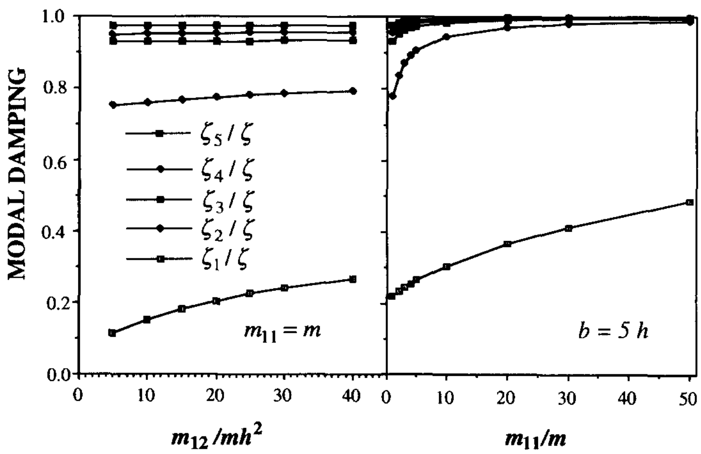

Fig. 4 Modal damping of a fixed-base structure with C_{free} damping matrix.

Table 1 Modal damping values (β=10%, $m_{11}=m$, $m_{12}=20\ mh^2$)

Mode	Fixed-base structure with C_{free} damping		Free-base structure with C_{fix} damping	
	Frequency	Damping	Frequency	Damping
j	$(m/k)^{1/2}\omega_j$	ζ_j/ζ_{free}	$(m/k)^{1/2}\omega_j$	ζ_j/ζ_{fix}
(1)	(2)	(3)	(4)	(5)
1	0.1997	0.2070	0.6674	1.178
2	0.5458	0.7760	0.8322	2.682
3	0.8880	0.9325	1.102	2.605
4	1.210	0.9551	1.323	1.387
5	1.504	0.9765	1.601	1.322
6	1.759	0.9841	1.828	1.159
7	1.972	0.9901	2.028	1.131
8	2.168	0.9917	2.221	1.115
9	2.393	0.9921	2.450	1.118
10	2.693	0.9924	2.758	1.124
11	–	–	0	0
12	–	–	0	0

is, the modal damping values do tend to ζ_{free} as m_{11} becomes large enough to fix the base of the structure. However, it is noted that m_{11} must be huge before the damping in the fundamental distortional mode grows significantly from the value in Table 1. The situation is somewhat different as m_{12}/mh^2 becomes large for $m_{11}=m$. In this case the foundation of the structure becomes fixed against rocking, but not against translation. Thus, the curves in the left-hand portion of Fig. 4 do not tend to unity as m_{12} approaches infinity. Probably the most important conclusion to be drawn from Fig. 4 is the fact that the trends shown in Table 1 are typical, rather than being strongly dependent on the choices for m_{11} and m_{12}. Use of the C_{free} damping instead of C_{fix}, can always be expected to give an unusually low damping value in the fundamental distortional mode, and also to produce somewhat low damping values in other low frequency modes.

Another way in which to present the effects of the choice of damping is in terms of the

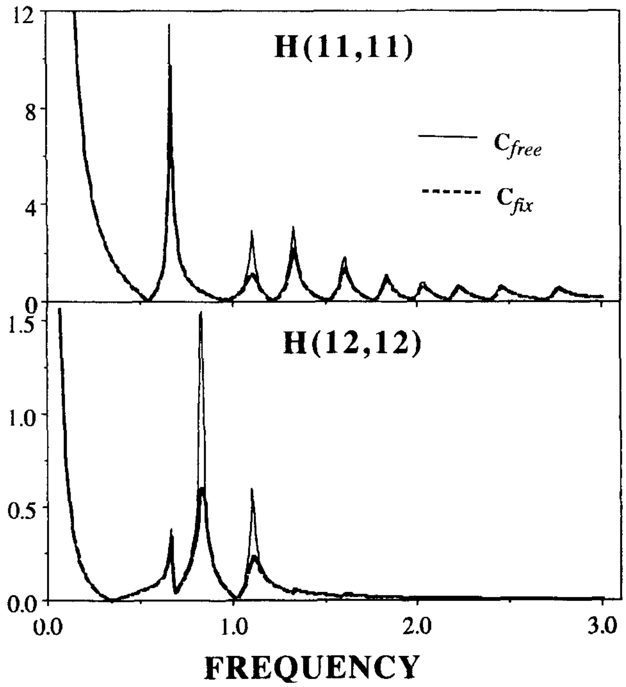

Fig. 5 Harmonic response for the base of the structure.

effect on the $H_{struc}(\omega)$ matrix which would be used in an analysis of soil-structure interaction (see Eq. (5)). Fig. 5 shows the absolute values of the two diagonal elements of this subset of the $H(\omega)$ matrix for the system with C_{free} giving 1% damping in all free-base modes, as well as for a system with C_{fix} giving 1% damping in all fixed-base modes. It is noted that the H terms are sometimes significantly larger for C_{free} damping than for C_{fix} damping. The greatest difference between the two situations, of course, is near some of the resonance peaks of the free-base structure. Stated another way, using the C_{fix} matrix with $\zeta = 1\%$ seems to give significantly more than 1% damping in some of the free-base modes. This has been confirmed by non-classical modal analysis of the free-base structure including C_{fix} damping for the situation with $\beta = 10\%$, $m_{11} = m$, $m_{12} = 20\ mh^2$ and $\zeta_{fix} = 1\%$. These values are shown, along with the modal frequencies, in the right-hand portion of Table 1. The increased damping in modes 2 and 3 is particularly evident, but it should be noted that mode 2 does not contribute in any significant way to the translational component $H_{11,11}$ in Fig. 5.

As shown in Eq. (3), the $H_{struc}(\omega)$ components represent dynamic structural flexibilities at the frequency ω. Use of C_{free}, insteaded of C_{fix}, in a situation for which it gives significantly larger H components, will result in a significant overprediction of the dynamic flexibility of the structure. Even when the effect of the soil is included, as in Eqs. (4) and (5), one can anticipate that such an overprediction of the H_{struc} structural flexibility will result in an overestimation of the seismic response of the soil-structure system. The stiffer the foundation soil, the stronger will be the dependence of the total flexibility $H_T(\omega)$ on $H_{struc}(\omega)$.

If one were to analyze a fixed-base structure with damping given by the matrix C_{fix}, and a soil-structure interaction system with a damping matrix C_{free}, then it would be very difficult to separate the effects of interaction from the effect of the damping matrix. As Fig. 5 shows, the system with C_{free} damping would have increased flexibility at some frequencies due solely to the damping. Failure to recognize this fact could lead one to an erroneous conclusion that all the increased flexibility was due to soil-structure interaction. In order to properly quantify the effects of soil-structure interaction it seems important to use the same structural damping matrix in the analysis of fixed-base and interacting systems.

It can be observed that the general conclusions drawn from Fig. 5 would be unchanged by the use of a different damping value ζ in the identification of the C_{free} and C_{fix} damping matrices, even though the numerical values would be altered. In particular, the height of the high resonance peaks in the H components are nearly proportional to ζ^{-1}, so that all these peaks would be scaled down by nearly the same amount if ζ were increased. The one significant difference which would result from an increased value of ζ relates to the smaller peaks. For example, one might consider all but the first two or three resonance peaks to be insignificant if ζ were increased beyond about 5%.

7. Conclusions

The damping matrix C_{fix} which gives a specified damping value ζ to each mode of a fixed-base structure is significantly different from the matrix C_{free} which gives damping ζ to each mode of the free-base structure. In particular, C_{fix} corresponds to larger dashpots between the foundation and the other masses. Use of C_{free} in a fixed-base structure results in a system with non-classical damping, and with a damping value considerably smaller than ζ in the fundamental distortional mode. Use of C_{fix} in a free-base structural analysis also gives non-classical damping and gives modes with damping much greater than ζ, but the larger damping is not particularly evident in the fundamental distortional mode.

If one wishes to study the effect of soil-structure interaction by comparing the response of structures with fixed and flexible bases, then it is important to use the same damping in each structure. Generally this will mean that one should use the damping matrix C_{fix} in studying a system with soil-structure interaction. Use of C_{free} instead would give a structure with increased dynamic flexibility in the lower frequency modes, and this damping induced flexibility could be easily confused with the flexibility introduced by the soil itself.

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Appendix

Fixed-base structure as limit of free-based structure

Without loss of generality, let the coordinates x be chosen such that M is diagonal. Furthermore, let $x = \{x_r^T, u^T\}^T$ and consider a similar partition of M and K and the matrices Φ of eigenvectors and Λ of eigenvalues. The eigen problem can then be written as

$$\begin{bmatrix} M_r^{-1} & 0 \\ 0 & M_u^{-1} \end{bmatrix} \begin{bmatrix} K_{rr} & K_{ru} \\ K_{ur} & K_{uu} \end{bmatrix} \Phi = \Phi \Lambda \tag{22}$$

in which the subscript u refers to degrees of freedom of the rigid foundation, and r refers to the reduced set of all other degrees of freedom (which would be adequate to described a fixed-base structure).

The object here is to relate the modes of the free-base system to those of the fixed-base structure described only by M_r and K_{rr}. The eigen equation for that system will be written as $M_r^{-1} K_{rr} \Phi_{rr} = \Phi_{rr} \Lambda_r$.

It is useful to note at this point that there are some restrictions on the K matrix for a free-base structure. In particular, for any foundation displacement u of such a structure, there must exist some superstructure displacement x_r such that the vector Kx of forces on the structure is zero. From the upper portion of $Kx = 0$ one can solve for x_r as $x_r = -K_{rr}^{-1} K_{ru} u$. Substituting this result into the lower portion of $Kx = 0$ gives $[K_{uu} - K_{ur} K_{rr}^{-1} K_{ru}] u = 0$. Obviously this can be satisfied for all choices of u only if $K_{uu} - K_{ur} K_{rr}^{-1} K_{ru} = 0$.

(1) Fixing the base by using large M_u

In order to analyze the fixed-base structure we will choose M_u to be very large, so that in most modes of vibration the base of the structure will not move, even though it is still technically a free-base structure. In particular, assume that $M_u = O(\varepsilon^{-1})$ with ε being very small. This then makes $M_u^{-1} = O(\varepsilon)$, so that the lower portions of Eq. (22) are very small. We will try writing the eigen solution as

$$\Phi = \begin{bmatrix} \Phi_{rr} & \Phi_{ru} \\ \Phi_{ur} & \Phi_{uu} \end{bmatrix} \tag{23}$$

$$\Lambda = \begin{bmatrix} \Lambda_r & 0 \\ 0 & 0 \end{bmatrix} \tag{24}$$

with Φ_{rr} and Λ_r representing the eigen solution for $M_r^{-1} K_{rr}$. The additional eigenvalues are taken as zero because the rigid body modes of motion of a free-base structure must have zero eigenvalues.

Not surprisingly, it is not possible to choose Φ_{ru}, Φ_{ur} and Φ_{uu} in such a way as exactly to solve Eq. (22). However, it is possible to choose them such that the equation for each term in Eq. (22) is approximately satisfied, with the discrepancy being $O(\varepsilon)$ times the terms not neglected. An approximate solution of this type is found by taking $\Phi_{ru} = -K_{rr}^{-1} K_{ru} \Phi_{uu}$ and $\Phi_{ur} = M_u^{-1} K_{ur} \Phi_{rr} \Lambda_r^{-1}$. In verifying the level of the approximation, it should be noted that $\Phi_{ur} = O(\varepsilon)$ since $M_u^{-1} = O(\varepsilon)$. It is also found, in substituting the expressions into Eq. (22), that Φ_{uu} appears as the matrix of eigenvectors of $M_u^{-1}(K_{uu} - K_{ur} K_{rr}^{-1} K_{ru})$, which is exactly a zero matrix for the free-base structure, as previously noted. Without loss of generality, one can take Φ_{uu} as $\Phi_{uu} = I_u$, an identity matrix with the same dimension as u.

In order to assign equal modal dampings to the fixed base modes we will use $\Gamma = 2\zeta \Lambda^{1/2}$, and solving Eqs. (11) and (13) gives $C = M\Phi \Gamma D^{-1} \Phi^T M$. The lower right-hand portion of the diagonal matrix D^{-1}, which can be written as D_u^{-1}, is $O(\varepsilon)$. Substituting for Φ_{ru} and Φ_{ur} gives

$$C = \begin{bmatrix} C_{rr} & C_{ru} \\ C_{ur} & C_{uu} \end{bmatrix} = 2\zeta \begin{bmatrix} M_r \Phi_{rr} \Lambda_r^{1/2} D_r^{-1} \Phi_{rr}^T M_r & M_r \Phi_{rr} \Lambda_r^{-1/2} D_r^{-1} \Phi_{rr}^T K_{ur}^T \\ K_{ur} \Phi_{rr} \Lambda_r^{-1/2} D_r^{-1} \Phi_{rr}^T M_r & K_{ur} \Phi_{rr} \Lambda_r^{-3/2} D_r^{-1} \Phi_{rr}^T K_{ur}^T \end{bmatrix} + O(\varepsilon) \tag{25}$$

One of the properties of this damping matrix is that its $O(1)$ parts represent only "internal" dashpots in the same way that any system satisfying $K_{uu} - K_{ur} K_{rr}^{-1} K_{ru} = 0$ has only internal springs. In particular, for any velocity $\dot{u}$ of the base, one can choose the superstructure velocity as $\dot{x}_r = -C_{rr}^{-1} C_{ru} \dot{u}$ in order to have a system with no forces resisting these velocities.

(2) Disadvantages of fixing the base by using large K_{uu}

In principle one can also effectively fix the rigid foundation of the structure by choosing K_{uu} to be very large. This seems to be less desirable from a theoretical point of view, as well as presenting some numerical difficulties. To see this let $K_{uu} = O(\varepsilon^{-1})$ with ε being very small. The lower right portion of Eq. (22) is now very large. Note that this change means that the K matrix no longer satisfies $K_{uu} - K_{ur} K_{rr}^{-1} K_{ru} = 0$ — the structure is no longer free-based. We will now find an approximate solution of Eq. (22) with $\boldsymbol{\Phi}$ being written the same as in Eq. (23) and with

$$\boldsymbol{\Lambda}=\begin{bmatrix}\boldsymbol{\Lambda}_r & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Lambda}_u\end{bmatrix} \tag{26}$$

in which $\boldsymbol{\Lambda}_u=O(\varepsilon^{-1})$. An approximate solution can be found by taking $\boldsymbol{\Phi}_{uu}$ and $\boldsymbol{\Lambda}_u$ as solutions of the eigen equation $\boldsymbol{M}_u^{-1}\boldsymbol{K}_{uu}\boldsymbol{\Phi}_{uu}=\boldsymbol{\Phi}_{uu}\boldsymbol{\Lambda}_u$ and using $\boldsymbol{\Phi}_{ru}=\boldsymbol{M}_r^{-1}\boldsymbol{K}_{ru}\boldsymbol{\Phi}_{uu}\boldsymbol{\Lambda}_u^{-1}=O(\varepsilon)$, $\boldsymbol{\Phi}_{ur}=-\boldsymbol{K}_{uu}^{-1}\boldsymbol{K}_{ur}\boldsymbol{\Phi}_{rr}=O(\varepsilon)$. Choosing equal modal dampings in the fixed base modes will now give

$$\boldsymbol{C}=\begin{bmatrix}\boldsymbol{M}_r\boldsymbol{\Phi}_{rr}\boldsymbol{\Gamma}_r\boldsymbol{D}_r^{-1}\boldsymbol{\Phi}_{rr}^T\boldsymbol{M}_r & \boldsymbol{M}_r\boldsymbol{\Phi}_{ru}\boldsymbol{\Gamma}_u\boldsymbol{D}_u^{-1}\boldsymbol{\Phi}_{uu}^T\boldsymbol{M}_u\\ \boldsymbol{M}_u\boldsymbol{\Phi}_{uu}\boldsymbol{\Gamma}_u\boldsymbol{D}_u^{-1}\boldsymbol{\Phi}_{ru}^T\boldsymbol{M}_r & \boldsymbol{M}_u\boldsymbol{\Phi}_{uu}\boldsymbol{\Gamma}_u\boldsymbol{D}_u^{-1}\boldsymbol{\Phi}_{uu}^T\boldsymbol{M}_u\end{bmatrix}=\begin{bmatrix}O(1) & O(\varepsilon^{1/2})\\ O(\varepsilon^{1/2}) & O(\varepsilon^{-1/2})\end{bmatrix} \tag{27}$$

One disadvantage of the C matrix in Eq. (27) is that it does not correspond to only internal dashpots within the structure. In particular, $\boldsymbol{C}_{uu}-\boldsymbol{C}_{ur}\boldsymbol{C}_{rr}^{-1}\boldsymbol{C}_{ru}\neq\mathbf{0}$ so that for a given velocity $\dot{\boldsymbol{u}}$ of the base, there is generally no possible choice of $\dot{\boldsymbol{x}}_r$ such that there are no external forces required to produce the velocities. Note that the $\boldsymbol{C}_{rr}$ sub-matrix is the same in Eq. (27) as in Eq. (25). Thus, the magnitudes of the dashpots connected to each mass point within the structure (the location of each component of $\boldsymbol{x}_r$ or $\dot{\boldsymbol{x}}_r$) are the same in the two formulations. The difference is that Eq. (27) corresponds to some dashpots being connected to "absolute ground", whereas for Eq. (25) these same dashpots are connected to the foundation of the structure. Both situations give the same damping of the fixed-base structure (corresponding to either $\boldsymbol{M}_u=\infty$ or $\boldsymbol{K}_{uu}=\infty$), but they will not give the same damping if the base of the structure is allowed to move by flexibility of the ground beneath the foundation. The "internal dashpot" damping matrix of Eq. (25) seems to be the more physically meaningful.

The computational disadvantage of using the large $\boldsymbol{K}_{uu}$ approach to find the eigen solution for the fixed-base structure comes from the relative magnitudes of the terms in

$$\boldsymbol{M}^{-1}\boldsymbol{K}=\begin{bmatrix}\boldsymbol{M}_r^{-1}\boldsymbol{K}_{rr} & \boldsymbol{M}_r^{-1}\boldsymbol{K}_{ru}\\ \boldsymbol{M}_u^{-1}\boldsymbol{K}_{ur} & \boldsymbol{M}_u^{-1}\boldsymbol{K}_{uu}\end{bmatrix} \tag{28}$$

The $\boldsymbol{M}_r^{-1}\boldsymbol{K}_{rr}$ portion which describes the fixed-base structure is $O(1)$. In the large $\boldsymbol{M}_u$ approach this significant portion is among the larger terms in Eq. (28), with the lower portions of the matrix being $O(\varepsilon)$. For a large $\boldsymbol{K}_{uu}$, though, Eq. (28) contains $O(\varepsilon^{-1})$ terms in the lower right portion, and these are much larger than the significant $\boldsymbol{M}_r^{-1}\boldsymbol{K}_{rr}$ portion. Obviously this has the possibility of introducing significant numerical errors.

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