Structural Engineering and Mechanics, Vol. 3, No. 1 (1995) 91-103
DOI: http://dx.doi.org/10.12989/sem.1995.3.1.091

Shear center for elastic thin-walled composite beams

Gerry D. Pollock†, Adam R. Zak‡, Harry H. Hilton‡ and M. Fouad Ahmad‡†

Aeronautical and Astronautical Engineering Department (AAE) and National Center for Supercomputing Applications (NCSA), University of Illinois at Urbana-Champaign, Urbana, IL 61801-2935, U.S.A.

Abstract. An analysis to determine shear centers for anisotropic elastic thin-walled composite beams, cantilevered and loaded transversely at the free end is presented. The shear center is formulated based on familiar strength of material procedures analogous to those for isotropic beams. These procedures call for a balancing of torsional moments on the cross sectional surface and lead to a condition of zero resultant torsional couple. As a consequence, due the presence of anisotropic coupling, certain non-classical effects are manifested and are illustrated in two example problems. The most distinguishing result is that twisting may occur for composite beams even if shear forces are applied at the shear center. The derived shear center locations do not depend on any specific anisotropic bending theories *per se*, but only on the values of bending and shear stresses which such theories produce.

Key words: anisotropic elasticity; composites; shear center; thin-walled beams.

1. Introduction

Thin-walled beams with both open and closed sections are frequently exploited in aerospace structures, and the use of advanced composites is becoming commonplace. The location of the shear center for thin-walled laminated beams is of particular interest due to non-classical effects associated with composite beam theory.

The focus here is on the class of beams, with constant cross sectional properties, fixed at one end and loaded in transverse shear at the opposite end. This problem has been studied in-depth based on St. Venant's solution from the theory of elasticity. Several early works in this area sought to define the shear center for isotropic beams. For sections with the abscissa as an axis of symmetry for both loading and geometry, Griffith and Taylor (1917) arrived at a shear center formula, based partially on soap film methods, in which shear stress distributions were determined so as to produce zero rate of twist at the free end. This formula was interpreted by Duncan (1953) as the centroid with the ordinates cubed. Duncan (1932, 1953) replaced Griffith's and Taylor's soap film analogies with a series representation of stress functions and found a shear center which coincided with Griffith's and Taylor's only when Poisson's ratio equaled zero. Trefftz (1935) defined the shear center in terms of total strain energy versus strain energy from the separate torsion and bending solutions. Since, as Goodier (1944) explained, it is impossible for the relative rotations of all corresponding elements of area between two cross sections

† Research Associate, University of Virginia-NASA Langley, Hampton, VA
‡ Professor Emeritus
‡† Research Scientist

to vanish, Goodier enforced a condition of zero average rate of twist over the section. Osgood (1943) asserted that the "usual" definition of shear center–that point through which transverse shear loads must pass so as to cause no rotation of the free end section–is inadequate to produce a unique location which is the reason why various formulations, based on this definition, yield differing results.

For isotropic beams, it is the torsional couple caused by tranverse forces resolved about some reference point (e.g., the shear center) which leads to twist, while for anisotropic beams, twist may result with or without torsional couples through bending and extension coupling. As a consequence, care must be taken when using shear center definitions such as those above for anisotropic beams. Trefftz' definition, for example, may not be suitable since, for a general laminate, the bending, torsion and extension problems are coupled. Recently, Reissner and Tsai (1972), through the use of influence coefficients based on St. Venant's solution, have obtained a definition for shear center, and Reissner (1989, 1991, 1992) has extended his influence coefficients to account for orthotropy and anisotropy. Using anisotropic plate theory, Mansfield (1979) has taken an alternate view of applying conditions on torsional moments rather than deformations to locate the shear center.

To avoid the complexities associated with theory of elasticity, methods analogous to those of classical thin-walled isotropic theories are applied in this paper to obtain shear center positions. The formulation of the shear center is based on familiar strength of material (engineering) procedures but extended to account for orthotropic coupling found in composite laminates. The use of the familiar simplifying assumptions inherent in the classical, engineering theory provides a clear, intuitive approach to the anisotropic shear center problem. In this context, no literature has been found on shear center formulations for arbitrary section geometry and lay-ups although for the special case of orthotropic materials (single ply), Bauchau (1985) has been able to decouple bending and torsion, and the shear center is determined from this condition.

The "engineering" shear center is often defined in two different ways, and though either definition produces the same result in the isotropic case, significant differences arise in the anisotropic case. Many authors (e.g., Allen and Haisler 1985, Peery and Azar 1982, Bisplinghoff, *et al.*, 1957, Donaldson 1993, Faupel and Fisher 1981) define shear center as the point on the cross sectional plane at which the application of shear forces causes no twist of section. Libove (1988) has considered this definition in terms of anisotropic closed sections. His recommendation is to "···formulate a shear center definition that is nearly analogous to that of the classical theory···" by finding a position through which the shear loads cause no *average* rate-of-twist over the beam length. In the present investigation, however, it is shown that the "analogous" definition comes directly from the other common shear center definition (described next), and no re-defining is necessary.

In the second definition (e.g., Oden and Ripperger 1981, Gjelsvik 1981, Niles and Newell 1938), the shear center is defined as the point on the cross sectional plane at which the application of shear forces causes a shear stress distribution which balances the torsional moment due to resultant shear forces. Under such a condition, no resultant torsional couple is generated by the shear loads.

For isotropic, homogeneous elastic sections, twist can only arise from resultant torsional couples: zero twist implies a zero torsional couple, and vice versa, and both definitions yield identical results. In the anisotropic case, zero torsional couple does not necessarily imply zero twist. For example, a shear load may be applied at a point on the cross section such that a zero net torsional couple is generated. However, due to bending moments caused by shear loads, twist may exist since bending and twist are coupled. Similarly, zero twist does not guarantee zero

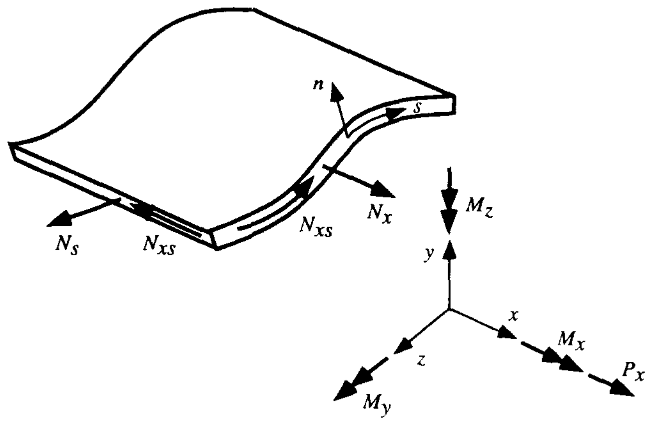


Fig. 1 Cross sectional stresses and resultants.

torsional couple. Consequently, either the resultant torsional couple can be made zero or the twist can be zeroed, but not both simultaneously. Bauld and Tzeng (1984) have made a similar observation that, except for certain cross-ply lay-ups, "···equilibrium configurations for which bending and twisting deformation modes are uncoupled do not exist". (The shear center problem is not considered in that work.) If the second definition (in terms of balanced torsional moments) is used, procedures analogous to the isotropic ones can be followed to determine shear centers. Hence, the second definition is seen as more natural and is used herein.

2. General results from composite beam theory

In general, due to anisotropy, full coupling between extension, bending and twist is exhibited in composite beams. This behavior can be represented symbolically through the following generalized constituitive relationship:

$$\begin{Bmatrix} P_x \\ M_z \\ M_y \\ M_x \end{Bmatrix} = \begin{bmatrix} SF_{11} & SF_{12} & SF_{13} & SF_{14} \\ SF_{21} & SF_{22} & SF_{23} & SF_{24} \\ SF_{31} & SF_{32} & SF_{33} & SF_{34} \\ SF_{41} & SF_{42} & SF_{43} & SF_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ a \\ b \\ \theta \end{Bmatrix} \tag{1}$$

where (see Fig. 1) P_x is the resultant force in the x direction, $M_z(M_y)$ is the resultant bending moment about the $y(z)$ axis caused by normal stresses in the $z(y)$ direction defined such that tensile stresses in the positive $x-y(y-z)$ quadrant produce a positive moment. M_x is the resultant torsional couple whose sense is given by the right hand rule about an axis parallel to the x--axis, ε_0 is the strain along the x-axis (beam axis), a and b are cross sectional constants representing the bending deformations (e.g., in Euler-Bernoulli theory, the curvatures of the beam axis in the y and z directions, respectively), θ is the rate of twist due to coupling and applied pure torsional moments (couples), and SF_{ij} are elements of the "stiffness" matrix $[SF]$.

The location of the origin of the (x, y, z) system is arbitrary, but it is understood that the transverse shear forces pass through the shear center so that (as will be demonstrated subsequently) no net torsional couple is produced by them. General assumptions leading to relations of the type given by Eq. (1) are only that the normal strain varies linearly over the section and that

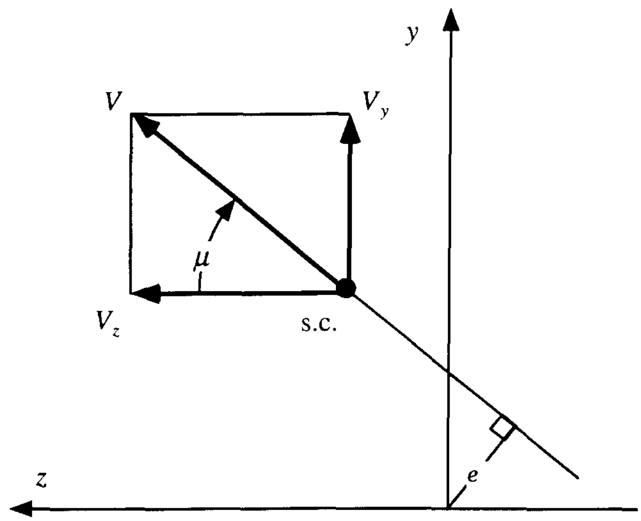


Fig. 2 Line-of-action of shear resultant.

the resultant shear stresses N_{xs} arising from applied shear loads can be ignored as producing insignificant deformations (Euler-Bernoulli theory is a special case incorporating these assumptions). Recall, also, that only beams of constant cross sectional properties are considered herein. Transverse shear and warping degrees of freedom may be explicitly appended to the above relations as desired. The specific values SF_{ij} depend on the particular beam theory used in addition to material properties and geometry and thus will be left general in the subsequent development. Also, separate stress-strain relations may be required due to the fundamentally different mechanical behavior between open and closed sections.

In short form, Eq. (1) may be written as:

$$\{\sigma\}=[SF]\{\varepsilon\} \tag{2}$$

where $\{\sigma\}=[P_x\ M_z\ M_y\ M_x]^T$, $\{\varepsilon\}=[\varepsilon_0\ a\ b\ \theta]^T$ and $[SF]$ is described by Eq. (1). Inversion gives:

$$\{\varepsilon\}=[CF]\{\sigma\} \tag{3}$$

where $[CF]=[SF]^{-1}$ and is the compliance matrix.

Spanwise equilibrium yields:

$$V_y=V\sin\mu=-\frac{dM_z}{dx}$$

$$V_z=V\cos\mu=-\frac{dM_y}{dx} \tag{4}$$

where V_y and V_z are the y and z components of the transverse shear force V. In general, V may be aligned along any direction oriented at an angle μ with respect to the z axis as shown in Fig. 2. Since only cantilevers loaded by transverse forces at the free end are considered, $P_x=M_x=0$.

Although N_{xs}, due to V is ignored in obtaining Eq. (1), the shear center formulation requires the shear stress due to applied transverse shear loads (i.e. in the absence of applied pure bending and torsional moments). This shear stress is denoted by $\bar{N}_{xs}$ where the overbar delineates it

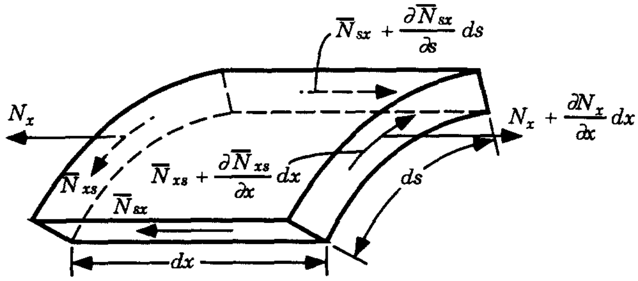


Fig. 3 Forces on an infinitesimal beam element.

from other shear stresses that may be envisioned. $\overline{N}_{xs}$ can be obtained through force and moment equilibrium of the forces per unit sectional length shown acting on an infinitesimal beam element in Fig. 3. To the first order:

$$\overline{N}_{xs}=-\int_0^s \frac{\partial N_x}{\partial x}ds+\overline{N}_{xs_0} \tag{5}$$

where $\overline{N}_{xs}$ is the shear force per unit sectional length, N_x is the normal force in the axial direction per unit sectional length, and $\overline{N}_{xs_0}$ denotes the values of $\overline{N}_{xs}$ at $s=0$. In open sections, $\overline{N}_{xs_0}=0$ because of the traction free edge at $s=0$. In what follows, an overbar on any quantity indicates that it is caused solely by $\overline{N}_{xs}$.

In general, N_x is related to all beam generalized displacements (ε_0, a, b and θ), and this relationship, consistent with the assumptions of Eq. (1), may be expressed as:

$$N_x=\begin{bmatrix}\rho_1 & \rho_2 & \rho_3 & \rho_4\end{bmatrix}\begin{Bmatrix}\varepsilon_0\\ a\\ b\\ \theta\end{Bmatrix} \tag{6}$$

where actual values of the coefficients ρ_i depend on the beam theory used and on whether the section is open or closed. Combining Eqs. (3)-(6) yields the shear force per unit sectional length required for sectional equilibrium:

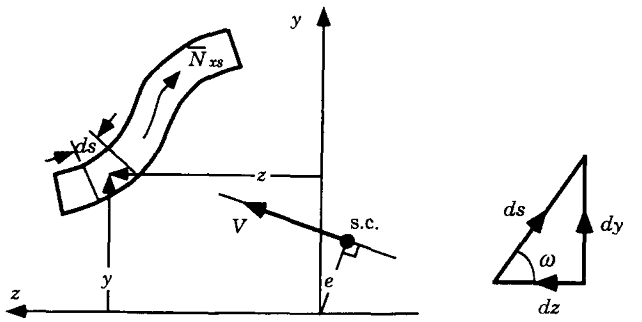
$$\overline{N}_{xs}=\int_0^s (\overline{n}_{xs_y}V\ \sin\mu+\overline{n}_{xs_z}V\ \cos\mu)ds+\overline{N}_{xs_0} \tag{7}$$

where

$$\begin{Bmatrix}\overline{n}_{xs_y}\\ \overline{n}_{xs_z}\end{Bmatrix}=\begin{bmatrix}CF_{12} & CF_{22} & CF_{32} & CF_{42}\\ CF_{13} & CF_{23} & CF_{33} & CF_{43}\end{bmatrix}\begin{Bmatrix}\rho_1\\ \rho_2\\ \rho_3\\ \rho_4\end{Bmatrix} \tag{8}$$

It will be useful to obtain relationships between twist and torque for the following analysis. Recalling that only transverse end forces are considered here (simple shear), then with the application of Eq. (4), the twisting deformation θ from Eq. (3) is:

$$\theta=CF_{42}\left(Lx-\frac{1}{2}x^2\right)V_y+CF_{43}\left(Lx-\frac{1}{2}x^2\right)V_z \tag{9}$$



(a) Shear stresses and forces (b) Differentials

Fig. 4 Balancing of moments on a segment of a section.

It is emphasized that this twist is caused by coupling with bending in which the bending moment is generated by the transverse shear forces. Consistent with previous notation, if $\overline{M}_{x_{s.c.}}$ is the torque about the shear center generated by $\overline{N}_{xs}$ (the shear center reference location is chosen for convenience), then the corresponding additional rate of twist $\overline{\theta}$ may be expressed as:

$$\overline{\theta}=\overline{M}_{x_{s.c.}}\overline{A} \tag{10}$$

The sectional parameter $\overline{A}$ may be termned an "anisotropic torsional rigidity" and depends on the beam theory used but should reduce to the isotropic torsional rigidity if isotropic material properties are used (see, for example, Pollock 1993).

According to the first shear center definition in terms of zero twist, θ must vanish as well as the additional contribution due to $\overline{\theta}$. Using the second definition based on resultant loads, the net torsional couple generated by $\overline{N}_{xs}$ must be zero. The latter condition is automatically satisfied by adopting isotropic procedures.

3. Shear center formulation

Preventing applied shear forces from generating a net torsional couple requires a moment balance between the moments due to V and those due to $\overline{N}_{xs}$ on the cross sectional surface. This is analogous to the method for determining isotropic shear center positions except for stress averaging through discontinuous layers in the case of composite laminates. Shear force resultants applied to the cross sectional plane and shear stresses per unit thickness on the cross section itself are shown in Fig. 4 along with a diagram relating differentials. Moment balance requires that:

For open sections:

$$Ve=-\int_0^S(\cos\omega)y(\overline{N}_{xs}ds)+\int_0^S(\sin\omega)z(\overline{N}_{xs}ds) \tag{11}$$

For closed sections:

$$Ve=-\oint(\cos\omega)y(\overline{N}_{xs}ds)+\oint(\sin\omega)z(\overline{N}_{xs}ds) \tag{12}$$

where $s=S$ is the contour coordinate at the end of the section. Satisfaction of the above equations ensures that when the shear force passes through the point (shear center, *s.c.*) located by the intersection of the shear force line-of-action and a perpendicular line emanating from the origin, moment balance is satisfied yielding zero net torque (zero torsional couple). Note that despite the zero torsional couple, θ is not zero as seen from Eq. (9) which is the major distinction between isotropic and anisotropic shear center formulations.

In the open section case, recognizing that $\bar{N}_{xs_0}$ is zero, the trigonometric relationships between the differentials shown in Fig. 4(b) can be used in Eq. (7) to give:

$$Ve=+\int_{y0}^{yS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}V\ \sin\mu+\bar{n}_{xs_z}V\ \cos\mu)ds\right]zdy$$

$$-\int_{z0}^{zS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}V\ \sin\mu+\bar{n}_{xs_z}V\ \cos\mu)ds\right]ydz \tag{13}$$

where (y_0, z_0) and (y_S, z_S) are the beginning and end coordinates, respectively, of the section. Neither V nor μ are functions of cross sectional coordinates, and so Eq. (13) can be written as:

$$e=+\int_{y0}^{yS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}\ \sin\mu+\bar{n}_{xs_z}\ \cos\mu)ds\right]zdy$$

$$-\int_{z0}^{zS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}\ \sin\mu+\bar{n}_{xs_z}\ \cos\mu)ds\right]ydz \tag{14}$$

where μ terms are left in the integrands for notational compaction. Note that e does not depend on V and all quantities on the right-hand-side of Eq. (14) are known.

Following a similar procedure, the shear center for closed sections is found to be:

$$e=+\int_{y0}^{yS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}\ \sin\mu+\bar{n}_{xs_z}\ \cos\mu)ds+\frac{\bar{N}_{xs0}}{V}\right]zdy$$

$$-\int_{z0}^{zS}\left[\int_{s0}^{sS}(\bar{n}_{xs_y}\ \sin\mu+\bar{n}_{xs_z}\ \cos\mu)ds+\frac{\bar{N}_{xs0}}{V}\right]ydz \tag{15}$$

In the closed section case, moment equilibrium alone is insufficient to provide the shear center offset e since $\bar{N}_{xs_0}$ is as yet undetermined. In the isotropic case, zero torsional couple implies zero twist, and therefore the additional condition of zero twist permits determination of $\bar{N}_{xs_0}$. As discussed earlier, for anisotropic sections, zero torsional couple does not necessarily imply zero twist. Referring to Eq. (10), the analogous anisotropic condition is that $\bar{\theta}=0$ (not $\theta=0$). This guarantees that the shear stresses $\bar{N}_{xs}$ generated by V induce no resultant torsional couple since V must pass through the shear center for the zero $\bar{\theta}$ condition to hold. The additional constraint provides a means of determining $\bar{N}_{xs_0}$ in terms of V. This relationship depends on the particular beam theory of choice but will be of the form:

$$\bar{N}_{xs_0}=\bar{n}_{xs_0}V \tag{16}$$

where $\bar{n}_{xs_0}$ is a section parameter depending on material properties and cross sectional geometry (see, for example, Pollock 1993). The form of this relationship can be seen as follows. If the right hand side of Eq. (15) is multiplied through by V, it represents the torsional moment generated by shear stresses in closed sections. Thus, the torsional moment is a linear function of V and this dependence is carried through in satisfying Eq. (10) for $\bar{\theta}=0$.

The final relation needed to determine the shear center offset e for closed sections is obtained by substituting Eq. (16) into Eq. (15):

$$e=+\int_{y0}^{ys}\left[\int_{s0}^{s}(\bar{n}_{xs_y}\sin\mu+\bar{n}_{xs_z}\cos\mu)ds+\bar{n}_{xs_0}\right]zdy$$

$$-\int_{z0}^{zs}\left[\int_{s0}^{s}(\bar{n}_{xs_y}\sin\mu+\bar{n}_{xs_z}\cos\mu)ds+\bar{n}_{xs_0}\right]ydz \tag{17}$$

Note that e is again dependent only on the shear stress distribution and independent of the magnitude of the external load V.

4. Discussion and numerical examples

The location of the shear center in both elastic isotropic and anisotropic sections is independent of the magnitude but not direction of the shear force V. There are, however, several differences between anisotropic shear center and its isotropic homogeneous counterpart. These differences are the result of non-classical effects related to material property coupling. First, while the isotropic homogeneous shear center depends only on section geometry, the anisotropic one depends additionally on material parameters (this was also observed for nonhomogeneous, isotropic elastic and viscoelastic bending by Hilton and Piechocki 1962).

If an isotropic homogeneous cross section has a geometric axis of symmetry parallel to the line-of-action of the shear load, the shear center must lie on that axis. This is governed by balancing moments and is a purely geometric effect. Since anisotropic shear centers depend on material parameters as well, the shear center need not lie on geometric axes of symmetry unless they are also material lines of symmetry.

The most unique distinction for anisotropic sections is that if shear center locations are evaluated with a procedure that is completely analogous to the isotropic one, then twist may still be produced. The residual twist is due to coupling resulting from anisotropic material properties.

For the purpose of providing numerical results, a laminated thin-walled, elastic beam theory developed by Zak, *et al.* (1985, 1987) is employed.

Using this beam theory, the present shear center formulation has been coded and extensively checked against known isotropic values (Pollock 1993). In every case the results reduce to the isotropic case, as required, when isotropic material properties are used. Two illustrative example with various cases are presented to demonstrate effects associated with anisotropic shear centers.

4.1. Example 1: Laminated uneven channel section

To study the effects of point of application of shear loads, an uneven channel section is exploited. The section is shown in Fig. 5 with a vertical shear load of 100 lbs acting at both the centroid (c.g.) and the shear center.

The shear center is calculated for the three lay-ups given in Table 1. The material denoted by M1 in Table 2 is used for each ply. Unsymmetric bending characterized by two shear load

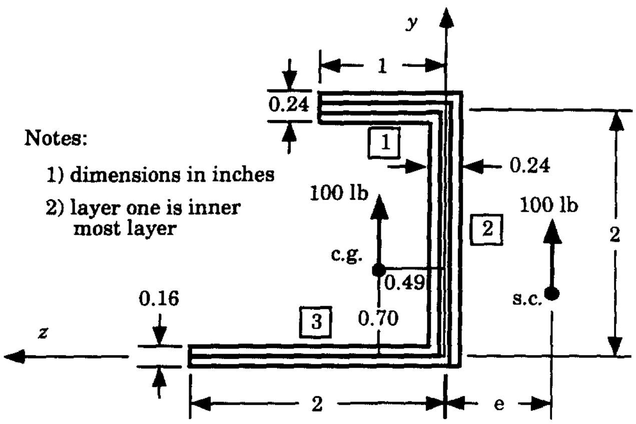


Fig. 5 Laminated uneven channel section.

Table 1 Configuration of uneven channel section

Case	Segment	Layer	Orientation (degrees)	Thickness (inches)
		1	−45	0.08
	1	2	0	0.08
		3	45	0.08
		1	−30	0.08
1	2	2	0	0.08
		3	30	0.08
	3	1	60	0.08
		2	−60	0.08
		1	−45	0.08
	1	2	0	0.08
		3	45	0.08
		1	30	0.08
2	2	2	0	0.08
		3	30	0.08
	3	1	60	0.08
		2	−60	0.08
		1	45	0.08
	1	2	0	0.08
		3	45	0.08
		1	30	0.08
3	2	2	0	0.08
		3	30	0.08
	3	1	60	0.08
		2	60	0.08

lines-of-action (horizontal and vertical) are considered for shear center calculations. In Table 3, the shear center offsets are listed for the three cases and compared to the equivalent isotropic case (i.e., an isotropic section of identical geometry and since the elastic, homogeneous, isotropic

Table 2 Material properties of unidirectional plies

Material	E_1 (Msi)	E_2 (Msi)	G_{12} (Msi)	ν_{12}
*M*1	3.99	1.26	0.51	0.260
*M*2	8.00	2.00	3.84	0.300

Table 3 Shear center locations for example 1

μ	*e* (in)			
(deg)	Case 1	Case 2	Case 3	Isotropic
0	0.497	0.496	0.575	0.421
90	0.451	0.453	0.437	0.410

Table 4 Effect of shear force position on tip deformation for example 1

Case	Transverse deflection (in)	Lateral deflection (in)	Transverse bending slope (rad)	Lateral bending slope (rad)	Twist (rad)
1					
c.g.	0.492	0.174	0.0191	0.00592	−0.109
s.c.	0.491	0.209	0.0190	0.00766	−0.001
2					
c.g.	0.493	0.169	0.0190	0.00565	−0.0878
s.c.	0.493	0.211	0.0191	0.00778	0.0019
3					
c.g.	0.554	0.181	0.0217	0.00577	−0.0785
s.c.	0.554	0.181	0.0217	0.00577	0.0021

shear center depends only on geometry, the isotropic material properties are irrelevant).

Although the only difference between the three anisotropic sections lies in the lay-up scheme, the shear center offsets vary by up to 16 percent and by as much as 37 percent compared to the isotropic case. It is also evident that the shear center position is extremely sensitive to fiber orientation, i.e. degree of anisotropy.

Effects of applying shear loads at points other than the shear center are investigated by moving the shear loads from the c.g. to the *s.c.* In Table 4, tip deformations due to vertical shear load at the shear center (*s.c.*) are compared to their counterparts with the shear load at the centroid (*c.g.*). Although changes in the lateral and transverse deflections are noted, twisting deformations, as also expected in the isotropic case, are severely affected by shear load location. The significance of this example is that, although reduced by one to two orders of magnitude, anisotropic twist does not vanish when the shear force acts through the shear center.

4.2. Example 2: Diamond shape section

The purpose of this example is to numerically demonstrate additional non-classical effects discussed earlier. A diamond section symmetric about both the z and y (horizontal and vertical) axes is chosen and shown in Fig. 6. A fairly complex lamination scheme (Table 5) including layer-and segment-wise material and geometric discontinuities is incorporated to demonstrate

Table 5 Details of diamond shape section

Segment	Layer	Orientation (rad)	Thickness (in)	Material
1	1	30	0.06	*M*1
	2	−30	0.08	*M*2
	3	30	0.08	*M*2
	4	−30	0.06	*M*1
2	1	60	0.08	*M*2
	2	0	0.12	*M*1
	3	−60	0.08	*M*2
3	1	45	0.08	*M*2
	2	−45	0.06	*M*1
	3	45	0.06	*M*1
	4	−45	0.08	*M*2
4	1	0	0.14	*M*1
	2	90	0.14	*M*2

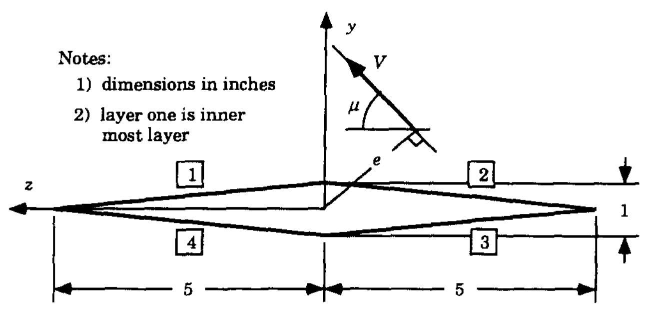


Fig. 6 Laminated diamond shape section.

Table 6 Shear center offsets for diamond section

μ (deg)	*e* (in)			
	Anisotropic		Isotropic	
	Open	Closed	Open	Closed
90	7.268	0.0767	7.500	0.000
60	6.362	0.0596	6.495	0.000
0	0.136	0.0137	0.000	0.000

the capabilities of the analysis. The properties of materials M1 and M2 are listed in Table 2. Both open and closed sections are considered where, in the open case, a small gap is imagined to exist between segments 1 and 4. Isotropic results are given as a reference.

Table 6 summarizes the results of the shear center locations for open and closed, isotropic and anisotropic cases with shear load line-of-action oriented at three different angles. In the open section case, due to the break between segments 1 and 4, only one axis of geometric symmetry exists, i.e. the z axis. As required, the isotropic shear center lies on this axis for the vertical shear load case (μ=0), but since this axis is not also an axis of material symmetry, the anisotropic shear center lies elsewhere.

For the closed section, two lines (y and z axes) of geometric symmetry exist, and thus the isotropic shear center coincides with this origin. Neither axis, however, is an axis of material symmetry, and hence the anisotropic shear center lies away from the origin. It is interesting that while the isotropic shear center remains fixed at the origin (to within the indicated accuracy), the anisotropic shear center changes with the angle μ of the line-of-action. It is important to emphasize that the isotropic shear center formula fails to predict the correct anisotropic shear center.

5. Conclusions

Shear centers for anisotropic elastic thin-walled beams can be determined using strength of material procedures analogous to those of isotropic beams which lead to a condition of zero resultant torsional couple over the section. As a consequence, due to the presence of bending/twisting coupling in composite beams, twist may occur even if shear forces do not produce torsional couples. Moreover, unlike isotropic beams, the shear center for anisotropic beams is dependent upon material properties in addition to section geometry. Two example problems numerically confirm these non-classical predictions which do not depend implicitly on any preselected anisotropic beam theory.

References

Allen, D. H. and Haisler, W. E. (1985), *Introduction to Aerospace Structural Analysis*, John Wiley & Sons, New York.

Bauchau, O. A. (1985), "A beam theory for anisotropic materials, " *Journal of Applied Mechanics*, **52**, 416-422.

Bauld, Jr. N. R. B. and Tzeng, L. S. (1984), "A Vlasov theory for fiber-reinforced beams with thin-walled open cross sections", *International Journal of Solids and Structures*, **20**(3), 277-297.

Bisplinghoff, R. L., Ashley, H. and Halfman, R. L. (1955), *Aeroelasticity*, Addison-Wesley Publishing Company, Boston.

Donaldson, B. K. (1993), *Analysis of Aircraft Structures, An Introduction*, McGraw-Hill Book Company, New York.

Duncan, W. T. (1932), "Torsion and flexure of cylinders and tubes", *Reports and Memoranda, National Advisory Committee on Aeronautics* (1444), 54-77.

Duncan, W. T. (1953), "The flexural centre or centre of shear", *Journal of the Royal Aeronautical Society*, **57**, 594-597.

Faupel, J. H. and Fisher, F. E. (1981), *Engineering Design*, 2nd Edn, John Wiley & Sons, New York.

Gjelsvik, A. (1981), *The Theory of Thin Walled Bars*, John Wiley & Sons, New York.

Goodier, J. N. (1944), "A theorem on the shearing stress in beams with applications to multicellular sections", *Journal of the Aeronautical Sciences*, **11**, 272-280.

Griffith, A. A. and Taylor, G. I. (1917), "The problem of flexure", *Reports and Memoranda, National Advisory Committee on Aeronautics* (399), 1-21.

Hilton, H. H. and Piechocki, J. J. (1962), "Shear center motion in beams with temperature dependent linear elastic or viscoelastic properties", *Proceedings of the Fourth U.S. National Congress on Applied Mechanics*, pp. 1279-1289, ASME.

Libove, C. (1988), "Stresses and rate of twist in single-cell thin-walled beams with anisotropic walls", *AIAA Journal*, **26**(9), 1107-1118.

Mansfield, E. H. (1979), "On the deflection of an anisotropic cantilever plate with variable rigidity", *Proceedings of the Royal Society of London*, **A. 366**, 491-515.

Niles, A. S. and Newell, J. S. (1938), *Airplane Structures*, **1**, 2nd Edn. John Wiley & Sons, New York.
Oden, J. T. and Ripperger, E. A. (1981), *Mechanics of Elastic Structures*, 2nd Edn. Hemisphere Publishing Corporation, McGraw-Hill Book Company, New York.
Osgood, W. R. (1943), "The center of shear again", *Journal of Applied Mechanics*, pp. A-62-A-64.
Peery, D. J. and Azar, J. J. (1982), *Aircraft Structures*, 2nd Edn. McGraw-Hill Book Company, New York.
Pollock, G. D. (1993), "*Thin-walled orthotropic laminated beam theory*", Ph.D dissertation, Department of Aeronautical and Astronautical Engineering, University of Illinois at Urbana-Champaign.
Reissner, E. (1989), "The center of shear as a problem of the theory of plates of variable thickness", *Archive of Applied Mechanics* (*Ingenieur-Archiv*), **59**, 325-332.
Reissner, E. (1991), "Approximate determination the center of shear by use of the st. venant solution for the flexure problem of plates of variable thickness", *Archive of Applied Mechanics* (*Ingenieur-Archiv*), **61**(8), 555-566.
Reissner, E. (1992), "On some ramifications of the center of shear problem", *Zeitschrift fur Angewandte Mathematik und Mechanik*, **72**(8), 315-319.
Reissner, E. and Tsai, W. T. (1972), "On the determination of the centers of twist and of shear for cylindrical shell beams", *Journal of Applied Mechanics*, **39**, 1098-1102.
Trefftz, E. (1935), "Über den Schubmittelpunkt in einem durch eine Einzellast gebogenen Balken", *Zeitschrift fur Angewandte Mathematik und Mechanik*, **15**(4), 220-235.
Zak, A. R. (1987), "Finite element limitations for orthotropic, laminated beam analysis", *Journal of Engineering for Industry*, **109**, 34-38.
Zak, A. R. and Nagarajan, S. (1985), "Finite element model for orthotropic beams", *Composite Structures*, **20**(1-3), 443-449.