# Harmony search algorithm for optimum design of steel frame structures: A comparative study with other optimization methods

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**Abstract.** In this article, a harmony search algorithm is presented for optimum design of steel frame structures. Harmony search is a meta-heuristic search method which has been developed recently. It is based on the analogy between the performance process of natural music and searching for solutions of optimization problems. The design algorithms obtain minimum weight frames by selecting suitable sections from a standard set of steel sections such as American Institute of Steel Construction (AISC) wide-flange (W) shapes. Stress constraints of AISC Load and Resistance Factor Design (LRFD) and AISC Allowable Stress Design (ASD) specifications, maximum (lateral displacement) and interstorey drift constraints, and also size constraint for columns were imposed on frames. The results of harmony search algorithm were compared to those of the other optimization algorithms such as genetic algorithm, optimality criterion and simulated annealing for two planar and two space frame structures taken from the literature. The comparisons showed that the harmony search algorithm yielded lighter designs for the design examples presented.

Keywords: optimum design; harmony search; steel frame structures.

## 1. Introduction

Many optimization techniques have been used to solve structural optimization problems. Some of them are optimality criteria (OC), simulated annealing (SA) and genetic algorithm (GA) whose numerical solutions will be compared with harmony search (HS) algorithm in this study.

OC methods are developed from indirectly applying the Kuhn-Tucker conditions of nonlinear mathematical programming combined with Lagrangian multipliers (Camp *et al.* 1998). OC methods were used in many structural optimization problems (Arora 1980, Tabak and Wright 1981, Lin and Liu 1989, Saka and Hayalioglu 1991, Rozvany and Zhou 1991, Chan 1992, Soegiarso and Adeli 1997).

SA was inspired by the analogy between the annealing of solids and searching the solutions to optimization problems. It was originally put forward by Kirkpatrick *et al.* (1983) for optimization problems. A broad and useful information can be found in the book by van Laarhoven and Aarts (1987). SA was applied to the size and/or topological optimization of metal structures, i.e., plane frames, plane and/or space trusses subjected to static or dynamic loading using discrete and/or

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continuous variables (Elperin 1988, Kincaid 1992, Kincaid 1993, Topping *et al.* 1993, Bennage and Dhingra 1995, Dhingra and Bennage 1995, Manoharan and Shanmuganathan 1999, Pantelidis and Tzan 2000, Chen and Su 2002, Hasancebi and Erbatur 2002, Park and Sung 2002). SA was also applied to the optimum design of concrete retaining structures (Ceranic *et al.* 2001) and shell structures (Barski 2006).

Regarding the use of SA in the optimum design of steel frames under the actual design constraints and loads of code specifications, the following articles can be considered: Huang and Arora (1997) employed SA in the optimum design of steel plane frames subjected to design constraints of AISC-ASD specification. Balling (1991) applied SA to the optimum design of steel space frames for the design constraints of AISC-ASD specification using discrete W steel sections and compared the results with the ones of branch and bound method. The behaviour of the structures in the last two articles was assumed to be linear-elastic. Degertekin (2007) applied SA to the optimum design of geometrically non-linear steel space frames under the design constraints of AISC-LRFD (1995) and AISC-ASD (1989) specifications and compared the results with the ones of genetic algorithm method. Rama Mohan Rao and Arvind (2007) proposed a SA algorithm in which a tabu search algorithm is embedded in the algorithm in order to prevent recycling of recently visited solutions. They used this algorithm for optimal stacking sequence design of laminate composite structures.

GAs which are applications of biological principles into computational algorithms, have been used to solve optimum structural design problems in recent years. They apply the principle of survival of the fittest into the optimization of structures. They are also able to deal with discrete optimum design problems and do not need derivatives of functions, unlike classical optimization. However, the optimum design procedures based on the genetic algorithm are time consuming and the optimum solutions obtained may not be global ones, though they are feasible both mathematically and practically. Genetic algorithms have been employed to solve many structural optimization problems. They were used for the optimum design of planar/space trusses and frames (Rajeev and Krishnamoorthy 1992, Camp *et al.* 1998, Shrestha and Ghaboussi 1998, Pezeshk *et al.* 2000, Hayalioglu 2000, 2001, Kaveh and Kalatraji 2002, 2004, Kaveh and Rahami 2006). Genetic algorithms were also employed to obtain optimum design of semi-rigid steel frames under the actual constraints of design codes (Kameshki and Saka 2001, 2003, Hayalioglu and Degertekin 2004, 2005).

A new meta-heuristic search algorithm called harmony search has been developed recently. Harmony search (HS) bases on the analogy between the performance process of natural music and searching for solutions to optimization problems. HS was proposed by Geem *et al.* (2001) for solving combinatorial optimization problems. It can be easily programmed and adopted for engineering optimization problems. Although HS is a new heuristic algorithm, it has been applied to some different engineering problems. These are river flood model (Kim *et al.* 2001), optimal design of dam drainage pipes (Paik *et al.* 2001), and design of water distribution networks (Geem 2006). HS was also applied to the optimal design of planar and space trusses (Lee and Geem 2004, 2005, Lee *et al.* 2005).

The main differences between HS and GA are summarized as: (i) HS generates a new design considering all existing designs, while GA generates a new design from a couple of chosen parents by exchanging the artificial genes; (ii) HS takes into account each design variable independently. On the other hand, GA considers design variables dependently with building block theory (Goldberg 1989). (iii) HS does not code the parameters, whereas GA codes the parameters. That is, HS uses real value scheme, while GA uses binary scheme (0 and 1). The main differences between HS and SA are summarized as: (i) HS obtains a new design considering all existing designs as mentioned

above, while SA generates a new design considering few neighbour designs of current design. (ii) HS preserves better designs in its memory whereas SA has not memory facility. The main differences between HS and optimality criterion method (OC) are also summarized as: (i) HS does not require derivative information while derivative information is required by OC. (ii) HS imposes much less mathematical requirements to solve optimum design problems than OC. (iii) The probability of becoming entrapped in a local optimum is prevented in HS, because HS has a memory and it is not hill-climbing algorithm whereas OC has not memory facility. These differences provide more flexible and powerful approach for HS than GA, SA and OC.

This paper aims at introducing HS into the optimal design of a different structural system, i.e., steel frame structures, under the actual design constraints of code specifications AISC-LRFD (1995) and AISC-ASD (1989) and comparing the results of HS algorithm with the ones of GA, SA and OC based algorithms using two planar and two steel space frame structures taken from current literature.

### 2. The formulations of the optimum design problem

The discrete optimum design problem of steel frame structures where the minimum weight is considered as the objective can be stated as follows

Minimize 
$$W(x) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i$$
 (1)

subjected to the stress constraints of AISC-LRFD (1995), displacement and size constraints. In Eq. (1), mk is the total numbers of members in group k,  $\rho_i$  and  $L_i$  are density and length of member i,  $A_k$  is cross-sectional area of member group k, and ng is total numbers of groups in the frame.

All the constraints are given in normalized forms which are suitable for HS whose objective functions can be arranged in an unconstrained manner.

The displacement constraints are

$$g_{jl}(x) = \frac{\delta_{jl}}{\delta_{ju}} - 1 \le 0, \quad j = 1, ..., m, \ l = 1, ..., nl$$
<sup>(2)</sup>

$$g_{jil}(x) = \frac{\Delta_{jil}}{\Delta_{ju}} - 1 \le 0, \quad j = 1, \dots, ns, \ i = 1, \dots, nsc, \ l = 1, \dots, nl$$
(3)

where  $\delta_{jl}$  is the displacement of the *j*-th degree of freedom due to loading condition *l*,  $\delta_{ju}$  is its upper bound, *m* is the number of restricted displacements, *nl* is the total number of loading conditions,  $\Delta_{jil}$ is interstorey drift of *i*-th column in the *j*-th storey due to loading condition *l*,  $\Delta_{ju}$  is its limit, *ns* is the number of storeys in the frame, *nsc* is the number of columns in a storey.

The stress constraints taken from AISC-LRFD (1995) are expressed in the following equations. For members subject to bending moment and axial force

for 
$$\frac{P_u}{\phi P_n} \ge 0.2$$
  
 $g_{il}(x) = \left(\frac{P_u}{\phi P_n}\right)_{il} + \frac{8}{9} \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)_{il} - 1.0 \le 0, \quad i = 1, ..., nm, \ l = 1, ..., nl$ 
(4)

for 
$$\frac{P_u}{\phi P_n} < 0.2$$
  
 $g_{il}(x) = \left(\frac{P_u}{2\phi P_n}\right)_{il} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right)_{il} - 1.0 \le 0, \quad i = 1, ..., nm, \ l = 1, ..., nl$  (5)

where *nm* is total number of members in the frame,  $P_u$  = required axial strength (compression or tension),  $P_n$  = nominal axial strength (compression or tension),  $M_{ux}$  = required flexural strengths about the major axis,  $M_{uy}$  = required flexural strengths about the minor axis,  $M_{nx}$  = nominal flexural strength about the minor axis,  $M_{uy}$  = 0,  $\phi = \phi_c$  = resistance factor for compression (equal to 0.85),  $\phi = \phi_t$  = resistance factor for tension (equal to 0.90),  $\phi_b$  = flexural resistance factor (equal to 0.90).

When the stress constraints of AISC-ASD (1989) are used the following equations are required to be included in the design formulation.

For members subjected to both axial compression and bending stresses

$$g_{i}(x) = \left[\frac{f_{a}}{F_{a}} + \frac{C_{mx}f_{bx}}{\left(1 - \frac{f_{a}}{F_{ex}'}\right)F_{bx}} + \frac{C_{my}f_{by}}{\left(1 - \frac{f_{a}}{F_{ey}'}\right)F_{by}}\right]_{i} - 1.0 \le 0, \quad i = 1, \dots, nc$$
(6)

$$g_i(x) = \left[\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}}\right]_i - 1.0 \le 0, \quad i = 1, \dots, nc$$
(7)

When  $f_a/F_a \le 0.15$ , Eq. (8) is permitted in lieu of Eqs. (6) and (7)

$$g_{i}(x) = \left[\frac{f_{a}}{F_{a}} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}}\right]_{i} - 1.0 \le 0, \quad i = 1, \dots, nc$$
(8)

For members subjected to both axial tension and bending stresses

$$g_i(x) = \left[\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}}\right]_i - 1.0 \le 0 , \quad i = 1, \dots, nb$$
(9)

where *nc* is total number of members subjected to both axial compression and bending stresses and *nb* is total number of members subjected to both axial tension and bending stresses.

In Eqs. (6)-(9), the subscripts x and y, combined with subscripts b, m and e, indicate the axis of bending about which a particular stress or design property applies, and  $F_a$  = axial compressive stress that would be permitted if axial force alone existed,  $F_b$  = compressive bending stress that would be permitted if bending moment alone existed,  $F'_e$  = Euler stress divided by a factor of safety,  $f_a$  = computed axial stress,  $f_b$  = computed compressive bending stress at the point under consideration,  $C_m$  = a coefficient whose value is taken as 0.85 for compression members in frames subject to sidesway. In Eq. (9),  $f_b$  is the computed bending tensile stress,  $f_a$  is the computed axial tensile stress,  $F_b$  is the allowable bending stress and  $F_i$  is the governing allowable tensile stress. Definitions of the allowable and Euler stresses and the other details are given AISC-ASD (1989) specification and therefore will not be repeated here.

The same normalized displacement constraints are used for AISC-ASD code as in Eqs. (2) and (3) with the exception that the subscript l is omitted due to single load condition.

The size constraint employed for constructional reasons given as follows

$$g_n(x) = \frac{d_{un}}{d_{bn}} - 1.0 \le 0 \qquad n = 1, ..., ncl$$
(10)

where  $d_{un}$  and  $d_{bn}$  are depths of steel sections selected for upper and lower floor columns, *ncl* is the total number of columns in the frame except the ones at the bottom floor.

The unconstrained objective function  $\varphi(x)$  is then written for AISC-LRFD code as

$$\varphi(x) = W(x) \left[ 1 + C \left( \sum_{j=1}^{m} \sum_{l=1}^{nl} v_{jl} + \sum_{j=1}^{m} \sum_{i=1}^{nsc} \sum_{l=1}^{nl} v_{jil} + \sum_{i=1}^{nm} \sum_{l=1}^{nl} v_{il} + \sum_{n=1}^{ncl} v_n \right) \right]$$
(11)

where C is a penalty constant to be selected depending on the problem.  $v_{jl}$ ,  $v_{jil}$ ,  $v_{il}$  and  $v_n$  are violation coefficients which are calculated as

$$\begin{array}{lll} \text{if} & g_{jl}(x) > 0 & \text{then} & v_{jl} = g_{jl}(x) \\ \text{if} & g_{jl}(x) \leq 0 & \text{then} & v_{jl} = 0 \\ \text{if} & g_{jil}(x) > 0 & \text{then} & v_{jil} = g_{jil}(x) \\ \text{if} & g_{jil}(x) \leq 0 & \text{then} & v_{jil} = 0 \\ \text{if} & g_{il}(x) > 0 & \text{then} & v_{il} = g_{il}(x) \\ \text{if} & g_{il}(x) \leq 0 & \text{then} & v_{il} = g_{il}(x) \\ \text{if} & g_{n}(x) \geq 0 & \text{then} & v_{n} = g_{n}(x) \\ \text{if} & g_{n}(x) \leq 0 & \text{then} & v_{n} = g_{n}(x) \\ \text{if} & g_{n}(x) \leq 0 & \text{then} & v_{n} = 0 \end{array}$$

For AISC-ASD code, the subscript l is omitted in Eqs. (11) and (12) together with the summation symbol associated with l, and the stress constraints in Eqs. (6)-(9) are used instead of Eqs. (4) and (5).

The minimum of the unconstrained function  $\varphi(x)$  will be searched by HS. It is clear that computation of  $\varphi(x)$  for HS requires the values of displacements and stresses in the frame. This is achieved by carrying out the analysis of frame structures.

## 3. Harmony search

The HS algorithm mimics music improvisation process where the musicians try to find a better harmony. A musician always desires to reach the best harmony, which can be obtained by numerous practices. The pitches of the instruments are adjusted after the each practice.

Fig. 1 demonstrates the analogy between harmony memory and steel design. Harmony memory (HM) is the most important part of HS, which is shown in Fig. 1. Jazz improvisation is the best example for clarifying the harmony memory. Many jazz trios consist of a guitarist, double bassist and pianist. Each musician in the trio has different pitches: guitarist [Fa, Mi, Sol, Re, Si]; double bassist [Si, La, Re, Sol, Do]; pianist [Do, Fa, Sol, Re, Mi]. Let guitarist randomly play Sol out of its pitches [Fa, Mi, *Sol*, Re, Si], double bassist Si out of [*Si*, La, Re, Sol, Do] and pianist Re [Do, Fa, Sol, Pa, So



Fig. 1 Analogy between harmony memory and steel design

Sol, *Re*, Mi]. Therefore, the new harmony [Sol, Si, Re] becomes another harmony (musically G-chord). If the new harmony is better than existing worst harmony in the HM, new harmony is included in the HM and the existing worst harmony is excluded from the HM. The process is repeated until the best harmony is obtained.

We consider a steel design process, which consists of three different list of design variables: The first, second and third design variables are selected from [W24×62, W18×76, W18×35, W14×82, W12×50], [W30×90, W27×84, W21×62, W16×50, W14×90], [W33×130, W27×102, W24×55, W14×61, W8×67], respectively. Let us assume W24×62, W14×90 and W33×130 are selected from the first, second and third list randomly. Thus, a new steel design is created [W24×62, W14×90, W33×130]. If the new design is better than existing worst design (the worst design is the one with the highest objective function value), the new design is included and worst design is excluded from the steel design process. This procedure is repeated until terminating criterion is satisfied.

An analogy between the music improvisation process and the optimum design of steel frames can be established in the following way: The harmony denotes the solution vector (weight of steel design) while the different harmonies during the improvisation represent the different solution vectors throughout the optimum design process. Each musical instrument denotes the design variables (steel sections) of objective function. The pitches of the instruments represent the selection of design variables (steel section number). A better harmony represents local optimum and the best harmony is the global optimum.

# 4. Optimum design using harmony search algorithm

The optimum design algorithm using HS algorithm consists of following steps: Initializing the harmony search parameters; initializing harmony memory; improvising a new harmony, updating the harmony memory, terminating the process (termination criterion).

## 4.1 Initializing the harmony search parameters

The HS algorithm parameters are assigned in this step. These are harmony memory size (HMS), harmony memory consideration rate (HMCR), pitch adjusting rate (PAR) and termination criteria (number of improvisation). These parameters are selected depending on the problem.

## 4.2 Initializing harmony memory

The Harmony memory (HM) matrix is filled with randomly generated designs as many as the size of the harmony memory (HMS).

$$HM = \begin{bmatrix} x_{1}^{1} & x_{2}^{1} & \dots & x_{ng-1}^{1} & x_{ng}^{1} \\ x_{1}^{2} & x_{2}^{2} & \dots & x_{ng-1}^{2} & x_{ng}^{2} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ x_{1}^{HMS-1} & x_{2}^{HMS-1} & \dots & x_{ng-1}^{HMS-1} & x_{ng}^{HMS-1} \\ x_{1}^{HMS} & x_{2}^{HMS} & \dots & x_{ng-1}^{HMS} & x_{ng}^{HMS} \end{bmatrix} \xrightarrow{\rightarrow} \varphi(x^{HMS})$$
(13)

Each row denotes a steel design in the HM.  $x^1, x^2, ..., x^{HMS-1}, x^{HMS}$  and  $\varphi(x^1), \varphi(x^2), ..., \varphi(x^{HMS-1}), \varphi(x^{HMS})$  are designs and the corresponding constrained-included objective function value, respectively. The steel designs in the HM are sorted by the unconstrained objective function values. (i.e.,  $\varphi(x^1) < \varphi(x^2) < ... < \varphi(x^{HMS})$ ). The aim of using HM is to preserve better designs in the search process.

# 4.3 Improvising a new harmony

A new harmony  $x^{nh} = (x_1^{nh}, x_2^{nh}, \dots, x_{ng}^{nh})$  is improvised from either the HM or entire section list. Three rules are used for the generation of the new harmony. These are HM consideration, pitch adjustment and random generation.

In the HM consideration, the value of first design variable  $x_1^{nh}$  for the new harmony is chosen from the HM (i.e.,  $\{x_1^1, x_1^2, ..., x_1^{HMS-1}, x_1^{HMS}\}$ ) or from entire section list  $(X_{SL})$ . The other design variables of new harmony  $(x_2^{nh}, ..., x_{ng-1}^{nh}, x_{ng}^{nh})$  are chosen by the same rationale. HMCR is applied as follows

$$\begin{cases} x_i^{nh} \in \{x_i^1, x_i^2, \dots, x_i^{HMS-1}, x_i^{HMS}\} & \text{if } rn \le HMCR \\ x_i^{nh} \in X_{SL} & \text{if } rn > HMCR \end{cases}$$
(14)

At first, a random number (rn) uniformly distributed over the interval [0,1] is generated. If this random number is less than the HMCR value, *i*-th design variable of new design  $(x_i^{nh})$  selected from the current values stored in the *i*-th column of HM. If rn is higher than HMCR, *i*-th design variable of new design  $(x_i^{nh})$  is selected from the entire section list  $(X_{SL})$ . For example, an HMCR of 0.90 shows that the algorithm will choose the *i*-th design variable (i.e., steel section) from the current stored steel sections in the *i*-th column of the HM with a 90% probability or from the entire section list with a 10% probability. A value of 1.0 for HMCR is not appropriate because of the

possibility that the new design may be improved by the values that are not stored in the HM.

Any design variable of the new harmony,  $x^{nh} = (x_1^{nh}, x_2^{nh}, \dots, x_{ng}^{nh})$ , obtained by the memory consideration is examined to determine whether it is pitch-adjusted or not. Pitch adjustment is made by pitch adjustment ratio (PAR). PAR investigates better design in the neighbouring of the current design. PAR is applied as follows

Pitch adjusting decision for 
$$x_i^{nh} \leftarrow \begin{cases} yes & \text{if } rna \le PAR \\ no & \text{if } rna > PAR \end{cases}$$
 (15)

A random number (*rna*) uniformly distributed over the interval [0,1] is generated for  $x_i^{nh}$ . If this random number is less than the PAR,  $x_i^{nh}$  is replaced with its neighbour steel section in the section list. If this random number is not less than PAR,  $x_i^{nh}$  remains the same. The selection of neighbour section is determined by neighbouring index. A PAR of 0.4 indicates that the algorithm chooses a neighbour section with a 40%×HMCR probability. For example, if  $x_i^{nh}$  is W24×84, neighbouring index is ±2 and the section list is [W24×55, W30×108, W24×84, W18×55, W14×30]. A random integer number is generated over the interval [+2, -2]. Let us assume +2 is generated randomly. The algorithm will choose W14×30 with a 40%×HMCR probability, or remain the same section (W24×84) with a (1 - 40%×HMCR) probability. HMCR and PAR parameters are introduced to allow the solution to escape from local optima and to improve the global optimum prediction of the HS algorithm (Lee and Geem 2005).

# 4.4 Updating the harmony memory

If the new harmony  $x^{nh} = (x_1^{nh}, x_2^{nh}, ..., x_{ng}^{nh})$  is better than the worst design in the HM, the new design is included in the HM and the existing worst harmony is excluded from the HM. In this process, it should be noted that HM matrix is sorted again by unconstrained objective function and the same design is not permitted in the HM more than once.

## 4.5 Terminating the process (termination criterion)

3-rd and 4-th steps are repeated until a termination criterion is satisfied. In the present work, two termination criteria were adopted for HS. The first one stops the algorithm when a predetermined total number of searches (i.e., no of improvisations) are performed. The second criterion stops the process before reaching the maximum search number, if more economical design (lighter frame) is not found during a definite number of searches.

## 5. Optimum design of steel frames using harmony search algorithm

The optimum design of steel frame structures using HS algorithm is graphically shown in Fig. 2. HS algorithm are given according to the stress constraints of AISC-LRFD code and it is the same algorithm for AISC-ASD code providing that the stress are used constraints of AISC-ASD and the displacement constraints for single loading condition as mentioned in Section 2. In Fig. 2,  $\varphi(X_w)$ ,  $\varphi(X_{new})$  and  $\varphi(X_{opt})$  denote the unconstrained objective function values of the worst design, new design and current optimum design, respectively in HM. The current optimum design  $(X_{opt})$  at the end of the last search is defined as the final optimum design.



Fig. 2 HS algorithm for optimum design of steel frame structures

# 6. Benchmark examples

In this section, HS is applied to the optimum design of four different steel frames, which are taken from current literature. The optimum design of these frames has already been performed using GA, SA and OC based algorithms in the literature. They were designed again using HS algorithm presented in this study and the design results were compared with the ones of the aforementioned algorithms.

# 6.1 Design of one-bay, eight-storey frame

Fig. 3 shows configuration, dimensions, number of grouping and the applied loads of the one-bay, eight-storey. This frame was optimized by Khot *et al.* (1976) using optimality criteria (OC). It was also designed by Camp *et al.* (1998) using GA. Young's modulus of E = 200 GPa and the material density  $\rho = 76.8$  kN/m<sup>3</sup>. The value of beam and column element was selected from 268 W sections of the AISC-ASD (1989) specification. The lateral drift at the top of structure was the only performance constraint which was no more than 2 inch (5.08 cm).



Fig. 3 One-bay, eight-storey frame

For HS algorithm, 30 different optimum frames were obtained generated from randomly selected 30 different initial designs and the lightest one of those was reported in Table 1. Design history of the best optimum and average frame weight for one-bay, eight-storey frame was also shown in Fig. 4.

The following parameter values were chosen for the HS algorithm: The size of harmony memory (HMS) was selected as 100 that the algorithm is sensitive to the value of it. When HMS was selected greater than 100, HS did not improve the optimal solutions, while for HMS < 50 it resulted in premature convergence. Another parameter affecting the results is the harmony memory consideration rate (HMCR), which was selected as 0.8. The higher values of HMCR tended to reach local optima, while the lower values of HMCR caused the non-optimal solutions. HS is also sensitive to the value of pitch adjusting rate (PAR) which was taken as 0.4. Using higher values for PAR caused non-optimal designs, while lower values for it resulted in local optima. The neighbouring index used in the pitch-adjustment selected as  $\pm 3$ . Lower values of it caused the local

Group no. –	OC	GA	HS
	Khot et al. (1976)	Camp et al. (1998)	The proposed method
1	W 14×34	W 18×46	W 18×40
2	W 10×39	W 16×31	W 16×31
3	W 10×33	W 16×26	W 16×26
4	W 8×18	W 12×16	W 12×19
5	W 21×68	W 18×35	W 18×35
6	W 24×55	W 18×35	W 18×35
7	W 21×50	W 18×35	W 16×31
8	W 12×40	W 16×26	W 16×26
Weight $(kN^*)$	41.01	32.83	31.93

Table 1 Design results for one-bay, eight-storey frame

1 KIN-101.972 Kg
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Fig. 4 Design history of the one-bay, eight-storey frame

Group no	Neighbouring index		
Group no.	±1	±2	
1	W 18×40	W 21×44	
2	W 18×40	W 21×44	
3	W 16×40	W 18×35	
4	W 16×31	W 12×26	
5	W 18×40	W 18×35	
6	W 16×31	W 16×40	
7	W 16×31	W 14×26	
8	W 14×26	W 12×26	
Weight (kN)	38.27	37.83	

Table 2 Design results for lower values of neighbouring index for one-bay, eight-storey frame

optima, whereas higher values of it yielded divergence from the optimal designs. Design results for lower values of neighbouring index were also given in Table 2. A value of 1.0 was found suitable for the penalty constant C. It was found from different runs that the lower values for C led to local optima, while higher values of it caused premature convergence. The HS is also sensitive to the values of the maximum number of searches (number of frame analyses). Computational experience gained after different optimum designs indicated that if the optimum design remains the same during the execution of 20% of the maximum search number, no further improvement is made in the HS process afterwards. Therefore, the first and second termination criteria, explained in Section 4.5, were selected as 5000 and 1000 in this example, respectively.

The best HS design yielded 22.1% lighter frame than OC and it also obtained 2.7% lighter frame than GA. Top storey drift calculated as 5.08 cm, which is equal to the limit value, in the best optimum design. HS developed the best optimum design at the 1042-th analysis and it did not change during 1000 frame analyses afterwards, and thus, HS terminated the search process after 2042 frame analyses, which is less than the 2500 frame analyses required by GA.

The average weight of 30 runs was calculated as 33.17 kN, with a standard deviation of 1.27 kN. HS developed the optimum designs with an average of 985-th frame analyses for 30 runs.

Design results for neighbouring index  $\pm 1$  and  $\pm 2$  were yielded 16.6% and 15.6% heavier frames than the one with neighbouring index  $\pm 3$ . Top storey drifts were calculated as 4.61 cm and 4.67 cm for neighbouring indexes  $\pm 1$  and  $\pm 2$ . Both designs converged at the local optima in comparison with the previous design in Table 1.

#### 6.2 Design of two-bay, three-storey frame

The two-bay, three-storey frame under a single-load case with vertical loads shown in Fig. 5 is the second benchmark example. This frame was optimized by Pezeshk *et al.* (2000) in accordance with the AISC-LRFD specification using GA.

Young's modulus of E = 29,000 ksi (1 ksi = 6.895 MPa) and a yield stress of  $f_y = 36$  ksi were used. Displacement constraints were not imposed for the design. The beam members were selected from a list with 256 W sections and W10 sections were used for column members. The member effective length factors  $K_x$  is calculated from the approximate equation proposed by Dumonteil



Fig. 5 Two-bay, three-storey frame (1 kip = 4.448 kN, 1 ft = 30.48 cm)

Table 3 Design results for two-bay, three-storey frame

Element group	GA	HS
Element group	Pezeshk et al. (2000)	The proposed method
Beam	W 21×62	W 24×62
Column	W 10×100	W 10×88
Weight (lb*)	22,392	21,266

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*1 lb=0.4536 kg
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Fig. 6 Design history of the two-bay, three-storey frame

(1992). For each column, the out-of-plane effective length factor  $(K_y)$  was considered as 1.0. The out-of-plane effective length factor for each beam member was specified to be 0.167. The parameters used in HS were selected the same as the ones of the first example.

For HS algorithm, 30 different optimum frames were also obtained and the lightest one of those was listed in Table 3. Design history for the best optimum design and average frame weight of 30 designs for two-bay, three-storey frame was also depicted in Fig. 6.

Floment group	Neighbouring index		
Element group –	±1	±2	
Beam	W 24×84	W 21×62	
Column	W 10×100	W 100×112	
Weight (lb)	27,160	23,530	

Table 4 Design results for lower values of neighbouring index for two-bay, three-storey frame

HS algorithm resulted in 5.0% lighter frame than GA. Stress constraints were active at the best optimum. HS found the best optimum design at the 1092-th analysis and it did not change during 1000 frame analyses afterwards, and thus, HS converged at the optimum after 2092 frame analyses. It was more than the 1800 analyses performed by GA.

The average weight of 30 runs was calculated as 22,240 lb, with a standard deviation of 595 lb. HS developed the optimum designs with an average of 1008-th frame analyses for 30 runs. The optimum designs were obtained with an average of 900 frame analyses in GA for 30 runs. In this case, HS achieved lighter frame than the one of GA with near analysis number. Design results for neighbouring index  $\pm 1$  and  $\pm 2$  were also reported in Table 4.

Design results of neighbouring index  $\pm 1$  and  $\pm 2$  were obtained 21.7% and 9.6% heavier frames than the one with neighbouring index  $\pm 3$ . Stress constraints were passive for both designs. In this case, lower values for neighbouring index also caused the local optima.

#### 6.3 Design of single-storey, 8 member space frame

The single-storey, 8 member space frame shown in Fig. 7 is the third benchmark example. It was optimized by Degertekin (2007) using SA and GA in accordance with the AISC-LRFD and AISC-ASD specifications. A36 steel grade with a modulus of elasticity of 200 GPa and shear modulus of 83 GPa was used in the space frame structures. The yield stress and unit weight of material are 248.2 MPa and 7850 kg/m<sup>3</sup>, respectively. Four different types of loads are employed: dead load (*D*), live load (*L*), roof live load (*L<sub>r</sub>*), and wind loads (*W*). Four load combinations were taken into account, per AISC-LRFD specification: I: (1.4*D*), II: (1.2*D* + 1.6*L* + 0.5*L<sub>r</sub>*), III: (1.2*D* + 1.6*L<sub>r</sub>* + 0.5*L*), and IV: (1.2*D* + 1.3*W* + 0.5*L* + 0.5*L<sub>r</sub>*).



Fig. 7 Single-storey, 8-member space frame

The values of 3.12 kPa for dead load (D), 2.4 kPa for live load (L) and roof live load ( $L_r$ ) were considered in the space frame structure. Wind loading was obtained from Uniform Building Code (1997) using the equation  $p = C_e C_q q_s I_w$ , where p is design wind pressure;  $C_e$  is combined height, exposure and gust factor coefficient;  $C_q$  is pressure coefficient;  $q_s$  is wind stagnation pressure; and  $I_w$  is wind importance factor. Exposure D was assumed and the values for  $C_e$  were selected depending on the frame height and exposure type. The  $C_q$  values were assigned as 0.8 and 0.5 for inward and outward faces. The value of  $q_s$  was selected as 0.785 kPa assuming a basic wind speed of 129 km/h (80 mph) and the wind importance factor was assumed to be one. The horizontal loads due to wind act in the x-direction at each unrestrained node.

The maximum drift of the top storey was restricted to H/400, where H is the total height of structure; the interstorey drift was also limited to  $h_c/300$ , where  $h_c$  is the height of the considered storey (Ad Hoc Committee 1986) These limits were increased by 30% to include the effect of the coefficient 1.3 in the LRFD wind load combination.

Two discrete design sets for both algorithms and design codes comprised 64 W sections each were used in the examples. The first one is beam section list taken from AISC-ASD (1989)-Part 2, "Beam and Girder Design"- Allowable stress design selection table for shapes used as beams. The boldface type sections (lighter ones) were selected starting from W36×720 to W12×19. The second one is column section list taken from the same code, Part 3, "Column Design"- Column W shapes tables. They were selected from W14×283 to W6×15. The effective length factor K, for unbraced frames were calculated from the approximate equation proposed by Dumonteil (1992). Geometrically nonlinear analysis, which was explained in Degertekin (2007), was performed for SA and HS algorithms.

The members of the frame were divided into three groups organized as follows: 1-st group: the beams in x-direction, 2-nd group: the beams in y-direction, 3-rd group: the all columns. The horizontal loads due to wind act in the x-direction at each unrestrained node. The maximum top storey drift and interstorey drift were restricted to 1.3 cm. The first and second termination criteria were set 1250 and 250 for the AISC-ASD. They were set 5000 and 1000 for the AISC-LRFD. The others parameters were the same as the ones of the first example.

This frame was optimized again using HS algorithm in accordance with the AISC-LRFD (1995) and AISC-ASD (1989) specifications. 10 different optimum frames were obtained and the design results of the lightest ones were summarized in Table 5. The design history for the single-storey 8-member space frame in accordance with AISC-LRFD was given in Fig. 8.

Group no	Degertekin (2007)		The proposed method	
Oroup no.	SA-LRFD	SA-ASD	HS-LRFD	HS-ASD
1	W 12×30	W 12×30	W 12×26	W 12×30
2	W 12×30	W 12×30	W 12×26	W 12×26
3	W 8×24	W 8×28	W 8×28	W 8×28
Weight (kg)	1728	1828	1675	1757
Top storey drift (cm)	1.24	1.0	1.26	1.0
Max. interstorey drift (cm)	1.24	1.0	1.26	1.0
Number of analyses	6120	1530	4412	1126

Table 5 Design results of single-storey, 8-member space frame

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Fig. 8 Design history of the single-storey 8-member space frame for AISC-LRFD

HS-LRFD produced 3.1% lighter frame than SA-LRFD. HS-LRFD obtained the best optimum design after 4412 searches (i.e., 4412 frame analyses). This indicates that HS-LRFD found the optimum design after 3412 searches and it did not change during 1000 searches afterwards. Both displacement and stress constraints active in the best optimum design. The average weight of 10 runs was calculated as 1737 kg, with a standard deviation of 68 kg. HS-LRFD converged at the optimum designs with an average of 4528 frame analyses for 10 runs while the optimum designs were obtained within 6120 frame analyses in SA-LRFD.

HS-ASD obtained 3.9% lighter frame than SA-ASD. It required 1126 frame analyses. This indicates that HS-ASD converged at the optimum design after 876 searches and it did not change during 250 searches afterwards. Both displacement and stress constraints active in the best optimum design. The average weight of 10 runs was calculated as 1812 kg, with a standard deviation of 61 kg. HS-ASD converged at the optimum designs with an average of 1195 frame analyses for 10 runs while the optimum designs were obtained after 1530 frame analyses for SA-LRFD. In this case, HS achieved lighter frame than SA with less analysis number.

## 6.4 Design of 4-storey, 84-member space frame

The last benchmark example is the 4-storey space frame with a square plan and side view shown in Fig. 9. The structure consists of 84 members divided into 10 groups. It was designed by Degertekin (2007) using SA. This frame designed again using HS in accordance with the AISC-LRFD (1995) and AISC-ASD (1989) specifications.

The groups were organized as follows: 1-st group: outer beams of 4-th storey, 2-nd group: outer beams of 3-rd, 2-nd and 1-st storeys, 3-rd group: inner beams of 4-th storey, 4-th group: inner beams of 3-rd, 2-nd and 1-st storeys, 5-th group: corner columns of 4-th storey, 6-th group: corner columns of 3-rd, 2-nd and 1-st storeys, 7-th group: outer columns of 4-th storey, 8-th group: outer columns of 3-rd, 2-nd and 1-st storeys, 9-th group: inner columns of 4-th storey, 10-th group: inner columns of 3-rd, 2-nd and 1-st storeys. The wind loads act in the *x*-direction at each node on the sides AB and CD. For the maximum and interstorey drift constraints, the values of 3.5 cm and 1.17 cm for the ASD, and 4.55 cm and 1.52 cm for the LRFD were imposed on the frame. The first and second termination criteria were set 20000 and 4000 for the LRFD. They were set 5000 and 1000 for the ASD. The others parameters were the same as the previous example.

For HS algorithms, 10 different designs were executed and the lightest ones of those were listed



Fig. 9 Four-storey 84-member space frame (a) plan, (b) side view

Group no	Degertekin (2007)		The proposed method	
Group no. –	SA-LRFD	SA-ASD	HS-LRFD	HS-ASD
1	W 18×35	W 16×31	W 16×31	W 16×31
2	W 18×35	W 16×40	W 16×31	W 16×40
3	W 18×35	W 16×40	W 16×31	W 16×31
4	W 18×35	W 18×40	W 16×40	W 18×40
5	W 8×31	W 8×35	W 8×31	W 8×40
6	W 12×40	W 8×40	W 10×39	W 10×39
7	W 10×39	W 8×31	W 8×40	W 8×35
8	W 12×45	W 14×48	W 10×39	W 14×48
9	W 8×28	W 8×48	W 8×28	W 10×39
10	W 12×58	W 14×82	W 10×77	W 14×74
Weight (kg)	23105	25222	22235	24863
Top storey drift (cm)	4.43	2.80	4.30	3.01
Max. interstorey drift (cm)	1.52	0.81	1.37	0.89
Number of analyses	20400	5100	14276	4178

Table 6 Design results of 4-storey 84-member space frame

in Table 6. Design history for the 4-storey, 84-member space frame in accordance with AISC-LRFD was shown in Fig. 10.

HS-LRFD yielded 3.8% lighter frame than SA-LRFD. Stress constraints were active while displacement constraints were not critical at the best optimum. The best optimum design was found after 14276 frame analyses. The best optimum was achieved at the 10276-th analysis and this design did not change during 4000 frame analyses afterwards. The average optimal weight for the 10 runs was 23165 kg, with a standard deviation of 644 kg. According to the average of the results of 10 runs, HS-LRFD required 15532 frame analyses which was less than the 20400 frame analyses



Fig. 10 Design history of the 4-storey 84-member space frame for AISC-LRFD

## required by SA-LRFD.

HS-ASD developed 1.4% lighter frame than SA-ASD. Stress constraints were active at the best optimum. The best optimum design obtained after 4178 frame analyses. This indicates that HS-ASD converged at the optimum design after 3178 searches and it did not change during 1000 searches afterwards. The average optimal weight for the 10 runs was 25118 kg, with a standard deviation of 289 kg. According to the average of the results of 10 runs, HS-ASD required 4296 frame analyses which was less than the 5100 frame analyses required by SA-ASD.

## 7. Conclusions

The following conclusions are drawn from the design examples presented when the harmony search algorithm is used and compared to the other methods in the optimum design of steel frame structures:

- 1. HS yielded 2.7%-5.0% lighter frames than GA. HS yielded 22.1% lighter frame than OC. HS also found 3.8%-3.9% lighter frames than SA. The reason for this is that HS uses more flexible approach in searching for optima than GA, OC and SA as explained in Section 1.
- 2. With regards to the number of analyses, HS required less number of analyses than SA and equal or less number of analyses than GA.
- 3. The average weights of the frames in the examples were close to the best optimum weights for HS. Standard deviations of the frames weights were also quite small in comparison with the frame weights, which was less than 4% in the all examples. These indicate that HS is able to find the global optima and it could be accepted as a powerful optimization technique for steel frame design when discrete and real design variables are used.
- 4. The most suitable tuning parameters for HS algorithm were found by computational experience, which are explained in Section 6.1.

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