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Technical Note

Application of continuous wavelet transform to detect damage in thin-walled beams coupled in bending and torsion

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1. Introduction

The continuous wavelet transform (CWT), which provides redundant scale information, is a suitable vibration-based damage detection tool in that it can easily detect subtle changes or local discontinuities due to damage in mode shapes or in their derivatives (Gentile and Messina 2003). While an extensive literature review is available in the work of Kim and Melhem (2004), the significant works appeared recently such as Gentile and Messina (2003), Douka *et al.* (2003), Chang and Chen (2005), Rucka and Wild (2006) can be given as current examples of CWT applications. In these works one of the bending vibration modes of damaged beam is analyzed with CWT to obtain transform coefficients which include damage information. Damage is generally modeled as notch, crack, or decrease in modulus of elasticity. Next, the wavelet coefficients, which are functions of scale and translation (spatial variable of beam) parameters, are plotted for different scales to seek sharp changes possibly caused by damages. Once damage location is determined its extent can be estimated, for the magnitudes of wavelet coefficients are proportional to loss of resistance.

Apart from the previous studies this note is interested in damage assessment in thin-walled beams using CWT. Because of asymmetric cross section, slightly different mode shapes will be measured for different offsets from shear centre of beam[1]. Therefore, that magnitude of wavelet coefficients for the same distance from beam root point will be different for various offsets from shear center is emphasized, and practical consequence of this reality is discussed.

2. Theory

The CWT of a function $f(x)$ is defined as $T_{a,b} = a^{-1/2}\int_{-\infty}^{\infty} f(x)\Psi^{*}_{a,b}(x)dx$, where a and b are

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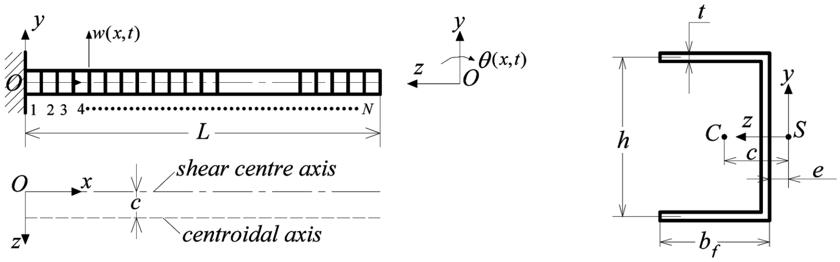


Fig. 1 The beam geometry

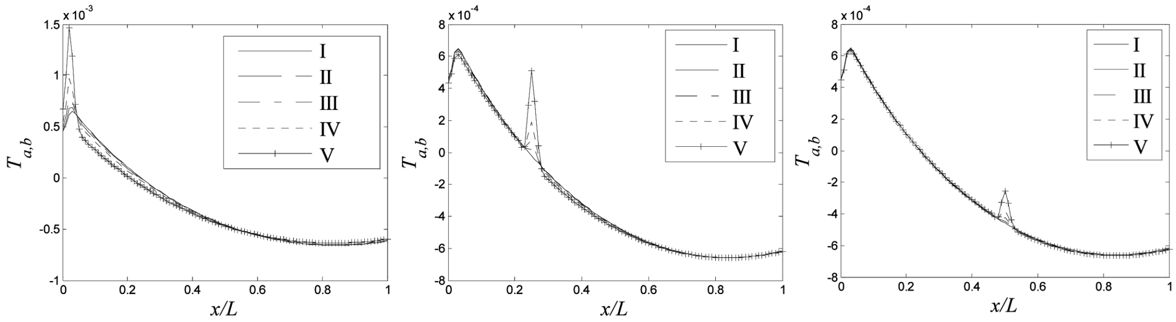
positive real numbers called scale and translation parameters, respectively. $\Psi_{a,b}(x)$ is the wavelet function obtained by a mother wavelet $\Psi(x)$ with the scale and translation parameters; $\Psi_{a,b}(x) = \Psi((x-b)/a)$, and overstar stands for complex conjugate. The mother wavelet should 1) have finite energy; $E = \int_{-\infty}^{\infty}|\Psi(x)|^2 dx < \infty$, 2) satisfy the admissibility condition; $\int_{-\infty}^{\infty} f^{-1}|\hat{\Psi}(x)|^2 df < \infty$, where $\hat{\Psi}$ is Fourier transform of Ψ (Addison 2002). Singularities, sharp change locations in $f(x)$ can be accurately determined by dilating and translating the wavelets with a and b parameters. On the other hand, a wavelet satisfying the equation $\int_{-\infty}^{\infty} x^k \Psi(x) dx = 0$, ($k$ = 0, 1, 2,…, n − 1) is known to have n vanishing moments, i.e., it is orthogonal to the polynomials up to order of (n − 1) (Douka *et al.* 2003).

The beam geometry is depicted in Fig. 1, where L is the length, c is the shear centre offset, w and θ are vertical displacement and rotation about the shear centre of beam cross section located at x at time t, respectively. S and C, respectively, denote the shear centre and the centroid of cross section. Following the well-known finite element modeling procedure one comes out with $4N$+4 (N: number of finite elements) natural frequencies and corresponding eigenvectors. The elements of ith eigenvector $\mathbf{Q}_i$ are the nodal deformation components, that is $\mathbf{Q}_i = \{w_j(t)\ \ \psi_j(t)\ \ \theta_j(t)\ \ \varphi_j(t)\}_i^T$, where $\psi_j = dw_j/dx$, $\varphi_j = d\theta_j/dx$ (j = 1, 2, …, N+1). Using the elements of i^{th} eigenvector, one can extract the i^{th} modal shape of elastic curve, Y_i, and i^{th} torsional mode Θ_i. However, in an experiment one will attach displacement transducer to a point somewhere on beam's flange, so that the modal function to be determined is $U_i = Y_i + d\Theta_i$, where $e < d < e + b_f$ (see Fig. 1, 3(a)). On the other hand, damage is modeled as decrease in the modulus of elasticity. Hence, showing damage extent by D, D = 10 means there is 10% decrease in bending, uniform and warping stiffness, respectively.

3. Results and discussions

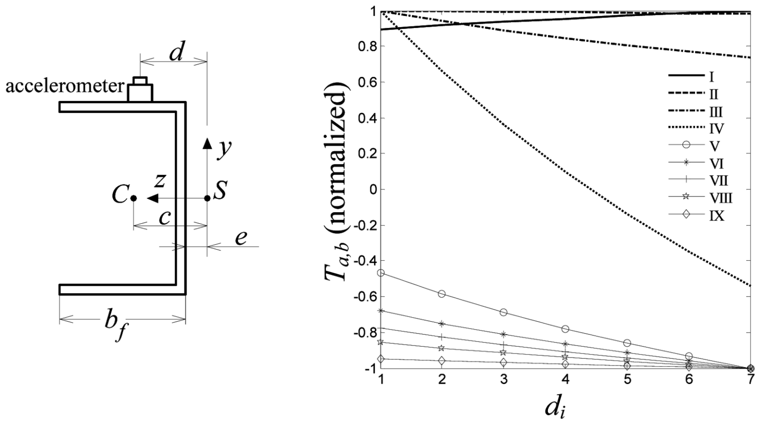
The following geometric and material properties are employed in this work: t = 2 mm, h = 50 mm, b_f = 40 mm, c = 16.4 mm, e = 3.7 mm, L = 1.5 m, $\Gamma = 2.0513 \times 10^{-11}$ m^6 (warping constant), $I_{zz} = 1.1941 \times 10^{-7}$ m^4 (second moment of inertia), $J = 3.6768 \times 10^{-10}$ m^4 (polar moment of inertia). The selected material is Aluminum 2024-T3 whose properties are: $E = 7.31 \times 10^{10}$ Nm^{-2} (modulus of elasticity), ρ = 2770 kgm^{-3} (density), ν = 0.33 (Poisson's ratio). Finite element number is set as N=100, and only the first mode shape is considered for the computations.

The relation between damage and wavelet coefficients is shown in Fig. 2, where three locations and five cases are considered for damage. The sharp picks in the figure correspond to damage positions whereas their magnitudes are proportional to loss of resistance. Generally wavelet coefficients get extremely high values at the boundaries since mode function is bounded, i.e., it does



(a) DL=0.02L (b) DL=0.25L (c) DL=0.5L

Fig. 2 Variation of wavelet coefficients wrt damage location and extent. I: $D = 0$, II: $D = 20$, III: $D = 40$, IV: $D = 60$, V: $D = 80$. Scale parameter: $a = 3$, DL: Damage Location, $d = 0.5b_f + e$



(a) (b)

Fig. 3 (a) Transducer location d. (b) Normalized wavelet coefficients versus d_i distances. $d_i = e + i(b_f/8)$, $i = 1, 2, \ldots, 7$. $D = 80$, I: DL = 0.1L, II: DL = 0.2L, III: DL = 0.3L, IV: DL = 0.4L, V: DL = 0.5L, VI: DL = 0.6L, VII: DL = 0.7L, VIII: DL = 0.8L, IX: DL = 0.9L

not extent to infinity. Hence damage in the proximity of edge is difficult to observe from the coefficients. In this work this deficiency is overcome by extending mode function from free end by cubic polynomial and from fixed end by its symmetry. The fixed end can also be extended by curve fitting, however using mode shape's symmetry is experienced to be more efficient. After several trials, *symlet* wavelet with two vanishing moments is preferred to enable good visual presentation. Furthermore, damage location is best determined when the scale parameter is fixed to 3. Since damage is modeled as decrease in modulus of elasticity, curvature discontinuity, which is proportional to loss of resistance, will occur between damaged and undamaged elements. Therefore, magnitudes of sudden picks tend to decrease as damage location gets closer to the free end, for curvature has smaller value in these locations.

Mode shapes of thin-walled beam vibrating out of symmetry plane will be slightly different for various d values (see Fig. 3(a)), thus wavelet coefficients for the same damage location and extent will vary depending on d. Then, what is the suitable offset d to measure the first mode shape so that wavelet coefficients have more information about damage? The answer can be extracted by examining Fig. 3, where variation of the magnitudes of wavelet coefficients wrt d is illustrated for ten damage locations. In view of the figure, d should be maximum if damage is in the vicinity of fixed end (case I). For cases III and IV, however, the smallest distance d will produce the most sensitive wavelet coefficients to damage. One should avoid measuring especially from the vicinity of the half of flange if damage is around the beam midpoint (case IV). On the other hand, coefficients appear to be less prone to d for the cases II and IX. For damages located on the half of free end side, large d values seem to be more suitable to determine the first mode shape since the magnitudes of wavelet coefficients get higher values as d becomes wider (cases V, VI, VII, VIII). In fact, damage location is not known a priori in most practical applications, so how can this figure be useful? In this case, mode shape can be determined through an arbitrary distance d, so that damage location is approximately estimated via CWT in the first step. Then, to get more damage-sensitive coefficients, this graph can be used to re-measure the most suitable mode shape.

In most of the practical applications certain amount of noise will contaminate measured data. The knowledge that coefficients are sensitive to d will particularly be useful when noisy data is handled. Generally lower scale wavelet coefficients will be affected the most by the presence of noise. However, as already indicated the damage existence is realized through lower scale coefficients if damage is modeled as decrease in the modulus of elasticity. Therefore, the mode shape derived from properly selected d distance will enable the wavelet coefficients with maximum amplitude, so that location and extent of damage can be more effectively evaluated. Although the discussions regarding the Fig. 3 are valid for the selected beam geometry, similar conclusions for various beam type structures coupled in bending and torsion can be extracted through the procedure in this note. Then, the significance of the current study becomes obvious. The analysis can be extended to other mode shapes and various kinds of damage, as well.

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