DOI: http://dx.doi.org/10.12989/sem.2008.29.2.237
Structural Engineering and Mechanics, Vol. 29, No. 2 (2008) 237-240

Technical Note

Discrete singular convolution method and applications to free vibration analysis of circular and annular plates

Ömer Civalek†

Division of Mechanics, Civil Engineering Department, Faculty of Engineering, Akdeniz University, Antalya, Turkiye

(Received September 12, 2006, Accepted August 17, 2007)

1. Introduction

Circular plates have many applications in civil, aerospace, petroleum, nuclear and, mechanical engineering. This paper deals with the application of the DSC method for the free vibration analysis of thin circular and annular plates with clamped and simply supported boundary conditions. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$. This can be expressed as (Wei 2000)

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x_i - x_k) f(x_k); \quad (n = 0, 1, 2, \ldots) \tag{1}$$

where superscript n denotes the nth-order derivative with respect to x. The Shannon's kernel is regularized given below is used

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{2}$$

where Δ is the grid spacing.

2. Free vibration analysis of circular and annular plate

The governing equation for circular plate under the axisymmetric motion is given

$$\frac{\partial^4 U}{\partial R^4} + \frac{2}{R}\left(\frac{\partial^3 U}{\partial R^3}\right) - \frac{1}{R^2}\left(\frac{\partial^2 U}{\partial R^2}\right) + \frac{1}{R^3}\left(\frac{\partial U}{\partial R}\right) - \Omega^2 U = 0 \tag{3}$$

where $R = r/a$, a is the outside radius of the plate, h thickness of plate, D the flexural rigidity, and Ω is the dimensionless frequency and its given $\Omega^2 = \rho h \omega^2 a^4 / D$. Eq. (3) can be given by applying the DSC as

† Associate Professor, E-mail: civalek@yahoo.com

$$\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(4)}(k\Delta x)U_{k,j} + \frac{2}{R_i}\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(3)}(k\Delta x)U_{k,j} - \frac{1}{R_i^2}\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(2)}(k\Delta x)U_{k,j}$$
$$+\frac{1}{R_i^3}\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta x)U_{k,j} = \Omega^2 U_{k,j} \tag{4}$$

Similarly, the boundary and regularity conditions can be written. Consequently, we solve the remaining eigenvalue problem to obtain the natural frequencies.

3. Results and discussions

The following combinations of boundary conditions have been considered: both inner and outer edges clamped (C-C), inner edge clamped, outer edge simply supported (C-S). Table 1 summarizes numerical results of frequencies by DSC for clamped circular plates. The frequency obtained by Du *et al.* (1995) using the GDQ method is also presented in Table 1 for comparison. Leissa (1969) gives the fundamental frequency for clamped support conditions. It is shown that, the results compare very well with the analytical (Blevins 1984), solution of Leissa (1969) and GDQ solutions (1995). As observed from Table 1, the obtained natural frequencies found by DSC are very accurate. The effect of Poisson's ratio on fundamental frequency for simply supported circular plates is shown in Table 2. Differential quadrature (DQ) results (Bert *et al.* 1994) are presented in this table, together with existing theoretical results (Leissa 1969). For the fundamental frequency, $N = 16$ grid points give acceptable results for simply supported boundary conditions. It can be seen that, the DSC results are generally in agreement with the results produced from the analytical (Leissa 1969) and the DQ results (Bert et *al.* 1994). First four modes of natural frequencies obtained for simply supported circular plates are presented in Table 3 together with the analytical solutions (Leissa 1969). The natural frequencies obtained by the present author (Civalek 2004) using the harmonic differential quadrature (HDQ) method are also presented in Table 3 for comparison. The DSC results are generally in agreement with the results produced from the analytical (Leissa 1969) and the HDQ results (Civalek 2004). It is found that the DSC method possesses both the advantages of HDQ and the flexibility of the DQ. Table 4 tabulates the frequencies obtained by DSC method for

Table 1 First two frequencies of the clamped circular plates

Frequencies	Leissa (1969)	GDQ (Du *et al.* 1995)	Exact (Blevins 1984)	DSC ($N = 17$)
Ω_1	10.21	10.20	10.22	10.23
Ω_2	39.77	44.68	39.77	40.01

Table 2 Comparison for fundamental non-dimensional frequencies of circular plates

ν	Simply supported					
	DQ (Bert *et al.* 1994)	Leissa (1969)	DSC ($N = 7$)	DSC ($N = 9$)	DSC ($N = 11$)	DSC ($N = 16$)
0.0	4.443	4.444	5.857	5.112	4.584	4.451
0.1	4.619	4.619	5.996	5.147	4.673	4.620
0.4	5.078	5.078	7.253	6.117	5.286	5.084

Table 3 Natural frequencies of simply supported circular plate

Method	Mode			
	1	2	3	4
DSC (N = 16)	5.02	30.37	74.88	140.52
Leissa (1969)	4.93	29.72	74.15	138.31
HDQ (Civalek 2004)	4.94	29.85	74.96	139.83

Table 4 First three frequencies of simply supported circular plate

Mode	Leissa (1969)	Present results		
		HDQ (Civalek 2004)	FEM (Civalek 1998)	DSC
1	4.93	4.86	4.78	5.02
2	13.89	13.85	13.81	14.03
3	25.61	26.01	24.96	26.11

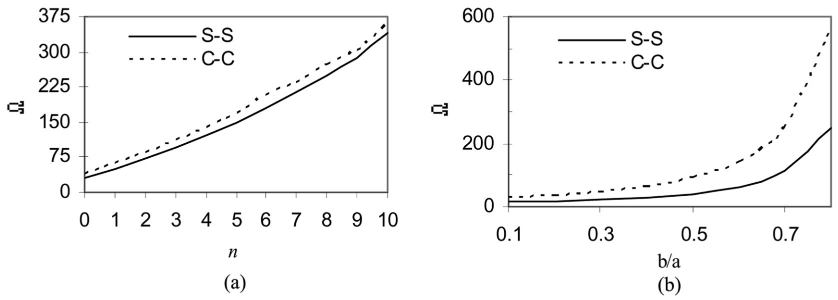

Fig. 1 Variation of frequency parameters (a) circular plate, (b) annular plate

thin circular plates with simply supported boundary conditions. These tables also include both the different numerical solutions and the analytical solutions. Author used the finite element (Civalek 1998) and harmonic differential quadrature method (Civalek 2004) for this problem before. From the table, the convergence of the DSC method is seen to be very good. Fig. 1(a) describes the manner of variation of frequency parameter with respect to circumferential wave number. It is shown that the increasing value of radial or circumferential wave always increases the frequency parameter. As expected, the C-C plate has the highest frequency parameter, followed by the S-S plate. With the increase of circumferential wave number, the effect of the boundary conditions on the frequency parameter is significant.

In case of the annular plate; the obtained results are presented in Table 5 for different radius ratios of b/a. Four different type plate configurations (S-C, S-S, C-C, and C-S) are taken into consideration. Table 5 presents the non-dimensional fundamental frequencies for annular plates. In this table, a is the inner radius and b is the outer radius of annular plates. The C-C annular plate has the highest frequency parameter, followed by the S-C, C-S, and S-S plates. It is shown that the increasing value of radius ratio b/a always increases the frequency parameter. With the increase of b/a ratio, the effect of the boundary conditions on the frequency parameter is significant. In other words, the effect of radius ratio on frequency is much more significant than that of boundary condition. Axisymmetric frequency parameters of annular plates are given in Table 6, it is observed that a good agreement between the present calculated results and the results of literature (Leissa 1969, Civalek 2004) has

Table 5 Non-dimensional fundamental frequency of annular plate

b/a	S-C	S-S	C-C	C-S
0.1	22.85	14.56	28.02	18.42
0.2	27.11	16.85	35.14	23.87
0.3	34.49	21.11	46.02	32.63
0.4	45.17	28.17	62.17	42.57
0.7	175.64	110.87	570.04	173.34

Table 6 Axisymmetric frequency parameters for annular plates ($\nu = 0.33$, $b/a = 0.5$)

Support conditions	Exact (Vogel *et al.* 1965)	Leissa (1969)	DSC
C-C	89.30	89.42	90.24
C-S	64.06	65.17	66.07
S-C	59.91	61.81	60.33
S-S	40.01	43.19	42.58

been obtained. The C-C annular plate has the highest frequency parameter. From the results presented in this table, it is clear that the present DSC results are in excellent agreement with those obtained using a variety of numerical methods. Variation of frequency parameter with radius ratio b/a for annular plates depicted in Fig. 1(b). The frequency parameter increases slowly for small b/a ratio ($b/a \le 0.4$) and then rapidly increases with increasing radius ratio b/a ($b/a > 0.4$).

It is found that the convergence of DSC approach is very good and the results agree well with those obtained by other researchers.

Acknowledgements

The financial support of the Scientific Research Projects Unit of Akdeniz University is gratefully acknowledged.

References

Bert, C.W., Wang, Z. and Striz, A.G. (1994), "Static and free vibrational analysis of beams and plates by differential quadrature method", *Acta Mechanica*, **102**, 11-24.

Blevins, R.D. (1984), *Formulas for Natural Frequency and Mode Shapes*. Malabur-Florida: Robert E. Krieger.

Civalek, Ö. (1998), *Finite Element Analyses of Plates and Shells*. Elaziğ: Fırat University, (in Turkish).

Civalek, Ö. (2004), "Application of Differential Quadrature (DQ) and Harmonic Differential Quadrature (HDQ) for buckling analysis of thin isotropic plates and elastic columns", *Eng. Struct.*, **26**(2), 171-186.

Du, H., Lim, M.K. and Lin, R.M. (1995), "Application of generalized differential quadrature method to vibration analysis", *J. Sound Vib.*, **181**(2), 279-293.

Leissa, A.W. (1969), *Vibration Analysis of Plates*. NASA, SP-160.

Lim, C.W., Li, Z.R. and Wei, G.W. (2005), "DSC-Ritz method for high-mode frequency analysis of thick shallow shells", *Int. J. Numer. Meth. Eng.*, **62**, 205-232.

Vogel, S.M. and Skinner, D.W. (1965), "Natural frequencies of transversely vibrating uniform annular plates", *J. Appl. Mech.*, **32**, 926-931.

Wei, G.W. (2001), "A new algorithm for solving some mechanical problems", *Comput. Meth. Appl. Mech. Eng.*, **190**, 2017-2030.