Structural Engineering and Mechanics, Vol. 29, No. 2 (2008) 187-201
DOI: http://dx.doi.org/10.12989/sem.2008.29.2.187

A methodology to estimate earthquake induced worst failure probability of inelastic systems

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(Received July 29, 2005, Accepted December 5, 2007)

Abstract. Earthquake induced hysteretic energy demand for a structure can be used as a limiting value of a certain performance level in seismic design of structures. In cases where it is larger than the hysteretic energy dissipation capacity of the structure, failure will occur. To be able to select the limiting value of hysteretic energy for a particular earthquake hazard level, it is required to define the variation of hysteretic energy in terms of probabilistic terms. This study focuses on the probabilistic evaluation of earthquake induced worst failure probability and approximate confidence intervals for inelastic single-degree-of-freedom (SDOF) systems with a typical steel moment connection based on hysteretic energy. For this purpose, hysteretic energy demand is predicted for a set of SDOF systems subject to an ensemble of moderate and severe EQGMs, while the hysteretic energy dissipation capacity is evaluated through the previously published cyclic test data on full-scale steel beam-to-column connections. The failure probability corresponding to the worst possible case is determined based on the hysteretic energy demand and dissipation capacity. The results show that as the capacity to demand ratio increases, the failure probability decreases dramatically. If this ratio is too small, then the failure is inevitable.

Keywords: failure probability; hysteretic energy demand; hysteretic energy capacity; confidence interval; non-linear analysis.

1. Introduction

Structural failure will occur when the earthquake induced hysteretic energy demand for a structure

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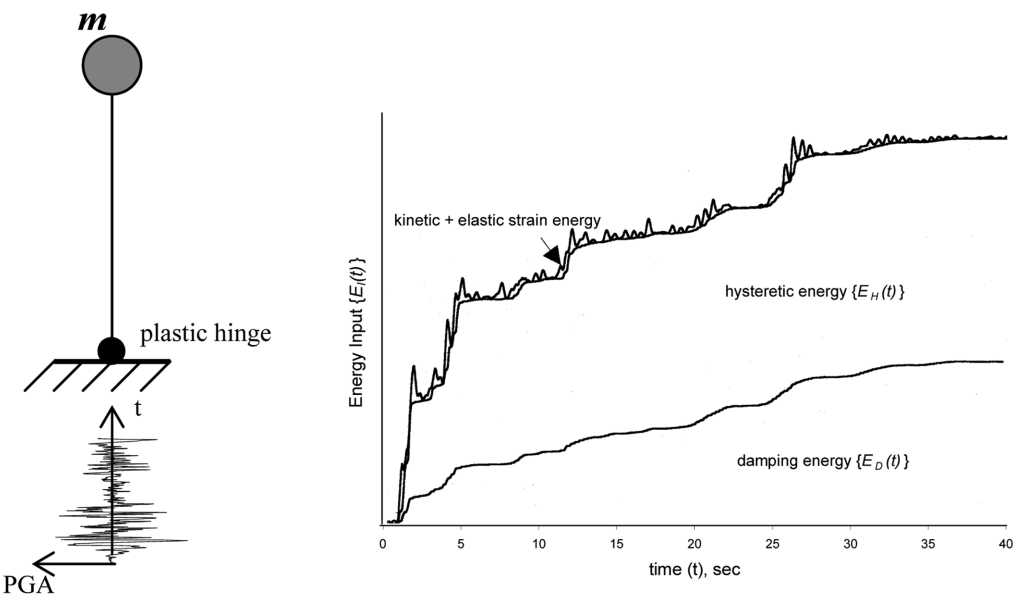
is larger than the hysteretic energy dissipation capacity of the structure. Structures in seismic regions are expected to dissipate seismic input energy through controlled inelastic deformations of the structure. Energy input to the structure subject to an earthquake ground motion (EQGM) is considered to be the most rational and reliable way to estimate damage of a structure, because energy parameters take into account not only the peak response of EQGM but also the ability of structure to absorb and dissipate energy. Part of the energy input consists of hysteretic energy dissipated through the hysteretic behaviour. Since the damage in structures is related to the hysteretic energy dissipated by the structure, it can be used as a seismic design parameter when the damage is expected not to exceed some specified limits (Bertero 2005). Hysteretic energy can be used as a limiting value of a certain performance level in seismic design of structures such as drift, ductility, structural damage and storey drift indices, etc. Determining the energy absorption and dissipation capacity and level of damage of the structure to a predefined EQGM is one of the questions involved in predicting the structure's response for low-performance levels (life safe, near collapse, collapse) in performance-based earthquake resistant design (PB-EQRD) (Vision-2000 1995, Bertero and Bertero 1999, Hamburger 1997, ATC-40 1996, FEMA-273 1997). To be able to select the limiting value of hysteretic energy for a particular earthquake hazard level, it is required to define the variation of hysteretic energy in terms of probabilistic terms. Performance-based earthquake resistant design based on energy approach have also gained great attention in recent years (Akbas *et al*. 2001, Choi and Shen 2001, Chai and Fajfar 2000, Chou and Uang 2000). Researches on estimating the hysteretic energy demand in single-degree-of-freedom (SDOF) as well as in multi-degree-of-freedom (MDOF) systems have also been carried out extensively (Sari 2003, Cruz and Lopez 2000, Tso *et al*. 1993, Leger and Dussault 1992, Uang and Bertero 1990). Chou and Uand (2003, 2004) proposed a procedure for evaluating the total energy demand through elastic dynamic analysis and its distribution throughout the height of multi-storey frames using inelastic energy spectra. Ordaz *et al*. (2003) presented a relationship between the the Fourier amplitude spectrum and the elastic energy input spectrum for SDOF systems. Manfredi *et al.* (2003) studied 128 near-fault earthquake ground motions using energy input and plastic cycles demand related parameters. Yamaguchi and El-Abd (2003) investigated the effect of earthquake characteristics on hysteretic dampers in multi-storey frames. Mollaioli *et al*. (2004) tried to establish a relation between the seismic energy and displacement for R/C multi-storey frames. They have conluded that the results obtained from SDOF systems could be extended to MDOF systems

This study focuses on the probabilistic evaluation of earthquake induced worst failure probability and approximate confidence intervals for inelastic single-degree-of-freedom (SDOF) systems with a typical steel moment connection based on hysteretic energy. For this purpose, hysteretic energy demand is predicted for a set of SDOF systems subject to an ensemble of moderate and severe EQGMs, while the hysteretic energy dissipation capacity is evaluated through the previously published cyclic test data on full-scale steel beam-to-column connections. The approximate confidence intervals for 95% and 90% degree of confidence levels are constructed. Then, the failure probability corresponding to the worst possible case is determined based on the hysteretic energy demand and dissipation capacity.

2. Probabilistic evaluation of SDOF systems based on hysteretic energy

For a SDOF system, energy dissipation capacity of the system requires the yielding and



(a) (b)

Fig. 1 Energy parameters for a SDOF system subjected to an EQGM

dissipating energy of only one plastic hinge at the base of the system (Fig. 1). The structure collapses when the earthquake induced hysteretic energy demand due to the plastic deformation (E_{ED}) is larger than the hysteretic energy dissipation capacity of the structure (E_{EC}) (Bertero and Bertero 1999). The hysteretic energy dissipation capacity (E_{EC}) of the plastic hinge and the hysteretic energy demand (E_{ED}) have unknown parametric probability density functions (*PDF*) (let us assume the hysteretic energy dissipation capacity of the plastic hinge (E_{EC}) and the hysteretic energy demand (E_{ED}) as statistically independent random variables). Let $\mu_{E_{EC}}$ and $\mu_{E_{ED}}$ are the population mean values of energy dissipation capacity at ultimate deformation of the plastic hinge and hysteretic energy demand, respectively; and $\sigma_{E_{EC}}$ and $\sigma_{E_{ED}}$ are the population standard deviations of energy dissipation capacity at ultimate deformation of the plastic hinge and hysteretic energy demand, respectively. Since it is practically impossible to determine the population means and standard deviations of the hysteretic energy demand and capacity, the estimators for these parameters, which are sample means ($m_{E_{ED}}$, $m_{E_{EC}}$) and standard deviations ($S_{E_{ED}}$, $S_{E_{EC}}$), can be used instead. $m_{E_{ED}}$ and $S_{E_{ED}}$ are the sample mean value and standard deviation of E_{ED}, respectively; $m_{E_{EC}}$, $S_{E_{EC}}$ are the sample mean value and standard deviation of E_{EC}, respectively.

Since the underlying distribution of E_{ED} and E_{EC} are unknown, Central Limit Theorem (CLT) can be applied to determine asymptotic failure probability when we have large samples of size (Ott 1993). The following steps are required for that:

a. First, the hysteretic energy dissipation capacity needs to be evaluated based on experimental studies,
b. Second, an analytical study is required to investigate the hysteretic energy demand,
c. Third, the asymptotic failure probability is determined based on the average hysteretic energy demand ($\bar{E}_{ED}$) and the average energy dissipation capacity ($\bar{E}_{EC}$).

3. Hysteretic energy dissipation capacity of steel moment connections

Steel moment connections are part of the moment-resisting frames, which are frequently used as lateral load resisting systems in many steel building structures. Cyclic behavior of a moment connection is of particular interest in structural engineering. A steel moment connection should dissipate the hysteretic energy induced by a severe EQGM to prevent partial or total collapse. The hysteretic energy dissipation capacity of a steel moment connection (the area enclosed by the hysteresis loop) system can be stated as (Uang and Bertero 1988)

$$E_{EC} = M_p(\theta_{pa}^{+} + \theta_{pa}^{-}) = (F_y Z_f)\theta_{pa} \tag{1}$$

where M_p is the plastic moment of the section, θ_{pa}^{+} is the accumulated positive deformation, θ_{pa}^{-} is the accumulated negative deformation, Z_f is the plastic section modulus of the element's flange, and F_y is the yield strength of the element.

E_{EC} is difficult to obtain due to the fact that it is not a constant value and depends on the deformation and/or loading path and on the type of connection (Shen and Akbas 1999, Uang and Bertero 1988). Shen and Akbas (1999) investigated E_{EC} of the predominantly used welded flange-bolted web connections in steel moment-resisting frames. A summary of their results is given in Fig. 2 that shows a variation of E_{EC} from 9.74×10^6 Nmm to as high as 383.73×10^6 Nmm. In most of the connections, E_{EC} varies between 0-130 × 10^6 Nmm and it is not likely for a connection to reach 400×10^6 of E_{EC}, unless it is repaired or upgraded. The statistical values $m_{E_{EC}}$ and $\sigma_{E_{EC}}$ are found to be 101.625×10^6 Nmm and 85.72×10^6 Nmm, respectively. The beam sizes used in the tests include small size beams (W410 and W460) as well as medium and large size beams (W530, W610, W760, and W920). A total of 70 test results are investigated. For the small and medium beam sizes, large variations in θ_{pa} were observed. The tests are carried out on typical exterior moment connections subject to quasi-static loading. θ_{pa} differs from 0.08% to 132.23%, depending on the connection type. Failure modes of the connections change from the fracture of the welded beam top flange to column flange connection to the fracture through the column flange and into the panel zone adjacent to the beam bottom flange.

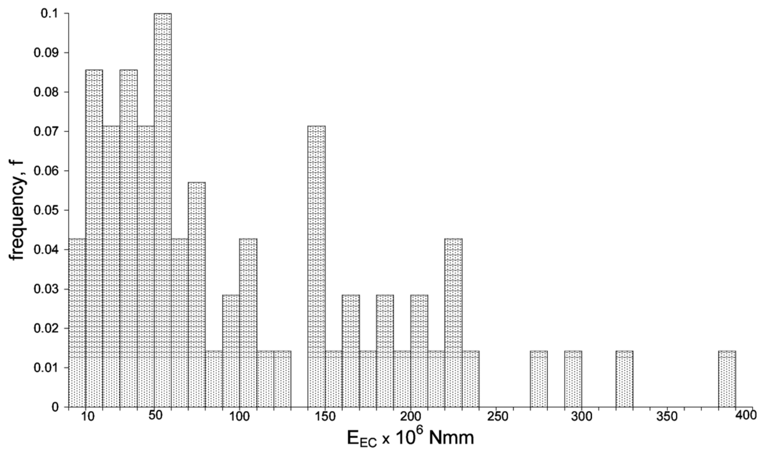


Fig. 2 Frequency distribution of the experimental results on the hysteretic energy dissipation capacity

4. Hysteretic energy demand

To determine the hysteretic energy demand (E_{ED}) on SDOF systems, non-linear dynamic time history analyses are carried out using DRAIN-2DX (Prakash *et al.* 1993). For this purpose, a set of SDOF system having natural periods (T) in the range of 0.1 sec to 3.0 sec are designed at 0.1 sec increments. The bilinear inelastic behaviour is assumed with no strain hardening. P-Δ effect is not included in the analysis. Damping ratio (ζ) is assumed to be 2% of critical damping. Stiffness proportional damping is used in the study.

The analyses are performed for three different strength indices: η = 0.1, 0.3, and 0.5. The strength index (η) is defined as the ratio of the base shear value (V_y) at which the structure begins its inelastic deformation to the seismic weight of the structure (W). A total of 27 representative EQGMs from significant earthquakes in USA and Turkey (1940 El Centro 1949, Olympia 1952, Taft 1966, Parkfield 1978 Miyagi 1994, Northridge Earthquake 1998, Ceyhan Earthquake 1999, Izmit Earthquake) are used for the analyses. These EQGMs are recorded at different types of soils, and have different peak ground accelerations, frequency contents, strong motion duration, distance and magnitude. Normalized response spectra of these EQGMs are plotted in Fig. 3. The peak ground accelerations (PGA) of the EQGMs are scaled to 0.3 g and 0.6 g to evaluate the hysteretic energy demand of SDOF systems subject to moderate and severe EQGMs.

The selected EQGMs are classified with respect to the predominant period of the EQGM record, T_g. The EQ Group (EQG) I, II, and III consist of the EQGMs having $0 < T_g \leq 0.7$ sec, 0.7 sec $< T_g \leq$ 1.0 sec, and $T_g \geq 1.0$ sec, respectively. To have a better understanding of the hysteretic energy demand on SDOF systems subject to the EQGMs in the EQG-I, -II, and -III, the SDOF systems are also classified with respect to T. The SDOF systems having natural periods $0 < T < 0.7$ sec (very short to short period systems), 0.7 sec $\leq T \leq 1.0$ sec (medium period systems), and $1.0 < T < 2.0$ sec (long to very long period systems) are referred to as Building Group (BG) I, II, and III, respectively. Then, the hysteretic energy demand for each building group subject to the EQGMs in each EQG is evaluated. The cell numbers for each possible case as well as the number of samples

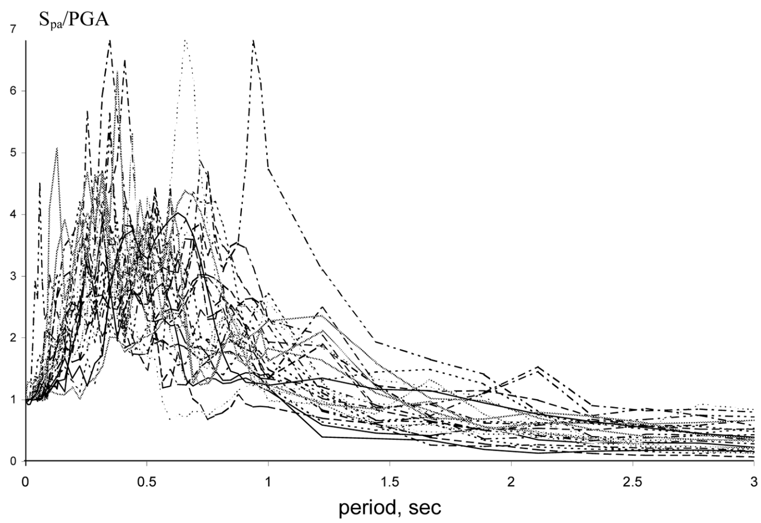


Fig. 3 Normalized response spectra of the EQGMs

Table 1 Classifying buildings and EQs and cell numbers

EQ group	Building group / Sample no.	I	II	III
		6	*4*	*10*
I $0 < T_g \leq 0.7$ sec	*10*	**I-I** (*60*)	**I-II** (*40*)	**I-III** (*100*)
II 0.7 sec $< T_g \leq 1.0$ sec	*8*	**II-I** (*48*)	**II-II** (*32*)	**II-III** (*80*)
III $T_g \geq 1.0$ sec	*9*	**III-I** (*54*)	**III-II** (*36*)	**III-III** (*90*)

Note: Numbers in parenthesis refer to the number of samples in that cell.

Table 2 Statistical values of the E_{ED}/m with respect to the cells, (cm/sec)^2

PGA (g)	η	Building group / EQ group	I $m_{E_{ED}}$	I $S_{E_{ED}}$	II $m_{E_{ED}}$	II $S_{E_{ED}}$	III $m_{E_{ED}}$	III $S_{E_{ED}}$
			(×1000)		(×1000)		(×1000)	
0.3	0.1	**I**	2.609	1.417	3.536	3.169	3.776	2.988
		II	2.841	1.299	4.618	4.016	4.779	3.372
		III	1.794	1.096	2.134	2.775	3.759	2.418
	0.3	**I**	2.241	2.232	2.167	2.583	2.767	2.933
		II	2.474	2.185	6.037	7.222	3.852	4.270
		III	0.616	1.296	1.249	3.282	1.914	2.943
	0.5	**I**	1.588	2.338	1.076	1.415	1.909	2.421
		II	1.520	2.503	4.810	7.962	2.306	3.264
		III	0.256	0.783	0.667	2.626	0.853	2.583
0.6	0.1	**I**	8.390	3.775	11.583	8.744	13.276	8.925
		II	9.260	3.649	12.800	9.982	15.697	9.727
		III	7.482	3.414	8.367	8.070	13.319	8.278
	0.3	**I**	10.403	6.899	13.257	13.582	14.396	13.016
		II	11.293	6.211	21.700	20.945	19.337	16.340
		III	5.106	5.046	7.600	12.639	13.855	10.613
	0.5	**I**	9.309	8.677	9.958	11.429	12.029	12.380
		II	10.419	8.139	24.373	27.020	17.161	17.238
		III	3.169	5.436	5.723	13.187	9.559	11.456

in each cell are given in Table 1, i.e. Cell No. I-III refers to the SDOF systems in BG-III subject to the EQGMs in EQG-I. The statistical values in each cell obtained from the non-linear dynamic time history analyses are given in Table 2.

When subject to moderate EQGMs (PGA = 0.3 g) in EQG-I and -III, the SDOF systems in BG-III have the highest $m_{E_{ED}}$ for any η. However, the systems in BG-II have the highest $m_{E_{ED}}$ when subject to EQGMs in EQG-II for η = 0.3 and 0.5. The systems in BG-II, in general, have the highest $S_{E_{ED}}$, while the systems in BG-I have the smallest $S_{E_{ED}}$ for any η, i.e. the E_{ED} is not much scattered and can be predicted more accurately for the buildings in this group. $m_{E_{ED}}$ is, in general, the highest for $\eta = 0.1$, except for $\eta = 0.3$ in Cell II-II, and decreases dramatically as η gets higher. However, the change in $S_{E_{ED}}$ is not that drastic and does not follow a uniform pattern of increasing or decreasing. It looks stable for the systems in BG-III for any η.

When subject to severe EQGMs (PGA = 0.6 g) in EQG-I and -III, the SDOF systems in BG-III have the highest $m_{E_{ED}}$ for any η. However, the systems in BG-II have the highest $m_{E_{ED}}$ when subject to EQGMs in EQG-II for $\eta = 0.3$ and 0.5. The systems in BG-II, in general, have the highest $S_{E_{ED}}$, while the systems in BG-I have the smallest $S_{E_{ED}}$ for any η, i.e. the E_{ED} is also not much scattered and can be predicted more accurately for the buildings in this group. $m_{E_{ED}}$ is, in general, the highest for $\eta = 0.3$, except for Cell II-II. The most dramatic increase in $m_{E_{ED}}$ occurs in BG-II subject to EQGMs in EQG-II. And the most dramatic decrease occurs in BG-I and II subject to EQGMs in EQG-III. The change in $S_{E_{ED}}$ is also very drastic and it even exceeds $m_{E_{ED}}$ in almost all the Cells for $\eta = 0.5$.

In cases where the population mean of hysteretic energy demand ($\mu_{E_{ED}}$) is not known, one may want to estimate it by constructing a (1-α)% confidence interval (or statistical interval) for E_{ED}. Considering of making no distributional assumption, one can construct an approximate (1-α)% confidence interval when having large sample size as (Wadsworth 1998)

$$m_{E_{ED}} - t_{\alpha/2}\frac{\sigma_{E_{ED}}}{\sqrt{n}} < \mu_{E_{ED}} < m_{E_{ED}} + t_{\alpha/2}\frac{\sigma_{E_{ED}}}{\sqrt{n}} \tag{2}$$

where α is a small fraction and $t_{\alpha/2}$ is standard normal distribution value exceeded by an area of $\alpha/2$. Note that when the population standard deviation of hysteretic energy demand, $\sigma_{E_{ED}}$, is not known, it can be replaced by $S_{E_{ED}}$ if there is large enough sample size. In this case an approximate (1-α)% confidence interval is

$$m_{E_{ED}} - t_{\alpha/2}\frac{S_{E_{ED}}}{\sqrt{n}} < \mu_{E_{ED}} < m_{E_{ED}} + t_{\alpha/2}\frac{S_{E_{ED}}}{\sqrt{n}} \tag{3}$$

When α is to be taken as 0.05 and 0.10, the corresponding $t_{\alpha/2}$ values are 1.96 and 1.65, respectively. Thus, approximate 90% and 95% confidence intervals can be stated as

$$m_{E_{ED}} - 1.65\frac{S_{E_{ED}}}{\sqrt{n}} < \mu_{E_{ED}} < m_{E_{ED}} + 1.65\frac{S_{E_{ED}}}{\sqrt{n}} \tag{4a}$$

$$m_{E_{ED}} - 1.96\frac{S_{E_{ED}}}{\sqrt{n}} < \mu_{E_{ED}} < m_{E_{ED}} + 1.96\frac{S_{E_{ED}}}{\sqrt{n}} \tag{4b}$$

Using Eqs. (4a) and (4b), one can easily obtain the 90% and 95% approximate confidence intervals of hysteretic energy demand, E_{ED}, for an inelastic SDOF system.

To construct the approximate confidence intervals, a parametric analysis of non-linear systems is done considering SDOF systems with 11 discrete fundamental natural periods (T) of 0.1, 0.2, 0.3, 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5 and 3.0 sec. For each period range, the sample mean ($m_{E_{ED}}$) and the standard deviation ($S_{E_{ED}}$) of hysteretic energy demand are obtained using the same EQGMs through non-linear dynamic time history analyses. The statistical values are given in Table 3. Among all the structures, the very short and short period systems have the smallest coefficient of variation (COV) (0.77-1.14) (Table 3), i.e. less variation in E_{ED} for these systems. COV is defined as the ratio of the

Table 3 Statistical values of E_{ED}/m

PGA (g)	T (sec)	η = 0.1 $m_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.1 $S_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.1 COV	η = 0.3 $m_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.3 $S_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.3 COV	η = 0.5 $m_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.5 $S_{E_{ED}}/m$ ×1000 (cm/sec)^2	η = 0.5 COV
0.3	0.1	1.42	1.19	0.84	0.10	0.15	1.50	0.03	0.07	2.33
	0.2	2.43	1.95	0.80	0.77	0.68	0.89	0.33	0.38	1.14
	0.3	3.15	2.56	0.81	2.13	2.02	0.95	1.30	1.69	1.30
	0.5	3.68	3.00	0.81	3.53	2.77	0.79	2.40	2.25	0.94
	0.6	3.69	3.14	0.85	3.74	3.20	0.86	2.60	2.92	1.12
	0.8	3.64	3.22	0.89	3.62	4.43	1.22	2.23	3.25	1.46
	1.0	3.55	3.22	0.91	3.40	5.25	1.54	2.01	4.74	2.36
	1.5	2.10	2.14	1.02	0.64	1.23	1.92	0.08	0.14	1.75
	2.0	1.91	2.16	1.13	0.50	1.27	2.52	0.36	1.10	3.03
	2.5	1.14	1.58	1.39	0.20	0.51	2.55	0.20	0.51	2.55
	3.0	1.00	1.57	1.56	0.23	0.62	2.72	0.34	0.75	2.24
0.6	0.1	6.64	5.32	0.80	3.35	2.62	0.78	0.74	0.75	1.01
	0.2	9.03	6.97	0.77	7.80	6.42	0.82	3.99	3.44	0.86
	0.3	9.99	8.05	0.81	11.92	9.83	0.82	9.42	8.59	0.91
	0.5	11.40	8.61	0.76	15.78	13.26	0.84	14.71	11.58	0.79
	0.6	11.38	8.69	0.76	15.81	14.16	0.90	15.68	13.61	0.87
	0.8	11.40	9.01	0.79	15.65	15.63	1.00	15.42	18.19	1.18
	1.0	11.13	8.77	0.79	14.54	16.07	1.11	14.46	19.88	1.38
	1.5	8.74	9.56	1.09	6.70	8.80	1.31	3.84	6.21	1.62
	2.0	7.41	6.56	0.89	6.25	9.61	1.54	3.24	7.29	2.25
	2.5	5.92	6.14	1.04	2.74	4.73	1.73	1.78	5.62	3.16
	3.0	5.21	6.08	1.17	2.63	5.96	2.26	0.66	1.97	2.97

sample standard deviation ($S_{E_{ED}}$) to the sample mean ($m_{E_{ED}}$). However, the very long period systems have the highest COV (2.92-3.03), i.e. higher variation in E_{ED} for these systems. The highest $m_{E_{ED}}$ is in the short and medium period systems as expected (Table 3). There is a dramatic reduction in $m_{E_{ED}}$ for very short and very long period structures.

Figs. 4 and 5 are plotted to see the approximate confidence intervals in graphical form. The highest approximate confidence intervals for E_{ED} always occur in the short- and medium-period range, while the lowest approximate confidence intervals occur in the very-long period range (Figs. 4 and 5). The approximate confidence interval decreases as η increases for PGA = 0.3 g, while it increases for PGA = 0.6 g. This means when a structure subject to a severe EQGM, the hysteretic energy input increases as the strength of the structure increases. However, when the structure subject to a moderate level EQGM, the structure may not yield as the strength of the system increases and this will decrease the hysteretic energy input to the structure.

5. Asymptotic failure probability based on hysteretic energy

If the underlying distribution of the hysteretic energy dissipation capacity and demand had known,

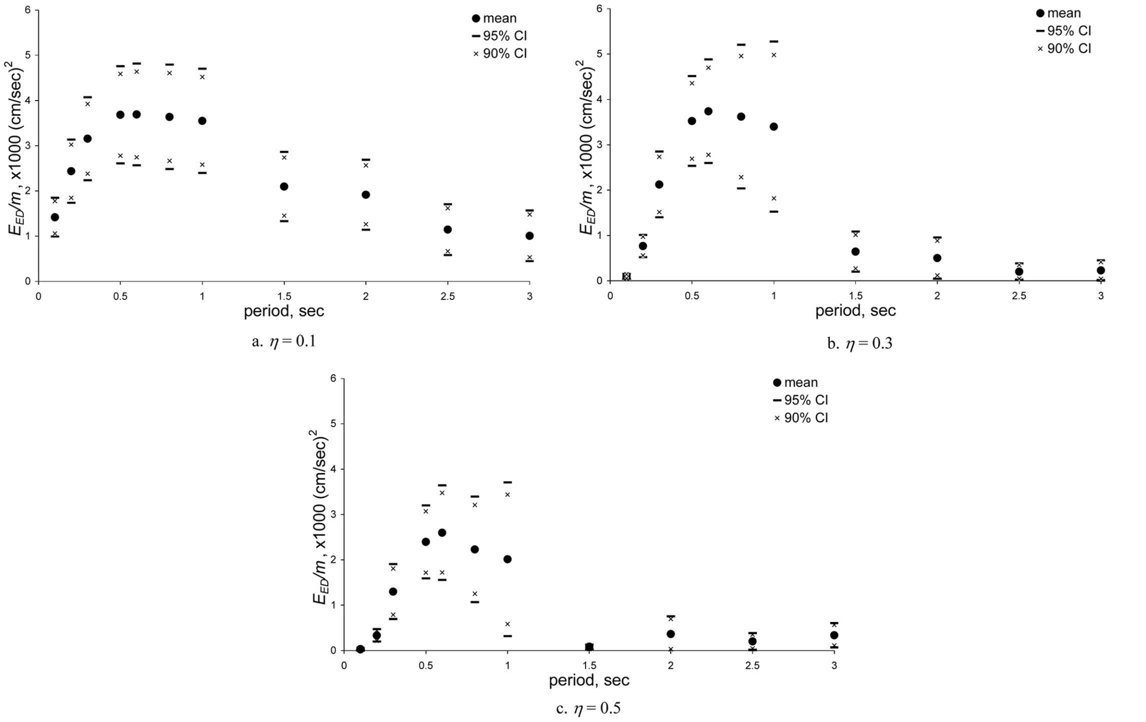


Fig. 4 Approximate confidence intervals (CI) of E_{ED}/m, PGA = 0.3 g

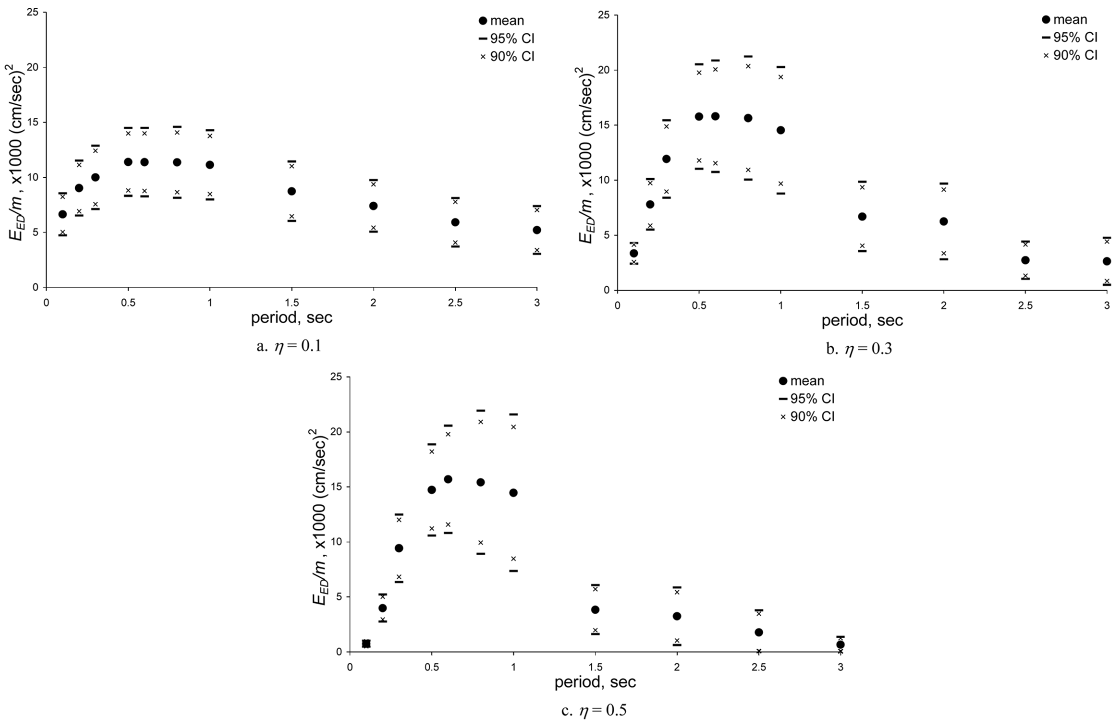


Fig. 5 Approximate confidence intervals (CI) of E_{ED}/m, PGA = 0.6 g

the exact failure probabilities could have been calculated as (Fig. 6(a)), i.e.

$$P_f = P(E_{EC} \le E_{ED}) = P(E_{EC} - E_{ED} \le 0) \tag{5}$$

Since, making any underlying distribution assumption (especially normal distribution) is not reasonable at all; asymptotic failure probability might be used instead that can be computed as

$$P_f = P(\overline{E}_{EC} \le \overline{E}_{ED}) = P(\overline{E}_{EC} - \overline{E}_{ED} \le 0) \tag{6}$$

where $\overline{E}_{E_{EC}}$ and $\overline{E}_{E_{ED}}$ are the sample average of hysteretic energy demand and capacity. Let safety margin be defined as $\overline{Z}_n = \overline{E}_{EC} - \overline{E}_{ED}$, then $f_{\overline{Z}}(z)$ is the probability density function of the safety margin (Fig. 6(b)). Failure will occur when $(\overline{Z}_n < 0)$ in this case. Thus, the asymptotic failure probability is

$$P_f = \int_{-\infty}^{0} f_{\overline{Z}}(z)dz = f_{\overline{Z}}(z) \tag{7}$$

Thus, the asymptotic distribution of $(\overline{Z}_n - \mu_{\bar{z}_n})/\sigma_{\bar{z}_n}$ will be a standard normal distribution. Defining $\mu_{\bar{z}_n} = \mu_{E_{EC}} - \mu_{E_{ED}}$ and $\sigma_{\bar{z}_n} = \sqrt{\dfrac{\sigma^2_{E_{EC}}}{n_{\overline{E}_{EC}}} + \dfrac{\sigma^2_{E_{ED}}}{n_{\overline{E}_{ED}}}}$, Eq. (7) becomes

$$P_f = F_{\overline{Z}}(0) \doteq \Phi\left(-\frac{\mu_{\bar{z}_n}}{\sigma_{\bar{z}_n}}\right) = \Phi\left(-\frac{\mu_{E_{EC}} - \mu_{E_{ED}}}{\sqrt{\dfrac{\sigma^2_{E_{EC}}}{n_{\overline{E}_{EC}}} + \dfrac{\sigma^2_{E_{ED}}}{n_{\overline{E}_{ED}}}}}\right) \tag{8}$$

where $F_{\overline{Z}}(0) \doteq \Phi(-\mu_{\bar{z}_n}/\sigma_{\bar{z}_n})$ means that the distribution function of $\overline{Z}_n$ has an asymptotic standard normal distribution function and $n_{\overline{E}_{EC}}$, $n_{\overline{E}_{ED}}$ are the number of samples for E_{EC} and E_{ED}, respectively. In this case, the asymptotic failure probability in Eq. (8) is equal to the area on the left side of

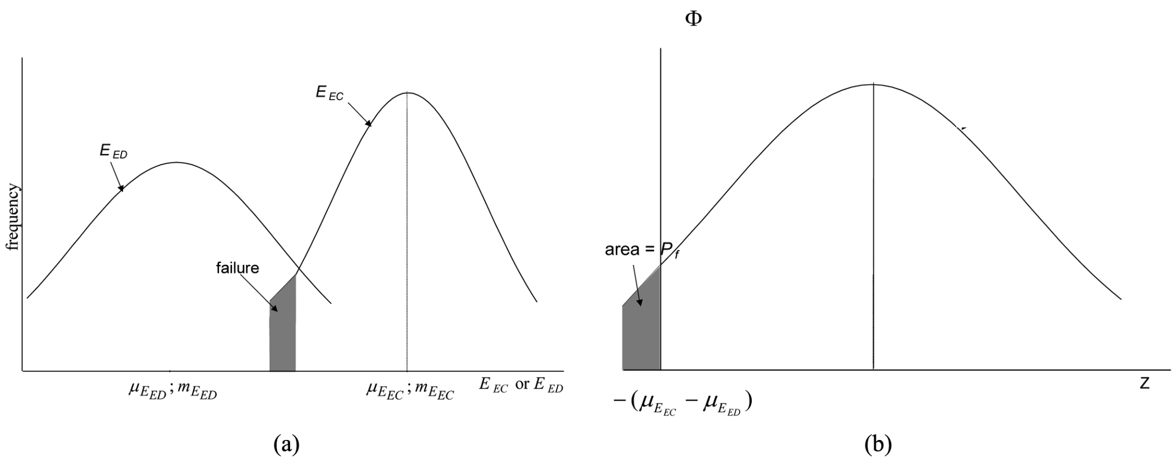


Fig. 6 Frequency distributions of E_{EC} and E_{ED} and probability density function of z

$-(\mu_{E_{EC}} - \mu_{E_{ED}})$ (shaded area in Fig. 6(b)). To investigate the P_f in terms of an over-energy capacity as the ratio between the population mean values of hysteretic energy dissipation capacity and hysteretic energy demand can be defined as follows

$$\Theta_E = \frac{\mu_{E_{EC}}}{\mu_{E_{ED}}} \tag{9}$$

And finally, Eq. (8) becomes

$$P_f = F_{\bar{Z}}(0) = \Phi\left(-\frac{\mu_{\bar{z}_n}}{S_{\bar{z}_n}}\right) = \Phi\left(-\frac{\mu_{E_{ED}}(\Theta_E - 1)}{\sqrt{S_{E_{EC}}^2/n_{\bar{E}_{EC}} + S_{E_{ED}}^2/n_{\bar{E}_{ED}}}}\right) \tag{10}$$

The failure probabilities can be obtained by using Eq. (10). Using Eq. (10) in the calculation of P_f can provide valuable information about the change in P_f in cases where the systems are over-designed. It is practically impossible to determine the population means and standard deviations of the hysteretic energy demand and capacity. However, $\mu_{E_{EC}} - \mu_{E_{ED}}$ can be estimated for instance by using a (1-α)% confidence interval. A (1-α)% confidence interval for $\mu_{E_{EC}} - \mu_{E_{ED}}$ is

$$m_{E_{EC}} - m_{E_{ED}} - t_{\alpha/2}\sqrt{\frac{S_{E_{EC}}^2}{n_{\bar{E}_{EC}}} + \frac{S_{E_{ED}}^2}{n_{\bar{E}_{ED}}}} \le \mu_{E_{EC}} - \mu_{E_{ED}} \le m_{E_{EC}} - m_{E_{ED}} + t_{\alpha/2}\sqrt{\frac{S_{E_{EC}}^2}{n_{\bar{E}_{EC}}} + \frac{S_{E_{ED}}^2}{n_{\bar{E}_{ED}}}} \tag{11}$$

Then, the extreme failure probabilities, $P_{f,\mathrm{lb}}$ and $P_{f,\mathrm{ub}}$, can be determined from Eq. (11) by using the boundary values of the confidence interval. The $P_{f,\mathrm{lb}}$ and $P_{f,\mathrm{ub}}$ should be evaluated as the worst (left hand side of Eq. (11)) and best (right hand side of Eq. (11)) ones. In this study, the worst failure probability ($P_{f,\mathrm{lb}}$) is of particular interest.

The $P_{f,\mathrm{lb}}$ is evaluated for the inelastic SDOF systems designed against moderate EQGMs (PGA = 0.3 g) and subject to the same level EQGM (Case 1), severe EQGMs (PGA = 0.6 g) but subject to a moderate level EQGM (PGA = 0.3 g) (Case 2), and severe EQGMs (PGA = 0.6 g) and subject to the same level EQGM (Case 3). α and the corresponding $t_{\alpha/2}$ value are taken as 0.05 and 1.96, respectively. Θ_E ratio is taken to be 1.25, 1.50, and 2.0. The results are given in Table 4 in which PGA = 0.3 g, 0.6 g(a), and 0.6 g(b) correspond to Case 1, Case 2, and Case 3, respectively. Note that for Θ_E = 1, P_f is equal to 0.5 (or 50%) and for $\Theta_E \le 1$, P_f is theoretically greater than 0.5 but it is assumed to be equal to 0.5, because it would not be rational to discuss the failure probability for cases in which the hysteretic energy demand is higher than the hysteretic energy dissipation capacity. Each case is evaluated for different strength ratios (η = 0.1, 0.3, and 0.5) with respect to the cell numbers given in Table 1.

In Cases 1 and 3 for Θ_E = 1.25, the $P_{f,\mathrm{lb}}$, is 50% indicating the failure for any η. For Θ_E = 1.50, in Case 1, the highest $P_{f,\mathrm{lb}}$ occurred in Cell II-II for η = 0.1 (34.2%), while the lowest occurred in Cell III-III (10.2%). For the same case, the highest $P_{f,\mathrm{lb}}$ occurred in Cells I-III, II-II, and II-III for η = 0.3 (50%), while the lowest occurred in Cell I-I (26.1%). However, for η = 0.5, failure ($P_{f,\mathrm{lb}}$ = 50%) is observed in many of the cells and the lowest $P_{f,\mathrm{lb}}$ is %40.1 in Cell III-I. For Θ_E=2.00, in Case 1, the $P_{f,\mathrm{lb}}$'s are 0% in all the cells for η = 0.1. For the same case, the highest $P_{f,\mathrm{lb}}$ occurred in Cell II-III for η = 0.3 (16.9%), while the lowest occurred in Cell I-I (0.1%). For η = 0.5, the highest $P_{f,\mathrm{lb}}$ occurred in Cell II-III (45.3%), while the lowest occurred in Cell III-I (1.1%). For Θ_E = 1.50, in Case 3, the highest $P_{f,\mathrm{lb}}$ occurred in Cell II-II for η = 0.1 (29.0%), while the lowest occurred in Cell I-III (7%). For the same case, the highest $P_{f,\mathrm{lb}}$ occurred in Cell II-III for η = 0.3

Table 4 The worst failure probabilities ($P_{f,\,\mathrm{lb}}$)

PGA	Cell No (Table 1)	$\Theta_E = 1.25$			$\Theta_E = 1.50$			$\Theta_E = 2.0$		
		$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.5$
		$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$	$P_{f,\,lb}$
0.3 g	I-I	0.500	0.500	0.500	0.106	0.261	0.428	0	0.001	0.016
	I-II	0.500	0.500	0.500	0.110	0.296	0.500	0	0.002	0.107
	I-III	0.500	0.500	0.500	0.091	0.500	0.500	0	0.038	0.193
	II-I	0.500	0.500	0.500	0.263	0.399	0.452	0	0.011	0.021
	II-II	0.500	0.500	0.500	0.342	0.500	0.500	0	0.039	0.167
	II-III	0.500	0.500	0.500	0.315	0.500	0.500	0	0.169	0.453
	III-I	0.500	0.500	0.500	0.197	0.311	0.401	0	0.003	0.011
	III-II	0.500	0.500	0.500	0.226	0.437	0.500	0	0.018	0.069
	III-III	0.500	0.500	0.500	0.102	0.368	0.500	0	0.007	0.221
0.6 g (a)	I-I	0	0	0	0	0	0	0	0	0
	I-II	0	0	0	0	0	0	0	0	0
	I-III	0	0	0	0	0	0	0	0	0
	II-I	0	0	0	0	0	0	0	0	0
	II-II	0	0	0	0	0	0	0	0	0
	II-III	0	0	0	0	0	0	0	0	0
	III-I	0	0	0	0	0	0	0	0	0
	III-II	0	0	0	0	0	0	0	0	0
	III-III	0	0	0	0	0	0	0	0	0
0.6 g (b)	I-I	0.500	0.500	0.500	0.086	0.139	0.235	0	0	0
	I-II	0.500	0.500	0.500	0.093	0.142	0.245	0	0	0
	I-III	0.500	0.500	0.500	0.070	0.172	0.398	0	0	0.011
	II-I	0.500	0.500	0.500	0.201	0.323	0.379	0	0.004	0.009
	II-II	0.500	0.500	0.500	0.290	0.395	0.470	0	0.010	0.025
	II-III	0.500	0.500	0.500	0.197	0.441	0.500	0	0.018	0.102
	III-I	0.500	0.500	0.500	0.153	0.243	0.297	0	0	0.003
	III-II	0.500	0.500	0.500	0.184	0.300	0.385	0	0.003	0.009
	III-III	0.500	0.500	0.500	0.098	0.128	0.253	0	0	0.001

(44.1%), while the lowest occurred in Cell III-III (12.8%). However, for η = 0.5, failure ($P_{f,\,\mathrm{lb}}$ = 50%) is observed in Cell II-III and the lowest $P_{f,\,\mathrm{lb}}$ is 23.5% in Cell I-I. For Θ_E = 2.00, in Case 1, the $P_{f,\,\mathrm{lb}}$'s are 0% in all the cells for η = 0.1. For the same case, the highest $P_{f,\,\mathrm{lb}}$ occurred in Cell II-III for η = 0.3 (1.8%), while the lowest occurred in Cells I-I, I-II, I-III, III-I, and III-III (0.0%). For η = 0.5, the highest $P_{f,\,\mathrm{lb}}$ occurred in Cell II-III (10.2%), while the lowest occurred in Cells I-I and I-II (0%). For Case 2, the $P_{f,\,\mathrm{lb}}$ is 0% in all the cells and for any Θ_E and η indicating that there is risk of failure for systems subject to a moderate EQGM if designed for a severe EQGM.

It was interesting to see that in all the cases, as η increases, the P_f increases as well. This result might seem irrational because of the intuitive feeling that the failure probability should decrease for systems having higher strength than for systems having lower strength. The reason for this result in this study was due to the fact that as the structure's strength increased, the COV increased as well (Tables 2 and 3). It should also be noted that the failure probabilities given in Table 4 are very high and not valid for engineering design, in which case they must be very small.

6. Conclusions and recommendations

For a given performance level, the variation of hysteretic energy in terms of probabilistic terms is required to define the limiting value of hysteretic energy for a particular earthquake hazard level. In this study, a methodology was developed for evaluating the failure probability of non-linear systems. Then, this proposed methodology was applied on inelastic SDOF systems by carrying out non-linear dynamic time history analyses .The failure probabilities for 95% degree of confidence level corresponding to the worst failure probability were determined and the approximate confidence intervals for hysteretic energy demand for 95% and 90% degree of confidence levels were constructed. From the results obtained in this study, the following observations are reached:

a. As the capacity to demand ratio increases, the failure probability decreases dramatically. If this ratio is too small, then the failure is inevitable.
b. In general, the highest P_f occurs for systems subject to EQGMs having the predominant ground motion period (T_g) between 0.7 sec-1.0 sec.
c. P_f decreases, in general, as the natural period of the systems gets far away from the predominant ground motion period for both $\ddot{u}_{g,\max} = 0.3$ g and 0.6 g.
d. Failure probability is very sensitive to strength of the system and demand to capacity ratio.
e. As can be seen from the frequency distributon of E_{ec}, the underlying distribution of energy dissipation capacity is not normal. If it were, we could have obtained the exact failure probabilities. That is why asympototic failure probaility is used in this paper.
f. It is practically impossible to determine a unique P_f for a structure. The P_f for a structure can only be evaluated for a lower and upper bound range.
g. The failure probability is generally higher for systems designed against moderate level EQGMs than for systems designed against severe EQGMs.

Based on the results, the following recommendations are proposed.

a. Statistical values for the hysteretic energy dissipation capacity of steel moment connections are assumed to be independent of the strength of the system, i.e. the same statistical values are taken for any η based on the test results. The validity of this assumption needs to be further investigated both experimentally and analytically.
b. It is obvious that P_f will change dramatically in MDOF systems due to the effect of earthquake redundancy degree of the system. That is why P_f in MDOF systems with different structural configurations should be investigated considering the effect of redundancy (for example, multi-varied normal distribution) and the results should be compared with that of SDOF systems.
c. This study has been carried on SDOF systems with typical steel moment connections, i.e. the results are only valid for the systems with these type of connections. Using the same principles, the same study can be repeated for the SDOF systems with a typical R/C moment connection.
d. Approximate confidence intervals can only give engineers a general idea about the energy input to the structure. To be able to develop an energy-based earthquake-resistant design method, not only the hysteretic energy demand should be known, but also the hysteretic energy dissipation capacities of the structure and structural elements.

Acknowledgements

This study was supported by Gebze Institute of Technology (GYTE), Scientific Research Project No: 02-B-02-01-05.

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