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Technical Note

Use of moving substructure element for vibration analyses of a structure due to a moving trolley carrying a swinging mass

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1. Introduction

Moving-load-induced vibration of structures is an important problem in structural engineering. Therefore, many researchers have devoted themselves in this field. However, from the existing literature, it is found that, due to the complexity of the mathematical expressions, only Wu (2004) has investigated the vibration characteristics of a three-dimensional framework due to a moving mass *carrying a swinging mass*. Although the formulations of Wu (2004) are available for determining the dynamic responses of the three-dimensional structure induced by the moving mass carrying a swinging mass, they are *not* available for determining the dynamic responses of the swinging motion of the swinging mass. To solve this problem, this paper presents a technique to replace the "moving trolley carrying a swinging mass" by a *moving substructure element*, such that the dynamic responses of the structure and the swinging motion of the swinging mass can be determined simultaneously.

2. Property matrices of the *moving substructure element*

Fig. 1 shows the mathematical model for the overhead crane hoisting a swinging mass with magnitude m_{sw}, where the trolley (with mass m_T) may move on the top beam of the overhead crane. Fig. 2 shows the instantaneous position of the moving trolley (m_T) located at the s^{th} beam element of the top beam. The position vectors of the trolley m_T and the swinging mass m_{sw} are given by

$$\vec{r}_T = (x_T + x_A)\vec{i} - y_A\vec{j}\,, \quad \vec{r}_{sw} = (x_T + x_A + \ell_r \sin\theta)\vec{i} - (y_A + \ell_r \cos\theta)\vec{j} \tag{1}$$

where y_A and x_A respectively represent the vertical (y) and horizontal (x) displacements of the

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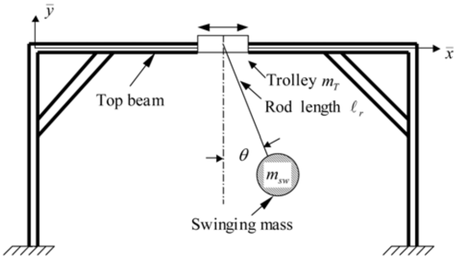

Fig. 1 Mathematical model for the overhead crane carrying a swinging mass m_{sw}

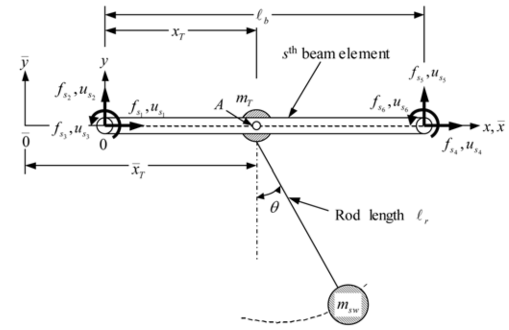

Fig. 2 A beam element subjected to a moving trolley carrying a swinging mass m_{sw}

contact point A given by

$$y_A \equiv w_{Ay} \equiv w_{Ay}(x_T, t), \quad x_A \equiv w_{Ax} \equiv w_{Ax}(x_T, t) \tag{2}$$

Time derivatives of Eq. (1) give

$$\dot{\vec{r}}_T = (\dot{x}_T + \dot{x}_A)\vec{i} - \dot{y}_A\vec{j}, \quad \dot{\vec{r}}_{sw} = (\dot{x}_T + \dot{x}_A + \ell_r\cos\theta\dot{\theta})\vec{i} - (\dot{y}_A - \ell_r\sin\theta\dot{\theta})\vec{j} \tag{3}$$

The total kinetic energy T and the total potential energy U of the moving trolley and the swinging mass are

$$T = \frac{1}{2}m_T[(\dot{x}_T + \dot{x}_A)^2 + \dot{y}_A^2] + \frac{1}{2}m_{sw}[(\dot{x}_T + \dot{x}_A + \ell_r\cos\theta\dot{\theta})^2 + (\dot{y}_A - \ell_r\sin\theta\dot{\theta})^2] \tag{4}$$

$$U = -(m_T + m_{sw})gy_A - m_{sw}g\ell_r\cos\theta \tag{5}$$

Substituting the last two equations into the following Lagrange's equations

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\eta}}\right) - \frac{\partial T}{\partial \eta} + \frac{\partial U}{\partial \eta} = 0 \qquad (\eta = x_A, y_A \text{ and } \theta) \tag{6}$$

one obtains

$$(m_T + m_{sw})\ddot{x}_A + m_{sw}\ell_r\cos\theta\ddot{\theta} = -(m_T + m_{sw})\ddot{x}_T = f_x \tag{7}$$

$$(m_T + m_{sw})\ddot{y}_A - m_{sw}\ell_r\sin\theta\ddot{\theta} = (m_T + m_{sw})g = f_y \tag{8}$$

$$m_{sw}\ell_r\cos\theta\ddot{x}_A - m_{sw}\ell_r\sin\theta\ddot{y}_A + m_{sw}\ell_r^2\ddot{\theta} = -m_{sw}\ell_r\cos\theta\ddot{x}_T - m_{sw}g\ell_r\sin\theta \tag{9}$$

where f_y and f_x represent the vertical (y) and horizontal (x) interaction forces at the contact point A, respectively, and $\ddot{y}_A$ and $\ddot{x}_A$ represent the associated accelerations. Besides, $\ddot{\theta}$ represent the angular acceleration of the swinging mass and $\ddot{x}_T$ represent the acceleration of the moving trolley. For convenience, we set

$$\dot{x}_T \equiv V_T, \quad \ddot{x}_T \equiv a_T \tag{10}$$

where $\dot{x}_T$ represents the velocity of the moving trolley.

Time derivatives of Eq. (2) yield

$$\ddot{y}_A = \ddot{w}_{Ay} + 2\dot{x}_T\dot{w}'_{Ay} + \dot{x}_T^2 w''_{Ay} = \ddot{w}_{Ay} + 2V_T\dot{w}'_{Ay} + V_T^2 w''_{Ay} \tag{11}$$

$$\ddot{x}_A = \ddot{w}_{Ax} + 2\dot{x}_T\dot{w}'_{Ax} + \dot{x}_T^2 w''_{Ax} = \ddot{w}_{Ax} + 2V_T\dot{w}'_{Ax} + V_T^2 w''_{Ax} \approx \ddot{w}_{Ax} \tag{12}$$

Based on the principle of mode superposition and the definition of shape functions, one has

$$w_{Ay} = N_2 u_{s_2} + N_3 u_{s_3} + N_5 u_{s_5} + N_6 u_{s_6}, \quad w_{Ax} = N_1 u_{s_1} + N_4 u_{s_4} \tag{13}$$

$$f_{s_2} = N_2 f_y, \quad f_{s_3} = N_3 f_y, \quad f_{s_5} = N_5 f_y, \quad f_{s_6} = N_6 f_y, \quad f_{s_1} = N_1 f_x, \quad f_{s_4} = N_4 f_x \tag{14}$$

In the last equations, u_{s_i} and f_{s_i} (i = 1 to 6) are, respectively, the nodal displacements and forces of the s^{th} beam element at which the moving trolley m_T is located (cf. Fig. 2), while N_k (k = 1 to 6) are the shape functions of the beam element given by

$$N_1 = 1 - \varsigma, \quad N_2 = 1 - 3\varsigma^2 + 2\varsigma^3, \quad N_3 = [\varsigma - 2\varsigma^2 + \varsigma^3]\ell_b$$

$$N_4 = \varsigma, \quad N_5 = 3\varsigma^2 - 2\varsigma^3, \quad N_6 = [-\varsigma^2 + \varsigma^3]\ell_b, \quad \varsigma = x_T/\ell_b \tag{15}$$

where ℓ_b is the length of the s^{th} beam element and x_T is the distance between the location of the moving trolley m_T and the left end of the s^{th} beam element, at time t.

Substituting Eqs. (10)-(15) into Eqs. (7)-(9), one obtains

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{f\} \tag{16}$$

$$\{\ddot{u}\} = [\ddot{u}_{s_1}\ \ \ddot{u}_{s_2}\ \ \ddot{u}_{s_3}\ \ \ddot{u}_{s_4}\ \ \ddot{u}_{s_5}\ \ \ddot{u}_{s_6}\ \ \ddot{\theta}]^T, \quad \{\dot{u}\} = [\dot{u}_{s_1}\ \ \dot{u}_{s_2}\ \ \dot{u}_{s_3}\ \ \dot{u}_{s_4}\ \ \dot{u}_{s_5}\ \ \dot{u}_{s_6}\ \ \dot{\theta}]^T$$

$$\{u\} = [u_{s_1}\ \ u_{s_2}\ \ u_{s_3}\ \ u_{s_4}\ \ u_{s_5}\ \ u_{s_6}\ \ \theta]^T \tag{17}$$

$$[m] = \begin{bmatrix} \alpha N_1 N_1 & 0 & 0 & \alpha N_1 N_4 & 0 & 0 & N_1 m_{sw}\ell_r \cos\theta \\ & \alpha N_2 N_2 & \alpha N_2 N_3 & 0 & \alpha N_2 N_5 & \alpha N_2 N_6 & -N_2 m_{sw}\ell_r \sin\theta \\ & & \alpha N_3 N_3 & 0 & \alpha N_3 N_5 & \alpha N_3 N_6 & -N_3 m_{sw}\ell_r \sin\theta \\ & & & \alpha N_4 N_4 & 0 & 0 & N_4 m_{sw}\ell_r \cos\theta \\ & & & & \alpha N_5 N_5 & \alpha N_5 N_6 & -N_5 m_{sw}\ell_r \sin\theta \\ & sym & & & & \alpha N_6 N_6 & -N_6 m_{sw}\ell_r \sin\theta \\ & & & & & & m_{sw}\ell_r^2 \end{bmatrix} \tag{18}$$

$$[c] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \beta N_2 N_2' & \beta N_2 N_3' & 0 & \beta N_2 N_5' & \beta N_2 N_6' & 0 \\ & & \beta N_3 N_3' & 0 & \beta N_3 N_5' & \beta N_3 N_6' & 0 \\ & & & 0 & 0 & 0 & 0 \\ & & & & \beta N_5 N_5' & \beta N_5 N_6' & 0 \\ & sym & & & & \beta N_6 N_6' & 0 \\ & & & & & & 0 \end{bmatrix} \tag{19}$$

$$[k] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & \gamma N_2 N_2'' & \gamma N_2 N_3'' & 0 & \gamma N_2 N_5'' & \gamma N_2 N_6'' & 0 \\ & & \gamma N_3 N_3'' & 0 & \gamma N_3 N_5'' & \gamma N_3 N_6'' & 0 \\ & & & 0 & 0 & 0 & 0 \\ & & & & \gamma N_5 N_5'' & \gamma N_5 N_6'' & 0 \\ & sym & & & & \gamma N_6 N_6'' & 0 \\ & & & & & & 0 \end{bmatrix} \tag{20}$$

$$\{f\} = \begin{Bmatrix} f_s \\ f_s \\ f_s \\ f_s \\ f_s \\ f_s \\ f_\theta \end{Bmatrix} = \begin{Bmatrix} -N_1(m_T + m_{sw})a_T \\ N_2(m_T + m_{sw})g \\ N_3(m_T + m_{sw})g \\ -N_4(m_T + m_{sw})a_T \\ N_5(m_T + m_{sw})g \\ N_6(m_T + m_{sw})g \\ -m_{sw}\ell_r \cos\theta a_T - m_{sw} g \ell_r \sin\theta \end{Bmatrix} \tag{21}$$

$$\alpha = m_T + m_{sw}, \quad \beta = 2(m_T + m_{sw})V_T, \quad \gamma = (m_T + m_{sw})V_T^2 \tag{22}$$

Eqs. (18), (19) and (20) are, respectively, the mass, damping and stiffness matrices of the *moving substructure element*. In addition, Eq. (21) is the equivalent nodal force vector induced by the moving trolley together with the swinging mass.

References

Wu, J.J. (2004), “Dynamic responses of a three-dimensional framework due to a moving carriage hoisting a swinging object”, *Inte. J. Numer. Meth. Eng.*, **59**(13), 1679-1702.