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A method for static and dynamic analyses of stiffened multi-bay coupled shear walls

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Abstract. In this study an approximate method based on the continuum approach and transfer matrix method for static and dynamic analyses of stiffened multi-bay coupled shear walls is presented. In this method the whole structure is idealized as a sandwich beam. Initially the differential equation of this equivalent sandwich beam is written then shape functions for each storey is obtained by the solution of differential equations. By using boundary conditions and storey transfer matrices which are obtained by these shape functions, system modes and periods can be calculated. Reliability of the study is shown with a few examples. A computer program has been developed in MATLAB and numerical samples have been solved for demonstration of the reliability of this method. The results of the samples show the agreement between the present method and the other methods given in literature.

Keywords: stiffened coupled shear wall; transfer matrix; static; dynamic.

1. Introduction

Shear walls are commonly used in high rise buildings to increase the resistance of the structure to the lateral loads. They are formed as coupled shear walls because of the presence of openings constituted for the architectural aspects such as windows, doors etc. The behavior of coupled shear walls can be improved by incorporating stiffening beams at various levels. A number of methods are available for the analysis of high rise buildings with coupled shear wall systems. These include the finite element method (Kwan 1993, Chaallal 1992, Kwan 1995, Kim and Lee 2003, Cengiz and Saygun 2007), continuous connection method (Rosman 1964, Basu *et al.* 1979, Coull and Smith 1983, Aksogan *et al.* 1993, Li and Choo 1996, Wang *et al.* 1999, Ha and Tan 1999, Aksogan *et al.* 2003, Arslan *et al.* 2004, Zeidabadi *et al.* 2004, Bozdogan *et al.* 2005, Aksogan *et al.* 2007), the equivalent frame method (Smith 1970, Chaallal and Ghamallal 1996), and the boundary element method (Rashed 2000).

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the static and dynamic analyses of stiffened coupled shear walls. It will be assumed for the analysis that the behavior of the materai is linear elastic and small displacement theory is valid and P-delta effects are negligible.

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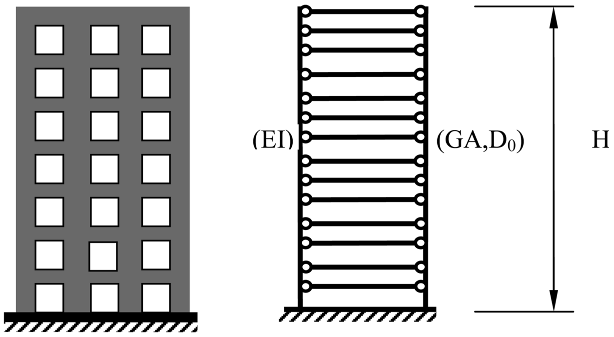

Fig. 1 Mathematical model of equivalent sandwich beam

2. Analyses

Under the horizontal loads, coupled shear walls demonstrate neither Timoshenko beam, nor Euler-Bernouilli beam behavior. The behavior of coupled walls is equivalent to the behavior of a sandwich beam which behaves as a combination of Timoshenko beam and Euler-Bernouilli beam (Fig. 1). Initially the differential equation of this equivalent sandwich beam can be written. The flexural rigidity of sandwich beam is equal to the sum of the flexural rigidities of shear walls near the openings. The shear rigidity of the sandwich beam is equal to the sum of shear rigidities of the connecting beams. The global flexural rigidity of the structural system can be calculated with the help of axial deformations of shear walls near the openings.

2.1 Storey transfer matrices

Under the lateral forces acting on the storey levels are considered equation of coupled shear wall of i.th storey can be written as Swaddiwudiphong *et al.* (2001)

$$EI_i\frac{d^4y_i}{dz^4} - GA_i\frac{d^2y_i}{dz^2} + GA_i\frac{d\psi_i}{dz} = 0 \tag{1}$$

$$GA_i\frac{dy_i}{dz} + D_i\frac{d^2\psi_i}{dz^2} - GA_i\psi_i = 0 \tag{2}$$

where y_i are the total shape functions, z is the vertical axis, ψ_i is the rotation angles of coupled shear wall due to bending, EI_i are the total bending rigidities of shear wall and D_i are the bending rigidities which represent the axial deformation and can be calculated from the equation below

$$D_i = \sum EA_j r_j^2 \tag{3}$$

where E is modulus of elasticity, A_j is the cross sectional area of the jth pierced wall r_j is the distance of the jth pierced wall from the centroid of cross-sections GA_i are the equivalent shear rigidities of connecting beams and can be calculate as in Eq. (4) (Potzta and Kollar 2003)

$$GA_i = \sum\frac{6EI_{bj}[(d_j+s_j)^2 + (d_j+s_{j+1})^2]}{d_j^3h\left(1+\dfrac{12\rho EI_{bj}}{GA_{bj}d_j^2}\right)} \tag{4}$$

where, d_j are the clear span lengths of coupling beams, s_j are the wall lengths, EI_{bj} and GA_{bj} are the flexural rigidities and the shear rigidity of connecting beams, respectively, h is the height of storey and ρ is a constant depending on the shape of cross-section of the beams (ρ = 1.2 for rectangular cross-sections).

With the solution of Eqs. (1) and (2) with respect to the z, total shape function and rotation angle can be obtained as

$$y_i(z) = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 \cosh(a_i z) + c_6 \sinh(a_i z) \tag{5}$$

$$\psi_i(z) = c_2 + 2c_3 z + (3z^2 + 6b_i)c_4 + \left(-\frac{EI_i}{GA_i}a_i^2 + a_i\right)c_5 \sinh(a_i z) + \left(-\frac{EI_i}{GA_i}a_i^2 + a_i\right)c_6 \cosh(a_i z) \tag{6}$$

where c_1, c_2, c_3, c_4, c_5, c_6 are integral constants. a_i and b_i can be calculated as follows.

$$a_i = \sqrt{\left(1 + \frac{D_i}{EI_i}\right)\frac{GA_i}{D_i}}, \qquad b_i = \frac{D_i}{GA_i} \tag{7}$$

With the help of Eq. (5), the total rotation angle, $y_i'(z)$, bending moment of shear wall, $M_{wi}(z)$, bending moment because of the axial deformation, $M_{axi}(z)$, and the total shear force, $V_i(z)$, can be obtained as follows.

$$y_i'(z) = c_2 + 2c_3 z + 3c_4 z^2 + c_5 a_i \sinh(a_i z) + c_6 a_i \cosh(a_i z) \tag{8}$$

$$M_{wi}(z) = EI_i y_i^{11} = EI_i(2c_3 + 6c_4 z + c_5 a_i^2 \cosh(a_i z) + c_6 a_i^2 \sinh(a_i z)) \tag{9}$$

$$M_{ai}(z) = -D_i \psi_i^1 = -D_i(2c_3 + 6c_4 z + c_6 f_i a_i \sinh(a_i z) + c_5 f_i a_i \cosh(a_i z)) \tag{10}$$

$$\begin{aligned} V_i(z) &= EI_i\frac{d^3 y_i}{dz^3} - GA_i\frac{dy_i}{dz} + GA_i\psi_i \\ &= c_4(6EI_i + 6GA_i b_i) + c_5 \sinh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) + c_6 \cosh(a_i z)(GA_i(f_i - a_i) + EI_i) \end{aligned} \tag{11}$$

where f_i

$$f_i = \frac{-EI_i}{GA_i}a_i^3 + a_i \tag{12}$$

When Eqs. (5), (6), (8), (9), (10) and (11) can be written in a matrix form as follows

$$\begin{bmatrix} y_i(z) \\ y_i'(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V(z) \end{bmatrix} = \begin{bmatrix} 1 & z & z^2 & z^3 & \cosh(a_i z) & \sinh(a_i z) \\ 0 & 1 & 2z & 3z^2 & a_i \sinh(a_i z) & a_i \cosh(a_i z) \\ 0 & 1 & 2z & (3z^2 + 6b_i) & f_i \sinh(a_i z) & f_i \cosh(a_i z) \\ 0 & 0 & 2EI_i & 6EI_i z & EI_i a_i^2 \cosh(a_i z) & EI_i a_i^2 \sinh(a_i z) \\ 0 & 0 & -2D_{0i} & -6D_{0i} z & -D_{0i} f_i a_i \cosh(a_i z) & -D_{0i} f_i a_i \sinh(a_i z) \\ 0 & 0 & 0 & (6EI_i + 6GA_i b_i) & \sinh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) & \cosh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{13}$$

At the initial point of the storey for z = 0, Eq. (13) can be written as

$$\begin{bmatrix} y_i(0) \\ y_i'(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} = \mathbf{A}_i(\mathbf{0}) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{14}$$

$$\underline{\mathbf{c}} = [c_1 \ \ c_2 \ \ c_3 \ \ c_4 \ \ c_5 \ \ c_6]^t \tag{15}$$

If vector $\underline{c}$ is solved out from formula (14) and is substituted in the Eq. (13), Eq. (16) will be obtained.

$$\begin{bmatrix} y_i(z) \\ y_i'(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V_i(z) \end{bmatrix} = A_i A_i^{-1}(0) \begin{bmatrix} y_i(0) \\ y_i'(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} \tag{16}$$

where $T_i = A_i A_i^{-1}(0)$ is the storey transfer matrix for $z = h$.

2.2 Static analysis

Static analysis of coupled shear walls under the horizontal loads can be done by the help of transfer matrices. When the lateral forces acting on the storey levels are considered, the relationship between displacement and internal forces can be written as follows

$$\begin{bmatrix} y_{(i+1)} \\ y_{(i+1)}' \\ \psi_{(i+1)} \\ M_{w(i+1)} \\ M_{ax(i+1)} \\ V_{(i+1)} \end{bmatrix} = \mathbf{T}_i \begin{bmatrix} y_i \\ y_i' \\ \psi_i \\ M_{wi} \\ M_{axi} \\ V_i \end{bmatrix} + \mathbf{F}_i \tag{17}$$

Lateral external force vector of ith storey is expressed as F_i.

$$\mathbf{F}_i = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ -f_i]^t \tag{18}$$

Here f_i is the horizontal load applied at the i.th storey level. When the Eq. (17) is written for each storey successively, expression which gives the structural relationship between base and top of structures can be written as follows

$$\begin{bmatrix} y_{top} \\ y^1{}_{top} \\ \psi_{top} \\ M_{wtop} \\ M_{axtop} \\ V_{tepe} \end{bmatrix} = \left[\prod_{k=n}^{1} \mathbf{T}_k\right] \begin{bmatrix} y_{base} \\ y^1{}_{base} \\ \psi_{base} \\ M_{wbase} \\ M_{axbase} \\ V_{taban} \end{bmatrix} + \sum_{s=1}^{n-1}\left[\prod_{k=n}^{s+1} \mathbf{T}_k\right]\mathbf{T}_s + \mathbf{F}_n \tag{19}$$

where t and p matrices can be written as

$$\mathbf{t} = \prod_{k=n}^{1} \mathbf{T}_k, \quad \mathbf{P} = \sum_{s=1}^{n-1}\left[\prod_{k=n}^{s+1} \mathbf{T}_k\right]\mathbf{F}_s + \mathbf{F}_n \tag{20}$$

Eq. (21) can be obtained by substituting Eq. (20) in Eq. (19)

$$t\begin{bmatrix} y_{base} \\ y^1{}_{base} \\ \psi_{base} \\ M_{wbase} \\ M_{axbase} \\ V_{taban} \end{bmatrix} + P = \begin{bmatrix} y_{top} \\ y^1{}_{top} \\ \psi_{top} \\ M_{wtop} \\ M_{axtop} \\ V_{top} \end{bmatrix}_n \tag{21}$$

When the boundary conditions are applied to the Eq. (21), it turns into the following form

$$\begin{bmatrix} t_{44} & t_{45} & t_{46} \\ t_{54} & t_{55} & t_{56} \\ t_{64} & t_{65} & t_{66} \end{bmatrix} \begin{bmatrix} M_{wbase} \\ M_{axbase} \\ V_{base} \end{bmatrix} = \begin{bmatrix} -P(4) \\ -P(5) \\ -P(6) \end{bmatrix} \tag{22}$$

By using Eq. (22) M_{wbase} (bending moment at the bottom of the structure), M_{axbase} (bending moment due to axial deformation at the bottom of the structure) and V_{base} (total shear force at the bottom of the structure) can be found. Then, the other unknowns can be found by the help of transfer matrices.

2.3 Dynamic analysis

For free vibration analysis the coupled shear walls are considered as a discrete lumped mass system.

The values of m_i can be approximated by

$$m_1 = \frac{m_T}{n} * 1.5 \tag{23}$$

$$m_i = \frac{m_T}{n} \quad (i = 2, 3, \ldots, n-1) \tag{24}$$

$$m_n = \frac{m_T}{2n} \tag{25}$$

where n is the number of storeys, m_T is the total mass of the system.

The storey transfer matrices obtained from Eq. (16) can be used for dynamic analysis of coupled shear wall. Therefore, when considering the inertial forces in storey levels, the relationship between, i.th and $(i + 1)$.th storey can be shown by the following matrix equation

$$\begin{bmatrix} y_{(i+1)} \\ y'_{(i+1)} \\ \psi_{(i+1)} \\ M_{w(i+1)} \\ M_{ax(i+1)} \\ V_{(i+1)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ m_i\omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T}_i \begin{bmatrix} y_i \\ y'_i \\ \psi_i \\ M_{wi} \\ M_{axi} \\ V_i \end{bmatrix} \tag{26}$$

where, m_i is the mass of i.th storey and ω are the natural frequencies of the system. Dynamic transfer matrix can be shown as T_{di}.

$$\mathbf{T}_{di} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ m_i\omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{T}_i \tag{27}$$

The displacements-internal forces relationship between base and top of the structure can be found as follows

$$\begin{bmatrix} y_{top} \\ y^1{}_{top} \\ \psi_{top} \\ M_{wtop} \\ M_{axtop} \\ V_{top} \end{bmatrix} = T_{dn}T_{d(n-1)}\ldots T_{d1} \begin{bmatrix} y_{base} \\ y^1{}_{base} \\ \psi_{base} \\ M_{wbase} \\ M_{axbase} \\ V_{base} \end{bmatrix} \tag{28}$$

When the boundary conditions are considered in Eq. (28), for nontrivial solution of $t_d = T_{dn}T_{dn-1}T_{dn-2}\ldots T_{d1}$ Eq. (29) can be gained.

$$\mathbf{f} = \begin{bmatrix} t_{44} & t_{45} & t_{46} \\ t_{54} & t_{55} & t_{56} \\ t_{64} & t_{65} & t_{66} \end{bmatrix} \tag{29}$$

The values of which set the determinant to zero are the natural frequencies of coupled shear wall.

2.4 Effect of stiffening beam

The effect of the stiffening beam is considered by

$$GA_{si} = r_i * GA_i \tag{30}$$

where r_i is a reduction factor and can be calculate by the following formula

$$r_i = \frac{I_s + I_b}{I_b} \tag{31}$$

where I_s is the second moment of are of stiffening beam and I_b is the second moment of are of connecting beams.

3. Procedure of computation

Procedure of computation of Transfer Matrix Method is presented below step by step

1. Calculation of the structural properties of each storey (GA, EI, m, ...).
2. Computation of storey transfer matrices for each storey using the structural properties obtained in step 1.
3. Computation of system transfer matrix with the help of storey transfer matrices.
4. Applying the boundary conditions and obtaining the nontrivial equation.
5. Determination of the angular frequencies by using numerical method.
6. Determination of modes with the help of storey transfer matrices using the angular frequencies.

4. Numerical examples

In this part of this study, to verify the presented method three numerical examples have been solved by a program written in MATLAB (2004). The results are compared with the ones which had been given in literature.

Example 1. In this example, a coupled shear wall with stiffened beam (Fig. 2) is analyzed. The stiffening beam is located at the mid-level of the structural height. The geometry and properties of the structure are as follows: total height H = 60 m; storey height h = 3 m; thickness of coupled shear wall = 0.30 m; d_1= 1.5 m, s_1 = 6.5 m, s_2 = 10 m; second moment of are of coupling beams= 0.00213314 m^4, and the second moment of are of stiffening beam = 0.073233 m^4; $E = 2.4 * 10^7$ kN/m^2 and the uniform load is w = 15 kN/m. Top displacement, base bending moment and base axial force calculated by this method are compared with those found in the literature (Arslan *et al.* 2001) (Table 1).

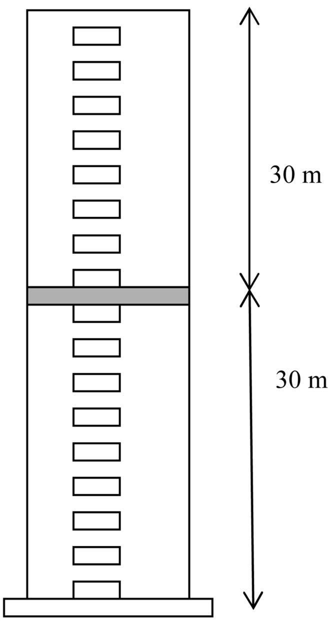

Fig. 2 Coupled shear wall of example 1

Table 1 Comparison of some results with those of Literature in Example 1

Displacement, Axial Force and Bending Moment		
	Arslan *et al.* (2001)	Present study
y(H) (mm)	6.66	6.60
T(0) (kN)	1422.49	1519.80
M(0) (kNm)	12775.1	12190.0

Example 2. A typical single bay coupled shear wall is analyzed as an example. The shear wall rests on a rigid foundation and has a thickness of 0.3 m as well as and the following properties: $d_1 = 2$ m, $s_1 = s_2 = 6$ m, $H = 95$ m, $h = 3.8$ m, coupling beam section: $0.3 * 0.3$ m, (density) = 24.00 kN/m^3 and $E = 2.76 * 10^7$ kN/m^2. The location of the stiffening beam is located at mid-level of the structural height and has a cross section of $0.3 * 1.5$ m. The first five natural frequencies are calculated in this method and are compared with those found in the literature Kuang and Chau (1998) (Table 2).

Table 2 Comparison of natural frequencies in Example 2 (Hz)

Mode	Kuang and Chau (1998)	Present method
1	0.76	0.75
2	2.93	2.84
3	8.12	8.03
4	13.30	12.90
5	22.46	22.01

Example 3. In this example the coupled shear wall shown in Fig. 3 is considered. The coupled shear wall with three bays consists of 12 storey, has the following properties: E = 2.10^7 kN/m^2, p = 24.05 kN/m^3. The height of connecting beams is 80 cm, the cross-sectional area of stiffening beams is 0.5136 m^2, second moment of are of stiffening beams is 0.441 m^4 and the thickness of the wall is 16 cm everywhere. The same coupled shear wall is considered in (Bikce and Aksogan) and analyzed using both continous connection and finite element methods. Free vibration analysis is carried out and the results are compared with those found in the literature (Bikce *et al.* 2000) (Table 3).

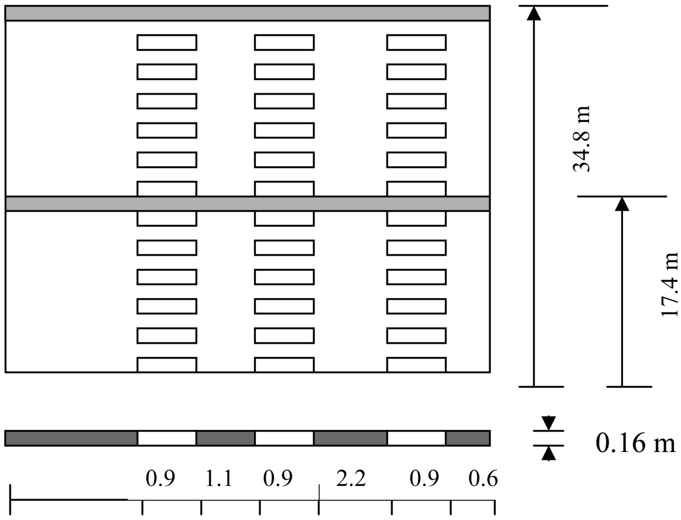

Fig. 3 12 storey coupled shear wall

Table 3 Comparison of natural frequencies in Example 3 (Hz)

Mode	SAP 2000 [20]	Bikce and Aksogan (2000)	Present method
1	3.1926	3.2138	3.1891
2	12.0504	12.2186	12.1494
3	30.7974	30.7603	33.7450
4	45.5791	45.6712	46.4534
5	71.4034	71.4489	72.2198

Example 4. A part of a 24 storey office building with coupled shear walls is investigated in this example (Fig. 4). The structure is designed for seismic loadings. The known dimensions are, H = 67.2 m, h = 2.8 m. The height of connecting beams is 0.35 m, the cross-sectional area of stiffening beams is 0.3 m^2, the height of stiffening beams is 1 m, the thickness of the wall is 0.3 m everywhere, E = 2.0 $*$ 10^7 kN/m^2 and p = 24.05 kN/m^3. In this example the first five natural frequencies of the building is found and compared with the values given in the literature (Emsen and Aksoan 2005) in Table 4.

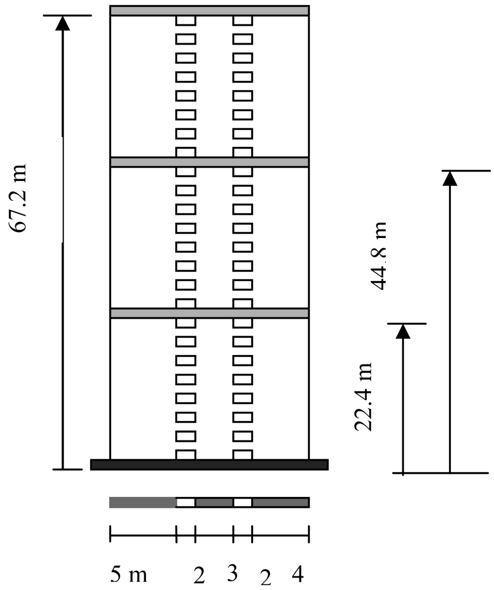

Fig. 4 Fourth example Structure

Table 4 Comparison of natural frequencies in Example 4 (Hz)

Mode	SAP 2000 (Finite Element)	Emsen and Aksogan (2005)	Present method
1	1.32823	1.33192	1.3501
2	5.02593	5.01227	5.1362
3	10.3234	10.2786	10.3279
4	20.8250	20.8289	21.6247
5	29.7554	29.6079	30.1175

5. Conclusions

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the static and dynamic analysis of stiffened coupled shear walls. In this method the whole structure is idealized as a sandwich beam. Examples demonstrate a good agreement with the finite element method solution given with literature. The proposed method is simple and accurate enough to be used both for the concept design stage and final analyses. This method is also suitable for implementation on any computer program.

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