Structural Engineering and Mechanics, Vol. 28, No. 4 (2008) 443-460

DOI: http://dx.doi.org/10.12989/sem.2008.28.4.443

Strength buckling predictions of cold-formed steel built-up columns

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(Received February 15, 2007, Accepted December 27, 2007)

Abstract. The aim of this paper is to propose a design procedure for predicting the buckling strength of built-up, cold-formed steel columns based on the two well known methods; the effective width method and the Direct Strength Method. Several design approaches, based on different elastic buckling solutions, were considered in this investigation. Traditional hand methods, without interaction effects between the different modes, and a new numerical spline finite strip method were used to predict the buckling stresses. All of the proposed methods were compared with experimental data on plain and lipped, built-up columns. Results have shown that the effective width approaches are more accurate than the Direct Strength Method. However, both methods can be investigated using more experimental data to assess a practical design method for built-up columns.

Keywords: buckling; built-up columns; DSM; effective width; spline finite strip.

1. Introduction

Structures such as cold formed, thin-walled sections usually rely upon the in-plane stiffness of the individual thin plates from which they are made up. The ease of construction offers a large variety of shapes and there is no significant limitations concerning the cross section geometry (Fig. 1). However, these plate elements present a relatively high out-of-plane flexibility which makes them susceptible to the various types of local and global buckling modes. The behaviour of such structural elements has been the research interest of so many researchers (Hancock *et al.* 1985, Al-Bermani and Kitipornchai 1990, Djafour *et al.* 1999, 2001). For local buckling design, the majority of current design codes (CEN 2004, AISI 1996, 2004) use the effective width concept combined

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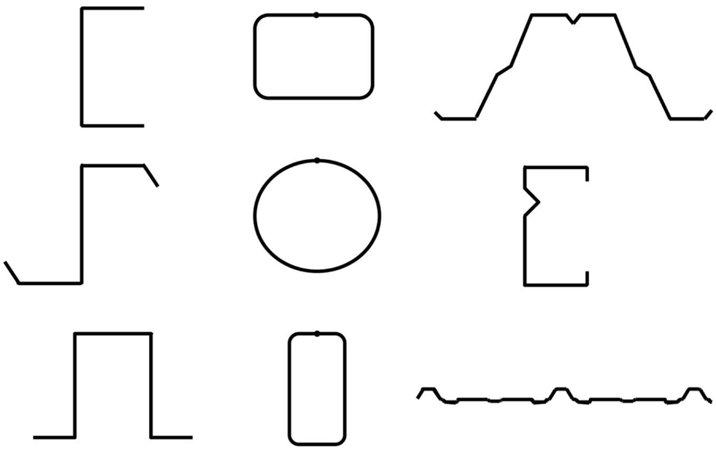

Fig. 1 Various cold-formed cross sections

with a Winter's type buckling curve. The essential idea behind this concept is that local plate buckling leads to reductions in the effectiveness of the plates that comprise a cross-section. That is, the effective plate under a simplified stress distribution can be used to approximate the real plate with a nonlinear stress distribution due to buckling. The effective width method is a useful design model, even though it ignores inter-element (between flange and web) equilibrium and compatibility when determining the buckling behaviour.

The actual trend in the design procedure of cold formed steel structural members (NAS 2004) is to use the Direct Strength Method (DSM) developed by Schafer and Pekoz (1998). It presents a competitive alternative to the existing effective section methods as it avoids lengthy effective width calculations (Hancock 2002). The DSM proposes a formal design procedure based on elastic buckling solutions for the complete cross section, which are obtained from a rational buckling analysis rather than the traditional solutions for each individual element. For isolated structural members the most efficient buckling analyses are the Finite Strip Method (FSM) (Papangelis and Hancock 1995, 2006, Schafer 2006) and the Generalized Beam Theory (GBT) (Davies and Leach 1994, Davies 2000, Silvestre and Camotim 2002). One additional design approach for cold-formed steel members worthy of mention is the Erosion of Critical Bifurcation Load approach championed by Dubina (Ungureanu and Dubina 2004, Szabo *et al.* 2004).

Sometimes, for economical purposes, cold-formed members are assembled by connecting elements (lacing bars or batten plates) to form a built-up column, as shown in Fig. 2. The moment of inertia of the cross section of the column, and thus its flexural stiffness in the plane of the connecting elements, increases with the distance between the chord axes. The existence of the connecting elements can make the whole cross section work as an entity. In addition to the two flexural buckling modes of the whole section (in a plane perpendicular to the battens and in a plane parallel to the battens), built-up columns with doubly symmetric cross-sections and discrete connecting elements have three other buckling modes by which the axially loaded, cold formed steel chords may fail:

- Long wave buckling, which can be flexural, torsional, or torsional-flexural. This is a global instability that involves translation and/or rotation of the entire cross section of the chord. Rational analysis hand solutions for long column buckling are available (Yu 2000). However,

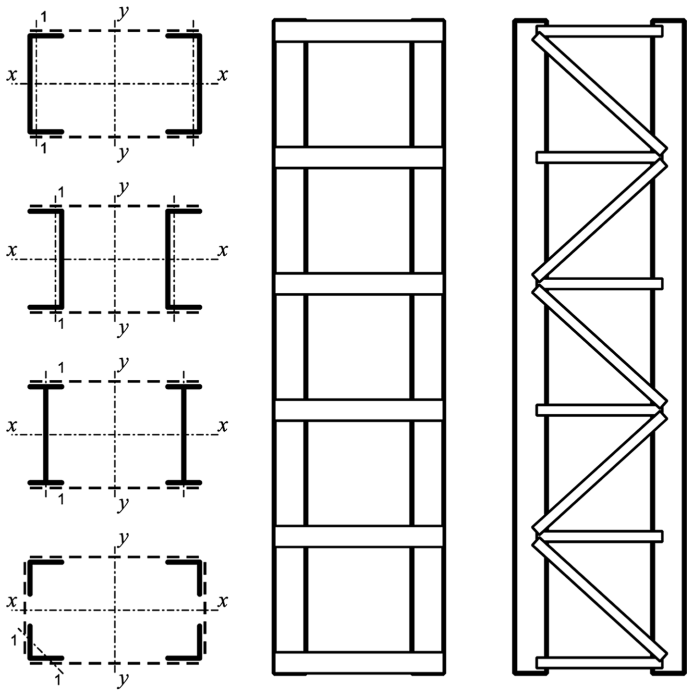

Fig. 2 Built-up columns

having the chords partially built-in to the battens may cause some additional difficulty.

- Local buckling minima, which occur at half-wavelengths that are less than the largest dimension of the chord. The cross section is distorted but the distortion involves only rotations at the fold-lines. It is well known that if the chord is long enough, local buckling involves several half-waves and the critical stress is independent of the boundary conditions in the longitudinal direction. Thus, the presence of the connecting elements will affect the local buckling behaviour of the chords only if their spacing is short.
- Distortional buckling, which involves distortion of one portion of the cross section and predominately rigid response of a second portion. It is characterized by relative movement of the fold-lines. The half-wavelength of distortional buckling is intermediate between that of local buckling and global buckling. It is typically several times larger than the largest dimension of the chord. If the connecting elements spacing is less than the critical distortional half-wavelength, their discrete restraining effect may retard distortional buckling and boost the strength.

All these modes can interact to produce unexpected complex buckling modes.

Very few studies have been carried to study built-up, cold formed columns (Niazi 1993, Stone and LaBoube 2005). Since the behaviour and design of thin-walled members are very sensitive to small details in the cross section, particularly those that serve to retard one or more cross-section instabilities, it becomes important to have a rational and efficient buckling analysis for the entire section of built-up members. This analysis must deal with all relevant buckling solutions such as: local, distortional, chords, Euler, whole global section, and their combinations. Currently, to predict the elastic buckling behaviour, two main groups of methods are available, the elastic buckling hand solutions and the numerical methods. In the former, closed-form predictions of the buckling stress in the local mode (including or excluding the interaction of the connected elements), the distortional mode (including consideration of the web/flange elastic and geometric stiffness), and the overall mode (including flexural and flexural-torsional buckling) are available. In the numerical methods group, a large variety of methods may be used to provide accurate elastic buckling solutions for cold-formed steel columns. If in practice, the finite element method is the most general and powerful numerical method, the time spent in data preparation and post-processing makes it very costly.

The Generalized Beam Theory (GBT) has been proven as an efficient and powerful approach to calculate the elastic buckling load of cold-formed steel structural members. The ability to separate the different buckling modes makes the method especially amenable for design methods. However, the GBT is hard to understand and only a few researchers use it. In the last decade, the finite strip methods, especially the Spline Finite Strip Method (SFSM) have been widely used to study the stability of thin-walled sections, taking into account all possible buckling modes: local, distortional, global, and their combinations. This method uses the uniform B3-Spline functions in the longitudinal direction and conventional interpolation functions in the transverse direction. Recently, Kim and Choi (2004) have improved the SFSM by introducing non-symmetrically spaced knots in the longitudinal direction.

Since a lot of work has been carried out for determining the elastic buckling load for isolated, thin-walled members, a tentative use of the two basic design methods, the effective width method and the Direct Strength Method, will be made to study the buckling behaviour of built-up columns. To the best knowledge of the authors, no study has been made in this area and a consistent integration of all types of buckling modes into the design of built-up, thin-walled columns is needed.

In this research, the effective width method philosophy and formulae used in EUROCODE 3 (CEN 2004) for hot rolled built-up columns are explored. In the Direct Strength Method, several design approaches, starting from different elastic buckling solutions, are considered for investigation. Hand solutions and numerical solutions are used. In the hand solution methods, the classical "element" type solution and a semi-empirical type solution will be used. In the "element" type solution, the local buckling stress of the section is obtained from critical stresses of isolated plate elements with or without considering the interaction between the connected elements of the chords. On the other hand, a new method known as, "the Compound Spline Finite Strip Method" (CSFSM), (Djafour *et al.* 2007) was used to predict the elastic buckling behaviour incorporating all buckling modes and all structural details by numerical methods. This method adequately combines spline finite strips and beam finite elements to model built-up columns.

The performance of all the proposed design methods is assessed by comparison to gathered experimental data on built-up columns formed by two equally plain or lipped channels assembled by batten plates. The column tests, conducted in 1999 by Moldovan *et al.* (1999) at the Building Research Institute, INCERC, Timisoara, Romania, on U and C shaped channels with different types of connecting elements (battens, stitches, bolts, and welds) and those conducted by Niazi (1993) on built-up elements composed of cold-formed C-profiles with battened plates or C stitches, were used for comparison.

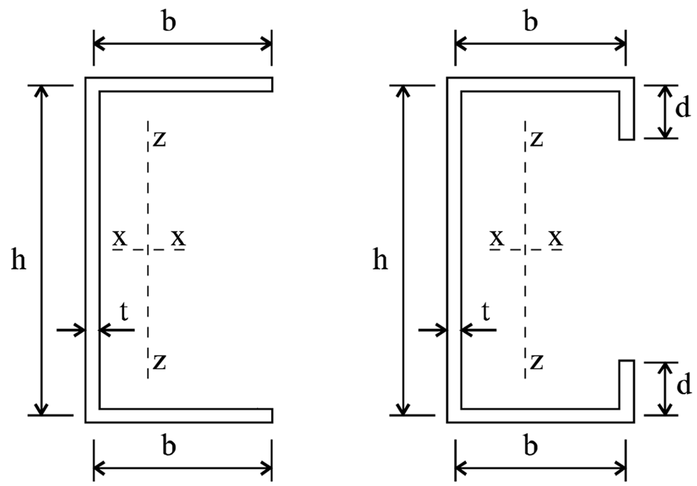

Fig. 3 Characteristics of U and C shaped cross-sections

2. Experimental data

2.1 INCERC program

The test program conducted by Moldovan at INCERC, Timisoara (Romania) provided experimental ultimate loads for built-up, cold-formed channels compressed between pinned ends. The test specimens were brake-pressed from high strength zinc-coated structural steel sheets having a nominal yield stress of 235 MPa. The test program comprised two series of built-up channels. The first series consisted of U shaped columns with a nominal thickness of 1.8, 3.0, and 4.0 mm and a nominal web depth of 85 and 100 mm. The nominal flange width was either 60 or 80 mm. The second series was the C shaped columns with a nominal thickness of 1.85 and 3.0 mm and a nominal web depth of 85, 100, and 120 mm. The nominal flange width was 55, 65, and 85 mm. The nominal width of the lip was 18, 20, and 25 mm. The average value of measure for the cross-sectional dimensions of the test specimens are shown in Table 1 using the nomenclature defined in Fig. 3.

The specimens were tested at various column lengths ranging from 1,220 to 2,000 mm. To form the built-up column from two U or C shaped channels, different types of connecting elements (battens, stitches, bolts, and welds) were used. They were placed at various distances ranging from 460 to 840 mm.

2.2 Niazi program

Niazi (1993) conducted eighteen tests on columns with battened plates and C stitches. The main dimensions of the elements are summarized in Table 1. The measured yield strength of the steel was 455 MPa for the profiles with a thickness of 2.5 mm and 428 MPa for the profiles with a thickness of 3 mm. The dimensions of the profiles were chosen in such a way that all the stability phenomena (local and global) could be met in the series of tests.

Table 1 Average measured specimen dimensions
From experimental work of Moldovan *et al*. (1999) and Niazi (1993).

	Test Spec.	*h* (mm)	*b* (mm)	*d* (mm)	*t* (mm)	*h/t*	*b/t*	*d/t*	*h/b*	*d/b*	*d/h*	$L^{(*)}$ (mm)	$a^{(**)}$ (mm)
U Shape. A. Moldovan *et al.* (1999)	P 5-1	86.50	58.00		1.80	48.06	32.22		1.49			2000	634
	P 5-2	87.30	57.00		1.80	48.50	31.67		1.53			2000	634
	P 5-3	88.00	61.00		1.75	50.29	34.86		1.44			2000	634
	P 7-1	82.20	60.00		3.05	26.95	19.67		1.37			1220	500
	P 7-2	82.30	60.50		2.95	27.90	20.51		1.36			1220	660
	P 7-3	82.50	59.70		3.00	27.50	19.90		1.38			1220	660
	P 10-1	102.00	81.20		3.00	34.00	27.07		1.26			2000	626
	P 10-2	98.50	81.50		2.95	33.39	27.63		1.21			2000	626
	P 12-1	84.40	81.00		4.00	21.10	20.25		1.04			1500	840
	P 12-2	86.00	80.00		4.05	21.23	19.75		1.08			1500	840
	P 12-3	85.00	81.30		3.95	21.52	20.58		1.05			1500	840
C Shape. A. Moldovan *et al.* (1999)	P 19-1	84.00	54.50	18.00	1.85	45.41	29.46	9.73	1.54	0.33	0.21	2000	634
	P 19-2	85.50	55.00	18.00	1.85	46.22	29.73	9.73	1.55	0.33	0.21	2000	634
	P 19-3	86.00	56.00	15.50	1.85	46.49	30.27	8.38	1.54	0.28	0.18	2000	634
	P 21-1	98.70	65.80	18.80	1.60	61.69	41.13	11.75	1.50	0.29	0.19	1480	810
	P 21-2	98.00	65.60	20.50	1.80	54.44	36.44	11.39	1.49	0.31	0.21	1480	810
	P 21-3	97.70	64.70	21.00	1.75	55.83	36.97	12.00	1.51	0.32	0.21	1480	460
	P 27-1	121.70	84.50	25.50	3.05	39.90	27.70	8.36	1.44	0.30	0.21	1700	530
	P 27-2	125.20	84.80	24.40	3.05	41.05	27.80	8.00	1.48	0.29	0.19	1700	530
	P 27-3	121.30	82.30	23.30	3.00	40.43	27.43	7.77	1.47	0.28	0.19	1700	530
C Shape. Niazi (1993)	120B1	120.0	60.0	20.0	2.50	48.00	24.00	8.00	2.00	0.33	0.17	4000	1300
	120B2	120.0	60.0	20.0	2.50	48.00	24.00	8.00	2.00	0.33	0.17	3000	960
	180B1	180.0	70.0	25.0	3.00	60.00	23.33	8.33	2.57	0.36	0.14	4000	1280
	180B2	180.0	70.0	25.0	3.00	60.00	23.33	8.33	2.57	0.36	0.14	4000	1920
	180B3	180.0	70.0	25.0	3.00	60.00	23.33	8.33	2.57	0.36	0.14	3000	950
	180B4	180.0	70.0	25.0	3.00	60.00	23.33	8.33	2.57	0.36	0.14	3000	1420

Note. (*) L: Length of the built-up columns
(**) a: Longitudinal spacing of connectors between elements.

3. Column design methods

3.1 Effective width method vs. direct strength method

Current design specifications, using either the Effective Width Method or the Direct Strength Method, employ the elastic local, distortional, or Euler buckling characteristics to calculate the ultimate strength of columns. The effective width approach is an element-based method since it looks at the elements forming a cross-section of each chord of the built-up columns in isolation. The local web/flange interaction is then ignored. The main idea behind this method is to determine the locations in a cross-section where material is really ineffective in carrying load at the full applied stress. The basic formulae commonly used in design specifications following this approach is

$$\frac{b_{eff}}{b} = \rho = \frac{1}{\bar{\lambda}_P}\left(1 - \frac{0.22}{\bar{\lambda}_P}\right) \le 1 \quad \text{for} \quad \bar{\lambda}_P > 0.673 \quad \text{otherwise} \quad \rho = 1 \tag{1}$$

where
$$\bar{\lambda}_P = \sqrt{\frac{f}{f_{cr}}} = \frac{1.052}{\sqrt{K}} \cdot \frac{b}{t}\sqrt{\frac{f}{E}}$$

b_{eff} is the effective width of an element (flange, web, lip); $f = f_y$ when interaction with other modes is ignored and f_{cr} is the critical elastic local buckling stress.

Because sections are more complex and the interaction between elements of the same section (e.g., web/flange) becomes more important, and the computation of the effective widths becomes complicated. The Direct Strength Method, proposed by Schafer and Pekoz (1998), avoids the calculation of the effective widths and employs the elastic buckling solutions for the entire member using real dimensions of the cross-section to determine the reduced strength of a column in a given mode due to buckling and/or yielding. This method was incorporated recently in the North American Specifications (AISI 2004).

For long columns, the nominal axial strength, P_{ne}, for flexural, torsional, or flexural-torsional buckling is given by AISI (2004)

$$P_{ne} = (0.658^{\lambda_c^2})P_y \quad \text{for} \quad \lambda_c \le 1.5 \tag{2}$$

Otherwise
$$P_{ne} = \left(\frac{0.877}{\lambda_c^2}\right)P_y$$

where $P_y = A_g f_y$ and $\lambda_c = \sqrt{P_y/P_{cre}}$. P_{cre} is the minimum of the critical elastic buckling load in flexural, torsional, or flexural-torsional buckling.

The idea was extended to other modes such as local and distortional buckling (Schafer and Pekoz 1998, Schafer 2002), the following forms are suggested:

For local buckling:

$$P_{nl} = \left[1 - 0.15\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} P_{ne} \quad \text{for} \quad \lambda_l > 0.776 \tag{3}$$

Otherwise
$$P_{nl} = P_{ne}$$

where $\lambda_l = \sqrt{P_{ne}/P_{crl}}$. P_{crl} is the critical elastic local column buckling load. When interactions between local and other modes are ignored, P_{ne} in Eq. (3) is replaced by P_y

For distortional buckling:

$$P_{nd} = \left[1 - 0.25\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} P_y \quad \text{for} \quad \lambda_d > 0.561 \tag{4}$$

Otherwise

where $\lambda_d = \sqrt{P_y/P_{crd}}$. P_{crd} is the critical elastic distortional column buckling load.

3.2 Elastic buckling analysis

Since the effective width method or the Direct Strength Method (DSM) are based on the determination of elastic buckling loads, two methods are used for predicting the nominal axial strength of built-up columns: the so-called conventional hand solutions, already applicable for single channels and a new proposed numerical method based on the spline finite strip method. The former method uses the classical element approach or the semi-empirical approach proposed by Schafer. For the latter method a Compound Spline Finite Strip Model (CSFSM) was recently developed (Djafour *et al.* 2007) to predict the elastic buckling loads and corresponding mode shapes for thin-walled built-up columns.

3.2.1 Elastic buckling hand solutions

Closed-form predictions of local, distorsional, or Euler buckling of built-up columns are examined using the element or semi-empirical approaches employed to study the behaviour of single channel, (not battened) thin-walled, cold-formed steel columns (Schafer 2002). For un-stiffened and stiffened elements with web depth h, flange width b, and lip length d, the critical local buckling stress, using the classical method and ignoring the interaction between modes, is

$$\sigma_{cr,web} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{h}\right)^2 \quad k = 4 \quad \text{for } U \text{ and } C \text{ shapes} \tag{5}$$

$$\sigma_{cr,flange} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad k = 4 \text{ for } C \text{ shapes and } k = 0.43 \text{ for } U \text{ shapes} \tag{6}$$

$$\sigma_{cr,lip} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{d}\right)^2 \quad k = 0.43 \text{ for } C \text{ shapes} \tag{7}$$

Taking the minimum of Eq. (5) through Eq. (7), we can calculate the approximate local buckling of the member. If interaction between modes is considered, (e.g., for C shaped, built-up columns), the semi-empirical approach is used, where the critical distorsional buckling stress is calculated using Eq. (6) with a buckling coefficient (k) taken as the minimum of the flange/lip local buckling or the flange/web local buckling given by the following empirical expressions given by Eqs. (8) and (9 or 10) (Schafer 2002)

$$k_{flange/lip} = -11.07\left(\frac{d}{b}\right)^2 + 3.95\left(\frac{d}{b}\right) + 4 \quad (\text{For } d/b < 0.6) \tag{8}$$

$$k_{flange/web} = 4\left(\frac{b}{h}\right)^2\left[2 - \left(\frac{b}{h}\right)^{0.4}\right] \quad \text{if} \quad \frac{h}{b} \ge 1 \tag{9}$$

Otherwise

$$k_{flange/web} = 4\left[2 - \left(\frac{b}{h}\right)^{0.2}\right] \tag{10}$$

The distorsional buckling load by hand solution, as proposed by Schafer is given by

$$P_{crd} = A_g f_{crd} \tag{11}$$

where A_g is the gross area of the member and, f_{crd} is the distorsional buckling stress given by

$$f_{crd} = \frac{k_{\phi fe} + k_{\phi we}}{k_{\phi fg} + k_{\phi wg}} \tag{12}$$

The k coefficients represent the elastic and geometric rotational stiffness of the flange and web. Details of these terms are given in (Schafer 2002). For Euler buckling columns, extensive hand expressions are available and may be used for hand calculation of the critical Euler buckling load.

3.2.2 Elastic buckling numerical solution - The proposed CSFS model

In the Spline Finite Strip Method, the elements of the thin-walled structures are longitudinal strips of a plate which are joined to one another along nodal lines running the total length of the structure. The use of B3-spline fits perfectly with the displacement function in the longitudinal direction, which has a C^2 continuity. In the transverse direction a polynomial displacement function is used. For each nodal line (q), four displacement functions having C^2 continuity can be evaluated for any coordinate point (Z): $U_q(Z), V_q(Z), W_q(Z)$ and $\theta z_q(Z)$ (Fig. 4).

Using energy methods, the linear stability eigen-problem can be obtained (Hancock and Lau 1985)

$$([K] - \lambda[G])\{\Delta\} = \{0\} \tag{13}$$

where $[K]$ and $[G]$ are the stiffness and stability matrices for the overall system, respectively. and $\{\Delta\}$ is the vector for all degrees of freedom defined in the global coordinate system

The batten plates connecting the columns are considered to behave as a beam element with shear deformation neglected. A classical 3D Beam Finite Element is used to model these batten plates (Fig. 5). Fig. 6 shows a beam element connected to two nodal lines (q) and (r) at coordinates $Z = Z_i$ and $Z = Z_j$. The displacement compatibility at node i require that

$$U_i = U_q(Z_i), \quad V_i = V_q(Z_i), \quad W_i = W_q(Z_i) \quad \text{and} \quad \theta z_i = \theta z_q(Z_i) \tag{14}$$

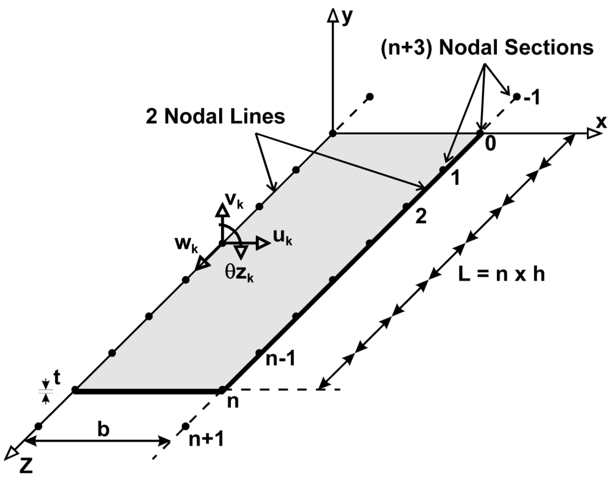

Fig. 4 A B3-spline strip

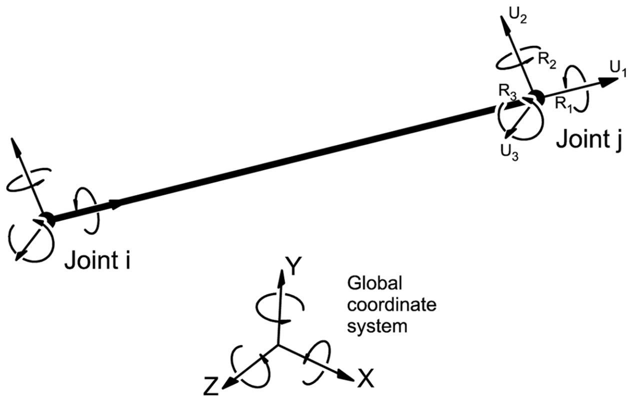

Fig. 5 A classical beam element

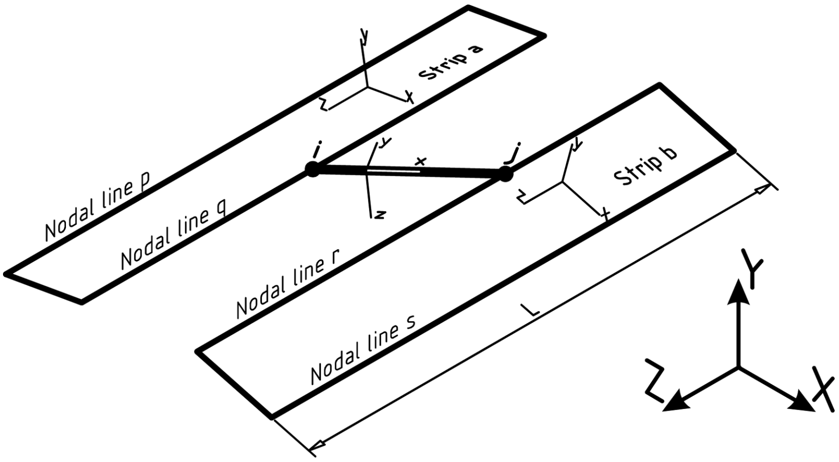

Fig. 6 Two strips connected by a beam

The rotational degrees of freedom can be related to the nodal line displacements by

$$\theta x_i = -\left.\frac{dV_q(Z)}{dZ}\right|_{Z=Z_i} \quad \text{and} \quad \theta y_i = \left.\frac{dU_q(Z)}{dZ}\right|_{Z=Z_i} \tag{15}$$

A transformation matrix is obtained when applying these equations to both ends of the beam. This matrix is used to transform the beam stiffness matrix which can be easily assembled to the system stiffness matrix.

A computer program (SHell Elastic Buckling Analysis, SHEBA) based on this "Compound Spline Finite Strip Model (CSFSM)" technique was developed to calculate the lowest elastic buckling loads and draw the corresponding failure modes. This program can take into account complex boundary conditions and local effects such as intermediate supports. More details about the proposed numerical model are given in References (Djafour *et al.* 1999, 2001, 2007)

4. Proposed design approaches

Based on existing work for the buckling analysis of single channels, some design approaches are proposed for predicting the strength design of built-up columns. The effective width method and the Direct Strength Method are investigated and the performance of the proposed design methods is assessed by comparison with experimental data obtained from the studies by Modovan *et al.* (1999) and Niazi (1993).

4.1 Effective width based design approaches

The effective width approach introduced in Eurocode 3 (CEN 2004) for evaluating the resistance of battened built-up members subjected to local buckling is used to verify its applicability to thin-walled, built-up columns. Two design approaches, starting from different elastic buckling solutions, are considered for this investigation, namely the classical "element" approach and the Compound Spline Finite Strip Model (CSFSM) numerical.

4.2.1 First approach (EC3-1)

i) Step1: The Euler buckling load is computed using the effective moment of inertia and the real dimensions of the chord.

$$N_{cr} = \frac{\pi^2 EI_{eff}}{l^2} \quad \text{where} \quad I_{eff} = 0.5h^2 A_{br} + 2\mu I_{yy} \tag{16}$$

ii) Step 2: The effective widths of the web, flanges, and lips of each chord forming the built-up column are determined by the formula

$$b_{eff} = \rho \cdot b = \frac{1}{\overline{\lambda}_P}\left(1 - \frac{0.22}{\overline{\lambda}_P}\right) \cdot b \quad \text{where} \quad \overline{\lambda}_P = \frac{1.052}{\sqrt{K_\sigma}} \cdot \frac{b}{t}\sqrt{\frac{f_y}{E}} \tag{17}$$

where $K_\sigma = 4.0$ for stiffened elements and 0.43 for un-stiffened elements.

iii) Step 3: Calculation of the shear stiffness coefficient (Sv) given by

$$\frac{1}{S_v} = \frac{a^2}{24EI_c} + \frac{ah}{12EI_b} \tag{18}$$

iv) Step 4: The critical buckling strengths about the minor and major axes are computed by the Eurocode approach using the effective width of the channels found in step 2.

$$N_{by,RD} = \frac{\chi_y A_{eff} f_y}{\gamma_{M1}} \quad \text{and} \quad N_{bz,RD} = \frac{\chi_z A_{eff} f_y}{\gamma_{M1}} \tag{19}$$

v) Step 5: The critical buckling load of the built-up column is calculated by solving the second order equation

$$0.5\psi N_{SD}^2 - (0.5 + 0.5\xi + \psi N_{R,SD})N_{SD} + N_{R,SD} = 0 \tag{20}$$

where:

$$\psi = \frac{1}{N_{cr}} + \frac{1}{S_v} \quad \text{and} \quad \xi = \frac{e_0 h_0 A_{eff}}{I_{eff}}$$

and $$N_{R,SD} = \min(N_{by,RD};N_{bz,RD})$$

vi) Comparison of calculated values with the experimental ones.

4.2.2 Second approach (EC3-2)

The same steps are followed as for the first approach except that in step 2 instead of using K_σ = 4.0 for stiffened elements and 0.43 for un-stiffened elements in the calculation of $\overline{\lambda}_P$, the buckling factors are determined from the equation

$$K_\sigma = \frac{\sigma_{cr}}{\sigma_E} = \sigma_{cr} \cdot \frac{12(1-\nu^2)(b/t)^2}{\pi^2 E} \tag{21}$$

where σ_{cr} is the elastic critical stress calculated by the proposed “Compound Spline Finite Strip Method (CSFSM)” program using the real dimensions (measured) of the channels. Following all these steps for the selected built-up columns, the obtained results are summarised in Tables 2 and 3 for the plain and lipped, built-up columns respectively.

It should be noted that these two approaches gave very close values of buckling load compared to the experimental ones. The mean value of the ratio $N_{u,\,expe}/N_{u,\,num}$ is about 0.99, 0.84, and 0.94 for the first approach and 0.83, 0.81, and 1.05 for the second approach, for U, C (Moldovan *et al.* 1999) and C (Niazi 1993) sections, respectively.

Figs. 7 and 8 show the variation of the web effective width ratio $\rho = h_{eff}/h$ with the slenderness factor $\lambda = \sqrt{\sigma_y/\sigma_{cr}}$. Herein, σ_y = 235 MPa, and σ_{cr} is the critical stress of the built-up columns obtained by multiplying the critical load (calculated by the two effective width approaches or the experimental critical load), by the effective area. For reference, the prediction from the (CEN 2004) method for plate local buckling calculated at σ_y, is included. In general, the results obtained by the two effective width approaches correlate very well with the experimental results.

Table 2 Experimental-to-predicted ratios for effective width solutions for plain built-up columns

	Col	$N_{u,\,expe}$ (KN)	$N_{u,\,Ec1}$ (KN)	$N_{u,\,Ec2}$ (KN)	$N_{u,\,expe}/N_{u,\,Ec1}$	$N_{u,\,expe}/N_{u,\,Ec2}$	$N_{u,\,Ec1}/N_{u,\,Ec2}$
U Shape. A. Moldovan *et al.* (1999)	P 5-1	80.00	94.168	118.22	0.85	0.68	0.96
	P 5-2	95.00	93.873	118.17	1.01	0.80	0.95
	P 5-3	70.00	91.141	114.81	0.77	0.61	0.96
	P 7-1	240.00	209.9	240.79	1.14	1.00	0.90
	P 7-2	225.00	200.25	233.63	1.12	0.96	0.90
	P 7-3	232.00	204.39	235.55	1.14	0.98	0.90
	P 10-1	175.00	236.99	297.81	0.74	0.59	0.95
	P 10-2	218.00	227.73	287.48	0.96	0.76	0.95
	P 12-1	323.00	324.31	382.62	1.00	0.84	0.89
	P 12-2	323.00	331.43	386.21	0.97	0.84	0.87
	P 12-3	392.00	319.96	380.01	1.23	1.03	0.89
			Mean value		0.99	0.83	0.83
			Standard deviation		0.15	0.15	0.03

Table 3 Experimental-to-predicted ratios for effective width solutions for lipped built-up columns

	Col	$N_{u,\,expe}$ (KN)	$N_{u,\,Ec1}$ (KN)	$N_{u,\,Ec2}$ (KN)	$N_{u,\,expe}/N_{u,\,Ec1}$	$N_{u,\,expe}/N_{u,\,Ec2}$	$N_{u,\,Ec1}/N_{u,\,Ec2}$
C Shape. A. Moldovan *et al.* (1999)	P 19-1	117.00	146.3	149.68	0.80	0.78	0.96
	P 19-2	125.00	147.77	151.66	0.85	0.82	0.95
	P 19-3	116.00	145.48	149.52	0.80	0.78	0.96
	P 21-1	140.00	150.23	167.59	0.93	0.84	0.90
	P 21-2	153.00	177.16	189.59	0.86	0.81	0.90
	P 21-3	140.00	171.66	184.8	0.82	0.76	0.90
	P 27-1	312.00	405.71	406.93	0.77	0.77	0.89
	P 27-2	334.00	405.66	409.15	0.82	0.82	0.87
	P 27-3	360.00	386.71	388.89	0.93	0.93	0.89
			Mean value		0.84	0.81	0.96
			Standard deviation		0.06	0.05	0.03

	Col	$N_{u,\,expe}$ (KN)	$N_{u,\,Ec1}$ (KN)	$N_{u,\,Ec2}$ (KN)	$N_{u,\,expe}/N_{u,\,Ec1}$	$N_{u,\,expe}/N_{u,\,Ec2}$	$N_{u,\,Ec1}/N_{u,\,Ec2}$
C Shape. Niazi (1993)	120B1	245.670	248.82	235.69	0.99	1.04	1.06
	120B2	346.34	342.26	340.48	1.01	1.02	1.01
	180B1	460.67	548.06	485.36	0.84	0.95	1.13
	180B2	447.34	462.37	396.67	0.97	1.13	1.17
	180B3	570.34	623.35	522.93	0.91	1.09	1.19
	180B4	549	597.16	508.27	0.92	1.08	1.17
			Mean value		0.94	1.05	1.12
			Standard deviation		0.06	0.06	0.07

Note:
$N_{u,\,Ec1}$=Local buckling load of the built-up columns from the first approach (EC3-1) of the effective width method.
$N_{u,\,Ec2}$=Local buckling load of the built-up columns from second approach (EC3-2) of the effective width method.
$N_{u,\,expe}$=Experimental buckling load of the built-up columns.

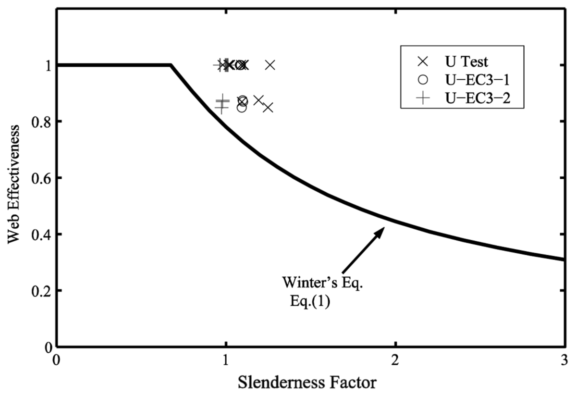

Fig. 7 Web effective width vs. CEN2004 Single Plain Channel Prediction

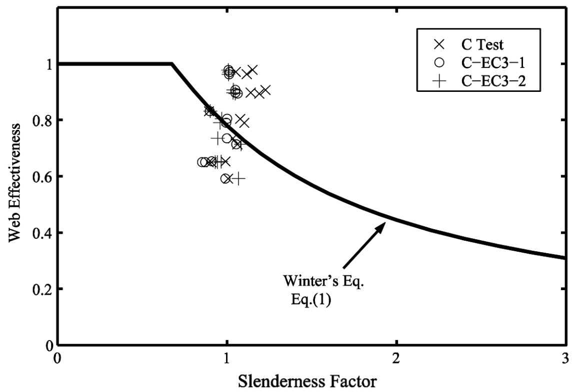

Fig. 8 Web effective width vs. CEN2004 Single Lipped Channel Prediction

4.3 Direct strength-based design approaches

The principles of the Direct Strength Method are used to determine the strength predictions of built-up columns. Depending on the calculation of the elastic buckling characteristics, four design approaches are considered for this investigation. These are as follows:

4.3.1 First and second approaches (DSM-1 and DSM-2)

Separate strength curves for local and distorsional buckling loads are considered in the Direct Strength Method (DSM). In the first approach (DSM-1), the local buckling load is simply calculated by the classical element method (i.e., Eqs. (5) through (7)). In the second approach (DSM-2), the closed-form hand method given by Eqs. (8) through (10) is used to predict the local buckling stress. No interaction between the local mode and the other modes is considered.

The predicted strengths for plain and lipped, built-up columns are given in Figs. 9 and 10 respectively. For plain channel, the semi-empirical method is not applicable. From these figures, we can see that the element method performs well for plain, built-up columns, while for lipped, built-up columns; the hand methods are more conservative for predicting the strength of columns.

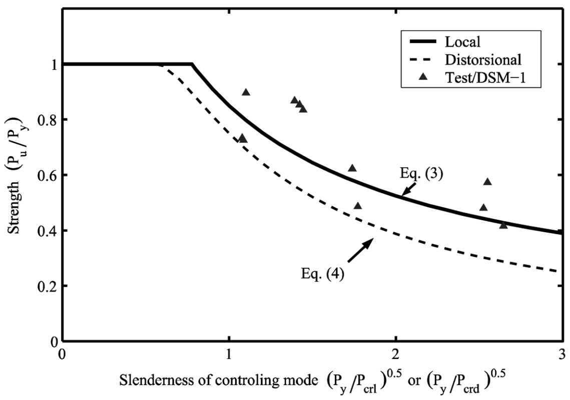

Fig. 9 Method DSM-1: Slenderness vs. Strength for Plain Built-up Columns

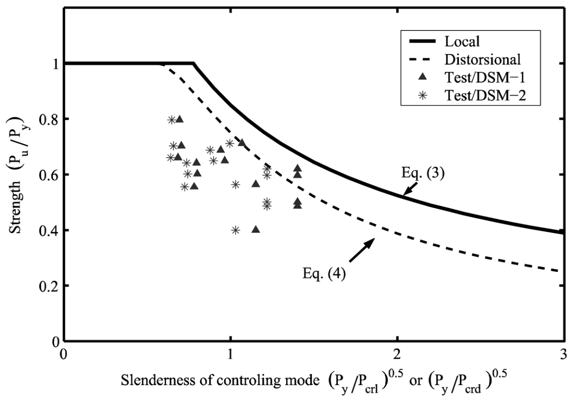

Fig. 10 Methods DSM-1 and DSM-2: Slenderness vs. Strength for Lipped Built-up Columns

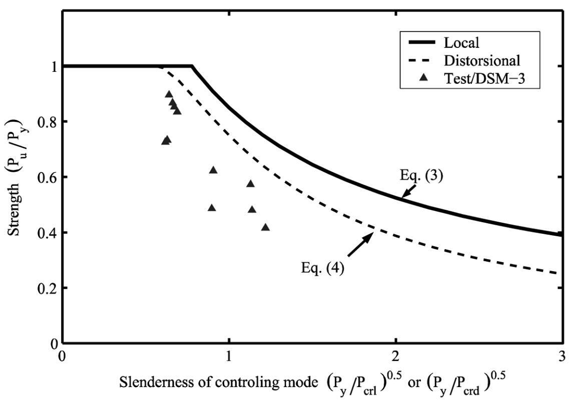

Fig. 11 Method DSM-3: Slenderness vs. Strength for Plain Built-up Columns

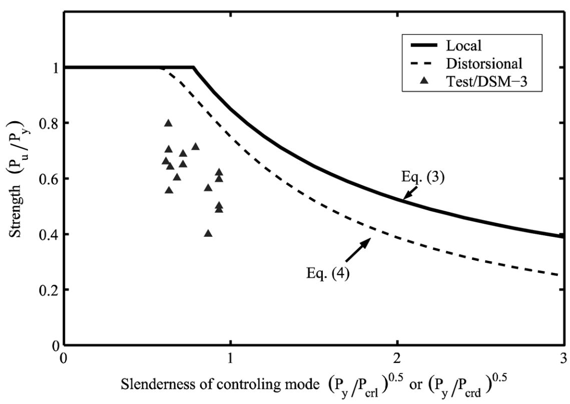

Fig. 12 Method DSM-3: Slenderness vs. Strength for Lipped Built-up Columns

4.3.2 Third approach (DSM-3)

In this approach, the elastic distorsional buckling load is given by Eqs. (11) and (12). The local buckling mode can still be calculated by the element approach as in DSM-1 or DSM-2. Figs. 11 and 12 provide a graphical representation of the direct strength equations for local and distorsional buckling loads along with the results for the buckling tests. The test buckling data shows more deviation for the lipped, built-up columns than the plain, built-up columns.

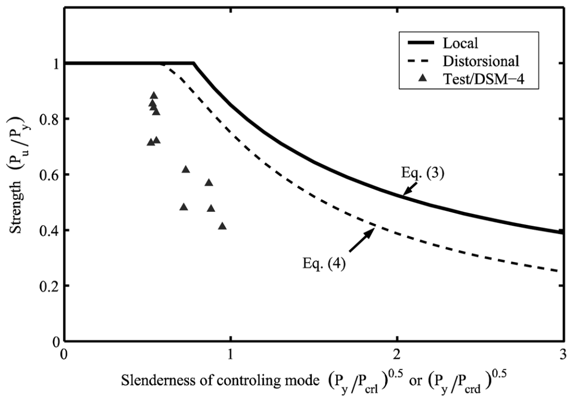

Fig. 13 Method DSM-4: Slenderness vs. Strength for Plain Built-up Columns

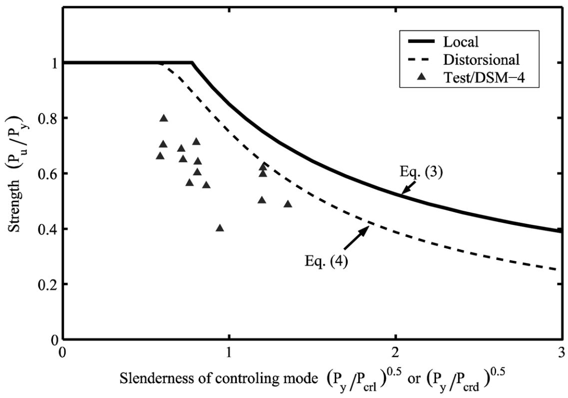

Fig. 14 Method DSM-4: Slenderness vs. Strength for Lipped Built-up Columns

Table 4 Experimental-to-predicted ratios for direct strength method solutions for plain built-up columns

	Col	$N_{u,\,expe}$ (KN)	$N_{u,\,DSM\text{-}1}$ (KN)	$N_{u,\,DSM\text{-}3}$ (KN)	$N_{u,\,DSM\text{-}4}$ (KN)	$N_{u,\,expe}/N_{u,\,DSM\text{-}1}$	$N_{u,\,expe}/N_{u,\,DSM\text{-}3}$	$N_{u,\,expe}/N_{u,\,DSM\text{-}4}$
U Shape. A. Moldovan *et al.* (1999)	P 5-1	80.00	36.898	56.151	69.428	1.08	0.71	0.58
	P 5-2	95.00	36.419	56.178	69.712	1.30	0.85	0.68
	P 5-3	70.00	36.011	53.364	66.346	0.97	0.66	0.53
	P 7-1	240.00	93.949	133.97	140.55	1.28	0.90	0.85
	P 7-2	225.00	89.267	128.4	136.85	1.26	0.88	0.82
	P 7-3	232.00	90.999	130.93	138.11	1.27	0.89	0.84
	P 10-1	175.00	103.17	146.82	170.3	0.85	0.60	0.51
	P 10-2	218.00	101.7	141.82	164.26	1.07	0.77	0.66
	P 12-1	323.00	178.12	216.78	224.1	0.91	0.74	0.72
	P 12-2	323.00	179.12	219.97	226.42	0.90	0.73	0.71
	P 12-3	392.00	174.48	214.19	222.5	1.12	0.92	0.88
				Mean value		1.09	0.78	0.71
				Standard deviation		0.17	0.11	0.13

Note:

$N_{u,\,DSM\text{-}1}$=Nominal axial strength of the built-up columns from the first approach (DSM-1) of the Direct Strength Method.

$N_{u,\,DSM\text{-}2}$=Nominal axial strength of the built-up columns from the second approach (DSM-2) of the Direct Strength Method.

$N_{u,\,DSM\text{-}3}$=Nominal axial strength of the built-up columns from the third approach (DSM-3) of the Direct Strength Method.

$N_{u,\,DSM\text{-}4}$=Nominal axial strength of the built-up columns from the fourth approach (DSM-4) of the Direct Strength Method.

$N_{u,\,expe}$=Experimental buckling load of the built-up columns.

Table 5 Experimental-to-predicted ratios for direct strength method solutions for lipped built-up columns

	Col	$N_{u, expe}$ (KN)	$N_{u, DSM\text{-}1}$ (KN)	$N_{u, DSM\text{-}2}$ (KN)	$N_{u, DSM\text{-}3}$ (KN)	$N_{u, DSM\text{-}4}$ (KN)	$N_{u, expe}/N_{u, DSM\text{-}1}$	$N_{u, expe}/N_{u, DSM\text{-}2}$	$N_{u, expe}/N_{u, DSM\text{-}3}$	$N_{u, expe}/N_{u, DSM\text{-}4}$
C Shape. A. Moldovan *et al.* (1999)	P 19-1	117.00	95.925	96.341	94.712	95.469	0.61	0.61	0.62	0.61
	P 19-2	125.00	95.92	97.427	95.477	97.363	0.65	0.64	0.65	0.64
	P 19-3	116.00	94.498	96.341	92.363	96.297	0.61	0.60	0.63	0.60
	P 21-1	140.00	80.022	83.876	87.241	98.309	0.87	0.83	0.80	0.71
	P 21-2	153.00	98.451	103.03	104.2	111.25	0.78	0.74	0.73	0.69
	P 21-3	140.00	93.812	98.305	101	107.79	0.75	0.71	0.69	0.65
	P 27-1	312.00	236.17	236.17	233.88	236.17	0.66	0.66	0.67	0.66
	P 27-2	334.00	237.53	237.53	233.72	237.53	0.70	0.70	0.71	0.70
	P 27-3	360.00	225.95	225.95	222.49	225.95	0.80	0.80	0.81	0.80
					Mean value		0.71	0.70	0.70	0.67
					Standard deviation		0.09	0.08	0.07	0.06

	Col	$N_{u, expe}$ (KN)	$N_{u, DSM\text{-}1}$ (KN)	$N_{u, DSM\text{-}2}$ (KN)	$N_{u, DSM\text{-}3}$ (KN)	$N_{u, DSM\text{-}4}$ (KN)	$N_{u, expe}/N_{u, DSM\text{-}1}$	$N_{u, expe}/N_{u, DSM\text{-}2}$	$N_{u, expe}/N_{u, DSM\text{-}3}$	$N_{u, expe}/N_{u, DSM\text{-}4}$
C Shape. Niazi (1993)	120B1	245.670	237.71	256.06	256.69	240.99	1.06	0.52	0.48	0.48
	120B2	346.34	237.71	256.06	256.69	278.18	1.01	0.73	0.68	0.67
	180B1	460.67	310.81	342.22	364.67	296.11	1.13	0.74	0.67	0.63
	180B2	447.34	310.81	342.22	364.67	264.36	1.17	0.72	0.65	0.61
	180B3	570.34	310.81	342.22	364.67	294.7	1.19	0.92	0.83	0.78
	180B4	549	310.81	342.22	364.67	294.75	1.17	0.88	0.80	0.75
					Mean value		0.75	0.69	0.66	0.78
					Standard deviation		0.14	0.13	0.11	0.18

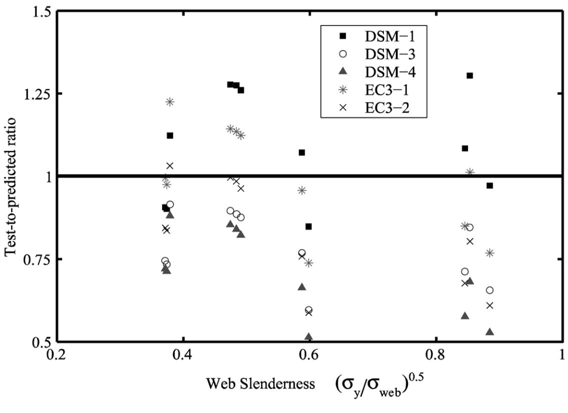

Fig. 15 Experimental-to-Predicted Ratios vs. Web Slenderness for Plain Built-up Columns

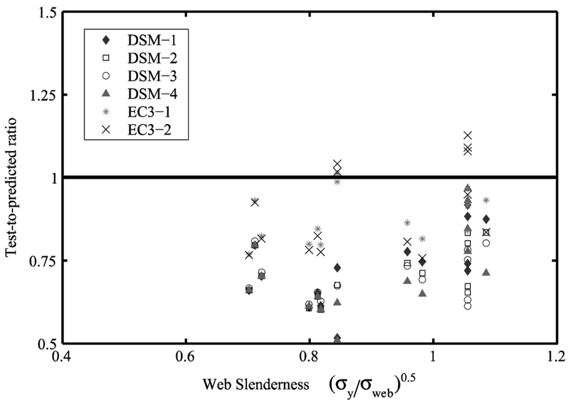

Fig. 16 Experimental-to-Predicted Ratios vs. Web Slenderness for Lipped Built-up Columns

4.3.3 Fourth approach (DSM-4)

The “Compound Spline Finite Strip Model” is used in this approach to predict the elastic buckling load. Variation in the predicted strength with respect to the slenderness, for the plain and lipped, built-up columns is depicted in Figs. 13 and 14, respectively. Again, the test data deviates from the local and distorsional buckling loads determined by the Direct Strength Method.

Finally, to show the accuracy of the proposed design methods for built-up columns, all the results are summarised in Tables 4 and 5 for plain and lipped, built-up columns respectively. The experimental-to-predicted ratios are plotted against the web slenderness for all the methods. Figs. 15 and 16 show this variation. In general, predictions of the strength buckling of built-up columns by the effective width approaches are better than the direct strength predictions. The best experimental-to-predicted ratio is related to the EC3-1 and EC3-2 approaches for U and C (Niazi 1993) built-up columns. Most of the direct strength approaches give less accurate ratios when compared to those of the effective width method. All comparison results are given in Tables 2 through 5.

5. Conclusions

Thin-walled, built-up columns may buckle in local, distorsional, chord overall, or Euler modes. For single channels, most of the current design specifications use either the effective width method or the Direct Strength Method. These methods employ the elastic local, distortional, or Euler buckling characteristics to calculate the ultimate strength of columns. Closed-form predictions of the buckling stress in the local, distorsional, or global modes are available for single channels. A new numerical method, developed by the authors, and based on the spline finite strip is proposed for predicting the elastic buckling stress.

Based on all these ideas, some design methods are proposed for the prediction of the strength of built-up columns. When using the effective width method, two approaches are considered. In the first method the buckling factor to evaluate the effective widths are taken as 4.0 and 0.43 while in the second approach this factor is calculated using the critical stress found by the proposed compound spline finite strip model. Comparisons with experimental data confirm a satisfactory degree of accuracy in the proposed design method. The experimental-to-predicted ratios obtained by this approach are almost equal to unity for both the plain and lipped columns. The numerical model demonstrates that numerical elastic local buckling solutions may be used as the key input to determining the strength of a large variety of built-up compression columns.

Whether implemented as a classical hand method or spline finite strip numerical method for elastic buckling determination, the Direct Strength Method provides significantly different strength predictions as compared to the experimental data. The experimental-to-predicted ratios obtained by this method are somehow not good. However, more experimental data is needed to reach a more convincing conclusion about the accuracy of the Direct Strength Method.

References

Al-Bermani, F.G.A. and Kitipornchai, S. (1990), "Non-linear analysis of thin-walled structures using least element/number", *J. Struct. Eng*., ASCE, **116**(1), 215-234.

American Iron and Steel Institute (1996), AISI Specification for the Design of Cold-Formed Steel Structural Members. American Iron and Steel Institute, Washington, D.C.

American Iron and Steel Institute (2004), Commentary on Appendix 1 Design of Cold-Formed Steel Structural Members with the Direct Strength Method. American Iron and Steel Institute, Washington, D.C.

CEN European Committee for Standardization (2004), "Eurocode 3: Design of steel structures, Part 1.3: General rules supplementary rules for cold-formed members and sheeting", *preEN-1993-Part.1-3*, Brussels, Belgium.

Davies, J.M. and Leach, P. (1994), "First-order generalised beam theory", *J. Constr. Steel Res.*, **31**(2-3), 187-220.

Davies, J.M. (2000), "Recent research advances in cold formed steel structures", *J. Constr. Steel Res.*, **55**(1-3), 267-288.

Djafour, M. and Megnounif, A. (1996), "Etude de la Stabilité des Colonnes à Etrésillons par la Méthode des Spline Bandes Finies, » Rapport de recherche", Université de Tlemcen.

Djafour, M., Megnounif, A. and Kerdal, D. (1999), "Elastic stability of built-up columns using the spline finite strip method", *6th Int. Colloquium on Stability and Ductility of Steel Structures SDSS'99.* Timisoara, Romania 9-11 Septembre 1999.

Djafour, M., Megnounif, A. and Kerdal, D. (2001), "The compound spline finite strip method for the elastic stability of U and C built-up columns", *Int. Conf. on Structural Engineering, Mechanics and Computation (SEMC 2001).* Cape Town, south Africa April 2nd-4th 2001.

Djafour, M., Megnounif, A., Kerdal, D. and Belarbi, A. (2007), "A numerical method for buckling analysis of built-up columns with stay plates", *Struct. Eng. Mech.*, **26**(4).

Hancock, G.J. and Lau, C.W. (1985), "Buckling of thin flat walled structures by a spline finite strip method", Research report N°R487, University of Sidney.

Hancock, G.J. (2003), "Cold formed steel structures", *J. Const. Steel Res.*, **59**(4), 473-487.

Kim, K.H. and Choi, C.K. (2004), "A non-symmetric non-periodic B3-spline finite strip method", *Struct. Eng. Mech.,* **18**(2).

Moldovan, A., Petcu, D. and Gioncu, V. (1999), "Ductility of thin walled members", Proceeding of the 6th International Colloquium on Stability and Ductility of Steel Structures – SDSS'99, Elsevier Science, 299-308.

NAS (2004), *Supplement to the North American Specification for the Design of Cold-formed Steel Structures*, American Iron and Steel Institute, Washington, DC.

Niazi, A. (1993), Contribution à l'étude de la stabilité des structures composées de profils à parois minces et section ouverte de type C. *Thèse de doctorat*, Université de Liège.

Papangelis, J.P. and Hancock, G.J. (1995), "Computer analysis of thin-walled structural members", *Comput. Struct.,* **56**(1), 157-176.

Papangelis, J.P. and Hancock, G.J. (2006), "Thin-wall: Cross-section analysis and finite strip analysis of thin-walled structures", Thin Wall v2.1. Centre for Advanced Structural Engineering, University of Sidney, <http://www.civil.usyd.edu.au/case/thinwall>

Schafer, B.W. and Pekoz, T. (1998), "Direct strength prediction of cold-formed steel members using numerical elastic buckling solutions", Thin Walled Structures, Research and Development, Eds. Shanmugan, N.E., Liew, J.Y.R. and Thevendran, V., Elsevier, pp.137-144.

Schafer, B.W. (2002), "Local, distortional, and euler buckling of thin-walled columns", *J. Struct. Eng.*, **128**(3).

Schafer, B.W. (2006), *CUFSM: Elastic Buckling Analysis of Thin-walled Members by Finite Strip Analysis, CUFSM v2.6*. <http://www.ce.jhu.edu/bschafer/cufsm> visited on 2 Oct. 2006.

Silvestre, N. and Camotim, D. (2002), "First-order generalised beam theory for arbitrary orthotropic materials", *Thin Wall. Struct.*, **40**(9), 755-789.

Stone, T.A. and LaBoube, R.A. (2005), "Behavior of cold-formed steel built-up I-sections", *Thin Wall. Struct.*, **43**, 1805-1817.

Szabo, I.F., Ungureanu, V. and Dubina, D. (2004), "Recent research advances on ECBL approach. Part II: Interactive buckling of perforated sections", *Thin Wall. Struct.*, **42**(2), 195-210.

Yu, W.W. (2000), *Cold-Formed Steel Design*. John Wiley & Sons, Inc.

Ungureanu, V. and Dubina, D. (2004), "Recent research advances on ECBL approach. Part I: Plastic-elastic interactive buckling of cold-formed steel sections", *Thin Wall. Struct.*, **42**(2), 177-194.