

**Technical Note**

# Elastic buckling of perforated plates subjected to linearly varying in-plane loading

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## 1. Introduction

The buckling behavior of plates has been studied by many researchers in structural mechanics for over a century (Timoshenko 1961). Steel plates are often used as the main components of steel structures such as deck and bottom of ship structures, plate girders and box girders. Openings in such steel plates are often needed to provide access for inspection, maintenance or simply to reduce weight. The presence of such openings in plate elements leads to change in stress distribution within the member and variations in buckling characteristics of the plate element (Shakerley and Brown 1996, Shanmugam *et al.* 1999). Several research studies have been conducted over the past two decades in order to investigate effects of the shape, size, location as well as the types of applied load on the performance and the buckling behavior of such perforated plates.

Shanmugam *et al.* (1999), and El-Sawy and Nazmy (2001) presented most of the previous work on the elastic buckling of perforated plates. In the literature a great deal of attention has been focused on studying the elastic buckling of perforated plates subjected to uniaxial, biaxial and shear loadings, but no work appears to have been related to the effect of the plate aspect ratio and the hole location on the buckling of rectangular plates subjected to linearly varying in-plane loading. The main objective of this paper is, therefore, to investigate the aforementioned effects on rectangular plates containing a circular hole.

## 2. Problem definition and analysis procedure

A rectangular plate containing a circular hole and its dimensions are given in Fig. 1. The plate is subjected to linearly varying in-plane loading in the longitudinal direction and its all edges are

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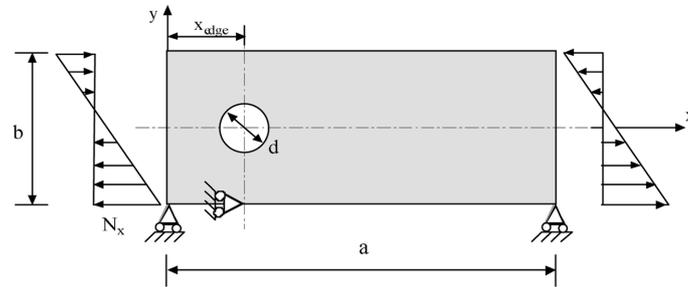


Fig. 1 Geometry of a plate with a circular hole

simply supported in out-of-plane direction. Three points on the edge at  $y = -b/2$ , are restrained from in-plane translation to prevent the plate from exhibiting rigid body motion. Plate aspect ratios ( $a/b$ ) are selected to have integer values, i.e., 1, 2, 3 and 4.

A linearly varying force is subjected to two opposite edges ( $x = 0$  and  $x = a$ ) as

$$N_x = -N \left( 1 - \frac{\alpha}{b} \left( y + \frac{b}{2} \right) \right) \quad (1)$$

where  $N$  is the intensity of the compressive force per unit length and  $\alpha$  is a numerical factor. By changing  $\alpha$  in Eq. (1), different particular cases may be obtained. For instance,  $\alpha$  is set to zero, the uniformly distributed compressive force is obtained. Pure in-plane bending is obtained by taking  $\alpha = 2$ . The other cases ( $0 < \alpha < 2$ ) give the combination of bending and compression. Herein, three loading cases  $\alpha = 0, 1$  and  $2$  are considered only.

The multipurpose finite element software program ANSYS (2005) is employed in this research. The general-purpose Elastic Shell63 element is used to model the perforated plate because it has the capacity to simulate both membrane and flexural behavior. This Shell63 element was selected for use in the parametric study based on its satisfactory performance in verification work described in the papers by El-Sawy and Nazmy (2001). An irregular discretisation in finite element modeling was employed. The mesh density of the plate was chosen based on the size of a circular hole. The default shell element size all over the plate is selected  $b/20$ . The shell element size along the hole perimeter was set to the smaller of  $b/50$  or  $\pi d/40$ . The center of circular holes was moved along the  $x$ -axis from the plate outer edge  $x_{edge} = 0.05b + d/2$  toward the center of the plate ( $x_{edge} = a/2$ ). Its Young's modulus  $E = 210$  GPa and poisson's ratio  $\nu = 0.3$  were selected. The linear buckling analysis is used to determine the critical buckling load of perforated plates.

In order to verify the method of analysis used in this study, a comparison with existing results in the literature on the elastic buckling of rectangular plates without a cutout has been performed. The

Table 1 Elastic buckling load  $N_{cr}$  for different loading cases

$\alpha$	$N_{cr}$ (kN/mm)		
	This study	Kang and Leissa	Ratio
0.0	0.7585	0.7523	1.008
1.0	1.4813	1.4823	0.999
2.0 for $a/b = 1$	4.8369	4.8360	0.991
2.0 for $a/b > 1$	4.5610	4.5327	1.006

values of  $N_{cr}$  obtained by ANSYS along with the corresponding values obtained from Kang and Leissa (2005) are listed in Table 1. They obtained these results by using an exact solution procedure based on the infinite power series. There is a good agreement between the two sets of results. The maximum deviation being less than 1% is within an acceptable level.

### 3. Discussion of results

A number of plates with different loading cases ( $\alpha$  ranging from 0 to 2), different normalized hole sizes ( $d/b$  ratios ranging from 0.1 to 0.7), and aspect ratio ( $a/b$  ranging from 1 to 4) were analyzed by using finite element package. The results obtained are plotted as shown in Fig. 2. The variations of the buckling load ratios  $N^*(=N/N_{cr})$  are plotted against the ratio of the distance between the plate outer edge and the center of the hole to the plate width ( $x_{edge}/b$ ).

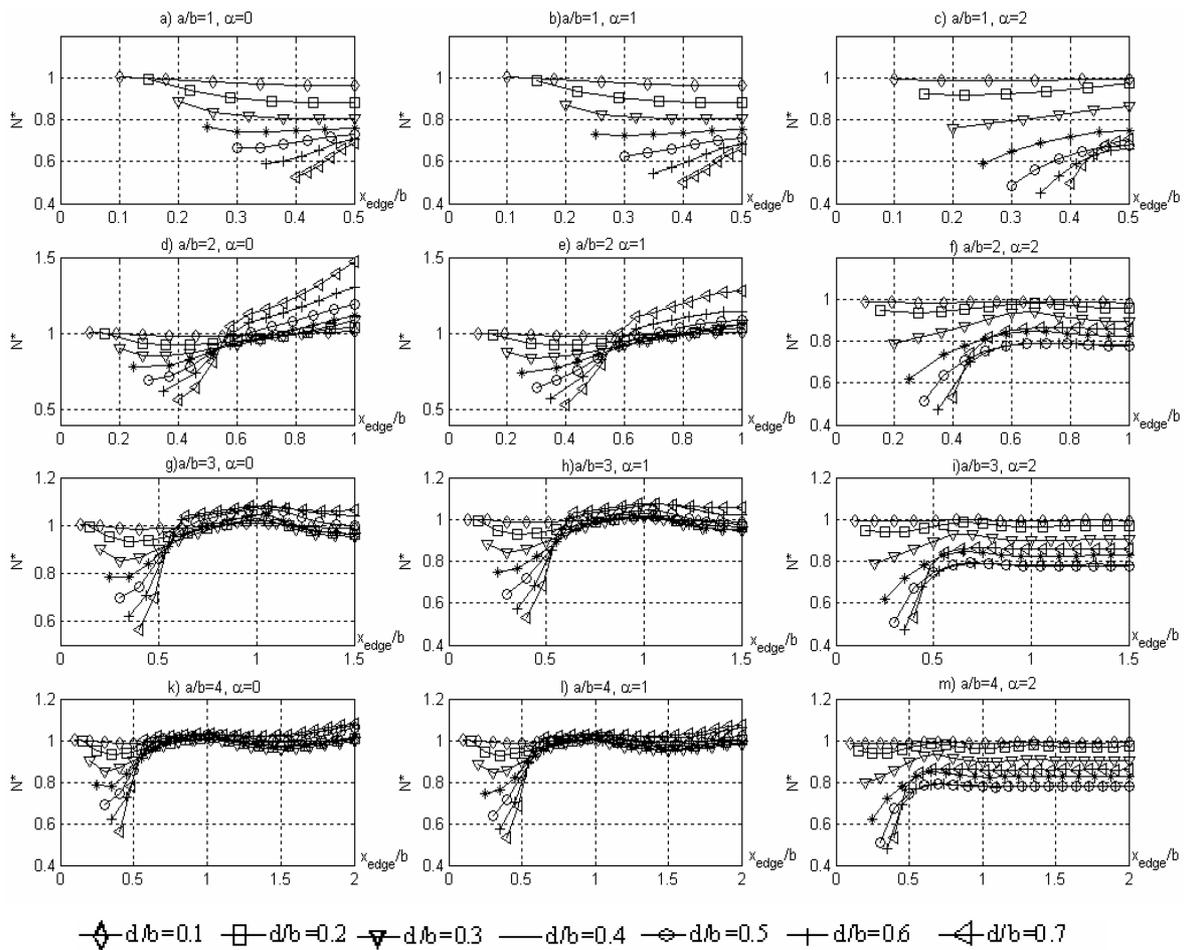


Fig. 2 Buckling load ratios  $N^*(=N/N_{cr})$  for square plates with circular holes moving along the major axis and subjected to different loading cases

Close observation of this figure shows clearly that the presence of a small hole (i.e.,  $d/b = 0.1$  and  $0.2$ ) has no considerable effect on the buckling load ratio of the rectangular plate. However, moving the holes with large diameter ( $d/b = 0.5-0.7$ ) along the  $x$ -axis causes a significant variation on buckling load ratios  $N^*$ . This figure also shows that the elastic buckling load ratio of a square plate is similar to the behavior of rectangular plates when a circular hole lies on the critical zone which is  $0 < x_{edge}/b \leq 0.5$ . The buckling load ratio  $N^*$  always decrease significantly if the hole is located in this zone. Perforations which lie on out of this zone do not cause the reduction on the buckling load of rectangular plates for  $\alpha = 0$  and  $1$  while perforations cause a decrease up to 20% in the buckling load for  $\alpha = 2$ .

#### 4. Conclusions

The buckling behavior of perforated rectangular plates subjected to linearly varying loading has been studied by using the finite element method. The elastic buckling loads  $N$  always decrease significantly if a large hole (i.e.,  $d/b = 0.4-0.7$ ) is located in the critical zone; therefore, it is recommended to have no large hole in this zone. For compression loading cases ( $\alpha = 0$  and  $1$ ), a circular hole which is located out of the critical zone has no considerable effect on the buckling load; on the other hand, the presence of a circular hole causes a reduction on the buckling load up to 20% for pure bending for  $\alpha = 2$ .

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